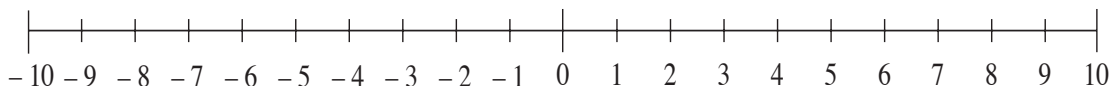


# 3 Negative Numbers

## 3.1 Negative Numbers

We extend our number system now to include negative numbers. It is useful to use a number line to illustrate this concept.



### Note: Inequality Signs

The sign  $<$  means 'less than' as in, for example,  $4 < 7$ , whilst  $>$  means 'greater than'; so  $8 > 4$ .

Using inequality signs you can see, for example, that

$$-2 < 4$$

$$-6 < -3$$

$$5 > -4$$

$$-1 > -7$$

You can check all these by looking at their positions on the number line.



### Example

Make each statement below true by using the symbols  $<$  or  $>$ .

(a)  $-5$    $4$

(b)  $3$    $7$

(c)  $-6$    $-9$

(d)  $2$    $-2$



### Solution

(a)  $-5 < 4$  (b)  $3 < 7$  (c)  $-6 > -9$  (d)  $2 > -2$



## Exercises

- What temperature is:
 

|                                                           |                                                            |
|-----------------------------------------------------------|------------------------------------------------------------|
| (a) $3^{\circ}\text{C}$ warmer than $-1^{\circ}\text{C}$  | (b) $6^{\circ}\text{C}$ colder than $-3^{\circ}\text{C}$   |
| (c) $5^{\circ}\text{C}$ warmer than $-5^{\circ}\text{C}$  | (d) $8^{\circ}\text{C}$ warmer than $-7^{\circ}\text{C}$   |
| (e) $5^{\circ}\text{C}$ colder than $-2^{\circ}\text{C}$  | (f) $3^{\circ}\text{C}$ colder than $1^{\circ}\text{C}$    |
| (g) $6^{\circ}\text{C}$ colder than $2^{\circ}\text{C}$   | (h) $8^{\circ}\text{C}$ warmer than $-12^{\circ}\text{C}$  |
| (i) $10^{\circ}\text{C}$ colder than $-2^{\circ}\text{C}$ | (j) $20^{\circ}\text{C}$ warmer than $-12^{\circ}\text{C}$ |
- What number is;
 

|                       |                       |
|-----------------------|-----------------------|
| (a) 3 more than $-2$  | (b) 6 less than 1     |
| (c) 5 more than $-7$  | (d) 6 more than $-10$ |
| (e) 5 less than $-4$  | (f) 16 less than 3    |
| (g) 5 more than $-20$ | (h) 6 more than 5     |
| (i) 12 less than 10   | (j) 20 more than $-8$ |
- Write each set of numbers in order with the smallest first.
 

|                                               |                                    |
|-----------------------------------------------|------------------------------------|
| (a) 6, $-7$ , 8, $-2$ , $-5$ , $-10$ , 3      | (b) 3, $-2$ , 8, 0, $-1$ , 1, $-3$ |
| (c) 5, $-7$ , $-20$ , 100, $-50$ , $-90$ , 60 |                                    |
- Put either a  $<$  or  $>$  sign between each pair of numbers to give a true statement.
 

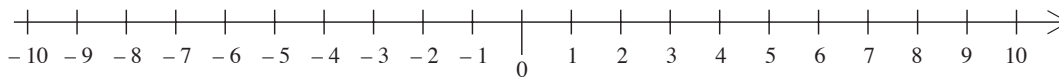
|               |               |
|---------------|---------------|
| (a) 4      2  | (b) $-6$ $-2$ |
| (c) $-3$ 4    | (d) 2 $-4$    |
| (e) $-6$ $-7$ | (f) $-6$ $-5$ |
| (g) 0      1  | (h) $-1$ 0    |
- Is each statement below true or false?
 

|               |               |
|---------------|---------------|
| (a) $6 > 7$   | (b) $4 > 3$   |
| (c) $8 > -1$  | (d) $5 > -6$  |
| (e) $-6 < -7$ | (f) $-1 > 0$  |
| (g) $-3 < 2$  | (h) $-7 < 6$  |
| (i) $-4 > -3$ | (j) $-5 < -2$ |
- Write down any integer that could go in the boxes below.
 

|                         |                         |
|-------------------------|-------------------------|
| (a) $5 < \square < 7$   | (b) $-5 < \square < -3$ |
| (c) $-3 > \square > -7$ | (d) $-6 < \square < 0$  |
| (e) $-1 < \square < 2$  |                         |

## 3.2 Addition and Subtraction

In this section we consider how to add and subtract when working with negative numbers. This is done using a number line.



To *add a positive number*, move to the *right* on a number line.

To *add a negative number*, move to the *left* on a number line.



### Example 1

Calculate:

(a)  $-3 + 8$

(b)  $4 + (-2)$

(c)  $-8 + 5$

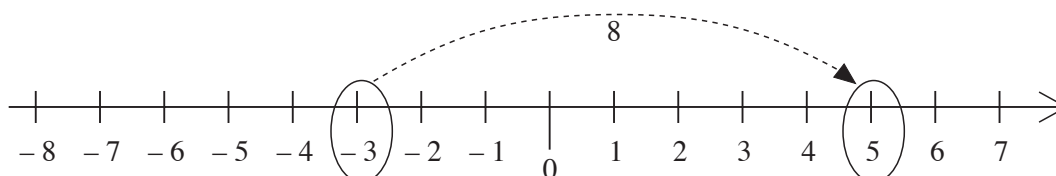
(d)  $-1 + (-3)$



### Solution

(a)  $-3 + 8$

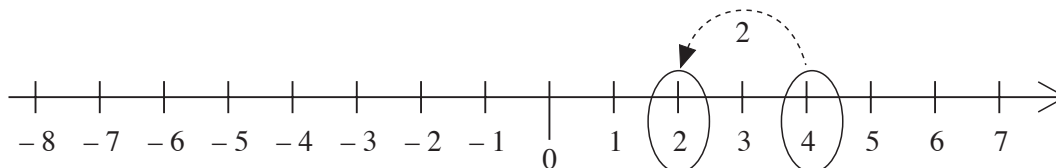
Start at  $-3$  and move 8 to the right:



So,  $-3 + 8 = 5$ .

(b)  $4 + (-2)$

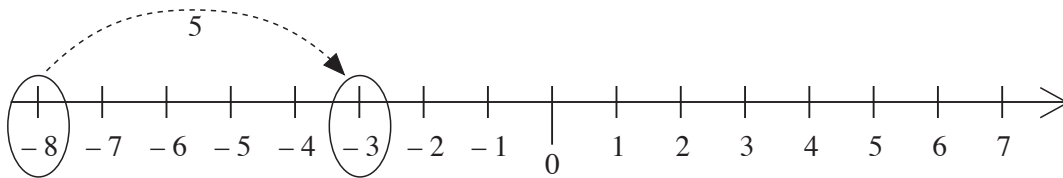
Start at 4 and move 2 to the left:



So,  $4 + (-2) = 2$ .

c)  $-8 + 5$

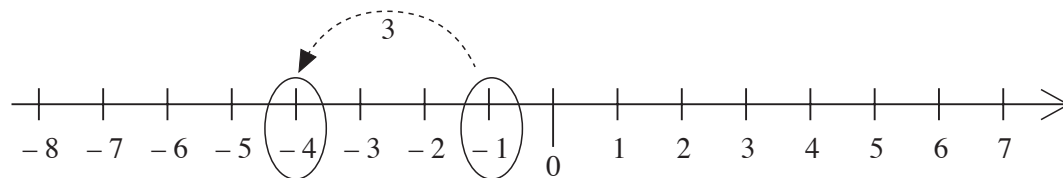
Start at  $-8$  and move 5 to the right:



So,  $-8 + 5 = -3$ .

(d)  $-1 + (-3)$

Start at  $-1$  and move 3 to the left:



So,  $-1 + (-3) = -4$ .

Subtraction is the *inverse* (or opposite) of addition, so

To *subtract a positive number*, move to the *left* on a number line.

To *subtract a negative number*, move to the *right* on a number line.



## Example 2

Calculate:

(a)  $5 - 7$

(b)  $-2 - 3$

(c)  $4 - (-2)$

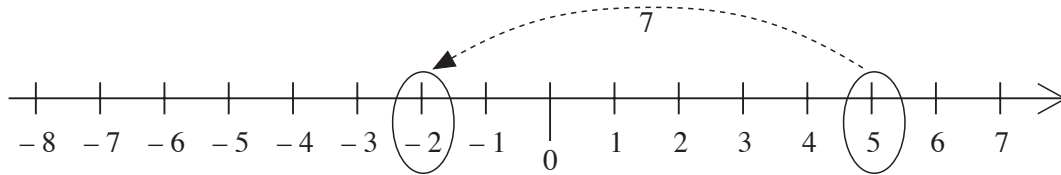
(d)  $-5 - (-3)$



## Solution

(a)  $5 - 7$

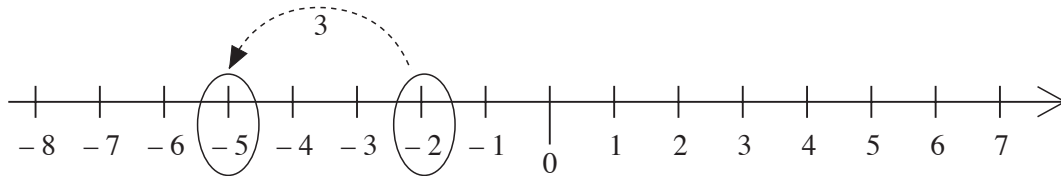
Start at 5 and move 7 to the left:



So,  $5 - 7 = -2$ .

(b)  $-2 - 3$

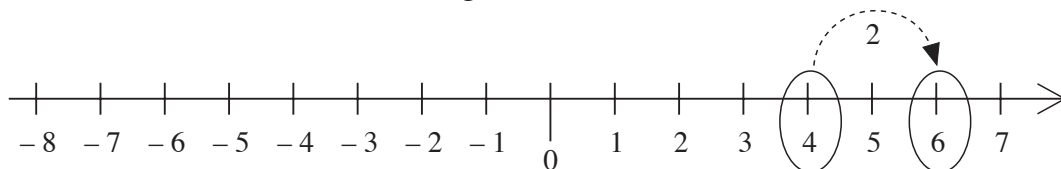
Start at -2 and move 3 to the left:



So,  $-2 - 3 = -5$ .

(c)  $4 - (-2)$

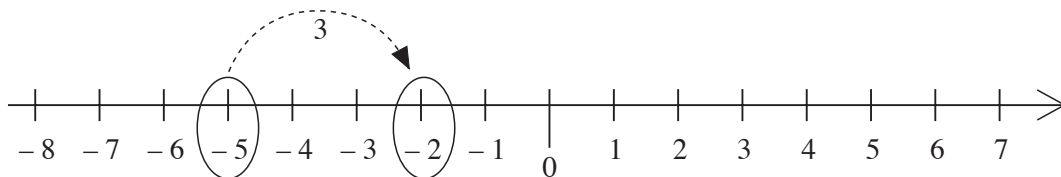
Start at 4 and move 2 to the right:



So,  $4 - (-2) = 6$ .

(d)  $-5 - (-3)$

Start at -5 and move 3 to the right:



So,  $-5 - (-3) = -2$ .



## Exercises

1. Use a number line to work out the following calculations:

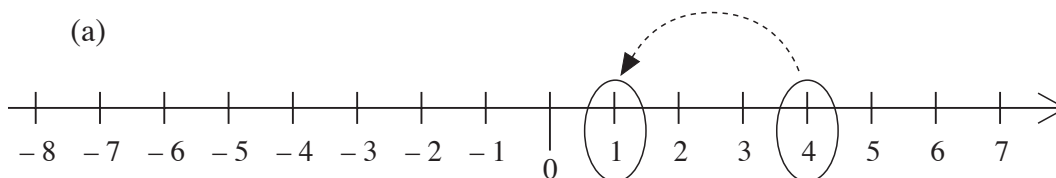
- |                 |                |                 |
|-----------------|----------------|-----------------|
| (a) $-4 + 6$    | (b) $-5 + 8$   | (c) $-1 + 3$    |
| (d) $-4 + 7$    | (e) $2 + (-3)$ | (f) $-1 + (-4)$ |
| (g) $-2 + (-3)$ | (h) $-6 + 6$   | (i) $-7 + 4$    |
| (j) $-6 + 2$    | (k) $-7 + 2$   | (l) $5 + (-5)$  |

2. Use a number line to calculate:

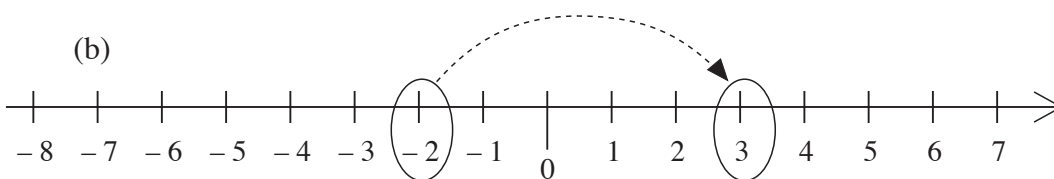
- |                 |                |                 |
|-----------------|----------------|-----------------|
| (a) $4 - 6$     | (b) $5 - 7$    | (c) $2 - 4$     |
| (d) $-1 - 1$    | (e) $-3 - 2$   | (f) $-4 - (-1)$ |
| (g) $3 - (-4)$  | (h) $5 - (-6)$ | (i) $8 - 12$    |
| (j) $-5 - (-1)$ | (k) $4 - 9$    | (l) $-4 - (-4)$ |

3. Write down the two possible sums that could be shown by each number line below:

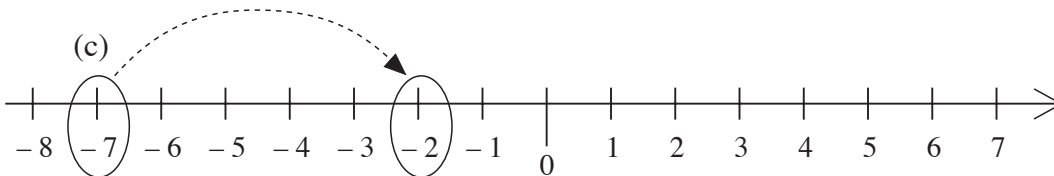
(a)



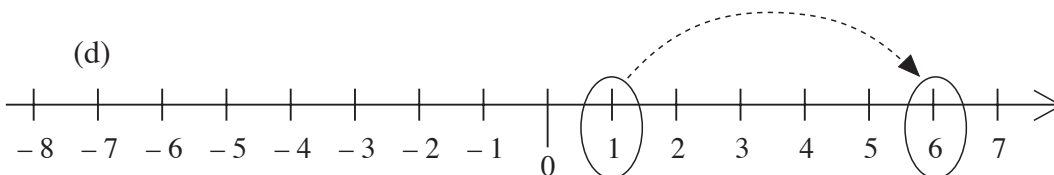
(b)



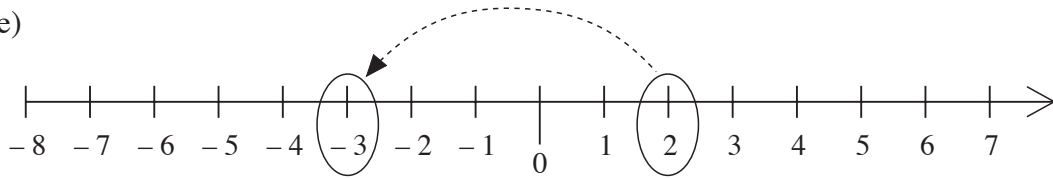
(c)



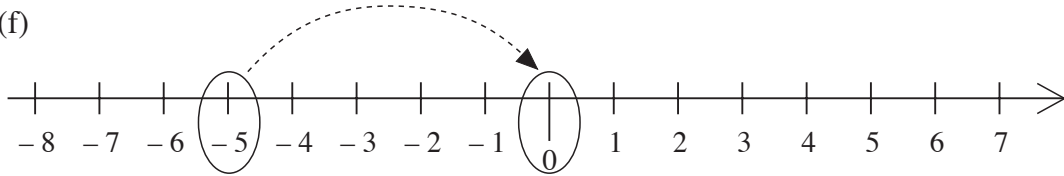
(d)



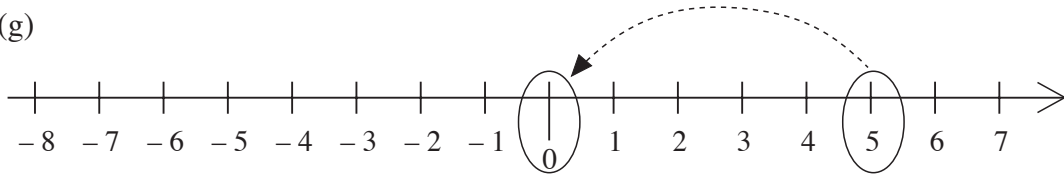
(e)



(f)



(g)



4. Copy and complete this addition table:

| +  | -4 | -2 | 0 | 2 | 4 |
|----|----|----|---|---|---|
| -3 |    |    |   |   |   |
| -1 |    |    |   |   |   |
| 1  |    |    |   |   |   |
| 3  |    |    |   |   |   |

5. Fill in the missing numbers on a copy of this addition table:

| +  |    | 1  |   | 3 |    |
|----|----|----|---|---|----|
| -5 | -5 |    |   |   | -1 |
|    |    | -3 |   |   |    |
|    |    |    |   | 0 |    |
|    | -2 |    | 0 |   |    |

6. Copy these equations and fill in the missing numbers:

(a)  $4 + \dots = 1$

(b)  $3 - \dots = -6$

(c)  $-6 + \dots = -7$

(d)  $\dots + 7 = 2$

(e)  $\dots - 7 = -8$

(f)  $4 - \dots = 7$

7. Write down the next 5 terms in each sequence:

(a) 10, 8, 6, 4, ...

(b) -10, -7, -4, -1, ...

(c) 19, 14, 9, 4, ...

(d) -1, -3, -5, -7, ...

(e) -20, -16, -12, -8, ...

(f) -5, -10, -15, -20 ...

(g) 18, 15, 12, 9, ...

(h) -16, -12, -8, -4, ...

8. Overnight, the temperature dropped from  $5^{\circ}\text{C}$  to  $-14^{\circ}\text{C}$ . By how many degrees did the temperature fall?

9. One day the level of the water in a river was 30 cm *above* its average level. One week later it was 12 cm *below* its average level. How far did the water level drop in the week?

10. A chest of treasure was hidden in the year 64 BC and found in 284 AD. For how long was the chest hidden?

## 3.3 Multiplication and Division

In this section we look at how to multiply and divide negative numbers.



### Example 1

(a) Calculate

$$(-2) + (-2) + (-2) + (-2) + (-2).$$

(b) Fill in the missing numbers:

$$(-2) + (-2) + (-2) + (-2) + (-2) = \dots \times (-2) = \dots$$



## Solution

(a)  $(-2) + (-2) + (-2) + (-2) + (-2) = -10$

(b)  $(-2) + (-2) + (-2) + (-2) + (-2) = 5 \times (-2) = -10$

In Example 1 we see that

*a positive number multiplied by  
a negative number gives a negative answer.*

This table shows what happens to the sign of the answer  
when positive and negative numbers are *multiplied*:

|   |   |   |
|---|---|---|
| × | + | - |
| + | + | - |
| - | - | + |

The same table can be used for *division* of positive and  
negative numbers.



## Example 2

Work out the following:

(a)  $5 \times (-7)$

(b)  $(-8) \times (-10)$

(c)  $(-42) \div 6$

(d)  $(-88) \div (-8)$



## Solution

(a) First calculate  $5 \times 7 = 35$ .

As a positive number is multiplied by a negative number, the answer will be negative:

$$5 \times (-7) = -35$$

(b) First calculate  $8 \times 10 = 80$ .

Here a negative number is multiplied by a negative number, so the answer will be positive:

$$(-8) \times (-10) = 80$$

(c) First calculate  $42 \div 6 = 7$ .

As a negative number is divided by a positive number, the answer will be negative:

$$(-42) \div 6 = -7$$

- (c) First calculate  $88 \div 8 = 11$ .

As a negative number is divided by a negative number, the answer will be positive:

$$(-88) \div (-8) = 11$$



## Exercises

1. Calculate:

- (a)  $(-7) \times 2$       (b)  $(-4) \times 8$       (c)  $(-2) \times (-5)$   
 (d)  $(-6) \times (-3)$       (e)  $(-3) \times 7$       (f)  $(-10) \times (-4)$   
 (g)  $8 \times 4$       (h)  $3 \times (-6)$       (i)  $(-7) \times (-2)$   
 (j)  $(-4) \times (-5)$       (k)  $(-7) \times 0$       (l)  $8 \times (-5)$

2. Calculate:

- (a)  $(-10) \div (-2)$       (b)  $(-15) \div 5$   
 (c)  $18 \div (-3)$       (d)  $14 \div (-7)$   
 (e)  $(-21) \div (-3)$       (f)  $(-45) \div 9$   
 (g)  $50 \div (-5)$       (h)  $(-100) \div (-4)$   
 (i)  $80 \div (-2)$       (j)  $26 \div (-13)$   
 (k)  $(-70) \div (-7)$       (l)  $(-42) \div 7$

3. Copy and complete these multiplication tables:

(a)

| $\times$ | 1 | 2 | 3 | 4 |
|----------|---|---|---|---|
| -1       |   |   |   |   |
| -2       |   |   |   |   |
| -3       |   |   |   |   |
| -4       |   |   |   |   |

(b)

| $\times$ | 1 | 0 | -1 | -2 | -3 |
|----------|---|---|----|----|----|
| -4       |   |   |    |    |    |
| -2       |   |   |    |    |    |
| 0        |   |   |    |    |    |
| 1        |   |   |    |    |    |

4. Copy and complete these multiplication tables:

(a)

|    |    |    |    |
|----|----|----|----|
| ×  |    | -1 |    |
| 2  |    |    |    |
|    | -2 | 2  |    |
| -3 |    |    | -9 |

(b)

|    |    |   |     |
|----|----|---|-----|
| ×  | -2 |   |     |
|    | 10 |   |     |
| -2 |    | 6 |     |
| 3  |    |   | -12 |

5. Copy these calculations, filling in the missing numbers:

- (a)  $\dots \times 5 = -20$                       (b)  $(-80) \div \dots = 4$   
 (c)  $16 \times \dots = -32$                       (d)  $(-4) \times \dots = 32$   
 (e)  $\dots \times (-3) = 12$                       (f)  $40 \div \dots = -8$   
 (g)  $-8 \times \dots = 48$                       (h)  $-32 \div \dots = 4$   
 (i)  $15 \times \dots = -60$                       (j)  $100 \div \dots = -25$

6. Write down the next 3 terms in each sequence:

- (a) 1, -2, 4, -8, 16, ...  
 (b) -1, 2, -4, 8, -16, ...  
 (c) 1, -10, 100, -1000, ...  
 (d) 1, -3, 9, -27, ...  
 (e) -1, 5, -25, 125, ...

For each sequence, describe the rule that is used to calculate the next term.

7. Make 2 copies of this multiplication table and fill in the missing numbers in 2 different ways:

|   |   |   |   |    |
|---|---|---|---|----|
| × |   |   |   |    |
|   | 1 |   |   |    |
|   |   | 4 |   |    |
|   |   |   | 9 |    |
|   |   |   |   | 25 |

8. Calculate:

(a)  $3 \times (-8) \times (-4)$

(b)  $(-4) \times (-8) \times (-2)$

(c)  $(-2) \times (-2) \times 2$

(d)  $4 \times (-7) \times 2$

(e)  $(-2) \times 8 \times (-4)$

(f)  $(-6) \times (-2) \times (-1)$

9. Calculate:

(a)  $\frac{(-3) \times (-4)}{(-2)}$

(b)  $\frac{5 \times (-6)}{(-2)}$

(c)  $\frac{(-7) \times (-5) \times (-2)}{5}$

(d)  $\frac{8 \times (-9) \times 6}{(-2) \times (-3)}$

(e)  $\frac{(-6) \times (-4)}{2}$

(f)  $\frac{(-4) \times (-7) \times 3}{(-12)}$

10. Calculate:

(a)  $(-6 + 10) \div (-2)$

(b)  $(12 - 24) \div (-2)$

(c)  $(6 + (-8)) \times (4 - 7)$

(d)  $((-2) + 8) \times ((-4) + 2)$

(e)  $((-4) \times 2) + (6 \times (-9))$

(f)  $(8 \times (-2)) - ((-4) \times 8)$

11. Calculate:

(a)  $(-6) \times (-3) + (-4)$

(b)  $(-5) \times 4 - (-3)$

(c)  $(-8) \times (-7) - 8 \times 7$

(d)  $(-11) \times 4 + (-8) \times (-3)$

(0, 5), (1, 5), (1, 4), (2, 4), (2, 5), (3, 5), (3, 4), (4, 4), (4, 5), (5,