3: Multiplying Decimals

***Question*: What is 0.1×0.1?**

Misconception	**Correct**
As $1 \times 1 = 1$ then by comparison $0.1 \times 0.1 = 0.1$	The answer is 0.01 as you are multiplying $\frac{1}{10}$ by $\frac{1}{10}$ which means $\frac{1}{10} \times \frac{1}{10}$. This has value $\frac{1}{100}$ or 0.01 as a decimal.

Further Explanation

Consider the simplest case as discussed above

$$0.1 \times 0.1$$

0.1 is the same as $\frac{1}{10}$ (a tenth), so 0.1×0.1 is the same as $\frac{1}{10}$ *times* $\frac{1}{10}$.

But what does this mean?

When we point at boxes containing 6 eggs each and say "3 *of* those boxes please", we walk out with 18 eggs, that is 3 *times* 6.

> The meaning of *times* can be therefore be interchanged with *of*.

Hence $\frac{1}{10}$ *times* $\frac{1}{10}$ can be seen as *one tenth of one tenth* which is one hundredth or $\frac{1}{100}$ or 0.01

To summarise:

$$0.1 \times 0.1 = \frac{1}{10} \textit{ times } \frac{1}{10} = \textit{one tenth of one tenth} = \textit{one hundredth}$$

$$= \frac{1}{100} = 0.01$$

So $\boxed{0.1 \times 0.1 = 0.01}$

Misconception 3

Generalization

The method that we have used for this simple case can be generalised to multiplying any two decimal numbers together. For example, what is the value of 54.321×0.06?

We know how to multiply 54321×6, but how do we cope with the decimal points in

$$54.321 \times 0.06 \text{ ?}$$

The rule is:

- first perform the multiplication as if there were no decimal points in it, giving here the number 325 926
- then count how many digits are behind (to the right of) the decimal point in both numbers, in this case 5 (3 + 2)
- insert the decimal point that many digits from the right to give the correct answer, in this case, move 5 places from the right to give 3.25926

To understand why the rule works we look at its individual component parts – it is best to tackle this by applying the principal of finding something similar which we can cope with, and then working out the difference between this and the case in hand.

Applying this method, we multiply first the numbers ignoring the decimal point, then examine the effect of having the decimal points as originally given.

(Remember that a number without a decimal point is the same as that number with a decimal point to its right, e.g. $139 = 139.0$)

Moving a decimal point one place to the left amounts to *dividing by 10*, e.g. 13.9 is the same as $\frac{139}{10}$. (Similarly, moving the point to the right is the same as *multiplying by 10.)*

Moving it again to the left means dividing by 10 again. So, having moved the point 2 places to the left amounts to dividing by 100. Moving it 3 places to the left amounts to dividing by 1000, and so on.

Note: What happens if one needs to move the point more positions than there are digits?

For example, if we have the result 76 from the first stage of multiplying 0.19 by 0.04; we now need to put the decimal point 4 positions from the right of 76? To do this we simply add zeros on the left of the number, as many as are needed. So for moving 4 positions to the left in 76, we first write it as 00076 and then move along the decimal point 4 positions from the left to get the answer 0.0076.

Returning to our example of multiplying 54.321 by 0.06, we used 54 321 instead of 54.321, which meant that the number that was 1000 times bigger than the given number, which consequently made the answer 1000 times too big. Further, we continued by using 6 instead of 0.06, a number 100 times bigger than was given, thus making the result yet another 100 times bigger. To bring the result back to what it should have been, we must divide the number 325 926 by 1000 and again by 100 – or doing it in one go, dividing by 100 000 (1000×100).

Referring back to our rule, dividing by 100 000 (5 zeros) is the same as moving the point 5 places from the right (remember this 5 came from the 3 digits to the right of the decimal point in 54.321 which were initially ignored and the 2 digits to the right of the decimal point in 0.06).

Follow-up Exercises

1. Calculate

   (a) 0.1×0.2 (b) 0.2×0.2 (c) 0.2×0.3

   (d) 0.5×0.5 (e) 0.8×0.2 (f) 0.6×0.9

2 Calculate

   (a) 0.01×0.1 (b) 0.02×0.3 (c) 0.04×0.6

   (d) 0.8×0.05 (e) 0.01×0.01 (f) 0.05×0.07

3. Calculate

   (a) 1.2×2.3 (b) 8.35×1.2 (c) 24.7×0.4

   (d) 1.5×1.5 (e) 3.45×2.7 (f) 54.3×0.04

Answers

1. (a) 0.02 (b) 0.04 (c) 0.06
   (d) 0.25 (e) 0.16 (f) 0.54

2. (a) 0.001 (b) 0.006 (c) 0.024
   (d) 0.04 (e) 0.0001 (f) 0.0035

3. (a) 2.76 (b) 10.02 (c) 9.88
   (d) 2.25 (e) 9.315 (f) 2.172