## UNIT 3  EAN Bar Codes  Lesson Plan 1

### 8-Digit EAN

<table>
<thead>
<tr>
<th>Notes</th>
</tr>
</thead>
<tbody>
<tr>
<td>T: Teacher   P: Pupil   Ex.B: Exercise Book</td>
</tr>
<tr>
<td>Many Ps will volunteer to answer; T chooses Ps to give suggestions.</td>
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<tr>
<td>Interactive discussion drawing on Ps’ experiences.</td>
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<tr>
<td>No system, except for the tagging of individual items (as often done in clothes shops), can solve the problem of shoplifting.</td>
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<tr>
<td>Again, T draws on experiences of Ps; they can give examples of local shops and supermarkets.</td>
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<tr>
<td>T explains that each bar code has a special final number which is a check digit. The check digit depends on the previous numbers in the bar code and will cause the bar code to be rejected if it is not correct.</td>
</tr>
<tr>
<td>T asks Ps for an 8-digit bar code from an item that they have and writes the bar code on the board, labelling the check digit.</td>
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<tr>
<td>T explains the calculation done by the computer, writing it on the board (or showing previously prepared OS).</td>
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<tr>
<td>Volunteer P works at board, explaining aloud each step of the calculation. Other Ps help if necessary.</td>
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</tbody>
</table>

### Activity Notes: Codes and Ciphers

#### Introduction

T: What codes can you think of that are used in everyday life?
T: What codes do supermarkets use all the time? (Bar codes)
T: Why do they use them? (Speedy reading of prices at check-outs, stock control, etc.)
T: In fact, bar codes not only speed up checkout times but also mean that the price of each product can be quickly altered, special offers (3 for the price of 2, etc.) can be automatically applied and stock control is immediate.
T: When might this system break down? (Power cuts, computer problems)
T: Where would it be ineffective with stock control? (Stolen items)

8 mins

#### Checking check digits

T: What does the computer do with items at the checkout? (Reads bar codes)
T: What happens if it reads a code incorrectly? (It alerts the checkout operator with a warning sound)
T: How does the computer know if it has read the bar code correctly?

<table>
<thead>
<tr>
<th>T: Consider the number</th>
</tr>
</thead>
<tbody>
<tr>
<td>0 0 3 6 8 1 2 4</td>
</tr>
</tbody>
</table>

↑ check digit

The final number is the check digit. The computer works out:

\[ 3 \times (1st + 3rd + 5th + 7th) + (2nd + 4th + 6th + \text{check digit}) \]
which must be divisible by 10.

T: Who would like to check this with our bar code?

P (at board): \[ 3 \times (0 + 3 + 8 + 2) + (0 + 6 + 1 + 4) \]
= \[ 3 \times 13 + 11 \]
= \[ 39 + 11 \]
= \[ 50 \]
T: Is this number divisible by 10? (Yes)
T: So will the computer accept this as a correct bar code? (Yes)
T: Well done.

(continued)
**Activity Notes**

### Unit 3: EAN Bar Codes

#### Lesson Plan 1

**8-Digit EAN**

**Notes**
- Individual work; allow 4 or 5 minutes with T monitoring and checking Ps' work.
- Volunteer P at board, carefully monitored, and helped if necessary.
- Give praise when deserved.

**Finding check digit**

T: We can also calculate a missing check digit.

For example, (writes on board, Exercise 1, part (a)):

$$\begin{array}{cccccc}
0 & 0 & 0 & 8 & 6 & 3 & 9 \_ \\
\end{array}$$

where $x$ is the unknown check digit.

T: How should I do this? (Use the method)

T: Who would like to work this out?

P (at board):

\[
3 \times (0 + 0 + 6 + 9) + (0 + 8 + 3 + x) \\
= 3 \times 15 + 11 + x \\
= 45 + 11 + x \\
= 56 + x \\
\]

T: What value of $x$ makes this divisible by 10?

Ps: 4

T: Good. So the check digit is 4.

T: Now you can try Exercise 1, problems (b) and (c).

**Solutions**

(b) \[
3 \times (5 + 2 + 4 + 1) + (0 + 1 + 2 + x) \\
= 3 \times 12 + 3 + x \\
= 36 + 3 + x \\
= 39 + x \\
\]

To make this divisible by 10, $x$ must be 1.
### Codes and Ciphers

#### UNIT 3  EAN Bar Codes  Lesson Plan 1

#### Activity

3  (continued)

(c) \( 3 \times (0 + 4 + 6 + 5) + (0 + 2 + 5 + x) \)

\[ = 3 \times 15 + 7 + x \]

\[ = 45 + 7 + x \]

\[ = 52 + x \]

To make this divisible by 10, \( x \) must be 8.

30 mins

### 4

Error correction  (Activity 2a)

T: We know how to detect one error in a bar code number but if the number is incorrectly read, can it be corrected?

T: Try this number

\[ 5 0 2 6 \ 8 0 2 0 \]

First show that it is not correct and then try to find what the number could have been.

It might not be the check digit that is wrong.

T: Who has some answers?

T: What can we conclude?

Ps: It is not possible to correct errors, so the computer needs to read the number again.

T: Well done.

45 mins

#### Homework

Find several examples of bar codes on products. Look at the actual design of the code and see what you can conclude about it.

Bring some examples of bar codes to the next lesson.
UNIT 3  EAN Bar Codes  Lesson Plan 2

Design
T: What have you noticed about your bar codes?  
(Various answers)
T: How many numbers are there on a bar code?  
(8 or 13; 7 on M & S products)

T: We’ll start with the 8-digit bar codes.
You should be able to see (writes on board):

• left hand guard rails (2 extended black lines)
• 4 numbers coded on the left
• centre guard (2 extended black lines)
• 4 numbers coded on the right
• right hand guard rails (2 extended black lines)

T: What else have you noticed?
Ps: Each number is coded by two lines.
T: Yes – what can you say about these lines?
Ps: They are of different widths/thicknesses.
T: Good; how many different widths/thicknesses are there?
Ps: Three?
T: No, look again.
Ps: Four.
T: Yes – you could refer to them as
very thin, thin, thick, very thick
We’ll see how to actually design the left hand numbers.

8 mins

Left hand codes
T: You will not know this, but the code for each digit is constructed
from a seven module system. Here is the code for 5 – of course,
you cannot see the construction lines.
T: To design each number we put either 0 (white) or 1 (black) into
code of the same width but with these rules
T: Can anyone show me another possible code for a number?

T: Now that we have 2 (3) examples you can work in pairs to find all
the possible solutions.

T: How many solutions have you found?  
(7, 8 or 9)
T: How many do we need?  
(10)
T: Who wants to show a solution?

Notes
Ps should have brought several examples of bar codes from
products. (T should have some ready in case they are needed.)

Ps will need to have 2 or 3 8-digit codes in front of them.

Use OS 3.1 to illustrate.

This should be very interactive
with Ps responding to the
answers given. Reluctant Ps
should be chosen to give
answers whenever possible.

Praising.

Use OS 3.2.

Use OS 3.3 here.

Either use the right hand side of
OS 3.2 or wait to see if Ps
volunteer answers.

If T has the the facilities the bar
code design program can be used
at this stage, or at the review stage
after Ps have had 5-10 minutes to
find solutions.

If any P has more than 10
solutions T will need to review the
errors with the whole class.

Volunteer Ps gives solutions; other
Ps agree/disagree.
<table>
<thead>
<tr>
<th>Codes and Ciphers</th>
<th>UNIT 3 EAN Bar Codes  Lesson Plan 2</th>
<th>Design of 8-Digit EAN</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Activity 2</strong></td>
<td><strong>Right hand codes</strong></td>
<td><strong>Notes</strong></td>
</tr>
<tr>
<td>(continued)</td>
<td>T: Well done!</td>
<td>T shows OS 3.3 or computer program to check answers. Continue until all 10 have been found; T gives praise, particularly to anyone has all the solutions. Review of systematic strategies for finding all solutions, e.g. for five 1s, move the right hand ‘0’ along to the left.</td>
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<tr>
<td></td>
<td>T: Well done!</td>
<td></td>
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<tr>
<td></td>
<td><strong>T</strong>: Well done!</td>
<td></td>
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<tr>
<td></td>
<td><strong>T</strong>: The full set of LH codes for 8-digit EANs is given on your sheet. The digits 0 to 9 are each arbitrarily given one of the 10 possible solutions.</td>
<td></td>
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<td><strong>T</strong>: The full set of LH codes for 8-digit EANs is given on your sheet. The digits 0 to 9 are each arbitrarily given one of the 10 possible solutions.</td>
<td></td>
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<tr>
<td></td>
<td><strong>T</strong>: Now what about right hand codes? What do you notice? (Similar system; 8 black bars, etc.)</td>
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<td></td>
<td><strong>T</strong>: Why is a different system needed for RH codes? (So that the light pen/scanner can detect whether it is reading from left to right or from right to left)</td>
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<td><strong>T</strong>: That’s right; a different but related design is used for RH numbers as you can see on your sheet.</td>
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<td></td>
<td><strong>T</strong>: Which number set is used for the LH code? (A)</td>
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<td></td>
<td><strong>T</strong>: Number set C is used for RH codes, but how is it related to number set A? (Each 0 becomes 1 and vice versa)</td>
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<td><strong>T</strong>: We call C the dual of A.</td>
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<td></td>
<td><strong>T</strong>: We have not yet used number set B, but can you see how it is obtained from C? (Reflection)</td>
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<td></td>
<td><strong>T</strong>: Could you now design another number set? (The dual of B)</td>
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<td></td>
<td><strong>T</strong>: We will see in the next lesson how number set B is used with 13-digit EANs.</td>
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<tr>
<td></td>
<td>30 mins</td>
<td>45 mins</td>
</tr>
<tr>
<td><strong>3</strong></td>
<td><strong>Homework</strong></td>
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<tr>
<td></td>
<td>Show that the check digit for the 13-digit EAN</td>
<td></td>
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<tr>
<td></td>
<td><strong>4 9 0 2 5 8 0 4 2 3 9 1 9</strong></td>
<td></td>
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<tr>
<td></td>
<td>is correct, using a method similar to that used for 8-digit EANs.</td>
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<td></td>
<td>Each P is given a copy of OS 3.4.</td>
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<td></td>
<td>Ps use their wrappers, etc. but restrict this to 8-digit EANs.</td>
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<td></td>
<td>T gives each pair of Ps a copy of OS 3.5.</td>
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<td></td>
<td>T gives Ps a few minutes to discuss this in pairs.</td>
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<td></td>
<td>Interactive discussion, preparing for next lesson.</td>
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<tr>
<td>Activity</td>
<td>Design of 13-Digit EAN</td>
<td></td>
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<tr>
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</tr>
<tr>
<td><strong>1</strong> Checking homework</td>
<td>Notes</td>
<td></td>
</tr>
<tr>
<td>T: Who could verify the check digit?</td>
<td>Volunteer P(s) show their method. Discuss with class.</td>
<td></td>
</tr>
<tr>
<td>Come and show us on the board.</td>
<td>Note that the calculation is 3 times ((3rd + 4th + \ldots + 12th)) digits and this time there are 13 digits, an odd number.</td>
<td></td>
</tr>
</tbody>
</table>
| \[
3 \times (9 + 2 + 8 + 4 + 3 + 1) + (4 + 0 + 5 + 0 + 2 + 9 + 9) \\
= 3 \times 27 + 29 \\
= 81 + 28 \\
= 110
\] This is divisible by 10. | T praises; sorts out any problems. |
| T: Good. | **8 mins** |

| **2** Practice | Individual or paired work. |
| T: What is the check digit for the number | T checks progress, intervening if necessary. |
| \[
\begin{array}{cccccccc}
5 & 0 & 0 & 0 & 4 & 3 & 6 & 7 \times
\end{array}
\] | A volunteer P shows working on board. T must ensure that all Ps have understood the method – the interactive program can be used to check this. |
| T: Who has the answer? | **15 mins** |
| \(8\) | |
| T: Who would like to show us their working? | |

| **3** Design of Code | If possible, Ps should each have their own example of 13-digit codes from products. |
| T: There is an added complication with the design of the 13-digit EAN. Has anyone spotted it? | T will need to give the rules here but the more that Ps can understand from their examples the better. |
| \(\text{(There is an odd number to code)}\) | **OS 3.5** can also be used to identify the number sets used if you look very carefully. |
| T: What else do you notice from your examples? | This is not easy to describe as it involves essentially stating the rules; T must make sure that Ps have understood. |
| \(\text{(The first number is outside the bar!)}\) | |
| T: Yes – the code shows the last 12 digits, 6 on each side of the guards; the first digit is found by the computer according to what number sets have been used for the LH numbers. | T allows at least 5 minutes for Ps to work in pairs on this activity. |
| What number set is used for the last 6 digits? \(\text{(Number set C)}\) | T checks that all answers are just 3 As and 3 Bs. Praising. |
| What could be used for the other 6 digits? \(\text{(A or B)}\) | |
| T: In fact, if you use only number set A for all 6 of these digits, this means that the first (uncoded) digit is 0. | |
| For the other 9 possibilities the left hand six numbers are coded using a combination of number set A or B. | |
| How many ways can you make using 3 As or 3 Bs? | |
| Here is one example: | |
| A A A B B B | |
| Now find all other possibilities but be systematic in your search. | |
| T: How many solutions have you found? \(7, 8, 9\) | |
| T: Show your solutions to the class. | |

(continued)
### Codes and Ciphers

**UNIT 3 EAN Bar Codes** Lesson Plan 3

<table>
<thead>
<tr>
<th>Activity 3 (continued)</th>
<th><strong>Design of 13-Digit EAN</strong></th>
</tr>
</thead>
<tbody>
<tr>
<td>T: In the same way as with the bar code design, these solutions are arbitrarily assigned to a digit, as shown on one side. Now the computer checks the check digit for the 13-digit EAN!</td>
<td></td>
</tr>
<tr>
<td><strong>30 mins</strong></td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>4 Practice</th>
<th><strong>Notes</strong></th>
</tr>
</thead>
<tbody>
<tr>
<td>T: Using the template on OS 3.7, illustrate the bar code for 9770049392077. Also verify that the check digit is correct.</td>
<td></td>
</tr>
<tr>
<td><strong>45 mins</strong></td>
<td></td>
</tr>
</tbody>
</table>

Each pupil is given a copy of OS 3.7, OS 3.6 and OS 3.5.

**Homework:** Activity 6
How could you design a bar code system that codes letters and digits?
**Key Stage:** 3 (and 2)

**Target:** Mainstream Year 7/8; high achieving Year 6

**Teaching Notes**

This is a comprehensive package for teaching the EAN-8 and EAN-13 bar codes used on most grocery products in Europe (and most of the world). Two distinct aspects are looked at here, namely

(a) the way in which the check digit works,
(b) the actual design of the bar code for each digit.

You might want to concentrate on just one of these aspects; indeed, the check digit algorithms could be easily used at Key Stage 2.

There are a number of Activities to make pupils think; these should be used for whole class, interactive discussion. While it is clear that pupils do not need to know how bar codes are designed or how they work in practice, it is motivating for them to look at this use of mathematics in an everyday context. (Just think what happens now if the electricity supply to a supermarket fails!) It would be helpful for the class to collect some examples of bar codes from products before starting this unit of work.

We have also designed computer programs that

(a) check pupils' answers when designing the sets of 10 distinct codes (for each of the 10 digits)

http://www.ex.ac.uk/cimt/resource/find-codes/

and

(b) illustrate the actual bar code (8-EAN) when the numbers are input

http://www.ex.ac.uk/cimt/resource/barcode-editor/

These are incorporated with the model lesson plans provided.

**Solutions and Notes**

**Activity 1**

The full set is given in Appendix 1. You can also use the computer programs to check answers.

**Exercise 1**

- (a) Check digit is 4
- (b) Check digit is 1
- (c) Check digit is 8

**Exercise 2**

- (a) Yes
- (b) Yes
- (c) No (as \(3 \times 16 + 14 = 62\) is not divisible by 10). Check digit should be 2.

**Activity 2**

If the first 7 digits are correct, then the number is

\[5 0 2 6 \ 8 0 2 3\]

However, any one of the digits could be wrong; for example, the correct number could be any of the following:

\[6 0 2 6 \ 8 0 2 0 \quad 5 3 2 6 \ 8 0 2 0\]
\[5 0 3 6 \ 8 0 2 0 \quad 5 0 2 8 \ 8 0 2 0\]
\[5 0 2 6 \ 9 0 3 0 \quad 5 0 2 6 \ 8 3 2 0\]
\[5 0 2 6 \ 8 0 3 0\]

All have just one change from the original number given, and all are correct as 8-digit EAN numbers.

Hence it is clear that the computer cannot correct even single errors and this bar code must be read again.
**Activity 3** Using an 8-module framework and keeping to the given rules, there are 8 possible patterns using a total of 5 black modules and also 8 using a total of 3 black modules. This gives 16 distinct patterns.

Changing the third rule in the list and, for example, adding 2, 4 or even 6 black modules will increase the number of patterns. (An additional 2 gives 5 more patterns; 4 gives 9 more and 6 gives 5 more.)

**Activity 4** Number set B is a reflection of number set C.

**Activity 5** There are 20 possibilities of which 10 are used for the coding (as shown in Appendix 3).

**Activity 6** Clearly this is not possible with the 7-module design.

The results in Appendix 3 show that you can increase the number of patterns by increasing the number of modules and/or changing the rules. There are many possibilities!