

Activities

G4.1 Symmetry About the Line $y = x$

Notes and Solutions (1 page)

ACTIVITY G4.1*Symmetry About the Line $y = x$*

Here are three functions that are 1 : 1 mappings on the domain given:

$$\text{A : } f : x \rightarrow 2x \quad (\text{all } x)$$

$$\text{B : } g : x \rightarrow x - 4 \quad (\text{all } x)$$

$$\text{C : } h : x \rightarrow x^2 \quad (x \geq 0)$$

1. For each of these functions, find their inverse function.
2. Draw coordinate axes for $-5 \leq x \leq 5$, $-5 \leq y \leq 5$ and sketch the three functions, f , g and h above.
3. Also, on the same graph, sketch the inverse functions, f^{-1} , g^{-1} and h^{-1} .
4. Draw (using a heavy line) the line $y = x$ on your graph.
5. What is the relationship between f , f^{-1} and the line $y = x$?
6. Confirm your answer to question 5 above, by also looking at g , g^{-1} and the line $y = x$ and similarly for h , h^{-1} and the line $y = x$.
7. State your conjecture that relates to functions, their inverses and the line $y = x$.
Verify this result by taking a new example for f (e.g. $f(x) = \frac{1}{1-x}$, $x \neq 1$).

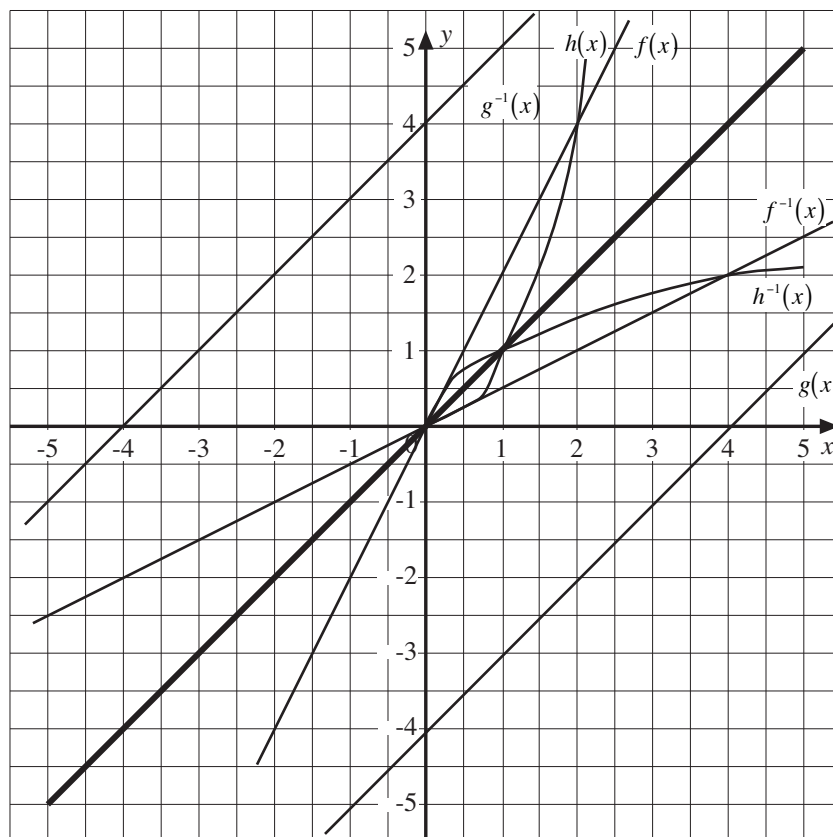
ACTIVITY G4.1

Notes and Solutions

Notes and solutions given only where appropriate.

G4.1 1. $f^{-1}(x) = \frac{1}{2}x$; $g^{-1}(x) = x + 4$; $h^{-1}(x) = +\sqrt{x}$ ($x \geq 0$)

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5. f^{-1} is the reflection of f in $y = x$.
7. In general, for any 1 : 1 mapping, $f(x)$ and $f^{-1}(x)$ are reflections of each other in the $y = x$ line.