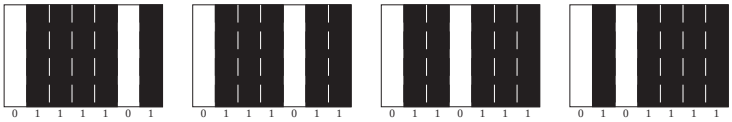
Codes and Ciphers	**UNIT 3** *EAN Bar Codes* Lesson Plan 1	*8-Digit EAN*
Activity		***Notes***
		T: Teacher P: Pupil Ex.B: Exercise Book
1	**Introduction** T: What codes can you think of that are used in everyday life? T: What codes do supermarkets use all the time? *(Bar codes)* T: Why do they use them? *(Speedy reading of prices at check-outs, stock control, etc.)* T: In fact, bar codes not only speed up checkout times but also mean that the price of each product can be quickly altered, special offers (3 for the price of 2, etc.) can be automatically applied and stock control is immediate. T: When might this system break down? *(Power cuts, computer problems)* T: Where would it be ineffective with stock control? *(Stolen items)* *8 mins*	Many Ps will volunteer to answer; T chooses Ps to give suggestions. Interactive discussion drawing on Ps' experiences. No system, except for the tagging of individual items (as often done in clothes shops), can solve the problem of shoplifting.
2	**Checking check digits** T: What does the computer do with items at the checkout? *(Reads bar codes)* T: What happens if it reads a code incorrectly? *(It alerts the checkout operator with a warning sound)* T: How does the computer know if it has read the bar code correctly? T: Consider the number 0 0 3 6 8 1 2 4 ↑ check digit The final number is the check digit. The computer works out: $3 \times (1\text{st} + 3\text{rd} + 5\text{th} + 7\text{th}) + (2\text{nd} + 4\text{th} + 6\text{th} + \text{check digit})$ which must be divisible by 10. T: Who would like to check this with our bar code? P (at board): $3 \times (0 + 3 + 8 + 2) + (0 + 6 + 1 + 4)$ $= 3 \times 13 + 11$ $= 39 + 11$ $= 50$ T: Is this number divisible by 10 ? *(Yes)* T: So will the computer accept this as a correct bar code? *(Yes)* T: Well done.	Again, T draws on experiences of Ps; they can give examples of local shops and supermarkets. T explains that each bar code has a special final number which is a *check digit*. The check digit depends on the previous numbers in the bar code and will cause the bar code to be rejected if it is not correct. T asks Ps for an 8-digit bar code from an item that they have and writes the bar code on the board, labelling the check digit. T explains the calculation done by the computer, writing it on the board (or showing previously prepared OS). Volunteer P works at board, explaining aloud each step of the calculation. Other Ps help if necessary.
(continued)		

Codes and Ciphers	**UNIT 3** *EAN Bar Codes* Lesson Plan 1	*8-Digit EAN*
Activity		***Notes***
2 *(continued)*	Now try the questions in Exercise 2.	Individual work; allow 4 or 5 minutes with T monitoring and checking Ps' work.
	T: Let's look at (a). Is the check digit correct? *(Yes)*	
	T: Good. Now (b). *(Yes)*	
	T: What about (c)?	
	Ps: It's not correct.	
	T: Who would like to volunteer to show us how to find the correct check digit here?	Volunteer P at board, carefully monitored, and helped if necessary.
	P (while writing on board):	
	$3 \times (5 + 1 + 8 + 2) + (0 + 6 + 6 + 2)$	
	$= 3 \times 16 + 14$	
	$= 48 + 14$	
	$= 62$	
	As this number has to be divisible by 10, it is too great by 2. The final number should have been 0.	
	T: Well done!	Give praise when deserved.
	20 mins	
3	**Finding check digit**	
	T: We can also calculate a missing check digit. For example, (writes on board, Exercise 1, part (a)):	A good opportunity to introduce simple algebra, using x for the unknown digit.
	$\boxed{0\,0\,0\,8\;\;6\,3\,9\,x}$	
	where x is the unknown check digit.	
	T: How should I do this? *(Use the method)*	
	T: Who would like to work this out?	A volunteer P works at the front of the class, with help from the class and/or T if needed.
	P (at board):	
	$3 \times (0 + 0 + 6 + 9) + (0 + 8 + 3 + x)$	
	$= 3 \times 15 + 11 + x$	Ps might need help here!
	$= 45 + 11 + x$	
	$= 56 + x$	
	T: What value of x makes this divisible by 10 ?	T asks whole class – they could display answers on individual whiteboards or volunteer to answer.
	Ps: 4	
	T: Good. So the check digit is 4.	
	T: Now you can try Exercise 1, problems (b) and (c).	T allows about 5 minutes for this.
	<u>Solutions</u>	T discusses any mistakes; shows that the answer is unique.
	(b) $3 \times (5 + 2 + 4 + 1) + (0 + 1 + 2 + x)$	T monitors Ps' work and reviews answers. T praises where deserved.
	$= 3 \times 12 + 3 + x$	
	$= 36 + 3 + x$	
	$= 39 + x$	
(continued)	To make this divisible by 10, x must be 1.	

Codes and Ciphers	**UNIT 3** *EAN Bar Codes* Lesson Plan 1	*8-Digit EAN*
Activity		***Notes***
3 *(continued)*	(c) $3 \times (0 + 4 + 6 + 5) + (0 + 2 + 5 + x)$ $= 3 \times 15 + 7 + x$ $= 45 + 7 + x$ $= 52 + x$ To make this divisible by 10, x must be 8.	
	30 mins	
4	**Error correction (Activity 2a)** T: We know how to detect one error in a bar code number but if the number is incorrectly read, can it be corrected? T: Try this number 5 0 2 6 8 0 2 0 First show that it is not correct and then try to find what the number could have been It might not be the check digit that is wrong. T: Who has some answers? T: What can we conclude? Ps: It is not possible to correct errors, so the computer needs to read the number again. T; Well done.	Paired or group work. Interactive discussion to start to set the scene. Allow about 10 minutes for pairs of Ps to consider possibilities; T should be ready to give prompts and hints. Ps give possible answers and explain their thinking. Remember that in real life situations, if the bar code cannot be read, the checkout operator can input the number manually.
	45 mins	
	Homework Find several examples of bar codes on products. Look at the actual design of the code and see what you can conclude about it. Bring some examples of bar codes to the next lesson.	

Codes and Ciphers	**UNIT 3** *EAN Bar Codes* Lesson Plan 2	*Design of 8-Digit EAN*
Activity		***Notes***
1	**Design** T: What have you noticed about your bar codes? *(Various answers)* T: How many numbers are there on a bar code? *(8 or 13; 7 on M & S products)* T: We'll start with the 8-digit bar codes. You should be able to see (writes on board): • left hand guard rails (2 extended black lines) • 4 numbers coded on the left • centre guard (2 extended black lines) • 4 numbers coded on the right • right hand guard rails (2 extended black lines) T: What else have you noticed? Ps: Each number is coded by two lines. T: Yes – what can you say about these lines? Ps: They are of different widths/thicknesses. T: Good; how many different widths/thicknesses are there? Ps: Three? T: No, look again. Ps: Four. T: Yes – you could refer to them as *very thin, thin, thick, very thick* We'll see how to actually design the left hand numbers. *8 mins*	Ps should have brought several examples of bar codes from products. (T should have some ready in case they are needed.) Ps will need to have 2 or 3 8-digit codes in front of them. Use **OS 3.1** to illustrate. This should be very interactive with Ps responding to the answers given. Reluctant Ps should be chosen to give answers whenever possible. Praising.
2	**Left hand codes** T: You will not know this, but the code for each digit is constructed from a *seven* module system. Here is the code for 5 – of course, you cannot see the construction lines. T: To design each number we put either 0 (white) or 1 (black) into code of the same width but with these rules T: Can anyone show me another possible code for a number? T: Now that we have 2 (3) examples you can work in pairs to find *all* the possible solutions. T: How many solutions have you found? *(7, 8 or 9)* T: How many do we need? *(10)* T: Who wants to show a solution?	Use **OS 3.2**. Use **OS 3.3** here. Either use the right hand side of **OS 3.2** or wait to see if Ps volunteer answers. If T has the the facilities the bar code design program can be used at this stage, or at the review stage after Ps have had 5-10 minutes to find solutions. If any P has more than 10 solutions T will need to review the errors with the whole class. Volunteer Ps gives solutions; other Ps agree/disagree.
(continued)		

Codes and Ciphers	**UNIT 3** *EAN Bar Codes* Lesson Plan 2	*Design of 8-Digit EAN*
Activity		***Notes***
2 *(continued)*	T: Well done!	T shows **OS 3.3** or computer program to check answers. Continue until all 10 have been found; T gives praise, particularly to anyone has all the solutions. Review of systematic strategies for finding all solutions, e.g. for five 1s, move the right hand '0' along to the left.
	 	So there are just four possible configurations using five 1s.
	30 mins	
3	**Right hand codes** T: The full set of LH codes for 8-digit EANs is given on your sheet. The digits 0 to 9 are each arbitrarily given one of the 10 possible solutions. T: Now what about right hand codes? What do you notice? *(Similar system; 8 black bars, etc.)* T: Why is a different system needed for RH codes? *(So that the light pen/scanner can detect whether it is reading from left to right or from right to left)* T: That's right; a different but related design is used for RH numbers as you can see on your sheet. T: Which number set is used for the LH code? *(A)* T: Number set C is used for RH codes, but how is it related to number set A? *(Each 0 becomes 1 and vice versa)* T: We call C the *dual* of A. T: We have not yet used number set B, but can you see how it is obtained from C? *(Reflection)* T: Could you now design another number set? *(The dual of B)* T: We will see in the next lesson how number set B is used with 13-digit EANs.	Each P is given a copy of **OS 3.4**. Ps use their wrappers, etc. but restrict this to 8-digit EANs. T gives each pair of Ps a copy of **OS 3.5**. T gives Ps a few minutes to discuss this in pairs. Interactive discussion, preparing for next lesson.
	45 mins	
	Homework Show that the check digit for the 13-digit EAN 4 9 0 2 5 8 0 4 2 3 9 1 9 is correct, using a method similar to that used for 8-digit EANs.	

Codes and Ciphers	**UNIT 3** *EAN Bar Codes* Lesson Plan 3	*Design of 13-Digit EAN*
Activity		***Notes***
1	**Checking homework** T: Who could verify the check digit? Come and show us on the board. $3 \times (9 + 2 + 8 + 4 + 3 + 1) + (4 + 0 + 5 + 0 + 2 + 9 + 9)$ $= 3 \times 27 + 29$ $= 81 + 28$ $= 110$ This is divisible by 10. T: Good. *8 mins*	Volunteer P(s) show their method. Discuss with class. Note that the calculation is 3 times $(3\text{rd} + 4\text{th} + ... + 12\text{th})$ digits and this time there are 13 digits, an odd number. T praises; sorts out any problems.
2	**Practice** T: What is the check digit for the number 5 0 0 0 4 3 6 7 8 4 6 7 *x* ? T: Who has the answer? *(8)* T: Who would like to show us their working? *15 mins*	Individual or paired work. T checks progress, intervening if necessary. A volunteer P shows working on board. T must ensure that all Ps have understood the method – the interactive program can be used to check this.
3	**Design of Code** T: There is an added complication with the design of the 13-digit EAN. Has anyone spotted it? *(There is an odd number to code)* T: What else do you notice from your examples? *(The first number is outside the bar!)* T: Yes – the code shows the last 12 digits, 6 on each side of the guards; the first digit is found by the computer according to what number sets have been used for the LH numbers. What number set is used for the last 6 digits? *(Number set C)* What could be used for the other 6 digits? *(A or B)* T: In fact, if you use only number set A for all 6 of these digits, this means that the first (uncoded) digit is 0. For the other 9 possibilities the left hand six numbers are coded using a combination of number set A or B. How many ways can you make using 3 As or 3 Bs? Here is one example: A A A B B B Now find all other possibilities but be systematic in your search. T: How many solutions have you found? *(7, 8, 9 ?)* T: Show your solutions to the class.	If possible, Ps should each have their own example of 13-digit codes from products. T will need to give the rules here but the more that Ps can understand from their examples the better. **OS 3.5** can also be used to identify the number sets used if you look very carefully. This is not easy to describe as it involves essentially stating the rules; T must make sure that Ps have understood. T allows at least 5 minutes for Ps to work in pairs on this activity. T checks that all answers are just 3 As and 3 Bs. Praising.

(continued)

Codes and Ciphers	**UNIT 3** *EAN Bar Codes* Lesson Plan 3	*Design of 13-Digit EAN*
Activity		***Notes***
3 *(continued)*	T: In the same way as with the bar code design, these solutions are arbitrarily assigned to a digit, as shown on one side. T: The computer first has to determine the number set used for the six LH digits; this gives the first digit. Now the computer checks the check digit for the 13-digit EAN! *30 mins*	Use **OS 3.6** here.
4	**Practice** T: Using the template on **OS 3.7**, illustrate the bar code for 9 7 7 0 0 4 9 3 9 2 0 7 7 Also verify that the check digit is correct. *45 mins*	Each pupil is given a copy of **OS 3.7, OS 3.6** and **OS 3.5**.
	Homework: Activity 6 How could you design a bar code system that codes letters *and* digits?	

UNIT 3 *EAN Bar Codes* Teacher Resource Material

Key Stage: *3 (and 2)*

Target: *Mainstream Year 7/8; high achieving Year 6*

Teaching Notes

This is a comprehensive package for teaching the EAN-8 and EAN-13 bar codes used on most grocery products in Europe (and most of the world). Two distinct aspects are looked at here, namely

(a) the way in which the check digit works,

(b) the actual design of the bar code for each digit.

You might want to concentrate on just one of these aspects; indeed, the check digit algorithms could be easily used at Key Stage 2.

There are a number of Activities to make pupils think; these should be used for whole class, interactive discussion. While it is clear that pupils do not need to know how bar codes are designed or how they work in practice, it is motivating for them to look at this use of mathematics in an everyday context. (Just think what happens now if the electricity supply to a supermarket fails!) It would be helpful for the class to collect some examples of bar codes from products before starting this unit of work.

We have also designed computer programs that

(a) check pupils' answers when designing the sets of 10 distinct codes (for each of the 10 digits)

http://www.ex.ac.uk/cimt/resource/find-codes/

and (b) illustrate the actual bar code (8-EAN) when the numbers are input

http://www.ex.ac.uk/cimt/resource/barcode-editor/

These are incorporated with the model lesson plans provided.

Solutions and Notes

Activity 1 The full set is given in Appendix 1. You can also use the computer programs to check answers.

Exercise 1 (a) Check digit is 4 (b) Check digit is 1 (c) Check digit is 8

Exercise 2 (a) Yes (b) Yes (c) No (as $3 \times 16 + 14 = 62$ is not divisible by 10). Check digit should be 2.

Activity 2 If the first 7 digits are correct, then the number is

5 0 2 6 8 0 2 3

However, any one of the digits could be wrong; for example, the correct number could be any of the following:

6 0 2 6 8 0 2 0	5 3 2 6 8 0 2 0
5 0 3 6 8 0 2 0	5 0 2 8 8 0 2 0
5 0 2 6 9 0 3 0	5 0 2 6 8 3 2 0
5 0 2 6 8 0 3 0	

All have just one change from the original number given, and all are correct as 8-digit EAN numbers.

Hence it is clear that the computer cannot correct even single errors and this bar code must be read again.

UNIT 3 *EAN Bar Codes* Teacher Resource Material (continued)

Activity 3 Using an 8-module framework and keeping to the given rules, there are 8 possible patterns using a total of 5 black modules and also 8 using a total of 3 black modules. This gives 16 distinct patterns.

Changing the third rule in the list and, for example, adding 2, 4 or even 6 black modules will increase the number of patterns. (An additional 2 gives 5 more patterns; 4 gives 9 more and 6 gives 5 more.)

Activity 4 Number set B is a reflection of number set C.

Activity 5 There are 20 possibilities of which 10 are used for the coding (as shown in Appendix 3).

Activity 6 Clearly this is not possible with the 7-module design.

The results in Appendix 3 show that you can increase the number of patterns by increasing the number of modules and/or changing the rules. There are many possibilities!