

1 Set Theory

1.1 Sets and Venn Diagrams

We use the idea of *sets* to classify numbers and objects and we use *Venn diagrams* to illustrate these sets.



Example

The sets A and B consist of numbers taken from the numbers $0, 1, 2, 3, \dots, 9$ so that

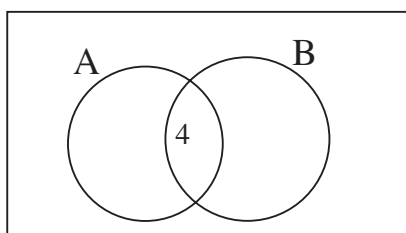
$$\text{Set A} = \{4, 7, 9\}$$

$$\text{Set B} = \{1, 2, 3, 4, 5\}$$

Illustrate these sets in a Venn diagram.

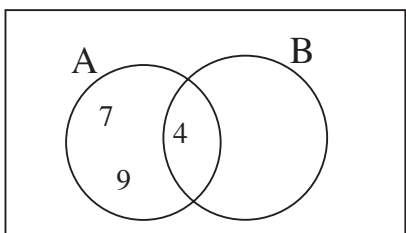


Solution

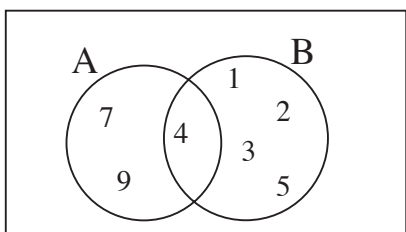


The framework for a Venn diagram is shown opposite, with the sets A and B indicated by the circles.

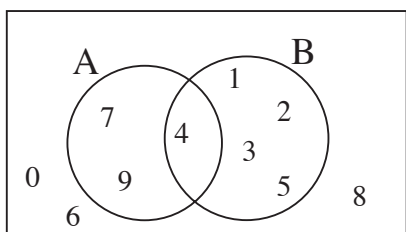
Since 4 is in both sets, it must be placed in the *intersection* of the two sets.



To complete set A, you put 7 and 9 in the part that does not intersect with B.



Similarly for B, you put 1, 2, 3 and 5 in the part that does not intersect with A.

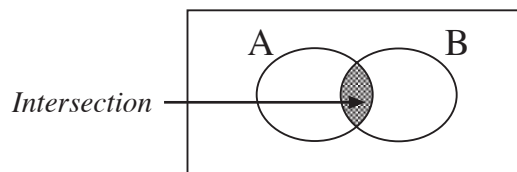


Finally, since the numbers 0, 6 and 8 have not been used in A or B, they are placed outside both A and B.

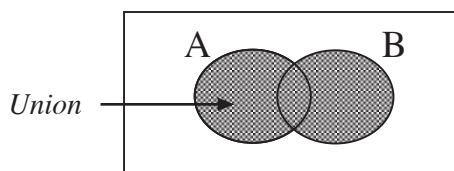


Note

The *intersection* of two sets consists of any numbers (or objects) that are in both A and B.



The *union* of two sets consists of any numbers (or objects) that are in A or in B or in both.



In the example above,

the intersection of A and B = { 4 }

the union of A and B = { 1, 2, 3, 4, 5, 7, 9 }

Note that, although the number 4 occurs in both A and B, it is *not* repeated when writing down the numbers in the union.

The complement of a set consists of any numbers (or objects) that are not in that set. In the example above,

the complement of A = { 0, 1, 2, 3, 5, 6, 8 }

the complement of B = { 0, 6, 7, 8, 9 }

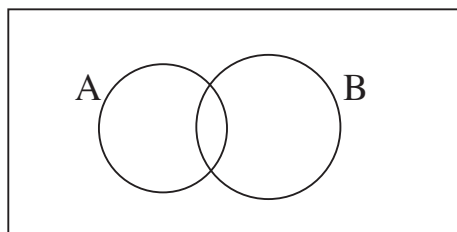


Exercises

1. Set A = { 1, 4, 5, 7, 8 }

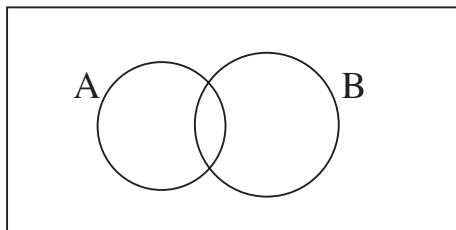
Set B = { 2, 6, 8, 10 }

- (a) Copy and complete the Venn diagram. Include all the whole numbers from 1 to 10.

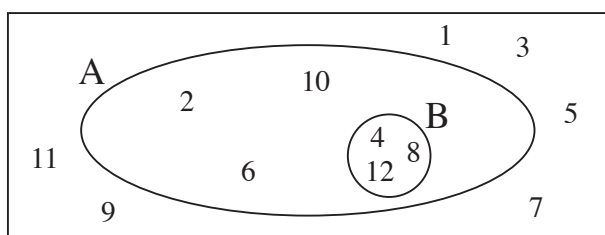


- (b) What is the intersection of A and B?
2. The whole numbers 1 to 10 are organised into 2 sets, set A and set B.
Set A contains all the odd numbers.
Set B contains all the numbers greater than 4.

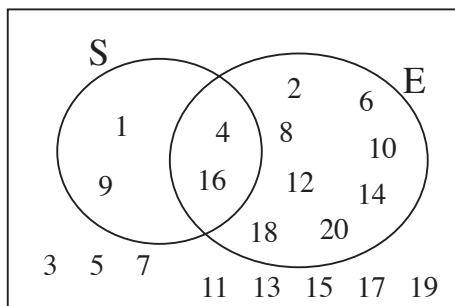
- (a) Copy and complete this diagram.



- (b) What is the union of A and B?
3. The whole numbers 1 to 12 are included in the Venn diagram.



- (a) List set A.
 (b) List set B.
 (c) Describe both sets in words.
 (d) What is the complement of A?
4. (a) Draw a Venn diagram to illustrate the sets P and Q. Include all the whole numbers from 1 to 15 in your diagram.
- $$P = \{3, 5, 7, 9\}$$
- $$Q = \{1, 3, 5, 7, 9, 11, 13, 15\}$$
- (b) What is the intersection of P and Q?
5. The whole numbers 1 to 20 are organised into sets as shown in the Venn diagram below.



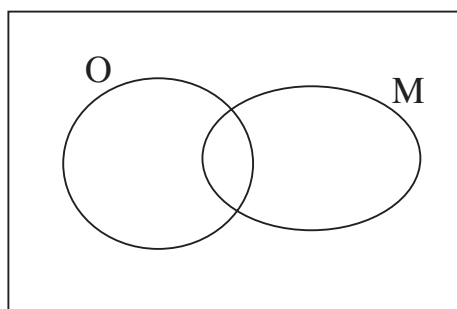
- (a) List set E.
 (b) List set S.
 (c) Describe each set in words.
 (d) What is the union of E and S?

6. The whole numbers 1 to 20 are organised into two sets,

O: Odd numbers

M: Multiples of 5

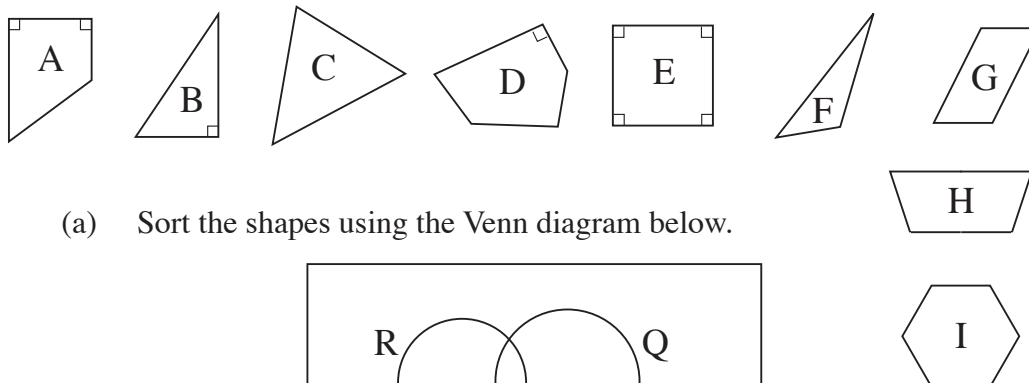
Copy and complete the Venn diagram, placing each number in the correct place.



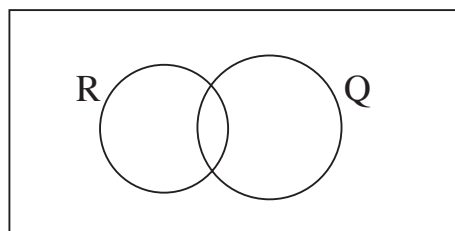
7. The shapes shown below are to be sorted into 2 sets, R and Q.

R contains shapes with a right angle.

Q contains shapes with four sides.



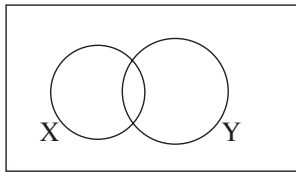
- (a) Sort the shapes using the Venn diagram below.



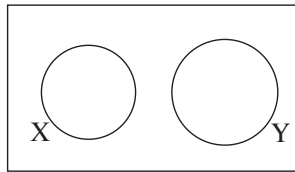
- (b) Which shapes are in both sets?
 (c) Which shapes are in R but not in Q?
 (d) Which shapes are not in R or Q?
8. Set P contains the letters needed to spell 'JENNY'.
 Set Q contains the letters needed to spell 'JEN'.
 Set R contains the letters needed to spell 'TED'.

- (a) Draw a Venn diagram for the two sets, P and R.
 (b) Draw a Venn diagram for the two sets, P and Q.

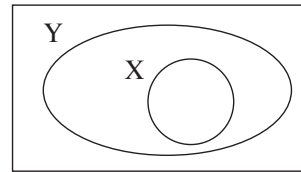
- (c) What is the union of P and R?
- (d) What is the intersection of P and R?
9. Set S contains silver coins in circulation in the UK.
Set R contains circular coins in circulation in the UK.
Draw a Venn diagram to illustrate these two sets. You should include *all* UK coins in the Venn diagram.
10. Which of these Venn diagrams would be best for the sets described below?



A



B



C

- (a) X is the set of all squares.
Y is the set of all rectangles.
- (b) X is the set of all triangles.
Y is the set of all squares.
- (c) X is the set of all quadrilaterals (4-sided shapes).
Y is the set of all triangles.
- (d) X is the set of all shapes containing a right angle.
Y is the set of all triangles.