UNIT G4 *Functions* — Data Sheets

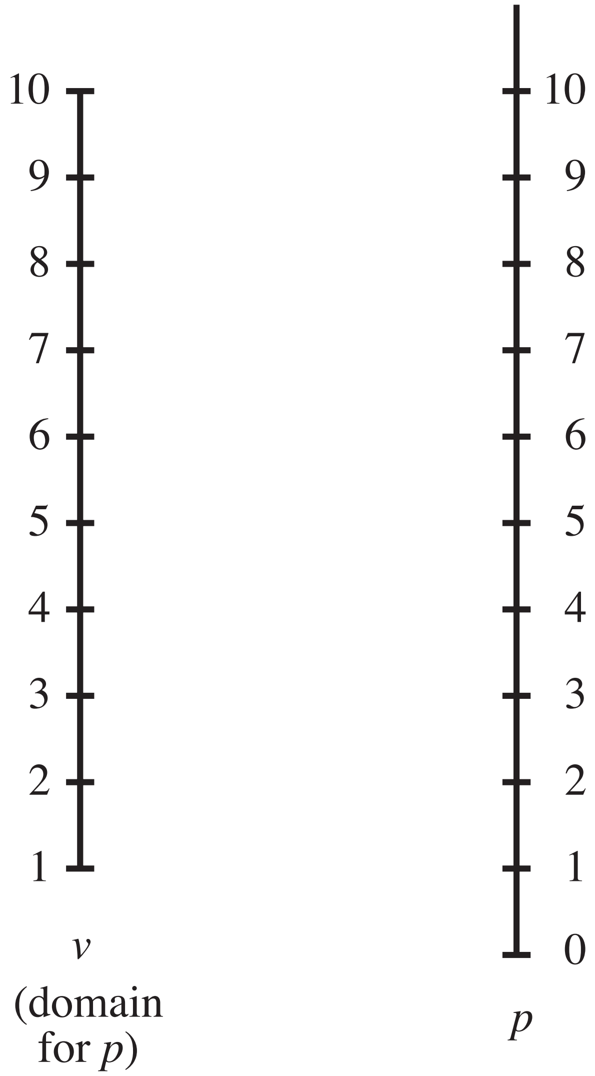
Data Sheets

Data Sheet G4.1 *Functions, Mappings and Domains 1*

Example If $p = \dfrac{10}{v}$, use the mapping diagrams below to show how v maps to p for $1 \le v \le 10$.

Consider integer values of v.

Solution



Extension Questions

What is the range for p ?

Is this a 1 : 1 mapping?

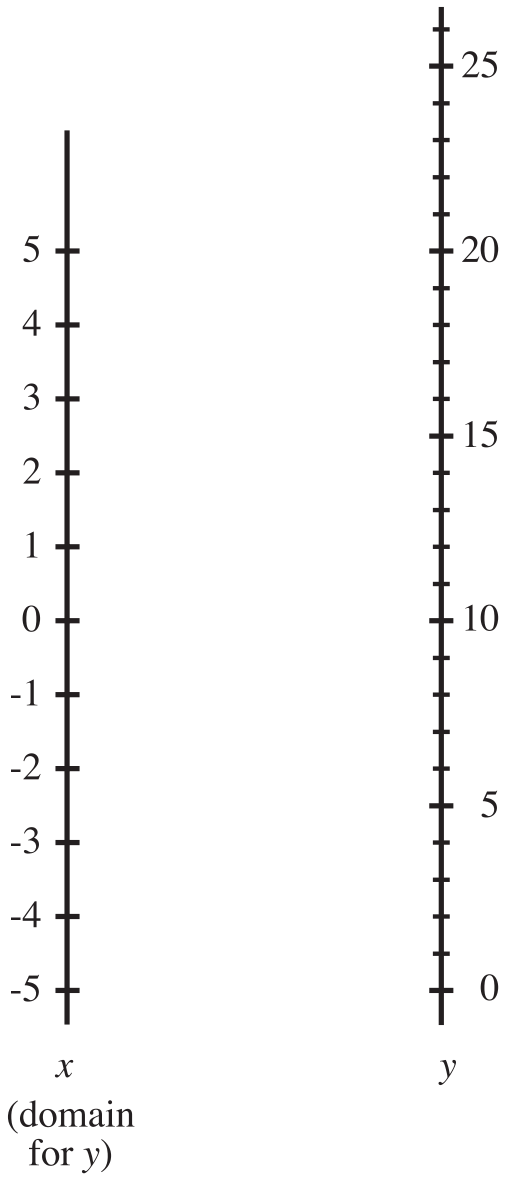
Data Sheet G4.2 *Functions, Mappings and Domains 2*

Example Complete the mapping diagram below for the function

$$y = f(x) = x^2, \quad -5 \le x \le 5$$

Consider integer values of x.

Solution



Extension Questions

What is the range for y ?

Is this a 1 : 1 mapping?

Data Sheet G4.3 *Functions, Mappings and Domains 3*

Example If f is defined by

$$f : x \to x^3 - x \text{ for all } x$$

what are the values of

(a) $f(-1)$ (b) $f(0)$ (c) $f(1)$ (d) $f(2)$?

Solution

(a) $f(-1) = \square^3 - \square = \square$

(b) $f(0) = \square^3 - \square = \square$

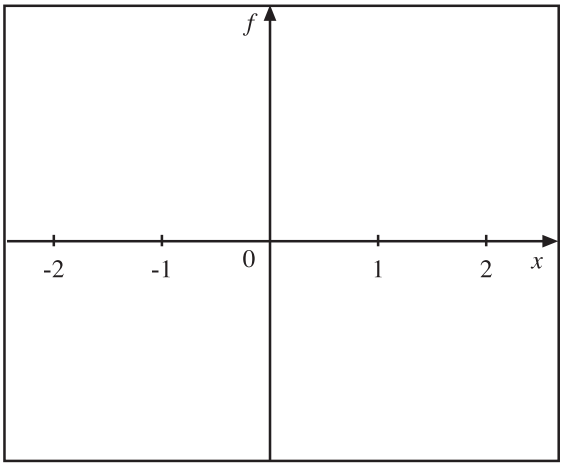
(c) $f(1) = \square^3 - \square = \square$

(d) $f(2) = \square^3 - \square = \square$

Extension Questions

What is the range for f ?

Sketch the function;



The function f is not a 1 : 1 mapping.

Explain why not.

Data Sheet G4.4 *Composite Functions*

The concept of a *function of a function* is introduced here.

Example The functions f and g are defined by

$$f : x \rightarrow 4x - 3$$
$$g : x \rightarrow 2x - 1$$

(a) Find $f(g(x))$ and $g(f(x))$.

(b) What are the values of $f(g(0))$ and $g(f(0))$?

Solutions

(a) $f(g(x)) = 4(g(x)) - 3$

$= 4(\square) - 3$

$= \square x + \square$

$g(f(x)) = 2(f(x)) - 1$

$= 2(\square) - 1$

$= \square x + \square$

(b) $f(g(0)) = \square \times \square + \square = \square$

$g(f(0)) = \square \times \square + \square = \square$

Data Sheet G4.5 *Inverse Functions 1*

If $F = \frac{9}{5}C + 32$, we can make C the subject of the equation by writing

$$\frac{5}{9}F = C + 32 \times \frac{5}{9}$$

or $$C = \frac{5}{9}(F - 32)$$

We say that F and C are *inverse functions*.

For inverse functions, f and g, then

$$\boxed{f(g(x)) = x = g(f(x))}$$

Example Show that if

$$f : x \rightarrow \frac{9}{5}x + 32$$

$$g : x \rightarrow \frac{5}{9}(x - 32)$$

then $f(g(x)) = x$

Solution

$$f(g(x)) = \frac{9}{5}\left(\boxed{}\right) + 32$$

$$= \boxed{}\,x - \boxed{} + \boxed{}$$

$$= \boxed{}\,x$$

Note: We write fg or $fg(x)$ to mean $f(g(x))$, etc.

Data Sheet G4.6 *Inverse Functions 2*

For functions that are 1 : 1 mappings, we can find their inverse functions.

Example If $f(x) = \dfrac{1}{1-x} + 2 \quad (x \neq 1)$, find its inverse function.

Solution

Let $y = \dfrac{1}{1-x} + 2$ and find x as a function of y.

i.e. $y - 2 = \dfrac{1}{(1-x)} + 2 - 2$

$$y - 2 = \frac{1}{\boxed{}}$$

$$1 - x = \frac{1}{\boxed{}}$$

$$x = \boxed{} - \frac{1}{\boxed{}}$$

$$f^{-1}(y) = \boxed{} - \frac{1}{\boxed{}}$$

So

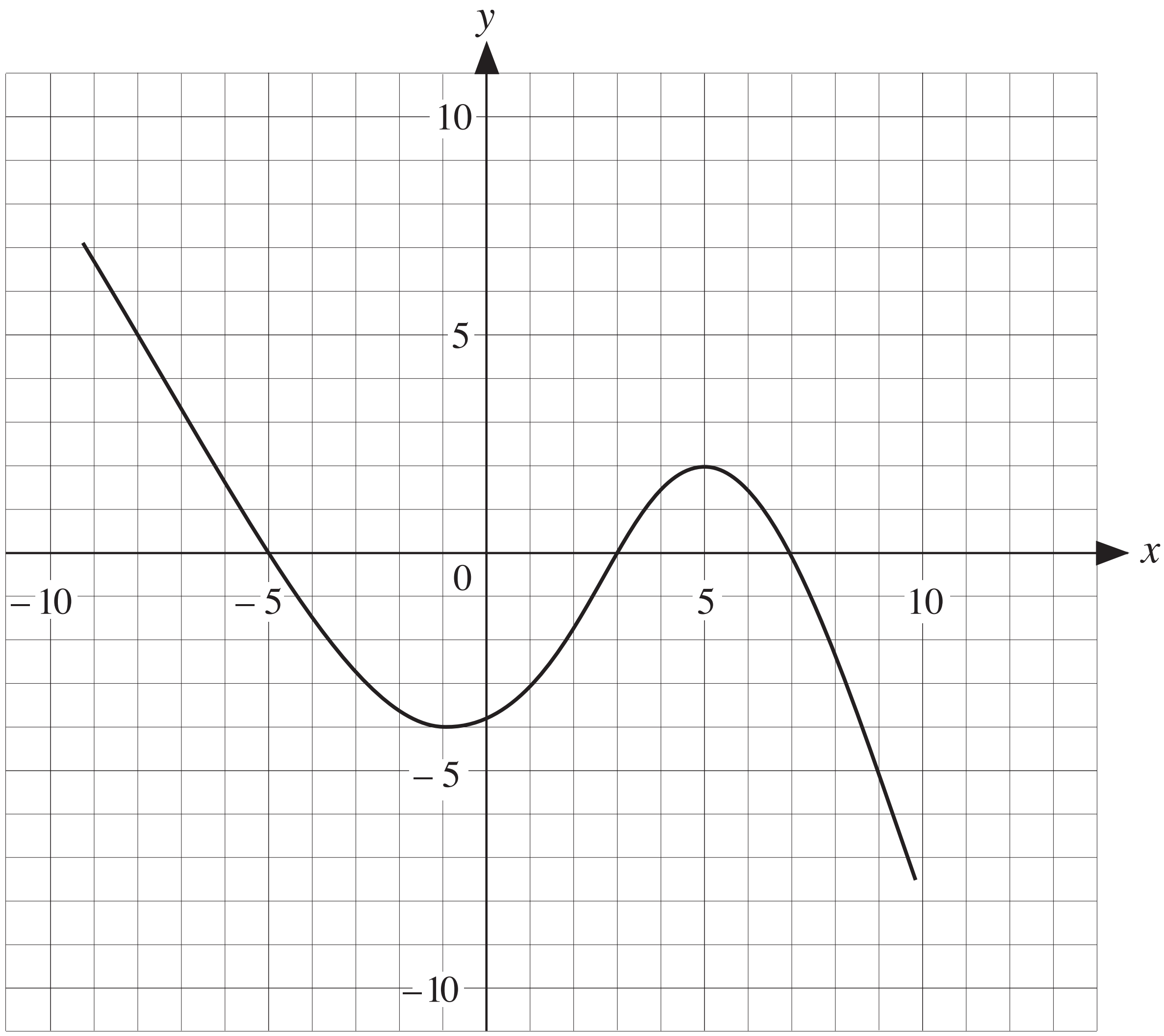
$$f^{-1}(x) = \boxed{} - \frac{1}{\boxed{}}$$

Check $f(2) = \dfrac{1}{1 - \boxed{}} + 2 = \boxed{}$

$$f^{-1}\left(\boxed{}\right) = \boxed{} - \frac{1}{\boxed{}} = \boxed{}$$

Data Sheet G4.7 *Graph Transforms 1*

The graph of $y = f(x)$ is shown below.



On the diagram, draw graphs of:

(a) $y = f(x + 3)$

(b) $y = f\left(\frac{x}{2}\right)$

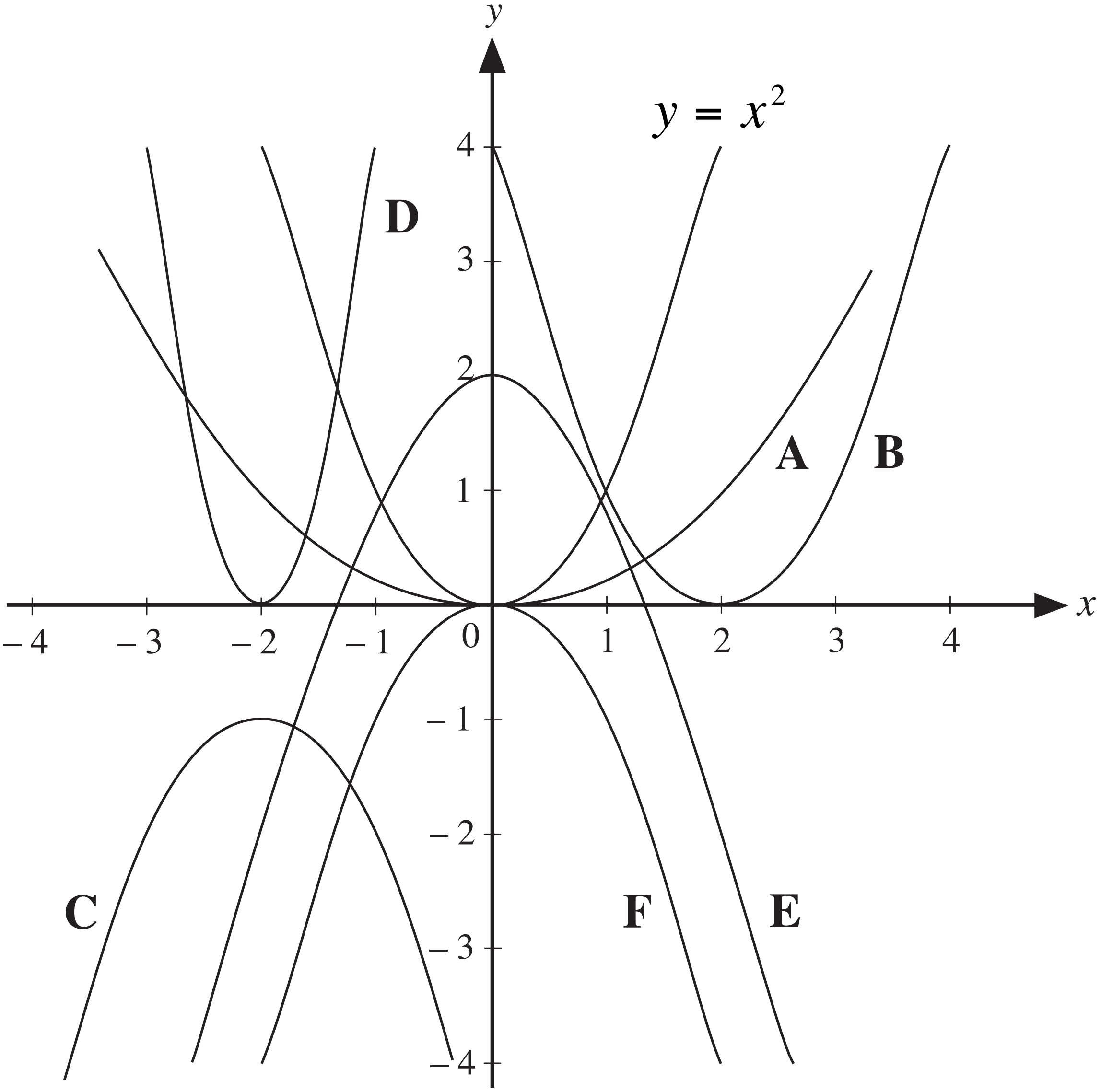
(c) $y = f(2x)$

(d) $y = f(x) - 2$

(e) $y = 2f(x)$

Data Sheet G4.8 *Graph Transforms 2*

The graph of $y = x^2$ is illustrated below, together with some transformations of this graph.



Suggest the possible forms of the transformation of $y = x^2$ to the functions with graphs labelled:

A ________________ **B** ________________ **C** ________________

D ________________ **E** ________________ **F** ________________