Developing Questioning Within a Secondary Mathematics Methods Course

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Teacher questioning impacts the level of student thinking that occurs during mathematical instruction. It is important to investigate how questioning is developed in secondary mathematics methods classes so that these environments can support the development of effective questioning strategies. This article explores how two preservice teachers use teacher noticing techniques to elicit, interpret, and respond to student thinking through a cycle of planning a lesson, practicing it with peers, revising it, teaching it in a high school classroom, and reflecting on it. The analysis explores how the preservice teachers use questioning to either privilege or minimize student thinking through focusing or funneling questioning structures and how flexible the preservice teachers were in asking questions based on what was noticed in student responses. While both participants stated the goal of creating student-centered learning environments, they varied in their ability to privilege student thinking to create such environments while teaching their lessons.

Questioning is one of the most important areas of preservice teacher preparation as it connects directly with eliciting, interpreting, and responding to student thinking (Ghousseini, 2015; Singer-Gabella et al., 2016; Sleep & Boerst, 2012; Stein et al., 2008). Preservice teachers bring various learning experiences into their teaching, which can create tensions around attending to and engaging with student thinking (Singer-Gabella et al., 2016). However, there is also a growing body of evidence that preservice teachers can learn and develop effective questioning practices through well-designed activities (Monson et al., 2020; Singer-Gabella et al., 2016; Sleep & Boerst, 2012; Star et al., 2011; Star & Strickland, 2008).

The study presented in this article explores how preservice teachers plan questions to ask during a lesson, how they use questioning in the lesson, and how they choose to act upon what they learn from student responses, specifically:

> How do preservice teachers use questioning to elicit, interpret, and respond to student thinking? In what ways does preservice teachers' questioning change through the plan-practice-teach cycle?

The study explores how preservice teachers use information about student thinking collected through the plan-practice-teach cycle, what challenges they encountered, and how they addressed them. Participants in the study were members of a secondary mathematics methods course. The focal activities for this study involved planning a high school mathematics lesson, practicing the lesson with their peers, and teaching it in a high school mathematics classroom.

Theoretical Background

This study examines how *teacher noticing* supports preservice teachers in developing how to use questioning effectively. Teacher noticing explores what teachers see during instruction, how they interpret what they see, and how they respond based on their interpretations (Bastian et al., 2021; Copur-Gencturk & Rodrigues, 2021; Sherin et al.,

2011). Teacher noticing relies on three key elements: "What is important here? Can I understand what is happening here? What is this a case of?" (Sherin & van Es, 2003). Teachers should use what they notice while teaching to ask the right questions at the right time and to adjust questioning based on perceived student understanding of a specific topic (Choy & Dindyal, 2021; Levin et al., 2009; Monson et al., 2020; National Council of Teachers of Mathematics (NCTM), 1991; Tienken et al., 2009; van den Kieboom et al., 2014). As such, teacher noticing should be a component of all teacher education and professional learning programs (Bastian et al., 2021; Copur-Gencturk & Rodrigues, 2021; Monson et al., 2020; Simpson & Haltiwanger, 2017; van Es & Sherin, 2002). When preservice teachers notice and attend to student thinking, they may struggle to move from an idealized version of what they envision while planning to the reality of enacting lessons in the classroom (Clift & Brady, 2005; van Es & Conroy, 2009). Throughout this study, teacher noticing provides a lens for how preservice teachers plan for and make sense of what students say and do.

Preservice teachers can learn, teach, and reflect on questioning practices (Blanton et al., 2001; Grosser-Clarkson, 2016; Singer-Gabella et al., 2016). Examining questioning patterns provides insight into preservice teachers' use of student thinking, moving away from the traditional Initiation-Reply-Evaluation (IRE) questioning structure of Mehan (1979) to considering whether they *focus* on student thinking to guide the direction of instruction or *funnel* instruction to match expectations, how they demonstrate their views about the role of the teacher, and whether the interpretations of student thinking formed during the lesson or those envisioned from the lesson plan guide the lesson. When focusing, teachers use questions to allow students to use their own and their peer's thinking to reason about mathematics; when funneling, teachers use their thinking to guide the lesson, and questions are used to have students create the teacher's explanation (Hagenah et al., 2018; Herbel-Eisenmann & Breyfogle, 2005; Wood, 1998). Focusing uses student ideas to move the lesson forward while funneling highlights students developing ideas similar to the teacher's way of thinking (Hagenah et al., 2018).

Effective questioning requires understanding how to notice what students are doing and how to elicit, interpret, and respond to student thinking (see Figure 1).

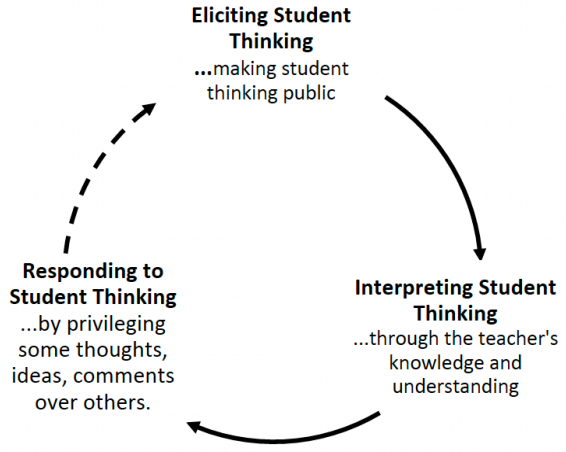

Figure 1. Eliciting, Interpreting, and Responding to Student Thinking

Eliciting student thinking involves making it public in the learning environment (Singer-Gabella et al., 2016), and teachers need to notice what students are doing. Teachers *interpret*

student thinking using their mathematical knowledge and experience without allowing outside factors or their interpretations to get in the way (Davis, 1997; Lambert, 1985; Singer-Gabella et al., 2016; Stein et al., 2008). When teachers *respond to students' thinking*, they act on what they have learned, choosing what to privilege and bypass (Singer-Gabella et al., 2016). Studies show that while leveraging student thinking is not straightforward (Moyer-Packenham & Milewicz, 2002), with the proper structure and support, preservice teachers can attend to student thinking while learning how to manage classroom instruction (Freese, 2006; Levin et al., 2009; Monson et al., 2020; Singer-Gabella et al., 2016; Windschitl et al., 2012).

Planning effective questions is an important part of eliciting, interpreting, and responding to student thinking (Corley & Rauscher, 2013; McCarthy et al., 2016; Moyer-Packenham & Milewicz, 2002). Teachers should "plan a route and strategy to use questions productively and develop students' thinking based on the learning objectives of their lessons" (Tienken et al., 2009, p. 42). How preservice teachers respond may be influenced by their vision of the lesson created during the planning process.

Teachers must link conceptual understanding of content with effective teaching practices focusing on student support (Ball & Forzani, 2009; Bransford et al., 2005). Teacher preparation programs are starting to balance content knowledge, teaching practices, and practical experiences best to prepare candidates for the complexities of the teaching profession (McDonald et al., 2013). An important direction advocated for in preservice teacher education is the use of practice-based activities, or "professional training that attempts to focus novices' learning more directly on the work of teaching rather than on traditional academic or theoretical topics that may have only marginal relevance to the realities of the classroom" (Forzani, 2014). Preservice teachers experience two worlds, one in their collegiate environment and one as a preservice teacher. As a result, there is a gap between how preservice teachers consider what to do (from their collegiate experiences) and what they can do (while teaching in the classrooms) (Kennedy, 1999).

Practicing lessons within a plan-practice-teach cycle provides preservice teachers the chance to develop as teachers by practicing new strategies with their peers in a safe environment (Kazemi et al., 2009; Lampert, 2010; Monson et al., 2020) before applying them in the classroom. This also allows preservice teachers to notice solution strategies and begin to plan how to address the needs they observe. Preservice teachers can then use student (i.e., peer) responses to interpret and inform their in-the-moment teaching decisions (Davis, 1997; Doerr, 2006), consistent with the development of teacher noticing (Bastian et al., 2021; Monson et al., 2020). Providing opportunities for novice teachers to experience teaching through rehearsal is beneficial as it approximates the act of teaching to problems of practice in controlled activities accompanied by coaching by teacher educators (Ghousseini et al., 2015). Preservice teachers receive feedback on their lessons from the course instructor and their peers, then revise their lesson plans to reflect this feedback. Preservice teachers share their thinking in a safe environment and improve alignment between instructional goals, planning the lesson, practicing with peers, and teaching the lesson in the classroom (Kazemi et al., 2016; Lampert et al., 2013). The coordination of such practices within a teacher preparation program allows preservice teachers to identify the benefit of visualizing and enacting lessons within the classroom experience (Ball & Cohen, 1999). There is currently little research on how preservice teachers use teacher noticing to elicit, interpret, and respond to student thinking, to use and develop questioning structures, and to use effective practices through the plan-practice-teach cycle.

Ghousseini and Herbst (2016) explored planning, practicing, and teaching in their study of secondary mathematics preservice teachers. Through investigation of planning, peer practice, teaching the lesson in a classroom, and reflection, preservice teachers could learn effective ways to enact effective teaching practices successfully. Teacher educators must focus on key instructional practices to maximize preservice teacher development (Ghousseini & Herbst, 2016). However, the pathway to developing teaching competence is not simple. Thompson et al. (2013) showed how secondary science preservice teachers developed in three patterns: one readily integrated new practices, one included new practices at specific times, and one used language from the new practices to label what they were doing without changing practice. This demonstrates the importance of teacher-educator feedback and intentionality in supporting preservice teachers differentially.

There is a gap in research and activities designed to help preservice teachers respond to student thinking and actions in the classroom (Kazemi et al., 2009; Matsko & Hammerness, 2014). Preservice teachers who participate in plan-practice-teach improve their teaching practices (Zembal-Saul et al., 2000) and connect their knowledge to important aspects of teaching (Lampert et al., 2013). What is missing is how the cycle supports preservice teachers in eliciting, interpreting, and responding to student thinking. This study focuses on how student thinking is privileged during instruction using different questioning structures and how the experience of the plan-practice-teach cycle may influence the noticing and actions of preservice teachers as they elicit, interpret, and respond to student thinking during the lesson.

Choice of Design

The design chosen for this study is a multiple-case study. The multiple case study allows for in-depth analysis of individual participants and comparison across cases to examine differences that stem from potential participant variances. This also allows for analysis of questioning and a richer understanding of the how and the why of questioning decisions, the phenomenon of interest in the study (Bernard & Ryan, 2010; Creswell, 2013; Yin, 2006). In this case study design, multiple lines of evidence (see the data elements below) combine to provide a rich understanding of questioning (Yin, 2006).

Participants

Participants were volunteers enrolled in a secondary mathematics methods course offered at a large mid-Atlantic public university. The methods course focused on developing practices for secondary mathematics teachers. It included activities that encouraged active classroom discourse and examined patterns and techniques of questioning through examination of lesson plans, classroom videos (of both self and others), and research-based practices. The course met once per week in the evenings and was a discourse-rich environment, modeling what was expected in high school classrooms. All the participants in the study were graduate students with full-time intern teaching placements (meaning that they were responsible for all aspects of instruction in the classroom). They were engaged in a program to achieve certification. The study started with six participants, and five continued throughout the study. Of these five participants, two are included in this discussion. These participants were selected as they represented the most significant differential development of questioning across the plan-practice-teach cycle and demonstrated what was possible and what constraints and challenges may need to be overcome to improve questioning practices for preservice teachers.

Design of the Study and Data Collection

The preservice teachers explored student thinking throughout their methods course in various activities provided by the course instructor (who was not the researcher), which are not included in this study. They analyzed student work to notice what was important in instruction and interpret student thinking, focusing on assessing and advancing questions (NCTM, 2014; Smith & Stein, 2018). As these activities were student work and not embedded in the teaching context, they could notice students thinking and interpreting what they noticed without worrying about assessing their instruction (Castro et al., 2005; Simpson & Haltiwanger, 2017). These activities provided the background experiences for the plan-practice-teach cycle. For this study, the data included reflections from a peer interview activity, draft lesson plans, transcripts and reflections from a peer practice, reflections on the peer practice, revised lesson plans, reflections on the instructed lesson, and transcripts from an interview with the researcher at the end of the semester.

Peer Interview

The peer interview was a clinical interview with the preservice teachers grouped into pairs, one acting as the teacher and a peer acting as a high school student. They used a common task, and "students" were asked to create graphs from short real-world scenarios, such as the cost of a stamp based on the weight of a letter. Preservice teachers were not allowed to plan for this activity but instructed the task extemporaneously. The focus of this activity was on how questioning was used. The interviews were recorded, and each preservice teacher reflected on their experience using the recording.

Draft Lesson Plan

Preservice teachers created a draft of a lesson plan for their targeted learning goal with commentary explaining their thinking for the design of their lesson. The lesson was taught in their field experience, mathematics classes at area high schools. The draft lesson plan represented preservice teachers' initial view of supporting students in developing a mathematical understanding of the learning goal. They had some experience writing lesson plans, but this study included a draft lesson plan, peer practice, revised lesson plan, taught lesson, and reflection through a revision cycle (described below) that was new for these preservice teachers. In addition, the course instructor expected that these lesson plans would include questions the preservice teacher anticipated asking, expected student responses, and possible teacher actions based on these responses and that they would work on developing their questioning throughout the plan-practice-teach cycle. Participants also included a reflection on their lesson's design, considered students' potential background knowledge, and anticipated how they would monitor student learning.

Peer Practice

The goal of this methods course activity was to practice implementing parts of their lesson (the time allotted for each practice (30 minutes) did not allow for the complete lesson to be taught, so preservice teachers were told to focus on important parts of the lesson). Each preservice teacher practiced teaching important elements of their lesson to a small group of peers (between four and six) acting as high school students. The lessons were recorded and transcribed. Transcripts were coded and linked to the corresponding elements of the draft lesson plan. While not an exact representation of what might occur with actual high school

students, it did provide insight into how the preservice teachers used questioning to elicit, interpret, and respond to student thinking and provided practice for their lessons that would soon take place in high school classrooms.

Peer Practice Reflection

Analysis of the peer practice allowed preservice teachers to deepen their knowledge about their teaching, as analyzing video recordings can support learning experiences (McDonald et al., 2013). The prompts for this reflection included asking how the preservice teacher elicited and built on student responses to promote teaching and what changes they would make to better support students in the lesson. In these reflections, preservice teachers developed claims about their teaching during peer practice and supported them with evidence from the lesson recording. These reflections provided data about their views on questioning, eliciting, interpreting, and responding to student thinking.

Revised Lesson Plan

After practicing and reflecting on their peer practice, participants completed a revised lesson plan, including annotations explaining significant changes. Data analysis comparing the draft and final lesson plans and practicing lesson implementation explored the development of questions and questioning sequences.

Teaching the Lesson

Each preservice teacher taught their lessons to students in a local high school, recorded them, and analyzed them. This analysis provided insight into what they noticed, how they interpreted what they noticed, and what actions they took to respond to student thinking. The prompts for their reflection included asking how the preservice teacher elicited and built on student responses to promote teaching and what changes they would make to better support students in the lesson. Questioning sequences were coded and aligned with the draft lesson plan and the peer practice.

Interview

A semi-structured interview was conducted between each participant and the researcher at the end of the semester. While there was a standard set of questions for all preservice teachers, some were explicitly focused on the data collected from each participant to allow the researcher to go deeper into the data (Bernard & Ryan, 2010). The common structure included questions about the design of questioning in the lesson plans, what changes the preservice teacher made, the impact of practicing with peers, and whether teaching the lesson with high school students matched expectations.

Process of Data Analysis

Data analysis investigated how preservice teachers used questioning through the plan-practice-teach cycle of a lesson. Collected data provides insight into how the participants privileged, elicited, interpreted, and responded to student thinking and their development about design and choice of questions through the plan-practice-teach cycle. It included preservice teacher analysis of the peer interview, the draft lesson plan, transcript of the peer practice, preservice teacher analysis of the peer practice, the final lesson plan, annotations for the final lesson plan, preservice teacher reflection of the instructed lesson, and transcript

of the preservice teacher interview at the end of the semester. Analyzing and coding the data around the evolution of questioning in the lesson organized the data "into meaningful segments and assigning names for the segments" (Creswell, 2013, p. 180), with codes coming from both inductive (from the specific data) and deductive (from the development of questioning) processes (Bernard & Ryan, 2010). The themes emerged from the coding that demonstrated commonalities and differences among the participants.

The themes showed how preservice teachers used either sequence of questions to elicit student responses to guide the course of the questioning (focusing) or used their thinking to determine the pathway (funneling), and sometimes a combination of both (Herbel-Eisenmann & Breyfogle, 2005; Wood, 1998)—identifying these questioning structures links to how each preservice teacher interpreted and responded to what they noticed during instruction. Sequences of questions were grouped to examine how preservice teachers used questioning to use or direct student thinking. Grouping questions allowed the researcher to consider how preservice teachers used eliciting, interpreting, and responding to student thinking, the underlying aspect of the study (Saldaña, 2011).

Groups of coded questions were "chunks." The creation of these chunks helped "develop a sense of the content of the theme" (Bernard & Ryan, 2010, p. 101) during the coding process. A chunk was an extended interaction or discussion between the teacher and students that explored the reasoning behind a mathematical topic. By requiring at least two student interactions in coding, the focus was on investigating student thinking with depth. For coding purposes, a chunk lasted through the end of a particular exploration of student thinking on a topic (based on the work of Franke et al., 2009).

Next, the chunks were analyzed to focus on and funnel questioning structures. In a focusing question structure, the teacher listens to student ideas and guides the conversation based on those ideas (Herbel-Eisenmann & Breyfogle, 2005). Students provide the cognitive activity around mathematics while the teacher determines which questions to ask to keep the focus on student thinking. Focusing allows students to think about mathematics and communicate their ideas to one another (Wood, 1998) and supports making connections (Herbel-Eisenmann & Breyfogle, 2005).

In a funneling structure, teachers continue until "a student gives a solution that is representative of the method that they [the teacher] intend the student to use" (Wood, 1998, p. 167) or directing student thinking to a predetermined path, based on the teacher views (Herbel-Eisenmann & Breyfogle, 2005). A funneling sequence may begin with a "probing question but allows little wait time and immediately follows up with questions that become more directed toward one particular answer" (NCTM, 2014, pp. 40-41). In funneling, the teacher tries to get students to view what the teacher sees, possibly through a series of indirect but increasingly directive questions (Mason, 2010), where students are responsible for providing the answers and reasoning the teacher seeks (Hagenah et al., 2018).

After coding, chunks of questions about the same topic and in the same aspect of the lesson were connected throughout the different aspects of the plan-practice-teach cycle: the draft and final lesson plans, the practice lesson with peers, and the reflection from teaching the lesson in the high school classroom. These common elements throughout the plan-practice-teach cycle examined the evolution of planning and enactment of questioning within a topic. They explored how the preservice teachers used teaching noticing to elicit, interpret, and respond to student thinking. This evolution is important as it shows what each preservice teacher learned from their experiences in peer and classroom lessons and what changes they made based on those experiences. Preservice teachers' reflections and interview responses

provided reasoning and support for some of the decisions made by the preservice teacher and helped explicate their choices.

Case Study #1: Beth: "Developing Active Listening"

Peer Interview: "Starting with a Blend"

In her peer practice teaching interview, Beth (a pseudonym) asked questions that led to focusing and funneling questioning structures. The students completed a task in which they created graphs for four real-world situations: the burning of a candle (A), the cost of a letter based on its weight (B), the cost of a bus per passenger as the number of passengers increases (C), and the depreciation of a car losing half of its value each year (D). Her initial question asked the 'student' (peer participant), "I'm wondering why you decided not to connect these points [points to graph B], but you connected them in this graph [points to graph A], this graph [points to graph C], and this graph [points to graph D]" (Beth Analysis #1, 2018, p. 1). Beth identified that this question aimed to learn what aspects of scenario B led to a different representation compared to the other three scenarios (Beth Analysis #1, 2018). She asked an inquiry question to elicit and focus on the student's thinking regarding the difference between discrete and continuous real-world situations. However, as Beth continued in her questioning, she guided the student to her interpretation rather than allowing the student to make connections, asking the student, "What happens when your graph intercepts the *x*-axis?"; "What made you decide to use the slope that you did?"; and "Are all of your graph continuous or are some of them not continuous?" (Beth Analysis #1, 2018, pp. 2-4). Each question introduced concepts the student had not yet introduced to the interview. Here, Beth shifted to a funneling questioning structure, working to guide the student to how she is making sense of the student's work (the graph). She asked the student to connect to her thinking rather than listen to her reasoning and provided questioning to uncover the student's thinking. This may represent her learning experiences in more teacher-centered learning environments and how teachers often guide students' thinking.

Planning (Draft Lesson Plan): "Developing A Work-in-Progress"

Beth acknowledged that parts of the lesson still need development. Her planned questions focused on reasons for student thinking and making connections within activities in the lesson, such as "Why did you put absolute value signs around that expression?" and "How would you explain your method to Kate so that she doesn't make this mistake again?" (Beth Lesson Plan Draft, 2018, pp. 6-7). Beth predicted how students would approach the task and what questions to ask to elicit student thinking. She demonstrated how she used planning to anticipate what would happen and how she would respond. Beth also wanted students to identify misconceptions, deciding she would scaffold questions to encourage students to connect to previous lessons. She showed how planning could support student connections across lessons, representing that she was focused on creating student-centered learning environments in her lessons. In her commentary, she planned to ask questions to allow students to share their thoughts so she could learn about them (Beth Lesson Plan Draft, 2018).

Peer Practice of Actual Lesson: "Expressing a Desire to Learn"

In the peer practice, Beth identified that her goal was to understand how learners would interact with the lesson. She knew there were places in her draft lesson plan that she needed

to develop further and wanted to use peer practice to help her develop those sections. Beth planned to notice her "students" thinking and consider ways to support them. She wanted to elicit students' reasoning and base her actions on her interpretation of their thinking, not guide students in her way of thinking. She adjusted her questions to address how she interpreted the students' thinking based on what she noticed during the lesson. For example, when a student discussed redoing the work represented in the task, she asked, "How did you redo it? What was different?" (Beth Transcript of Peer Practice, 2018, line 159). While this question was not included in her lesson plans, it showed how she used questioning to understand student thinking better. She was able to plan for and use student thinking to guide her lesson. Beth showed that the practice was a valuable exercise for her as she used it to model what might happen when she taught the lesson. However, she also acknowledged that the practice was with her peers and that it should help, but not dictate, how to make adjustments for her actual lesson, showing that the practice reinforced her views of how learning should occur.

Final Lesson Plan: "Focusing on the Goal"

Beth provided more reasoning behind her thinking in her final lesson plan based on student responses contributed during the peer practice. She had changed some questions to reflect her better understanding of how students might approach the learning goal. Beth explained that she wanted to focus on how students made sense of the problem rather than trying to anticipate all the ways that students may solve the problem (Beth Interview Transcript, 2018). She shared that she was worried that over-preparation would affect how she interpreted and responded to in-the-moment student thinking. This demonstrated how Beth used planning and practicing to help her prepare but not to overcome what she would notice during the lesson with high school students.

The questions in her final lesson plan reflected her vision of the lesson based on her experience with peer practice (see Table 1).

Table 1
Questioning Across the Draft and Final Lesson Plans (from Beth Lesson Plan Draft and Beth Lesson Plan Final)

Questions from the Draft Lesson Plan	Questions from the Final Lesson Plan
What are the restrictions for values that can be plugged into a logarithmic function?	How does the domain of logarithmic functions restrict solutions when solving logarithmic equations? How can you check for extraneous solutions?
How can you make sure that you don't "lose" a solution to a logarithmic equation?	Why do you have to be careful when applying properties of logarithms to logarithms with variable arguments?
How can you use logarithmic properties properly to solve logarithmic equations?	How can you make sure that you don't lose a solution when solving a logarithmic equation?
	What types of logarithmic equations have two possible solutions?

These revisions show Beth noticed from the practice with peers that some of her questions narrowed down key elements of the learning goal for the students, funneling them to her way

of thinking. Her revised questions show that she allowed students to explain more about the "whys" of the mathematical understanding of the learning goal.

Teaching the Lesson: "Listening Actively"

For her enacted lesson, Beth demonstrated how peer practice and lesson plan revisions can be enacted in the classroom. This illustrates how Beth used the planning and peer practice process to use the best student responses that aligned with her view of teaching. Beth highly valued student thinking and strove to meaningfully interpret and respond to "live" responses. Recall that she was hesitant to provide extensive anticipated responses in her plan so as not to sway her perceptions in any way. She listened to the responses of students (noticing student thinking) and directed a student to reconsider their thinking (acting on what she interpreted from what she noticed), as in advising a student to think more about their solution, "…maybe look back at your property notes and think about, what do we have to know about what's inside the logarithm in order to apply the property?" (Beth Analysis #3, 2018, p. 2). She demonstrated that while she learned from the peer practice, it did not funnel her into a particular questioning pattern. Her questioning prompted students to examine the meaning of rules and procedures involving logarithms. This demonstrates Beth's growth in using questioning to promote deeper mathematical understanding and focus on student-centered learning environments. As the students analyzed the sample solution that was included in the lesson, she shared that she "pushed them [students] to consider other possible errors by saying, "Do you think she ever saw anything being squared?" (Beth Analysis #3, 2018, p. 2). As she probed to clarify students thinking, she asked students to revisit answers when she wanted them to provide richer explanations or when she wanted students to consider the situation more deeply. Beth demonstrated the desire and ability to actively notice to, understand, interpret, and act on student thinking.

Summary

Initially, Beth elicited student thinking but funneled the student to her view of mathematics instead of focusing on how the student made sense of the task (in the peer interview). She was patient with her lesson development and actively listened during the peer practice and high school lessons. This showed that Beth could progress beyond this initial approach and develop her use of questioning to elicit, interpret, and respond to student thinking. She was consistent with the goals of her questioning throughout the evolution of her lesson. Beth modeled a clear understanding of student-centered instruction and used questioning to elicit, interpret, and respond to student thinking. She was careful not to put too much of her thinking into her questions and let student thinking guide the lesson. Beth showed how the role of purposeful questioning increased in her planning and teaching and paralleled her use of student thinking during instruction. She used more focusing questioning than funneling questioning in all but her initial experience, and her belief about the role of planning and lesson development, as well as her understanding of how to adjust her lesson during instruction during the plan-practice-teach cycle to maintain the student-centered learning environment, allowed her to learn to use questioning to learn how students made sense of tasks.

Case Study #2: Clara: "Stepping In to Help Out"

Peer Interview: "Showing Disconnected Interpretations"

After the peer student completed their solution to the task, Clara (a pseudonym) asked, "How did you come up with the form you wanted to put this graph in?" (Clara Analysis #1, 2018, p. 1). The "student" responded, "For zero letters, I have zero ounces. For one letter I have a bit more ounces and for each letter I increased by the same amount of ounces…or is it cost?" (Clara Analysis #1, 2018, p. 1). Clara shared in her analysis that she noticed how the student created their graph and interpreted the student's reasoning behind why the student graphed distinct points rather than a continuous function (Clara Analysis #1, 2018). Next, Clara reread the question asked in the task, explaining a different rate for one ounce compared to each ounce thereafter. She indirectly acknowledged the student's response by asking, "So are yours going up in an equal amount each ounce?" (Clara Analysis #1, 2018, p. 1). Clara funneled the student to her reasoning for the student's solution. She quickly moved from asking about the student's thinking to representing her thinking to the student. This may represent Clara's experiences with learning mathematics and how, in this case, the experiences have guided her into a more teacher-centered approach. The student explained why there are equal increases across the graph, "With each additional ounce, you pay the same amount of money" (Clara Analysis #1, 2018, p. 1). Clara repeated the student's answer to confirm the student's thinking. The student agreed, and the teacher replied, "Awesome" (Clara Analysis #1, 2018, p. 2). In this example from the peer interview, Clara modeled guiding the student toward the interpretation of the solution that she had developed. Clara moved toward the funneling questioning structure as her upcoming lesson developed through the plan-practice-teach cycle. This could reflect her experiences in teacher-centered learning environments and represent one of the challenges of moving toward a more student-centered approach. She provided her thinking in the form of a question and had the student confirm how they were interpreting the problem. This can be an approach that preservice teachers take to their questioning. It represents one strategy that may be used when a student is unsure about an explanation: the teacher provides suggestions about how the student determined the answer, and the student confirms it. This strategy tends to funnel students to the teacher's way of thinking.

Planning (Draft Lesson Plan): "Varying Expectations"

In her draft lesson plan and commentary, Clara varied in the questioning structures she anticipated for the lesson. The questions Clara created included multiple options for student thinking. There was a mixture of speculative questions that asked for student reasoning ("Do you agree with Cleo?"; "Why? Can you show me Cleo's and your work?" or "Why not? What did she do incorrectly?") and factual/procedural questions to determine how well students read and re-expressed equations ("What does it mean to take a base to the negative exponent? What happens?") (Clara Lesson Plan 1 Draft, 2018, p. 2). She sought a balance of questioning types and wanted to learn how students made sense of the task. In her activity on creating conjectures for exponential rules, questions focused on procedures more than concepts (such as "What were your steps when trying to figure out [the] rule?") and "what"/"how" questions ("What is the relationships between exponents and logarithms?") (Clara Lesson Plan 1 Draft, pp. 4-5). This line of questioning may reflect how Clara views and has experienced the development of mathematical procedures. She demonstrated an approach where the teacher models what is expected and expects students to follow – a

teacher-centered classroom. Clara planned open-ended structures in some places and guided in others. Such questions can lead to focusing or funneling questioning, depending on what the teacher notices, interprets, and responds to. This showed that her lesson could lead to structure during instruction depending on what was noticed and how she responded.

Peer Practice of Actual Lesson: "Allowing Exploration"

Clara allowed time for the peer "students" to work on the activity while she circulated, asking what the students were thinking, i.e., "Is there a reason you all did that?" and "Where did you get that from?" (Clara Transcript of Peer Practice, 2018, lines 48 and 52). She suggested that students check their answers to ensure they have solved the problems completely (Clara Transcript of Peer Practice, 2018). She focuses on students' responses but does not connect those responses to the provided task. After discussing the solution and briefly analyzing what background knowledge was expected of the high school students for the lesson (stepping out of their roles as students to discuss the task), Clara moved to the next part of the lesson, creating rules from examples. Clara circulated among the groups, asking questions and encouraging students to explore and explain their thinking. Examples included, "Now what's the next step that you want to do?" and "What would be an easier way to figure out what that is?" (Clara Transcript of Peer Practice, 2018, lines 423 and 438). She models what was represented in her lesson plan, and the teacher guides students through the procedure. A student responded to the second question with, "Great question," and Clara suggested the student could use order of operations (Clara Transcript of Peer Practice, 2018, lines 440 and 442). Here, Clara stepped in and suggested what the student could do. She presented her strategy and funneled the student toward analyzing the problem in a way like hers. She guided students toward her thinking and "helped" them to understand her thinking rather than focusing on their thinking. This again demonstrated how Clara drew upon a more teacher-focused environment when she made revisions in her lesson (especially when they occurred during teaching). Clara's reflections from the peer practice showed the impact of how her peers responded to open-ended questions and how they "struggled" with the content. She used these struggles to represent what her high school students would experience and took that thinking into her teaching experience.

Final Lesson Plan: "Having Unclear Expectations"

In the final lesson plan, Clara made changes to reflect both additional information that she learned about the background knowledge of the high school students and what she learned in the peer practice. She changed the warm-up to the opening activity of the lesson based on learning more about students' backgrounds in mathematics, making it more about conceptual understanding (finding errors in given student work) than calculation. She also made changes based on peer feedback that the exploration activity went "a little too fast from easy to hard" (Clara Lesson Plan Revision Explanations, 2018, p. 1). To address this, Clara changed some of the examples so they were "easier for them (the students) to make the connection with what the general rule may be" (Clara Lesson Plan Revision Explanations, 2018, p. 1). The only changes she made to her questioning from the draft lesson plan to the final one involved changing the wording to increase clarity (Clara Lesson Plan Revision Explanations, 2018). These minor adjustments indicate that Clara felt that the adjustments to the activity would not alter how she will need to notice, interpret, and respond to student thinking during the lesson. She showed that she still wanted a student-centered learning environment through the questioning plan that she created.

Teaching the Lesson: "Moving Away from Exploration"

Clara decided to create a "note sheet" to remind her of what was important about the lesson while she was teaching (instead of referring to the lesson plan) (see Table 2).

Table 2
Questioning in the Final Lesson Plan and Note Sheet (from Clara Lesson Plan Final and Clara Lesson Plan Annotations)

Questions from the Final Lesson Plan	Questions from the Note Sheet
What does it look like Cleo did?	What did Cleo do?
How do we read a logarithm?	Do you agree?
Do you agree with Cleo?	What did you do?
Why? Can you show me Cleo and yours work?	What does the negative 2 represent?
Why not? What did she do incorrectly?	What does ½ represent?
Can you write out how you would do the problem yourself?	What does a negative exponent represent?
Do you get what you expect?	Can we get a negative output? Why/Why not?
What does it mean to take a base to the negative exponent - what happens?	Question alterations:
You cannot have a negative exponent.	What do we do to ½ when it has a negative exponent?
Why am I not allowed to have a negative exponent?	How do we evaluate, what the base and exponent?
What does a negative exponent mean?	What's an asymptote?

In rewriting her questions on the note sheet, she shifted to more directive questions and asked for less thinking and reasoning from the students. Instead of letting students determine the strategy, Clara provided it and asked them to justify it. This showed the impact on Clara of the peer practice and perhaps on how she viewed the role of the teacher as she moved to a funneling questioning structure in her teaching, fearful of students "struggling" on the task. This fear establishes the teacher as the support and the one who rescues students when they are unsure what to do. Her takeaways from the peer practice showed her that students struggle to make sense of the task independently. Therefore, she supplied more information to students to make them more comfortable in the lesson. As scaffolding was added, it is difficult to know if students will be allowed to learn from their struggles or if the teacher's questions will provide them support whether they need it or not.

Clara shared that she believed the investigative problem needed a more straightforward entry point that allowed students to start with a specific value. As she was teaching, she wrote, "Should have reviewed exponent rules first" (Class Lesson Plan Annotations, 2018, p. 2). After the lesson, Clara reflected, "A lot of my students still struggle with thinking abstractly and therefore had trouble finding a starting place in the activity" (Clara Lesson Plan Annotations Commentary, 2018, p. 1). By scaffolding all students through her questioning, Clara created a role for the teacher to "save the day" and eliminated the challenge for students who may have been able to adapt to the situation.

Summary

In discussing what she noticed during the lesson, Clara shared throughout the semester that, she appreciated the importance of noticing and responding to student thinking. In her course reflection at the end of the semester, Clara stated, "It is important to plan for misconceptions, questions, and potential struggles" (Clara Final Reflection, 2018, p. 2). Her statements demonstrated that she believes in providing opportunities for students to struggle, but her actions showed she is unclear on how to support it during instruction. This represents the challenge some preservice teachers face in creating student-centered learning environments. Over the semester, she provided more and more guidance for students to help them understand what she wanted them to know, believing that all students would struggle so that she would help them through her questioning. While she originally planned for misconceptions, she used the plan-practice-teach cycle to decrease the role of student thinking, limiting student interaction with their misconceptions for fear that students would not be successful if they were responsible for the thinking. The reflection on her peer practice led her to expect the high school students to struggle. Throughout the semester, she decreased the role of student thinking in the lesson. Because of her experiences, Clara expected students to struggle and did not allow them to demonstrate what they could do. Clara also struggled with how to help students when uncertain, often leading to her providing her answers. This demonstrates the need for teacher educators to support preservice teachers in their development and understanding of questioning techniques. At the end of her reflection, she admitted that she sometimes had to stop herself from guiding students through a task. She "struggles with giving students the opportunity to productively struggle" (Clara Final Reflection, 2018, p. 5).

Cross-case Analysis

In their use of questioning, both preservice teachers used focusing and funneling questioning structures in different ways to demonstrate how they noticed and responded to student thinking. This reinforces that there are different trajectories that preservice teachers follow in their development (Thompson et al., 2013). Factors such as prior experiences, personal beliefs about the role of the teacher, and individual learning theories and style may have contributed to stark differences between Beth and Clara's development. While they often noticed what students were thinking at the start of a questioning sequence, they differed in how they interpreted and responded to what they noticed. Teacher responses ranged from questions to allow students to provide clarity or explore an error, questions to guide students to a particular way of thinking, statements to inform students how the teacher interpreted their thinking, or evaluative feedback to the students about their thinking. These teacher responses demonstrated many ways to notice and respond to student thinking (Copur-Gencturk & Rodrigues, 2021).

Throughout the semester, each preservice teacher in this study identified how they strove to use questions to learn about student thinking during mathematics instruction. Each believed they provided opportunities for students to showcase their thinking and diverse ideas, connecting to the meaning of eliciting student thinking (Singer-Gabella et al., 2016). In practice, they varied greatly, using questioning to elicit, interpret, and respond to student thinking (see Table 3).

Beth and Clara used different balances of focusing and funneling question structures in their teaching. Both tended to start questioning sequences with an open-ended, speculative question to elicit student thinking. Beth actively listened to multiple student responses before

asking another inquiry question, providing space and options for students to continue explaining their thinking. She identified the importance of students being able to explain their thinking, and her role was to redirect thinking, not provide it when they were in error. She used questions to allow students to deepen their understanding of the learning goal and make sense of errors. In this way, Beth focused on making sense of students' thinking, not hers, viewing the role of the teacher as eliciting and responding to student thinking.

Table 3
Typical Questioning Techniques for Each Preservice Teacher

Participant	Typical Questioning	Example
Beth	More Focusing than Funneling	Worked to help students think about how to solve problems. There were multiple examples where she would ask an initial question and then allow students to ask questions to one another before the interjecting again to spark student thinking (Beth Transcript of Peer Practice, 2018).
Clara	More Funneling than Focusing, guiding students to the answer for which she sought	Provided more structure through the revisions of her plan. She sometimes guided students to the answer she was looking for, "So if I look at this rule [points to product rule], is there some way I can split it up, so I can at least try to make sense of something helpful?" (Clara Transcript of Peer Practice, 2018, lines 491-492).

In contrast, Clara's actions often guided students to where she had planned the lesson or included her thinking about the students' responses. Four techniques led to funneling: leading the student to the teacher's view of the problem, getting students to agree, providing an answer to the question, or trying to move the lesson forward without students answering questions. After teaching her lesson to high school students, Clara reflected that she explained the steps, pushed them ahead, and provided the thinking for them. This is one purpose for funneling questioning structures; when students are not communicating well, she views the teacher's role as making connections for the students. Clara felt that she rushed them to the conclusion of the activity rather than allowing the students to reach their conclusions (Clara Analysis #3, 2018).

In exploring the plan-practice-teach cycle, the participants' questioning skills developed differently (see Table 4).

Table 4
Development of Questioning Through the Plan-Practice-Teach Cycle

Participant	Plan	Practice as Link to Planning and Teaching	Teach
Beth	Clarified and added detail to questions to better learn what students were thinking	Provided opportunities for students to explain thinking	Focused on using student thinking to move forward in the lesson

Clara	Did not know what to do with student errors	Provided more and more guidance to students	Guided students to her view of the task

Throughout the cycle, Beth made changes that allowed students to think of a more significant role in making sense of the learning goal, while Clara made changes that limited student thinking in the lesson. These perspectives were also exhibited when the participants taught their lessons in high school mathematics classrooms.

Beth improved her noticing of student thinking through the plan-practice-teach cycle. The more she planned and practiced, the more likely she was to delay providing her interpretation of student thinking, which encouraged the expansion of student ideas. By comparison, Clara's evolution increased the number of directive questions with fewer opportunities to elicit student thinking, allowing less opportunity to interpret and respond to student thinking as her lesson plan evolved.

Lesson plans guide what teachers think about how a lesson may proceed, but it is not always clear what type of questioning structure teachers will use. Beth initially created questions to elicit student thinking, possible correct and incorrect student responses to those questions, and her possible actions. Clara's lesson plan included both questions and possible student responses, but her vision involved only correct responses, guiding the lesson to the outcome she desired. Clara did not consider how to address a student who struggled. Throughout the semester, she decreased the cognitive demand she expected of her students, and her questions narrowed the opportunity for student thinking.

In her reflection, Beth described how she offered opportunities for students to express their thoughts. This allowed her a better understanding of how students thought. Clara also adjusted the questioning in her lesson as she developed her lesson plan. These questions led to more funneling questioning structures and provided students with her understanding of the task.

This study supports both the view that preservice teachers who participate in plan-practice-teach cycles may experience improvement (Zembal-Saul et al., 2000) and that it may be challenging to help preservice teachers enact effective practices (Kazemi et al., 2009). Beth demonstrated the ability to focus on student thinking consistently, while Clara provided fewer inquiry-based questions and increased the use of directive and procedural ones.

Discussion

The individual and cross-case analyses have four important implications for teacher educators.

Teacher Educators Must Provide Ways for Preservice Teachers to Address Their Beliefs

The preservice teachers involved in this study created different roles for themselves. These roles represented their views of teaching mathematics, both in the evolution of their lesson plans and their use of teacher noticing. In Clara's use of a funneling questioning structure, she guided students to her way of solving a problem. This belief in the role of the teacher contrasts with helping students make sense of problems and eventually solve the problems themselves (Singer-Gabella et al., 2016).

Preservice teachers need to learn more about their own beliefs and how they impact their teaching views. Teacher preparation programs need to create the conditions to allow

preservice teachers to benefit from uncovering and confronting their beliefs and facilitate multiple ways to use questioning to learn the thinking of students (building on the ideas of Philipp, 2007). Teacher educators must provide experiences for preservice teachers to notice what is important during instruction, analyze what they notice, discuss why it is important, and create strategies to respond to it. Once these strategies are created, implement them, review how students were engaged in their learning, and determine whether the teacher's and student's roles foster impactful learning environments.

Teacher Educators Must Encourage Preservice Teachers to Use Focusing Questioning Structures

Research shows that the use of focusing questioning structures allows students to create meanings of the mathematics they learn and better understand mathematics through explanation (Wood, 1998) and increases student responsiveness (Hagenah et al., 2018), while funneling questioning structures give "the illusion that learning is actually happening" (Wood, 1998, p. 172) and limits student opportunities for rigorous instruction (Hagenah et al., 2018). This study demonstrates that preservice teachers can learn to develop and use focusing questioning structures to notice student thinking, as evidenced by how Beth develops and uses questioning through the plan-practice-teach cycle. However, despite a desire to do so, Clara struggled to use questioning to create student-centered learning environments. This indicates the need for careful consideration of how to create peer practice environments and instructor feedback that will support the development of effective questioning. Teacher educators must provide clear feedback to preservice teachers, encouraging them to examine their questioning practices and providing opportunities for students to use their ideas to move the lesson forward. This would both encourage preservice teachers like Beth to move in the right direction and support those like Clara in using strategies in the ways that they planned rather than providing more guidance to help students get the answer.

Coursework and practice opportunities must include experiences that allow preservice teachers to consider multiple responses to student thinking and learn how to choose actions that maintain student-centered learning environments. Discussing during planning should address incorrect and incomplete responses and possible teacher actions. This is especially true for preservice teachers who may not have experienced these instructional environments in their learning.

Teacher Educators Need to Support Preservice Teachers to Notice Students Thinking

Teachers need to allow student thinking to diverge from their expectations and provide a space for students to offer solutions (Singer-Gabella et al., 2016). Teacher educators need to plan how to aid preservice teachers in moving from the vision in their lesson plan to the enactment of it, which is focused on noticing student thinking, interpreting that thinking, and considering effective ways to respond to it. This can be done within the plan-practice-teach cycle, with peer practice allowing for a safe space for preservice teachers to consider how to respond to different student answers. Recording practice lessons for review and feedback can support how preservice teachers can improve classroom interactions. Providing multiple strategies to support student discourse and encouraging the use of the role of the teacher to foster student-student interactions can allow students to retain control of their thinking in the lesson. Teacher educators can use these practice activities to instruct preservice teachers on

options that can be available to maintain students' thinking during instruction (Lampert et al., 2010).

Peer Practice Must be Carefully Tailored to be Effective

One of the key elements in this study is how peer practice impacted the use of questioning by preservice teachers. Preservice teachers benefit from practicing teaching with peers (Ghousseini et al., 2015; McDonald et al., 2013) and rehearsing in a safe, controlled environment (Ghousseini et al., 2015; Ghousseini & Herbst, 2016). Teacher educators must set up peer practices to focus on critical pieces in developing an understanding of a lesson. Both preservice teachers demonstrated the impact of the peer practice as they made changes to their lessons based on it. They adjusted the activities in their lesson plans based on the feedback from their peers.

When considering how to maximize the value of peer practice, teacher educators need to remember to coach both the preservice teachers acting as teachers and those acting as students to participate in ways that will benefit the lesson's implementation in the actual classroom. In addition, teacher educators need to support preservice teachers during lesson rehearsals actively and offer feedback that will support the development of targeted instructional practices.

Conclusion

This study demonstrates how preservice teachers with similar dispositions about using student thinking and understanding teacher noticing may develop unique questioning techniques. While both participants had similar experiences in the plan-practice-teach cycle, collaborated and shared ideas with peers, and had multiple opportunities to revise the vision of their lesson, their questioning within the lesson developed differently.

Preservice teachers can design lessons that help them notice, elicit, interpret, and respond to student thinking. While they may elicit student thinking, some may require more support and guidance to interpret that thinking and to allow students to develop understandings on their own. Peer practice is beneficial in the preparation of teachers and offers specific characteristics that benefit some preservice teachers more than others. This study is limited by the number of participants (two represented here, five in the completed study) and by the specific context and experiences of the participants. The two participants included in this discussion were selected based on the contrasts in their experience in the study. There were other experiences between these two, and teacher educators must consider how to support the differences that preservice teachers bring with them in considering activities and plan-practice-teach cycles. In addition, as all the participants were graduate students, they may not have represented the impact that there would be on undergraduate students in similar circumstances. However, the implications of this study do link to and build on the research of others. More study needs to occur to investigate how the role of the type or topic of the lesson may also be an impact on the development of questioning. It is also important to establish guidelines on peer practice so that the learning that occurs in these experiences supports classroom instruction and effective questioning practices. While preservice teachers can notice and identify the places where students are making sense of the mathematics within a lesson, they may need differentiated support to determine what to do with what they notice.

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