

The Role of Cognitive Conflict in Teaching Nonlinear Equations

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This study examines pre-service elementary mathematics teachers' understanding of nonlinear equations and explores how cognitive conflict relates to their conceptual understanding and performance in solving them. Data were collected from third-year pre-service mathematics teachers. An embedded mixed-methods design was used, and data-gathering tools included algebra achievement tests on nonlinear equations (administered as pre-tests and post-tests), open-ended cognitive conflict questionnaires, and whole-class discussions. Common misconceptions included accepting extraneous roots as valid solutions and failing to exclude values that render expressions undefined, particularly those that make denominators zero. Additionally, pre-service mathematics teachers demonstrated misunderstandings in concepts such as variables, domain, solution set, and equation roots. To foster cognitive conflict, the researcher used open-ended questionnaires presenting opposing student views. Paired-sample *t*-test results showed higher post-test scores among participants who completed all phases of the study. The findings also indicated that some misconception categories decreased following the instructional sequence, particularly those directly addressed in the cognitive conflict activities, whereas others persisted or increased.

Keywords: nonlinear equations, misconceptions, cognitive conflict, pre-service teachers, mathematics education.

Introduction

Although previous research has highlighted the role of misconceptions in students' learning and academic performance (Posner et al., 1982), fewer studies have examined instructional approaches that systematically address these misconceptions in classroom settings. One approach that has been frequently used to support students' conceptual understanding and address misconceptions is cognitive conflict (Mufit et al., 2020).

Despite its documented potential, cognitive conflict is not always easy to implement in classroom practice, as its effectiveness depends on how it is designed and enacted. Research has shown that cognitive conflict can support deeper conceptual understanding, even when it does not immediately result in complete conceptual change (Hewson & A'Beckett-Hewson, 1984; Murray, 1983). Studies in mathematics and science education also suggest that its impact varies depending on how conflicting situations are structured and how students are guided through them (Kang et al., 2010; Lee & Yi, 2013; Maharani & Subanji, 2018; Sari et al., 2022; Watson, 2002; Watson & Moritz, 2001). However, the literature also cautions that cognitive conflict alone does not necessarily lead to conceptual change, and its effectiveness depends on how conflicting situations are structured and supported instructionally.

In this context, cognitive conflict has been implemented through different instructional approaches, including diagnostic teaching, where carefully designed tasks are used to reveal inconsistencies in students' thinking and encourage them to reflect on and reconsider their ideas (Nussbaum & Novick, 1982; Tirosh & Graeber, 1990; Tirosh et al., 1998). Cognitive conflict refers to instructional situations in which students are confronted with inconsistencies between their existing ideas and new information. Students are presented with situations in which their existing knowledge is shown to be inadequate or in conflict

with their prior experiences (Delaney, 2001). The goal is to promote conceptual change by introducing conflict and unexpected information. In the Piagetian (or constructivist) approach, children are encouraged to interact with resources to create dissonance or cognitive conflict. In cognitive conflict instruction, the instructor poses various questions and presents differing viewpoints to help students recognize their mistakes independently, rather than pointing them out directly. When instructors engage students in mathematical situations that challenge some of their existing understandings of an idea or topic, students may initially view the contradictions as exceptions (Stylianides & Stylianides, 2008). To ground the instructional design theoretically, the study draws on conceptual change theory and cognitive conflict-based instructional principles, which are elaborated in the following section.

Conceptual Change and Cognitive Conflict

Conceptual change theory explains how learners revise deeply held misconceptions and reorganize their existing knowledge structures. Rooted in constructivist perspectives and influenced by Piagetian notions of assimilation and accommodation, conceptual change occurs when learners encounter situations that cannot be adequately explained using their existing conceptions. Assimilation refers to incorporating new information into existing cognitive structures without fundamentally altering them, whereas accommodation involves restructuring those cognitive structures to resolve inconsistencies. Cognitive conflict serves as a critical trigger in this process, creating a state of disequilibrium or cognitive dissonance that challenges learners' prior conceptions.

Within the conceptual change model proposed by Posner et al. (1982), meaningful conceptual change requires several interrelated conditions: dissatisfaction with existing conceptions, intelligibility and plausibility of alternative conceptions, and the fruitfulness of the new conception. Dissatisfaction arises when learners recognize the inadequacy of their current understanding. For change to occur, alternative conceptions must be understandable and appear reasonable within the learner's cognitive framework. Finally, the new conception must demonstrate explanatory or problem-solving power in novel situations.

Cognitive conflict-based instruction operationalizes these principles by deliberately exposing learners to contradictory ideas, unexpected results, or alternative solution strategies that highlight the limitations of their prior reasoning. However, conflict alone is insufficient; structured discussion and guided reflection are necessary to support accommodation rather than superficial adjustment. Nussbaum and Novick (1982) emphasize the importance of presenting contrasting viewpoints, encouraging learners to articulate and defend their reasoning, and allowing peer interaction to facilitate conceptual restructuring without immediate authoritative correction.

In the present study, these theoretical principles informed the design of the instructional intervention. The exposure phase introduced conflicting solution strategies, generating dissatisfaction. The conflict phase structured classroom discussions in which pre-service teachers compared and evaluated alternative reasoning, supporting intelligibility and plausibility. Finally, the resolution phase encouraged participants to apply revised conceptions to new problems, aligning with the condition of fruitfulness. In this way, the instructional design sought to facilitate accommodation and promote deeper conceptual change in understanding nonlinear equations. Although the study draws on conceptual change theory, the conditions proposed by Posner et al. (1982) were not used as explicit coding categories in the analysis. Rather, they are used as an interpretive lens to support the discussion of the findings.

Students' Understanding of Equations

Algebra is a foundational area of mathematics that prepares students for advanced study, making it a central focus of research in mathematics education (Knuth et al., 2006). Many studies have addressed misconceptions related to algebraic statements, as well as concepts of equality and variables. Consistent with this view, Stephens (2006) identifies four areas in algebra where students commonly struggle, as documented in the literature: “solving algebraic equations,” “interpreting symbolic expressions,” “translating from verbal to symbolic representations,” and “gaining a general understanding of the structural aspects of algebra.” Nonlinear equations include quadratic, radical, logarithmic, exponential, and rational equations that are commonly encountered in secondary school and teacher education mathematics.

Previous studies have examined students' misconceptions of first-degree equations with one unknown (Erbaş, 1999; Küchemann, 1981; Hall, 2002; Radford & Puig, 2007; Toka & Aşkar, 2002; Ucar-Sarımanoğlu, 2019). Generally, students' difficulties and misconceptions regarding equations fall into two main categories. The first category involves “characteristic” or “grammar” errors that occur when students manipulate operations incorrectly, such as in the cases of $39x - 4 = 35x$, $3(n + 4) = 3n + 4$, or solving $x - 2 = 5$ as $x = 3$ (Carry et al., 1980; Kieran, 1988). The second category involves conceptual difficulties, such as neglecting to verify the solution to an equation or focusing solely on the solution process without considering the values of x that satisfy the equation (Lee & Wheeler, 1987).

Beyond studies focusing on specific types of equations, research has also documented more general patterns of students' algebraic errors that are not restricted to linear or nonlinear equations. Becker's (1988) study examined students' errors in secondary-level algebra, particularly characteristic errors made by seventh- to ninth-grade students while learning elementary algebra. Rather than focusing on a specific equation type, Becker classified errors related to illegal operations into four categories: (1) “Erroneous application of an operator,” occurring when students disregard the conditions for applying an operator, such as $\frac{-x^2+xy^2}{y^2-x} = \frac{-x^2+x}{-x} = 1$; (2) “Erroneous execution of an operator”, when students incorrectly apply the operator, such as $2(2 + 3x) - 1 + 2x = 4 - 1 + 5x$; (3) “Erroneous matching of variables in the premise of an operator”, where a variable is incorrectly substituted within a formula, as in $16b^2 + 80bc + 25c^2 = (4b + 5c)^2$ and (4) “Incorrect identification of variables in the conclusion,” where variables are mismatched in the conclusion, such as $(4uw - 3v)(3v + 4uw) = 4uw^2 - 3v^2$.

However, studies that address misconceptions related to nonlinear equations as a primary focus of instruction, particularly through cognitive conflict-based instructional designs, remain limited.

While some studies have explored the use of cognitive conflict in the context of first-degree equations (e.g., Toka & Aşkar, 2002) or algebra learning more broadly (e.g., Maharani & Subanji, 2018), research that systematically applies cognitive conflict-based instructional designs to the teaching and learning of nonlinear equations remains notably limited.

This study focuses on pre-service elementary mathematics education students' understanding of nonlinear equations. It investigates how cognitive conflict-based instruction is associated with their conceptual understanding and academic performance in solving these equations. While existing research provides valuable insights into students' algebraic difficulties and the instructional use of cognitive conflict, there remains a need for

studies that examine how cognitive conflict-based instructional designs can support pre-service teachers' understanding of nonlinear equations. Unlike prior studies that have primarily examined cognitive conflict in the context of first-degree equations and middle school learners (e.g., Toka & Aşkar, 2002), the present study shifts the focus to pre-service elementary mathematics teachers and to nonlinear equations that involve domain restrictions, extraneous roots, and structurally more complex algebraic reasoning. By examining how cognitive conflict operates in this more advanced, conceptually demanding context, the study extends existing research and situates it within the field of mathematics teacher education. In particular, the study explores how carefully constructed, conflicting student viewpoints can serve as an instructional mechanism to surface and challenge pre-service teachers' misconceptions about nonlinear equations.

The specific research questions of the study are as follows:

1. What misconceptions and conceptual difficulties do pre-service elementary mathematics teachers exhibit in solving nonlinear equations?
2. How does pre-service elementary mathematics teachers' academic performance in solving nonlinear equations change following cognitive conflict-based instruction?

Method

Research Design and Participants

This study employed an embedded mixed-methods classroom intervention design (Creswell & Plano Clark, 2018), consisting of a one-group pre-test–post-test quantitative component and qualitative analyses of participants' written responses and classroom discussions. The design was intervention-based and sequential, involving a pre-test, cognitive conflict activities, whole-class discussions, and a post-test.

The quantitative component was used to examine changes in pre-service elementary mathematics teachers' academic performance following the instructional intervention. The qualitative component focused on identifying participants' misconceptions, conceptual difficulties, and shifts in reasoning through open-ended questionnaires, cognitive conflict-based activities, and classroom discussions.

The two strands were integrated at the interpretation stage, consistent with recommendations for mixed-methods research (Creswell & Plano Clark, 2018). Quantitative results were used to identify patterns of change in performance and variation in misconception frequencies, while qualitative findings were used to explain how these changes emerged by examining participants' reasoning and conceptual restructuring processes.

Data were collected from third-year pre-service mathematics teachers enrolled in a General Mathematics course via academic performance assessments and open-ended questionnaires administered during instruction. The number of participants varied across phases of the study. A total of 49 pre-service elementary mathematics teachers were enrolled in the course. Of these, 36 participated in the pre-test, 37 participated in the post-test, and 20 completed all phases of the study (pre-test, cognitive conflict activities, and post-test) and were therefore included in the paired-samples analyses. Participants in each phase were not necessarily the same individuals; therefore, the groups should not be interpreted as matched or longitudinal samples.

Because the number of participants differed across phases, frequency-based findings are reported by phase rather than using a fixed sample. This variation should be taken into account when interpreting both frequency-based findings and quantitative results.

The reduction in the sample was mainly due to routine absenteeism during the data-collection period, including scheduling conflicts and individual absences. Based on available course information, there was no indication that attrition was systematically related to participants' prior achievement or performance levels. Nevertheless, because participation in all phases was voluntary, self-selection bias should be considered when interpreting the findings.

Table 1 summarizes the variation in participant numbers across the different phases of the study.

Table 1.
Participation across study phases

Phase	Number of participants	Reason for inclusion/exclusion
Enrolled	49	Students enrolled in the course
Pre-test completed	36	Students who completed the pre-test
Cognitive-conflict activity	35	Students who participated in the intervention sessions
Post-test completed	37	Students who completed the post-test
Completed all phases	20	Students included in paired-samples analysis

As shown in Table 1, the number of participants varied across phases. Therefore, frequencies reported in the findings reflect phase-specific participation rather than a single consistent sample.

Data Collection

The researcher used an achievement test administered as a pre-test and post-test to assess students' academic performance in solving nonlinear equations. These written tests included five questions designed to elicit students' misconceptions and difficulties in solving nonlinear equations, as presented in Table 2. The items required participants to solve nonlinear equations designed to reveal common misconceptions, such as false roots or extraneous solutions.

Table 2.
Pre-test and Post-test Items

Item	Pre-test	Post-test
1	$(x + 4)(x^3 - 4)\sqrt{x} = 0$	$(x + 3)(x^3 - 1)\sqrt{x} = 0$
2	$x - 1 = \sqrt{a - x^2}$, where a is a given constant.	$\sqrt{x + 3} + \sqrt{2x - 1} = 4$
3	$\log \sqrt{1 + x} + 4\log \sqrt{1 - x} = \log \sqrt{1 - x^2} + 3$	$\log_2(x + 2) + \log_2(3x - 4) = 4$
4	$x^{\sqrt{x}} = \sqrt{x^x}$	$3x^{\sqrt{x}} = 3\sqrt{x^x}$
5	$\frac{x^2 - 7}{x - 4} = \frac{x + 5}{x - 4}$	$\frac{x^2 - 7}{x + 3} = \frac{x + 5}{x + 3}$

Note. All equations were solved over the set of real numbers.

The test was designed to examine students' understanding of various types of nonlinear equations involving radicals, logarithmic and exponential expressions, and rational equations. The achievement tests used in the study consisted of open-ended items designed to elicit students' underlying conceptions and misconceptions related to nonlinear equations. The pre-test and post-test were developed as parallel forms, each comprising conceptually matched items. The specific items are presented in Table 2, and their alignment with targeted misconception categories is shown in Table 3.

Content validity was assessed through expert review by mathematics education specialists, who evaluated the alignment among test items, targeted misconceptions, and instructional objectives. The credibility of the instrument was supported by expert validation, a parallel-forms design, and the triangulation of quantitative results with qualitative data from open-ended cognitive conflict questionnaires.

Table 3.
Alignment between test items and targeted misconception categories

Item	Targeted Misconception
1	Ignoring domain restrictions when solving via zero-product rule
2	Generating extraneous roots through algebraic manipulation
3	Incorrect application of logarithm rules; improper simplification
4	Overgeneralization of "equal bases-equal exponents" rule
5	Neglecting denominator restrictions

For the cognitive conflict-based instruction, the researcher used open-ended questionnaires that presented contrasting student views, derived from pre-service teachers' incorrect pre-test responses and the literature. These statements were designed to challenge common misconceptions in solving nonlinear equations, particularly those related to function domains, variable signs, and extraneous solutions, as illustrated in Table 4. Although the primary focus of the study was on nonlinear equations, a limited number of inequality items were included in the cognitive conflict activities to examine whether participants could transfer their reasoning about domain restrictions and solution validity across closely related algebraic contexts, such as between equations and inequalities. In each task, participants were presented with multiple student statements concerning the same mathematical task. Typically, one statement reflected a common misconception, while

another provided an alternative justification or challenged the earlier reasoning. Participants were asked to indicate which student's view they agreed with and to justify their choice. By presenting a misconception alongside a counterargument, the prompts were designed to highlight conflicting reasoning and encourage participants to critically evaluate the validity of the underlying mathematical reasoning.

Table 4.

Open-ended cognitive conflict prompts presenting contrasting student views

Student	Statement
Student 1	For the equation $x^2 - x + 7 = 0$, $-x$ represents a negative quantity and x^2 states a positive number.
Student 2	The roots that I obtain by taking the square of both sides of the equation to find the solution set of the equation $\sqrt{x+1} + \sqrt{2x-3} = 3$, satisfy the given equation.
Student 3	I don't agree with the 2 nd student's view, because I know that the domain of an equation is the set of values that satisfy both sides of the given equation. If the roots that I obtain by taking the square of both sides of the equation do satisfy all the statements of the equation, then these roots form the solution set.
Student 4	To solve the equation $\log\sqrt{1+x} + 2\log\sqrt{1-x} = \log\sqrt{1-x^2} + 3$, I can turn the multiplication inside the log into an addition outside the log ($\log\sqrt{1-x^2}$ can be expressed as $\log\sqrt{1+x} + \log\sqrt{1-x}$). So I can abbreviate the equation by subtracting $\log\sqrt{1+x}$ from both sides of the equation. The roots of the equation $\log\sqrt{1-x} = 3$ form the solution set.
Student 5	I do not agree with the 4th student. The roots of the equation $\log\sqrt{1-x} = 3$ may not make the statement of $\log\sqrt{1+x}$ defined. Accordingly, solving only the equation $\log\sqrt{1-x} = 3$ may lead to extraneous roots. Therefore, it is necessary to check whether the solutions of $\log\sqrt{1-x}$ also satisfy the domain restrictions of $\log\sqrt{1+x}$.
Student 6	To solve the equation $(x+2)\sqrt{x} = 0$, I solve the equations $x+2 = 0$ or $\sqrt{x} = 0$. The roots that I obtain form the solution set. These roots are 0 and -2.
Student 7	For an inequality of $x^2 + x - 12 \leq 0$, the statement $x^2 + x - 12$ could get both negative and positive values. Although the inequality is less than and equal to zero, positive values could be seen on the solution set.

First, the algebra achievement test was administered as a pre-test to the class. One week later, cognitive conflict was introduced over three weeks through open-ended questionnaires that presented opposing student views. To stimulate cognitive conflict, the opinions of seven students on nonlinear equations were shared with the participants, who were asked to

evaluate these conflicting perspectives in the questionnaire. Individually, each participant documented their own responses to the student views. A whole-class discussion followed, where participants examined these views and worked toward a consensus (the resolution phase in cognitive conflict theory). Finally, a post-test following the cognitive conflict-based instruction was administered. Among the participants who attended the cognitive conflict activities ($N = 35$), 20 volunteered to complete the full sequence of pre-test, post-test, and cognitive conflict activities; thus, the experimental part of the study was conducted with these 20 participants.

Data Analysis

The analysis of qualitative data from pre-service mathematics teachers' written responses and classroom discussions followed an iterative process that combined inductive, deductive, and abductive reasoning (Johnson & Onwuegbuzie, 2004).

In the inductive phase, two researchers independently examined participants' responses to identify recurring error patterns and conceptual difficulties related to nonlinear equations. Initial codes were generated directly from the data. In the deductive phase, these emerging patterns were compared with categories of misconceptions reported in the literature on algebraic reasoning. This step allowed the researchers to examine the consistency of the findings with established frameworks. In cases where responses did not fully align with existing categories, the researchers revisited the data and refined the coding framework through interpretive discussion (abductive phase). Discrepancies between coders were discussed until consensus was reached, and previously coded responses were re-examined to ensure consistency across categories.

Quantitative data were analyzed using SPSS software to examine differences between pre-test and post-test scores. After checking the assumptions for parametric testing, a paired-sample t-test was conducted, and the results were tabulated.

It should be noted that two different participant groups were considered in the analyses. Quantitative analyses (pre-test–post-test comparisons) were conducted only with the 20 pre-service teachers who completed all phases of the study. In contrast, the frequencies of misconceptions reported in Tables 5–8 are based on all students who participated in each respective phase (pre-test, cognitive conflict activities, and post-test). Therefore, the number of participants varies across phases and may exceed the sample used for quantitative analyses.

Ethical Considerations

This study was conducted in accordance with institutional ethical guidelines. Participants were informed about the purpose and procedures of the study, and informed consent was obtained. Participation was voluntary and had no effect on course grades or academic standing.

All data were anonymized to ensure confidentiality. Because the study was conducted within a course context and the researcher was also the instructor, participants were explicitly informed that they could withdraw at any time without any consequences.

Findings

The following findings were derived from the analysis of pre-test, post-test, and cognitive conflict-based activities involving pre-service mathematics teachers enrolled in the General Mathematics course.

Conceptual difficulties and misconceptions identified in the pre-test

The pre-test was administered to 36 pre-service mathematics teachers. The solution methods they employed, their success rates, and the reasons for their mistakes and failures were recorded. Conceptual difficulties (misconceptions) and their frequency, as identified in the pre-test, are summarized in Table 5.

Table 5

Frequencies of misconceptions among students who participated in the pre-test (N = 36)

Misconceptions	Frequency	Percentage
Failure to exclude values that make the denominator zero after clearing fractions.	14	38.9%
Generating extraneous roots by incorrectly converting multiplication within a logarithm into addition outside the logarithm.	21	58.3%
Including in the solution set values that make other factors undefined, resulting in extraneous roots.	32	88.9%
Failure to consider special base cases in exponential equations (e.g., base equal to 1, where equal-exponents reasoning may fail, or base equal to 0, which may lead to undefined expressions)	33	91.7%

Note. Frequencies and percentages are reported based on the number of students who participated in each phase of the study.

According to the pre-test results, the most common error involved the loss or misinterpretation of roots when solving exponential equations. As illustrated in Figure 1 (b), one participant equated the exponents after matching the bases, without considering that the base must be neither 0 nor 1, as these cases violate standard equal-exponent reasoning or may result in undefined expressions. In doing so, the participant included $x = 0$ in the solution set, even though substituting this value produced the indeterminate form 0^0 . At the same time, several participants failed to recognize that the root made the base equal to 1, even though this value satisfied the original equation. These responses suggest that participants treated algebraic equivalence mainly as a procedural rule, without attending to the structural conditions under which the rule was applied.

The second most common error involved generating extraneous roots by setting each factor equal to zero without verifying domain restrictions. As seen in Figure 1 (a), participants separated the factors and solved them independently, yet did not examine whether the resulting values satisfied the radicand condition. Consequently, roots that produced undefined expressions were still treated as valid solutions.

The third most frequent error involved logarithmic expressions. In Figure 1 (c), a participant converted multiplication inside the logarithm into addition outside it and then simplified the expression by canceling terms. However, the participant did not check whether the resulting values satisfied the domain restrictions of the logarithmic function. This shows that participants focused on applying rules rather than checking whether the transformed expression remained valid.

The fourth common error involved canceling common denominators in rational equations. As shown in Figure 1 (d), participants canceled the denominators at the outset and solved the resulting nonlinear equation. However, they retained values that made the denominator equal to zero. Fourteen participants included such values in their solution sets, suggesting that cancellation was treated as a neutral transformation rather than one that imposes domain constraints.

These examples show that participants often applied algebraic operations procedurally while overlooking how those operations affect the structure and validity of the solution set.

$$\frac{x^2-7}{x-4} = \frac{x-5}{x-4}$$

$$x^2-7 = x-5$$

$$x^2-x-12=0$$

$$(x+3)(x-4)=0$$

$$x_1=-3 \quad x_2=4$$

$$G.K = \{-3, 4\}$$

(a) Example illustrating failure to check denominator restrictions

$$\frac{(x+4)(x^2-4)}{\sqrt{x}} = 0$$

$$\frac{x+4}{1} \cdot \frac{x^2-4}{1} = 0$$

$$x+4=0 \quad x^2-4=0$$

$$x=-4 \quad x=2 \quad x=0$$

$$G = \{-4, -2, 0, 2\}$$

(b) Example illustrating acceptance of extraneous roots

$$3 \log \sqrt{1-x} + 4 \log \sqrt{1-x^2} + 3 = 3$$

$$\log \sqrt{1-x} = 1.5$$

$$2 \log \sqrt{1-x} = 3$$

$$\log \sqrt{1-x} = 1.5$$

$$\sqrt{1-x} = 10^{1.5}$$

$$1-x = 10^3$$

$$-33 = x$$

$$G.K = \{-3, 3\}$$

(c) Example illustrating incorrect logarithmic transformation

$$x \sqrt{x} = \sqrt{x^2}$$

$$x^{3/2} = x^{1/2}$$

$$x = \frac{1}{2}$$

$$x = 0$$

$$x = 4$$

$$x = \{0, 4\}$$

(d) Example illustrating misuse of equal-base reasoning

Figure 1. Examples of students' written responses illustrating common misconceptions in solving nonlinear equations

Few participants solved the rational equation correctly, primarily because many failed to exclude values that made the denominator zero. Participants generally managed to exclude the value that made the denominator equal to zero.

Misconceptions identified from cognitive conflict-based activities

Following the pre-test, the researcher aimed to generate cognitive conflict in the classroom by presenting an open-ended questionnaire that included differing and opposing student views. These views, which included conflicting opinions, were derived from the pre-service teachers' incorrect pre-test answers and from the literature. To stimulate cognitive conflict, seven student perspectives on nonlinear equations were introduced to 35 pre-service mathematics teachers one week after the pre-test, and they were asked to evaluate these conflicting views in the questionnaire. This was followed by a whole-class discussion about the conflicting perspectives. Presenting contrasting student viewpoints helped reveal participants' reasoning and underlying misconceptions that might otherwise remain implicit.

The misconceptions and their frequency, as identified from the cognitive conflict-based activity, are summarized in Table 6.

Table 6

Misconceptions identified during the cognitive conflict-based activities (N = 35)

Misconceptions	Frequency	Percentage
For a nonlinear equation of $x^2 - x + a = 0$ with $a \geq 0$, incorrectly believing that $-x$ must be a negative number in order to equal zero, due to the assumption that x^2 is always positive.	19	54.3%
Believing that performing operations on both sides of an equation does not change the solution set of the equation.	3	8.6%
Viewing extraneous roots as unrelated to domain restrictions.	1	2.9%
For a nonlinear inequality $x^2 - x + a \leq 0$ mistakenly believing that positive values cannot be included in the solution set because the inequality requires values to be less than or equal to zero.	12	34.3%
Neglecting the rule that the radicand must be greater than or equal to zero.	16	45.7%

Note. Frequencies and percentages are reported based on the number of students who participated in each phase of the study.

The conflicting views regarding the second, third, and fifth misconceptions, as shown in Table 6, were derived from the pre-service teachers' incorrect pre-test answers. When presented with both correct and incorrect perspectives, participants generally provided logical responses. Those who had made errors on the pre-test recognized their mistakes when confronted with differing student perspectives and during classroom discussions. For example, in the cognitive conflict-based activity, participants who had included extraneous roots in the solution set stated that roots should be included only if they made both sides of an equation defined over the real numbers.

Although the number of pre-service teachers exhibiting misconceptions decreased after the intervention, certain misconceptions persisted, particularly those drawn from the literature. One of the most persistent misconceptions concerns the interpretation of the quadratic form $x^2 - x + a = 0$ (where $a \geq 0$). Several participants argued that $-x$ must represent a negative quantity in order for the equation to equal zero, reasoning that x^2 is always positive. This response indicates confusion between interpreting an algebraic expression and interpreting an equation. In their reasoning, x^2 was treated as a fixed positive number rather than as a variable-dependent expression whose value depends on x . As a result, the parameter a was implicitly viewed as needing to "compensate" for the assumed positivity of x^2 , reflecting arithmetic balancing rather than structural reasoning about symbolic relationships.

Some participants further suggested that the sign of $-x$ might vary with the value of x , while others invoked the discriminant and concluded that the equation would be undefined in the real numbers when the discriminant is negative. These responses show that participants moved between procedural rules and informal reasoning about signs, without clearly distinguishing between properties of expressions and conditions for solutions of equations.

The other common misconception was the belief that positive values could not be included in the solution set for a nonlinear algebraic inequality $x^2 - x + a \leq 0$ because the inequality requires values to be less than or equal to zero.

Three of the 35 pre-service teachers believed that performing the same operation on both sides of an equation does not affect its roots. As illustrated in Table 4, one pre-service teacher explicitly rejected the 3rd student's domain-based argument and aligned with the 2nd student's procedural reasoning, stating that performing the same operation on both sides does not change the roots. In this case, squaring both sides introduced an extraneous root; however, the participant included it in the solution set without verifying that it satisfied the original equation. This example demonstrates that procedural equivalence was prioritized over the solution's conditional validity.

In the logarithmic task, the 5th student emphasized the need to verify whether the obtained roots satisfied the domain conditions of both logarithmic expressions. However, as illustrated in Table 4, one pre-service teacher rejected this domain-based reasoning, arguing that extraneous roots are inherently invalid. Although the participant acknowledged the notion of extraneous solutions, the explanation did not address the structural reason for their occurrence: algebraic simplifications may generate values for which logarithmic expressions are undefined. This suggests that the participant's reasoning relied more on an intuitive sense of solution validity than on explicit reference to formal domain restrictions.

Examples of pre-service mathematics teachers' considerations regarding conflicting student views are provided in Table 7.

Table 7

Some examples of pre-service mathematics teachers' considerations

Conflicting views	Pre-service mathematics teachers' considerations
<i>Student 1</i> For the equation $x^2 - x + 7 = 0$, $-x$ represents a negative quantity and x^2 states a positive number.	(Negative number of $-x$) "Since x^2 and 7 are strictly positive numbers, $-x$ should be a negative number so that the equation could be equal to zero..."
<i>Student 2</i> "The roots that I obtain by taking the square of both sides of the equation to find the solution set of the equation $\sqrt{(x+1)} + \sqrt{(2x-3)} = 3$ satisfy the given equation."	(Performing operations on both sides of an equation)
<i>Student 3</i> I don't agree with the 2nd student's view, because I know that the domain of an equation is the set of values that satisfy both sides of the given equation. If the roots that I obtain by taking the square of both sides of the equation do satisfy all the statements of the equation, then these roots form the solution set.	"No. I do not agree with the 3rd student's view. Because, when we perform operations on both sides of the equation, the roots of the equation do not change..."
<i>Student 4</i> To solve the equation $\log \sqrt{1+x} + 2 \log \sqrt{1-x} = \log \sqrt{1-x^2} + 3$ I can convert the multiplication inside the logarithm into addition outside the logarithm, since $\log \sqrt{1-x^2} = \log \sqrt{1+x} + \log \sqrt{1-x}$ then I can simplify the equation by subtracting $\log \sqrt{1+x}$ from both sides. The roots obtained from solving $\log \sqrt{1-x} = 3$ form the solution set.	(Viewing extraneous roots as meaningless rather than domain-related) "I do not agree with the 5th student's view. Because, extraneous roots do not make sense in the equation"
<i>Student 5</i> I do not agree with the 4th student. The roots of the equation $\log \sqrt{1-x} = 3$ may not make the statement of $\log \sqrt{1+x}$ defined. Accordingly, solving only the equation $\log \sqrt{1-x} = 3$ may lead to extraneous roots. Therefore we need to check whether the roots of the equation $\log \sqrt{1-x}$ are also defined for $\log \sqrt{1+x}$.	

Conceptual difficulties and misconceptions identified in the post-test

A week after implementing cognitive conflict, a post-test parallel to the pre-test was administered. The number of misconceptions identified in the post-test is shown in Table 8. Responses that reflected calculation errors or incomplete solutions were not included in this analysis.

Table 8

Frequencies of misconceptions among students who participated in the post-test (N = 37)

Misconceptions	Frequency	Percentage
Failure to exclude values that make the denominator zero after clearing fractions.	2	5.4%
Generating extraneous roots by incorrectly converting multiplication within a logarithm into addition outside the logarithm.	26	70.3%
Including in the solution set values that make other factors undefined, resulting in extraneous roots.	11	29.7%
Failure to consider special base cases in exponential equations (e.g., base equal to 1, where equal-exponents reasoning may fail, or base equal to 0, which may lead to undefined expressions)	35	94.6%

Note. Frequencies and percentages are reported based on the number of students who participated in each phase of the study.

As shown in Table 8, the categories of conceptual difficulties remained structurally similar in the post-test; however, the frequencies of specific misconceptions showed a mixed pattern of change. While “failure to exclude values that make the denominator zero after clearing fractions” and “generating extraneous roots through invalid algebraic transformations” decreased, the frequency of logarithmic misconceptions increased, and misconceptions related to exponential expressions remained consistently high. These two misconceptions were targeted in the cognitive conflict activities through contrasting student statements that foregrounded domain verification and the conditional validity of algebraic operations. These conflicting views are presented in Table 4. In contrast, the misconception about the loss of roots when the base of an exponential expression is zero or one was not directly reflected in the conflicting student views observed during the intervention. Because this specific conceptual issue was not challenged through structured cognitive conflict, its frequency remained largely unchanged in the post-test. This pattern suggests that reductions in misconception rates were more likely when the misconception had been directly confronted through the cognitive conflict prompts.

Although misconceptions related to logarithmic transformations were addressed during the cognitive conflict phase through contrasting views on the improper simplification of logarithmic expressions, as illustrated in Table 4, a substantial number of pre-service teachers continued to generate extraneous roots in the post-test when faced with a different logarithmic equation. In the post-test item requiring the application of the product rule for logarithms, 26 participants again produced extraneous solutions. This finding suggests that the intervention may have prompted local reconsideration of specific procedural errors, but provides limited evidence for a generalized understanding of the conditional validity of logarithmic transformations. The persistence of this misconception suggests that addressing it in only one type of task may be insufficient to support transfer across algebraically related contexts.

The quantitative part of the study

One week after the cognitive conflict activities, a post-test parallel to the pre-test was administered. Participation varied across phases, with 36 participants in the pre-test, 35 in the cognitive conflict activities, and 37 in the post-test. Only 20 participants completed all phases of the study and were therefore included in the paired pre-test–post-test analysis. Consequently, the quantitative analysis is based on this subset of participants.

Before the analysis, the normality of the difference scores was examined using the Kolmogorov–Smirnov test. As shown in Table 9, the test was not statistically significant, suggesting no strong evidence of departure from normality. The normality of the difference scores was also examined using the Shapiro–Wilk test, which was not statistically significant ($p > .05$). In addition, the distribution of the difference scores was inspected visually using Q–Q plots. Following the normality check, a paired-sample t -test was conducted to compare pre-test and post-test scores. Each test consisted of five open-ended items, with a maximum possible score of 100. Each item was scored dichotomously, with 20 points for a completely correct solution set and 0 points for an incorrect one. This approach was adopted to ensure consistency and objectivity in evaluating whether participants arrived at mathematically valid solution sets, particularly regarding issues such as loss of roots or extraneous roots.

Descriptive statistics indicated an increase in students' performance following the instructional intervention. The mean pre-test score was $M = 20.00$ ($SD = 15.89$), whereas the mean post-test score increased to $M = 38.00$ ($SD = 14.36$).

Table 9
Kolmogorov-Smirnov Test

	N	Mean	SD	p
Dif Scores	20	18	18.238	0.144*

* The normality test was not statistically significant, suggesting no strong evidence of departure from normality.

As shown in Table 10, the results of the paired-sample t -test revealed a statistically significant difference between pre-test and post-test scores, $t(19) = -4.41$, $p < .001$. The 95% confidence interval for the mean difference was $[-26.54, -9.46]$. To assess the magnitude of this difference, Cohen's d for paired samples was calculated using the mean difference divided by the standard deviation of the difference scores, indicating a large effect size ($d_z \approx 0.99$). These findings suggest a strong association between participation in the cognitive conflict-based activities and increased performance among the participants who completed all phases of the study.

Table 10
Paired samples t-test

Measure	N	M	SD	Mean difference	SDdiff	SEdiff	t	df	p
Pre-test	20	20.00	15.89						
Post-test	20	38.00	14.36	-18.00	18.238	4.078	-4.41	19	<.001

Note. Mean difference was calculated as pre-test minus post-test. d_z was calculated by dividing the mean difference by the standard deviation of the difference scores.

Discussion

This study provides insight into the misconceptions and conceptual difficulties that pre-service elementary mathematics teachers exhibit regarding nonlinear equations, as well as the role of cognitive conflict-based instruction in addressing these difficulties. The findings reveal persistent challenges in areas such as domain restrictions, solution verification, parameter interpretation, and the conditional validity of algebraic transformations. In particular, the tendency to accept extraneous roots and the failure to exclude values that make the denominator zero after clearing fractions suggest a reliance on procedural rule application without sufficient attention to the structural constraints embedded in algebraic expressions. This pattern is consistent with longstanding distinctions between procedural and conceptual knowledge in mathematics education (Hiebert & Lefevre, 1986; Star, 2005).

The observed reduction in the frequencies of certain misconceptions from pre-test to post-test suggests that structured cognitive conflict may be associated with the reconsideration of specific procedural errors. Presenting opposing student viewpoints created cognitive tension, encouraging participants to reconsider their existing ideas. In addition to shifts in the frequencies of misconceptions, the increase in post-test scores may indicate improvements in academic performance accompanied these conceptual reconsiderations. However, such gains varied depending on whether the misconception had been targeted during the intervention.

One of the central contributions of this study is that carefully constructed conflicting student views may serve as a mechanism for generating cognitive conflict and making pre-service teachers' misconceptions visible. The open-ended questionnaires that presented opposing student views proved effective in generating cognitive conflict and making participants' misconceptions explicit. This finding is consistent with prior research suggesting that the effectiveness of cognitive conflict depends on how contrasting viewpoints are deliberately constructed and pedagogically orchestrated in classroom settings (Kang et al., 2010; Lee & Yi, 2013; Maharani & Subanji, 2018; Sari et al., 2022; Watson, 2002; Watson & Moritz, 2001). In the present study, presenting a misconception alongside a counterargument appeared to encourage participants to reconsider their reasoning. However, the findings also suggest that simply presenting conflicting views is not sufficient; the design, sequencing, and facilitation of these tasks play a crucial role in determining whether learners engage in deeper structural reflection or remain at a procedural level.

Equations involving square roots were particularly challenging for participants. While radical expressions are often considered conceptually demanding, the difficulty observed in this study extended beyond computational complexity. In the equation $x - 1 = \sqrt{a - x^2}$, several participants interpreted the parameter " a " as an unknown variable to be solved for, attempting to isolate " a " on one side and express it in terms of x . This response reflects a

blurred distinction between interpreting an algebraic expression and interpreting an equation with a designated unknown. Rather than recognizing that the equation required solving for x under a given parameter constraint, participants procedurally redefined the task. Such responses indicate difficulties in coordinating symbolic manipulation with the structural roles of variables and parameters. From a conceptual perspective, this may reflect the incorporation of new information into existing procedural schemas rather than a restructuring of understanding (Chi, 2008; Posner et al., 1982).

The findings also indicate that the effects of cognitive conflict were mostly local. Although misconceptions related to zero solutions and extraneous roots decreased when targeted, the persistence of the logarithmic misconception in a structurally different post-test item indicates limited transfer across related algebraic contexts. Research on transfer has shown that successful generalization depends on recognizing deep structural features rather than relying on surface similarities (Chi et al., 1981; Gentner, 1983). In the present study, participants appeared to maintain a procedural interpretation of logarithmic equivalence, prioritizing rule application over structural verification of domain conditions. This suggests that while cognitive conflict facilitated local conceptual adjustments, it did not consistently produce broader structural reorganization.

Taken together, these findings can be interpreted in relation to theoretical perspectives on learning that emphasize the role of cognitive conflict in prompting the reconsideration of existing ideas. The structured exposure to conflicting solution strategies appeared to challenge participants' existing conceptions and was associated with increased intelligibility and plausibility of alternative reasoning. However, the uneven transfer of these shifts across structurally related contexts suggests that cognitive conflict may support local conceptual adjustments without necessarily leading to broader restructuring of understanding. In this respect, the study highlights both the potential and the limitations of cognitive conflict-based instruction for deeper algebraic understanding.

Educational Implications

Although research on algebraic equations is well established, existing studies indicate a continued need for instructional approaches that prioritize conceptual understanding over procedural proficiency. Cognitive conflict may be an effective instructional strategy for addressing specific algebraic misconceptions. It is observed that students often confuse the concept of variables with constants. By addressing these misconceptions, lecturers can help clarify the distinction between variables and constants, making it easier for students to understand variables.

Lecturers should also provide students with opportunities to understand why the roots they find after certain operations (such as squaring both sides of an equation) may produce extraneous solutions, new solutions that do not satisfy the original equation. Additionally, instructors may encourage students to consider variations in the domain and solution set when solving equations, as performing operations without regard to the domain can lead to the loss of roots and the introduction of extraneous roots.

From a curricular perspective, the limited attention to extraneous roots and domain restrictions in the current national mathematics curriculum (Ministry of National Education [MONE], 2023) may be consistent with some of the conceptual difficulties observed among the participants. While the present study does not allow causal claims about curriculum effects, this observation suggests that curriculum developers may consider embedding domain analysis and solution validation as explicit learning outcomes within equation-related standards. For example, students could be expected not only to solve equations

procedurally but also to justify the validity of their solutions with respect to the original equation and its domain. Learning objectives that emphasize checking solutions, identifying extraneous roots, and interpreting solution sets in relation to functional domains may support deeper conceptual understanding and reduce persistent misconceptions.

Limitations

Several limitations of this study should be acknowledged. Due to the one-group pre-test–post-test design, alternative explanations for the observed improvement cannot be ruled out. These may include practice effects, testing effects, maturation over the instructional period, and the influence of regular classroom discussions. In addition, the quantitative findings are based on a subset of participants who completed all phases of the study, which may have introduced selection bias. Therefore, the results should be interpreted as indicative of an association rather than a causal effect.

The sample consisted of a relatively small and homogeneous group of pre-service elementary mathematics teachers. Participation in the study was voluntary, and only 20 participants completed all phases of the instructional intervention. This attrition may have introduced self-selection bias, as participants who completed the study may have been more motivated or more interested in the instructional approach, thereby limiting the generalizability of the findings.

In addition, the scoring approach employed in the achievement test was dichotomous, assigning full credit only for completely correct solution sets. While this ensured scoring consistency, it may not have captured partial understanding or intermediate reasoning processes. Future studies may benefit from using more fine-grained rubrics that account for different components of algebraic reasoning.

The instructional intervention was implemented over a relatively short period of three weeks and focused exclusively on nonlinear equations. As the intervention was not embedded within a broader algebra curriculum, the findings should be interpreted with caution regarding ecological validity. The effectiveness of cognitive conflict-based instruction may differ when integrated into longer-term instructional sequences or across multiple algebraic topics.

Future research may consider implementing longer-term instructional interventions embedded within broader algebra curricula and involving more diverse participant groups to examine further the ecological validity and generalizability of cognitive conflict-based instructional approaches.

Conclusion

This study examined pre-service elementary mathematics teachers' understanding of nonlinear equations and investigated the role of cognitive conflict-based instruction in addressing common misconceptions. The findings indicate that pre-service teachers experience significant conceptual difficulties related to variables, domains, solution sets, and extraneous roots. Both quantitative and qualitative findings suggest that cognitive conflict-based activities may help identify specific misconceptions and promote local improvements in performance. However, these improvements were not uniform across all types of nonlinear equations. While some misconceptions decreased, others persisted, and in some cases increased, particularly those related to logarithmic and exponential expressions.

These findings indicate that short-term cognitive conflict activities may be effective in making certain misconceptions visible and encouraging reconsideration of specific

procedural errors, but may not be sufficient to support generalized conceptual restructuring across different equation types. In addition, the interpretation of the results should be considered with caution, given the limited matched sample size, the absence of a control group, and the exploratory nature of the design.

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