

Representational Coordination in Deaf and Hard-of-Hearing Mathematical Problem Solving

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Historically, mathematics education research regarding deaf and hard-of-hearing (D/HH) students has often relied on ‘modality-comparison’ designs, which treat formats such as printed English, annotated diagrams, and signed explanations as simple, interchangeable containers for mathematical content. This exploratory qualitative case study shifts the analytic focus from modality-based performance outcomes to a ‘representational coordination’ framework, investigating how two D/HH eighth-grade students construct meaning as they solve algebraic word problems. Using a hybrid deductive-inductive approach to analyze video-recorded sessions and ‘word-for-sign’ transcripts, this study examines four core analytic processes: representational entry, transformation, coordination, and externalization. Findings indicate that students do not simply receive information in a given modality; rather, they engage in ongoing, dynamic representational conversion and ‘representational repair’ (i.e., moments where errors become visible sites of semiotic negotiation and self-monitoring). The study highlights that D/HH students utilize diverse, interconnected strategies, ranging from silent written inscription to active signed explanation, gesture, and embodied action, to construct mathematical relationships. These results suggest that equitable instruction must move beyond ‘access’ defined as the provision of modalities, and instead foster representational competence through explicit instruction, multimodally attentive assessment, and collaborative interpretation practices. The findings underscore that ASL, visual-spatial reasoning, embodied action, and formal notation should be valued as a single, interconnected system essential for long-term mathematical success.

Introduction

Mathematical problem solving is a complex, multimodal activity that requires learners to interpret, transform, and connect various representational registers, such as written language, symbolic notation, diagrams, and graphs, to preserve mathematical relationships across problem-solving contexts (Duval, 2006). For all students, success in mathematics depends on the ability to coordinate these registers; however, for deaf and hard-of-hearing (D/HH) students, this challenge is inherently more complex. D/HH students frequently navigate a rich, multimodal communicative landscape that includes written English, American Sign Language (ASL), conventional algebraic notation, visual displays, and embodied forms of expression.

Historically, mathematics education research has often relied on “modality-comparison” designs, which treat formats such as printed English, annotated diagrams, and signed explanations as simple, interchangeable containers for mathematical content. This assumption, however, obscures the active, constructive nature of learning. As Ainsworth (2006) has argued, multiple representations are beneficial only when learners can interpret and integrate them; the translation among these systems is, in itself, a demanding learning task. Furthermore, Kelly and Mousley (2001) demonstrated that for D/HH students, word-problem solving is not merely a reading task but a complex act of interpreting relational language and mapping it onto mathematical situations. When the focus of research remains solely on whether one modality (e.g., ASL vs. text) yields better performance than another, researchers overlook the active processes (e.g., representational entry, transformation, and externalization) that define D/HH students’ mathematical reasoning.

Despite the breadth of literature documenting mathematics achievement gaps among D/HH students (Pagliaro & Kritzer, 2013), a critical gap remains: we lack empirical insight into the real-time cognitive mechanisms that drive these outcomes. Prior work has largely focused on achievement metrics (Pagliaro & Kritzer, 2013), word-problem difficulty (Kelly & Mousley, 2001; Kelly et al., 2003), or isolated vocabulary acquisition (Kelly & Gaustad, 2007; Lang & Pagliaro, 2007), providing little insight into how students actually navigate the interplay of ASL, English, symbolic notation, and visual displays during the moment of problem-solving. This study addresses this need by shifting the analytic lens from modality-based performance outcomes to the dynamic, multimodal processes of representational coordination. Grounded in a framework that views mathematical understanding as a constructive process, this research examines how D/HH eighth-grade students construct meaning while solving algebraic word problems. By analyzing the interplay of signed explanations, written work, gestures, and embodied actions, this study offers a nuanced understanding of how D/HH students enter tasks, repair representational breakdowns, and externalize reasoning, ultimately illuminating the complex relationship between representational access and mathematical success.

Ultimately, these findings may reshape how educators approach mathematics instruction for the D/HH community by moving beyond a narrow focus on access to modalities to a more refined pedagogical strategy that actively supports the cognitive coordination required to link ASL, written English, and symbolic representations. By identifying the specific points of breakdown and repair in students' real-time reasoning, this study offers teachers practical insights for designing interventions that build representational competence, ensuring that D/HH students are supported not just in arriving at a final answer, but in navigating the complex representational landscape essential for long-term mathematical success.

Conceptual Framework and Literature Review

This section synthesizes the theoretical underpinnings and existing literature relevant to this study. It establishes the representational coordination framework, conceptualizing mathematical understanding as a dynamic process of register coordination. It further reviews literature regarding multiple external representations, the representational challenges specific to deaf and hard-of-hearing (D/HH) students, and the role of American Sign Language (ASL) and embodied resources in mathematical reasoning. Finally, it outlines the analytic processes (i.e., entry, transformation, coordination, and externalization) that guide the present analysis.

Representational Coordination in D/HH Mathematical Problem Solving

This study is grounded in a representational coordination framework for understanding how D/HH students construct mathematical meaning. The framework begins with the premise that mathematical understanding is not encountered directly; rather, it is constructed through the interpretation, transformation, and coordination of representational registers. In the context of this study, students encounter three mathematical tasks through different initial representational forms: printed English, annotated diagrams with written English, and instructor-signed ASL. The analytic focus, therefore, is not whether one modality produces better performance, but how students enter, transform, coordinate, and maintain mathematical meaning as they move among these forms.

Because the three tasks entered through different initial representational forms (i.e., printed English, an annotated visual/table display with written English, and instructor-signed

ASL), the analysis attends to how students moved among forms rather than treating modality as an isolated variable. Students' explanations also moved across ASL, written notation, gesture, pausing, counting, and other embodied actions. Thus, the analytic focus is not simply whether one format produced better performance, but how students entered, transformed, coordinated, and sometimes lost mathematical meaning across representational forms.

Mathematical Understanding as Representational Coordination

Representations are recognized in mathematics education as cognitive tools that shape how students notice, organize, and explain ideas (Pape & Tchoshanov, 2001). Duval (2006) emphasized that mathematical comprehension depends on the ability to coordinate different registers, such as natural language, algebraic notation, diagrams, and graphical forms. From this perspective, solving a mathematics problem requires more than carrying out procedures within a single register; students must interpret one representation, transform it into another, and maintain the underlying mathematical structure across that transformation. For example, a word problem may need to be transformed into an equation; a table may need to be interpreted as a relationship among variables; and a signed explanation may need to preserve the structure of a written algebraic expression. Difficulty may arise not only from computation but also from the loss of mathematical structure during these transformations.

Research on multiple external representations supports this view. Ainsworth (1999, 2006) argued that multiple representations can be beneficial only when they complement one another or constrain interpretation, but these benefits are not automatic; translation among representational systems is itself a demanding learning task that requires cognitive effort (Adu-Gyamfi et al., 2019; Ainsworth et al., 2002; Bossé et al., 2014). Furthermore, pictorial representations can either support or interfere with a learner's construction of mental models, depending on how well they align with the task (Schnotz & Bannert, 2003). This framework cautions against assuming that adding a diagram or visual prompt automatically reduces cognitive load; students must explicitly learn how to read visual displays as mathematical representations (diSessa, 2004).

Word Problems, Representation, and the D/HH Context

For D/HH students, representational coordination is complicated by a rich communicative landscape that includes written English, American Sign Language (ASL), algebraic notation, visual displays, and embodied forms of expression. Word-problem solving is not merely a reading task; Kelly and Mousley (2001) demonstrated that D/HH students must interpret relational language and map it onto mathematical situations, a task heavily influenced by the complexity of relational linguistic features (Kelly, et al., 2003). Achievement gaps in mathematics (Pagliaro & Kritzer, 2013) are often shaped by instructional opportunity (Kelly et al., 2003) and the development of mathematical vocabulary through intricate networks of linguistic, visual, and conceptual associations (Kelly & Gaustad, 2007; Lang & Pagliaro, 2007).

Visual representations occupy an important but complex role in D/HH mathematics education. Because many D/HH students rely heavily on visual language and visual attention, diagrams, tables, and spatial displays are often assumed to be accessible. However, visual form alone does not guarantee mathematical clarity. A visual representation supports reasoning only when students can identify the relationships it encodes. In a fixed-fee/rate situation, for example, students must coordinate the written description, the table structure,

the variables representing fee and rate, and the algebraic relationship connecting text count to total cost. A diagram can therefore reduce some linguistic demands while adding representational demands.

ASL and Embodied Resources as Representational Tools

ASL should not be reduced to a support system for English, nor treated as merely another version of written text. Krause (2019, 2023) argued that ASL plays a fundamental role in mathematics classrooms as a visual-spatial resource that can spatialize relationships, contrast quantities, and sequence operations in ways that linear text cannot. However, signed presentation does not automatically eliminate difficulty; researchers have found that signed arithmetic story problems vary significantly in difficulty based on problem structure and the specific cognitive demands of reasoning (Ansell & Pagliaro, 2006). Thus, assessing the mathematical achievement of signing D/HH students requires approaches that are deeply attentive to their specific linguistic resources (Henner et al., 2021).

Mathematical thinking is further externalized through bodily action, including pointing, tracing, counting, and gesturing. Research has shown that gesture provides evidence of embodied mathematical knowledge and can contribute to the generation of new mathematical ideas (Alibali & Nathan, 2012; Goldin-Meadow et al., 2009). For signing D/HH students, this is critical, as conventional signs, classifiers, facial expressions, and spatial indexing interact during mathematical explanation (Goldin-Meadow et al., 2012). Consequently, in this study, actions such as air-writing or finger counting are treated as integral to the student's representational activity rather than as irrelevant behavioral noise.

Analytic Processes of the Framework

This framework directs attention to four core analytic processes: *Representational Entry* (how students initially access a task), *Representational Transformation* (the conversion of information from one form to another), *Representational Coordination* (the degree to which students maintain mathematical relationships as they move among representations), and *Representational Externalization* (the extent to which students make their reasoning visible), grounded in the work of Duval (2006) and Ainsworth (2006) on register coordination and multiple external representations.

By applying this framework, the study avoids the “modality-comparison” trap. It treats errors not as evidence of mathematical deficiency, but as moments where representational coordination becomes visible. This approach positions ASL and visual-spatial action as legitimate, constructive resources for mathematical reasoning, providing a more precise lens for examining how D/HH students navigate the representational landscape essential for long-term mathematical success.

Although prior research establishes that D/HH students' mathematics learning is shaped by language access, representational opportunity, ASL resources, visual displays, and embodied action, less is known about how D/HH middle-school students coordinate these resources in real time while solving algebraic word problems. Much existing work has focused on achievement outcomes, word-problem difficulty, vocabulary, assessment access, or broad instructional concerns. The present study addresses this gap by examining representational entry, transformation, coordination, and externalization across ASL, written English, algebraic notation, visual/table displays, gesture, pauses, and written work.

Guiding Research Questions

The following research questions guided the study:

RQ1. How do two D/HH eighth-grade students enter and interpret algebraic word problems presented through different initial representations: printed English, annotated diagram with written English, and instructor-signed ASL?

RQ2. How do students coordinate ASL, written English, algebraic notation, diagrams/tables, gestures, embodied actions, and written inscriptions while constructing mathematical meaning?

RQ3. Where do students' representational transformations appear productive, tentative, or disrupted as they move among problem statements, signed explanations, visual displays, written work, and symbolic notation?

These questions retain the original interest in problem format while shifting the analytic focus from modality comparison to representational coordination.

Research Methodology

Research Design and Collaboration

This study utilized an exploratory qualitative case study design to examine how two D/HH eighth-grade students constructed mathematical meaning. A qualitative case study is appropriate for investigating phenomena within their real-life context, particularly when the boundaries between phenomenon and context are not clear (Yin, 2018). The “bounded phenomenon” here was the students' representational activity: the real-time cognitive and communicative process of entering, transforming, coordinating, and externalizing mathematical reasoning (Merriam & Tisdell, 2016).

To ensure methodological rigor, the study employed researcher triangulation. The research design, data analysis, and manuscript preparation were conducted collaboratively between the teacher-researcher and a second researcher. This collaboration served as a peer-debriefing mechanism, requiring interpretive decisions (e.g., glossing ASL signs or coding representational events) to be subject to consensus, thereby mitigating potential teacher-researcher bias (Lincoln & Guba, 1985).

Site, Participants, and Researcher Positionality

The study took place at the South Carolina School for the Deaf. The participants were two D/HH eighth-grade students. As the teacher-researcher, I (the lead author) was a mathematics teacher at the school. The participating students were in my classes in previous years. However, due to structural changes in the school administration, I was no longer the participants' teacher at the time of this study and data collection. This separation was methodologically advantageous: it minimized instructional coercion and dual-role conflict, while leveraging my existing rapport with the students and deep familiarity with the school's linguistic environment (Tracy, 2010). My positionality as an insider enabled a nuanced interpretation of nonstandard ASL mathematical discourse, while collaboration with the second researcher provided the necessary distance to ensure confirmability.

Task Design and Representational Conditions

Over three sessions, students completed three distinct algebraic tasks. Each task was designed to elicit representational activity rather than to compare performance across

modalities, as “modality-comparison” designs often obscure the learner’s constructive processes. The initial representational conditions were:

Task 1: Written English only (evaluating an equation solution).

Task 2: Annotated table with written English (modeling a phone plan).

Task 3: Instructor-signed ASL only (constructing an equation with a solution of $x = -3$).

Students completed three open-ended algebraic tasks across three after-school sessions during one week. Each session lasted approximately 30–45 minutes, and one mathematics problem was presented per session. The written-English task asked students to evaluate a proposed solution to an algebraic equation. The annotated visual/table task asked students to model a fixed-fee and rate relationship using information distributed across a text and a table. The ASL-only task asked students to generate a multi-step equation with a specified solution. Because the tasks differed in both their initial representations and mathematical structures, they were treated as analytically distinct cases rather than as experimentally equivalent stimuli.

Data Collection and Preparation

The primary data sources were video-recorded problem-solving sessions and word-for-sign transcripts. Images of written work were used as supporting artifacts to contextualize and verify transcript references to writing, showing work, symbolic notation, and silent written activity; they were not treated as independent objects of analysis.

Video-based qualitative research is essential for capturing the multimodal nature of D/HH reasoning (ASL, facial expression, gesture, spatial referencing) (Derry et al., 2010). Transcripts were created using “word-for-sign” conventions, which prioritize preserving the student’s communicative form over a polished English translation (Bucholtz, 2000). Nonstandard signs, pauses, air-writing, and false starts were retained as analytic data to ensure that the “representational repair” processes identified in the framework remained visible.

Analytic Framework and Coding

Analysis was guided by the representational coordination framework, focusing on four processes: Representational Entry (accessing the task), Representational Transformation (converting information), Representational Coordination (maintaining mathematical relationships), and Representational Externalization (making reasoning visible), grounded in the work of Duval (2006) and Ainsworth (2006).

Coding proceeded via a hybrid deductive-inductive approach (Saldaña, 2021). Deductive codes were derived from the four framework processes, while inductive codes emerged from recurring patterns in the students’ work (e.g., “representational repair” moments). All coding was verified through a peer debriefing with the secondary researcher. This framework directs attention to how students enter tasks, repair representational breakdowns, and externalize reasoning, avoiding the “modality-comparison” trap by treating errors as visible sites of coordination. Table 1 outlines the coding categories derived from this approach.

Table 1.
Analytic Coding Categories for Representational Activity

Analytic Category	Definition	Examples of Evidence
Representational entry	How the student initially accesses the task	Sign-reading the written prompt; orienting to the table; receiving the prompt through ASL
Sign-reading and linguistic transformation	Movement from written English or symbols into ASL	Signing mathematical notation; fingerspelling words; using a sign that approximates a mathematical term
Symbolic representation	Use or transformation of algebraic notation	Writing expressions, equations, variables, constants, or operations
Visual/table representation	Interpretation or use of diagrams, tables, labels, or layout	Referring to table values; using rows or columns; showing the paper to the camera
Embodied representation	Bodily action used to represent mathematical meaning	Finger counting; air-writing; gesturing cancellation; tracing division
Coordination of quantities	How the student relates numbers, variables, and units	Linking 100 texts to \$25; identifying change from 100 to 200 texts
Coordination of operations	How the student represents operations and inverse operations	Distribution; subtraction; division; changing signs; moving terms
Coordination of equality	How the student preserves or disrupts equation balance	Adding or subtracting on one side; isolating x ; transforming both sides
Metacognitive regulation	Student monitoring, revising, or evaluating their reasoning	"My bad"; "I wrong"; "What next"; restarting
Externalization profile	Degree and form of visible reasoning	Extended signed explanation; long silent work followed by brief signed summary

Analysis proceeded in six phases. First, the researchers repeatedly viewed the videos and read the transcripts to become familiar with each problem-solving episode. Second, each task was mapped for its representational demands. Third, transcripts were segmented into representational events, defined as moments when students entered, transformed, coordinated, or externalized mathematical meaning through ASL, writing, gestures, actions, symbolic notation, or visual references. Fourth, events were coded through a hybrid deductive-inductive approach. Fifth, within-case analyses examined each student's

representational profile across tasks. Sixth, cross-case and cross-task analyses compared how different representational demands shaped students' reasoning pathways." This coding structure allowed the analysis to capture both mathematical content and communicative form without reducing the data to correctness.

Trustworthiness, Reflexivity, and Ethics

To ensure the study's rigor, we employed multiple strategies of trustworthiness (Lincoln & Guba, 1985): *Credibility*, achieved through prolonged engagement with the video data and multiple viewings of transcript events; *Confirmability*, addressed through reflexive memoing, which documented the basis for interpretive decisions regarding ambiguous signs or gestures (Tracy, 2010); and *Transferability*, supported by "thick description" of the participants, tasks, and analytic pathways, allowing readers to judge the applicability of findings to their own contexts.

Ethical considerations were managed through informed consent procedures, student assent, and the use of restricted, encrypted storage for video data. The study emphasized voluntary participation, ensuring students' responses reflected their own natural problem-solving processes rather than instructional compliance.

Several methodological boundaries shaped the study. The sample consisted of two students, so findings are exploratory and illustrative rather than representative. The three tasks differed in both mathematical structure and presentation format, so the study cannot make causal claims about modality effects. Word-for-sign transcripts preserve important features of ASL communication but cannot fully capture ASL's visual-spatial richness without reference to video. Long periods of silent work, especially for Student 2, also limit what can be inferred from signed explanations alone.

Researcher Positionality and Reflexivity

As this study relies on the interpretation of complex ASL mathematical discourse and embodied actions, researcher positionality is foundational to the study's credibility. The research was conducted through a collaborative partnership between the teacher-researcher and a second researcher.

The teacher-researcher is a mathematics teacher at the South Carolina School for the Deaf. Although no longer the participants' instructor at the time of data collection, this insider status provided critical rapport and a deep familiarity with the school's unique linguistic environment. This proximity allowed for a nuanced interpretation of nonstandard ASL signs, classifiers, and classroom-based gestural interactions that an external researcher might overlook. To minimize potential coercion or dual-role conflicts, the study was structured so that no instructional authority was exercised over the participants during the data collection period.

The second researcher served as a critical collaborator, providing the necessary external distance to ensure confirmability. By participating in peer-debriefing and consensus-building, the research team mitigated the risk of teacher-researcher bias. All interpretive decisions, particularly the glossing of ASL signs, the categorization of ambiguous gestures, and the identification of pauses as evidence of planning, were subjected to collaborative review. This study maintains reflexivity through consistent memoing, which documents the rationales for interpretive choices. By triangulating these perspectives, the researchers sought to balance the teacher-researcher's "insider" fluency with the second researcher's

analytical perspective, ensuring that findings remained grounded in the data rather than personal assumption.

Findings

Posed Problems and Transcripts

In the transcripts, comments within parentheses are explanatory interpretations of the students' signs. Comments within brackets denote a student's actions or pauses during signing. Ellipses indicate pauses. "." indicates a brief pause, and "..." or "...." indicate a longer pause. The images of student-written work are included to help readers contextualize and verify the accompanying word-for-sign transcripts. They are not analyzed as independent written products; analytic claims are based on the transcript record of students' signed explanations, visible actions, pauses, gestures, and transcript-documented references to writing.

Problem 1. This problem was provided in written form only with no additional teacher input.

A student solved the equation $3(2x - 5) + 4 = 2x + 13$ and got $x = 6$. Do you agree or disagree with their answer? Show all your steps and explain your thinking clearly.

Student 1:

[She sign-reads the math prompt.] Uhh, student solve problem equal. *What*. ["What" in italics is a transition word.] 3 topic (signs "topic" but means "parenthesis"). 2 x negative (rather than "minus") 5 topic (signs "topic" but means "parenthesis")... add 4 equal 2 x add 13 and got uh x equal 6. D O (each letter signed) I agree or not agree with their (signed as though there are many students) answer? Show A L L (each letter signed) my steps (signed like walking step not mathematical step) and explain my thinking clear.

So first I put-down 3 topic (signs "topic" but means "parenthesis") (...) 3 (x 2) negative (means "minus") (rather than "minus") 5 mean put first 3 times 5 distribute then 3 times uh 5.. No (shown by shaking her head) 3 times 2 then 3 times 5.. ok.... [thinks silently for 25 seconds]. So I put-down 6 x because 3 times 2 x means 3 times 2 is 6 add x. Then I put-down 3 times 5 is 3 15. O K. Now I put now put I put-down 6 x negative (means "minus") 15 add 4 equal 2 x add 13. What next? (signed like she is asking an audience) *What*.. [pauses to think]. Now I put-down... [pauses to think for 7 seconds] negative (means "minus") (hesitantly signs). No.. (thinking again) now I put-down 6 x subtract 2 x because 5 2 off x. So I put-down [writes without signing for 5 seconds] 6 subtract 2 is 4, [pauses to write for 3 seconds] now 4 x [pauses to think for about 6 seconds]. No my bad (signs "bad") my-bad (gestures the sign that means my bad). [Writes without signing for 6 seconds] My bad. I put-down 15 add 4. *What*.. [thinks for 6 seconds] 19 [writes without signing for 8 seconds]. Because add [writes for 6 seconds] now [pauses to think for 3 seconds] now I put-down... [pauses to think for 10 seconds] 13 19 subtract 3 is.... [pauses to think and write for 26 seconds]. Now I put-down now I put-down 19 subtract 13 is, *what*, 6. Then I put-down 6 x add 2 x equal 6. Then I put-down 6 add.. x add 2 x is, *what*, 8 x because add then 8x... [pauses to think for 7 seconds]. My no wrong [thinks for a couple of seconds] 6 9... (signs to herself like thinking aloud) [thinks for 12 seconds]. 1 (signs the number 1, which can mean "hold on" or "one second"). I wrong.... [pauses to write and think for 20 seconds]. I put-down 3 topic (the student signed "topic" when she should have signed "parenthesis") 2 x negative (means "minus") 5 then add 4 equal 2 x (signs "thinking") add 13. Now, *what*, I put-down topic (signs "topic" but means "parenthesis") always first 3 times 2 is 6 x because 3 2 x then 3 times 5 is 15... [pauses to write for 26 seconds]. Now I just put-down 6 x negative (means "minus") 15 plus 15 equal 2 x add 13. Now, what next? (asking herself) *What*. Now I thinking that I think I should put-down 13.. [thinking for 3 seconds]. 15 add 14 add... No my bad no. 15 add 4 add 13. *What* [pauses to think and write for 14 seconds]. Now I put-down 32. *What*. Now I should put-down... [pauses to write for 15 seconds]. Now 6 x negative (means "minus") 32 equal 2 x. Now I put-down 36 divide 2 is 3 [pauses to write for 6 seconds]. Now I put-down 63 x equal negative 32. *What*. Divide [pauses to write and

think for 27 seconds] 5 (signed like thinking aloud) times (signed like thinking aloud) no 3 times 5 is 15; then 3 times 6 is 18. Now.... (pauses to write and think for 26 seconds). I not think that is right answer because 3 negative 32 divide 3 can't nothing because decimal can't. So I start again.... (shakes her head disappointedly) [pauses to write for 23 seconds]. Now I just put-down 6 no 3 topic (signs "topic" but means "parenthesis") 2 x negative (means "minus") 5 close-parenthesis positi.. add 4 equal 2 x neg.. add 13. Now, *what*, I put-down 6.. 2.. 3 topic (signs "topic" but means "parenthesis") 3 2 x negative (means "minus") 6... 5 means. *What*. Add... times so 3 times 2 x means 6 x. 6 x. Now 3 times 5 is 15. Now, *what*, next [pauses to think for two seconds] 5. I put-down 6 divide negative 6 x negative (means "minus") 2 x. *What*... (pauses to write for 12 seconds). Now, *what*, 15 add 4 add 13, *what*, (pauses to write for 2 seconds). 15 add 4 add 3 13 is, *what*, 32 [pauses to write for 5 seconds]. Now, *what*, divide, *what*, 32 divide 4 is 7 [pauses to write for 3 seconds]. 3 x equal 7.

So I don't agree because they created answer they create answer number problem. But they create answer got wrong answer. See people problem. But my wisdom... opinion that they should like put-down always first topic (signs "topic" but means "parenthesis") always if not topic (signs "topic" but means "parenthesis") mess-up up. So first always topic (signs "topic" but means "parenthesis"), then should do 3 times 3 times 5 15. 3 times 2 is 6. 3 times no, *what*, answer 60. So then after that no topic (signs "topic" but means "parenthesis") nothing; then put-down 6 x negative (means "minus") 3 because should right hope. But after that put-down, *what*, says 15 plus put-down add add add add. Then 32 divide 4 put-down divide (air draws the division house thing for long division). Easy. So thank-you for watch bye.

Sta 1

Open-Ended Math Question
A student solved the equation $3(2x - 5) + 4 = 2x + 13$ and got $x = 6$.
Do you agree or disagree with their answer? Show all your steps and explain your thinking clearly.

$$3(2x - 5) + 4 = 2x + 13$$

$$6x - 15 + 4 = 2x + 13$$

$$6x - 11 = 2x + 13$$

$$6x + 2x = 6$$

$$4x = 6$$

$$3(2x - 5) + 4 = 2x + 13$$

$$6x - 15 + 4 = 2x + 13$$

$$4x = 32$$

$$x = 7$$

$$3(2x - 5) + 4 = 2x + 13$$

$$6x - 15 + 4 = 2x + 13$$

$$6x - 32 = 2x$$

$$3x = -32$$

$$\begin{array}{r} 15 \\ 13 \\ 4 \\ \hline 32 \end{array}$$

Figure 1. Student 1's written work on Problem 1.

Student 2:

[He sign-reads the math prompt.] A student solved equal (signed equal, but meant equation) 3 parenthesis 2 x negative (means "minus") 5 parenthesis, *uhhh* (twiddles fingers while searching for a

sign or what to say next. In ASL that is typically used to represent “uhhh” or “umm” like in English) add 4 equals 2 x add 13 got x equals 6. I look.... [writes and thinks for 3 minutes and 14 seconds]. Ok, I not sure right answer (uninterpretably tries to spell something but does not form correct hand shapes). I did 3 open-parenthesis 2 x negative (means “minus”) 5 close-parenthesis add 2 equal 2 x add 13. So I do-do (means do), *uhhh*. I add 2 x negative (means “minus”) 5 because parenthesis means add; SO I add but negative (uninterpretably tries to sign something but does not use the proper handshape or movement) subtraction so equal 3 3 x times 3 if 3-3 (puts a 3 with right and and 3 with left hand together meaning they interact with each other in some way) means 9 x. Then other equal says 2 x add 13 add 15 x x 15 x, *uhhh*. SO do-do answer, uhh, 15 x. Group now, *uhhh*, positive 4 change negative 4 subtract 15 on the left makes 11 9 times 11 times 9 equal x equal 99.

Stu 2

Open-Ended Math Question
A student solved the equation $3(2x - 5) + 4 = 2x + 13$ and got $x = 6$.
Do you agree or disagree with their answer? Show all your steps and explain your thinking clearly.

$$3(2x - 5) + 4 = 2x + 13$$

$$3 \cdot 2 x + 4 = 2x + 13$$

$$9x + 4 = 15 + 5x$$

$$\begin{array}{r} -4 \quad -4 \\ \hline x = 1 \end{array}$$

$$x = 9$$

Figure 2. Student 2’s written work on Problem 1.

Problem 2. This problem was provided in the form of Figure 3 with no additional teacher input.

A phone plan charges a monthly fee plus a cost per text message. One month, the total cost was \$25 for 100 texts. Another month, the total cost was \$40 for 200 texts. Write and solve a multi-step equation to determine the monthly fee and cost per text. Explain how you set up your equation.				
Month	Monthly Fee (<i>m</i>)	Cost per Text (<i>p</i>)	Number of Texts (<i>t</i>)	Total Cost (<i>C</i>)
Month 1	<i>m</i>	<i>p</i>	100	\$25
Month 2	<i>m</i>	<i>p</i>	200	\$40

Figure 3. Problem 2.

Student 1:

[She sign-reads the math prompt.] Picture math, *what* [shows the paper with the image for a few seconds to the camera]. Says A phone P plan cost a monthly FEE positive A cost before (correct use of “before”) per together message 1 month all-together cost, *what*, 25 for 100 texts another month all-together cost, *what*, 40 for 200 texts. Write and solve multiplication step (like “walking”) equal to DETERMINE THE monthly FEE and cost PER text. Explain how I set-up my equal [shows paper again]. SO this... [thinks for 15 seconds] paper is hard. OK... [thinks and writes for a minute] [shows her work to the camera], *what*.. [writes for 5 seconds] [shows work to the camera]. Now I try DO 200 subtract 100.. no 200 plus 100 is, *what*, 300.. [writes for 5 seconds] 300 texts. Now, *what*, how how 25 to 40 how. So I put-down 20 25 (self-corrected 20 to 25) plus 25. Hopefully works... [pauses for 9 seconds to write and think]. 25 plus 25 is 50. So that not answer, but means less-than... [writes and thinks for 10 seconds]. Maybe try [pauses for 5 seconds] 25 plus 20 is 45 no no 25 plus... [thinks for 7 seconds] thinking. I try 200 divide 40 see what happens.... [writes and thinks for 20 seconds] means 5 no no means 25; add 5 is [counts on her hands like thinking out loud] 30 answer. So I need add like (thinks while counting loosely on her hands and writes in the air with her finger instead of using ASL).... [thinks and writes for 27 seconds]. Write and solve a multiply step (like “walking”) equal.. [thinks for 3 seconds]. I try *m* for month 1 is 100 less-than 100/200 (signs it as a fraction)... [thinks and writes for 12 seconds]. Try... [thinks for 6 seconds]. Divide means.... [writes and thinks for 42 seconds]. Shhh (signs to other student in the room).... [thinks and writes for 36 seconds]. Understand (means “I understand”) means now see (signed like “we will see”).... [thinks and writes for 19 seconds]. Now now I just put-down... [thinks and writes for 10 seconds]. 300 *t* is texts then 25 plus 15 is, *what*, 40; so that means [thinks and writes for 5 seconds] so subtract 25 is 15 [thinks and writes for 23 seconds] that means I need like 200 minus 100 means 100 so girl now next month 3 is, *what*, 300 [thinks for 4 seconds] no 3-months, *what*, [thinks and writes for 30 seconds] “divide 300 divide do-do thinking” (signed like thinking out loud)... [thinks for 9 seconds]. That means 3-months off PER 300 texts all-together, *what*... [thinks and writes for 14 seconds] all-together, *what*, 55. So that means.... [thinks and writes for 21 seconds] [shows work to the camera]. 3-months per 300 texts off 55 dollars. I hope I got right. Thank-you. Bye. I-Love-you.

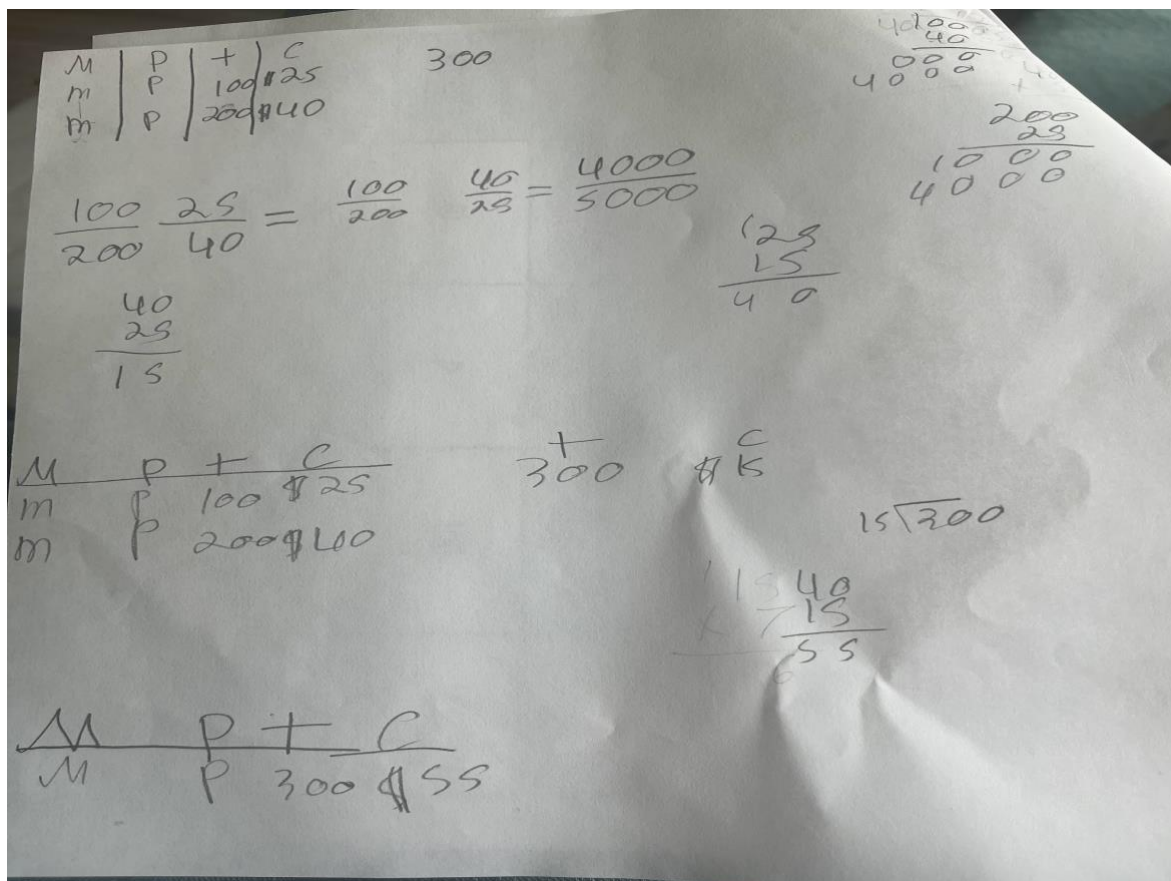


Figure 4. Student 1's written work on Problem 2.

Student 2:

[He sign-reads the math prompt.] Now I in picture math. OK a phone plan (tries to sign something; makes an x and then signs something like "insult", possibly trying to sign "cost" as he is sign-reading the problem). Month FEE plus a (gestures "each") text month T full T E before (like, "in the past"). 25 dollar for 100 TX (means "text", but did not complete the proper handshape for the full word). Another month T full the same T E before (signs a timeline, meaning "before"). 40 dollar for 200 T texts.... [begins thinking for 4 MINUTES AND 22 SECONDS]. OK. SO. OK answer, *uhhh*, how-much cost each month. 25 for 100 texts and 40 for 200 texts. Divide; get 0.50 for EACH text 1 TEXT add add add. Then get 100 texts for month. Get month FEE 25 dollar each everyday 1 1 E 100 text (flips hands in a gesture that means here you go (sort of)). Ok (thumbs-up).

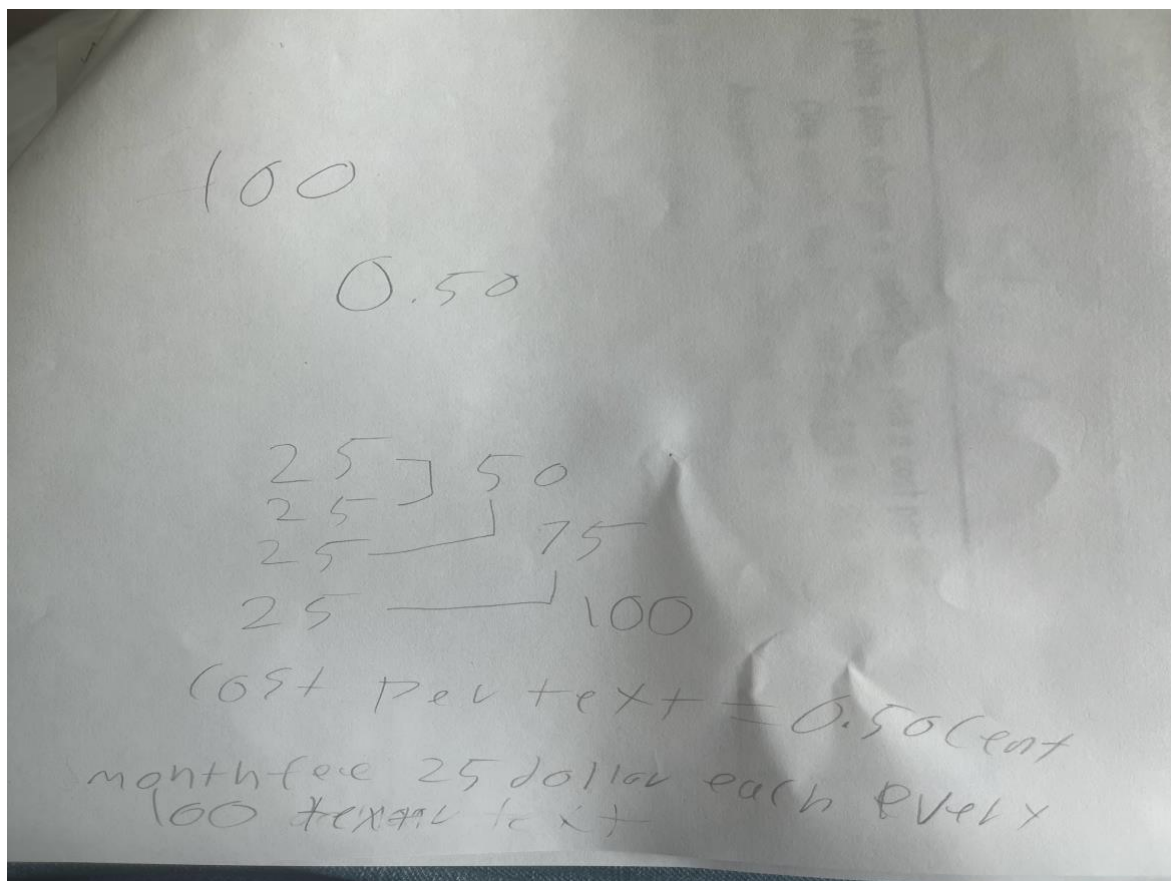


Figure 5. Student 2's written work on Problem 2.

Problem 3. The teacher signed this problem with no additional written form.

Instructor signs: Create your own multi-step equation that has a solution of $x = -3$. Then solve your equation step by step and explain how you know your answer is correct.

Student 1:

I need make-up my own word with x equal negative 3 how thinking [thinks for a few seconds]. I try use... [pauses to think for 15 seconds] thinking how hard [thinks for a few seconds] so hard. Need equal negative 3 how I have-to put-down equal 100-80 (signs as 100 (1C) and 80, but means 180) add subtract so [pauses to think for 10 seconds]. I need create that have like 8.. like 8 negative 8 divide (doesn't finish the sign for divide). No no negative 12 divide 4 is negative 3. That negative need so I put-down.. [thinks and writes for 7 seconds]. Think I need put-down 100-80; so I just put-down something [thinks for 6 seconds], 12 maybe [writes for 5 seconds] negative 12 $\times$... [thinks for 16 seconds] add... 6 negative 12 $\times$ positive (means "add" 6 equals, *what*, 10, then add 6 add 6 (shown as writing it under while solving an equation on paper); don't need *slash* (signs to indicate that the 6's would cancel out (italicized since it isn't an actual sign but is more of a classifier)). Put-down 10 minus 6 because positive 6 positive 6 (the second 6 signed below the other). Don't need so subtract 6 [writes for 6 seconds]; 10 subtract 6 is 9 8 7 6 5 (counts down on her hands like thinking aloud) 4. Now, *what*, negative 12 $\times$ equals 4 put-down [writes for 5 seconds]. Now, *what*, negative 12 divide 4 is, *what*, negative 3, but put-down x equal negative 3 always. Thank you, bye.

Handwritten work on a piece of paper:

$$\begin{array}{l}
 x = -3 \\
 -12x + 18 = 10 \\
 \quad \quad \quad +6 \quad \quad \quad = 6 \\
 -12x = 4 \\
 x = -3
 \end{array}$$

STU 1

Figure 6. Student 1's written work on Problem 3.

Student 2:

OK... [works without signing for 5 minutes and 35 seconds]. OK, I (signs "try" without proper handshape) x 18 (puts the x before the 18) subtract 30 equal negative 36. So I change subtract 30 negative 30 change positive then [writes for 2 seconds] change positive 30 negative 36 (held up like you would be able to write under it). Subtract positive 30 equal negative 6 x 8 (means 8xt) (moves his hands like he is trying to sign but does not form words. Signs something that could be "just" or simply scratching his ear. If it is the sign "just" it is not with the proper handshape). Multiply, *what*, opposite means divide 8 (signs 8, but presumably means 18). Negative 6 equal, *what*, negative 3 means; answer is x equal negative 3. Ok (thumbs up).

$$x18 - 30 = -36$$

$$+ 30 \quad + 30$$

$$18 \div -6 = -3$$

$$x = -3$$

$$x = -3$$

STU2

Figure 7. Student 2's written work on Problem 3.

Analysis of Findings

The analysis focuses on how students interpreted, transformed, coordinated, and revised mathematical representations across written English, ASL, algebraic notation, visual/table displays, gestures, finger-counting, spatial referencing, and written work. Because the three tasks differed in both presentation format and mathematical demand, the findings describe how students worked with, moved among, and sometimes struggled to coordinate these various representations.

Students converted the initial task representation into other working representations. Across the three tasks, students did not simply receive a problem in one representational form and solve it within that same form. Instead, they performed Representational Transformation, converting information into other working forms as they attempted to make sense of the problem.

Problem 1 was presented in written English only, but Student 1 immediately transformed the written prompt into a signed representation. She sign-read the equation and accompanying instructions, signing “topic” for what the transcript interprets as “parenthesis,” using “negative” where the written equation used subtraction, fingerspelling parts of the written directions, and signing “steps” in a way the transcript notes as resembling walking steps rather than mathematical steps. Her first encounter with the problem was therefore not simply silent reading; it was an active conversion of written English and algebraic notation into ASL-based and embodied forms.

As Student 1 continued working on Problem 1, she moved between the written equation, signed operations, and written algebraic inscriptions. For example, she signed through the distributive structure of $(3(2x - 5))$, wrote intermediate expressions, paused to think, and returned repeatedly to the original equation. Her written work and signed explanation were not separate products; they functioned as connected representations she used to maintain the relationship between the original equation and her transformations of it.

Student 2 also performed these transformations, though in a more compressed fashion. In Problem 1, he sign-read the equation, then wrote and thought silently for over three minutes, a period of intense Representational Externalization via written work, before producing a signed explanation. His primary working representation appeared to be written inscription, with ASL used primarily to report his process retrospectively.

Problem 3 showed conversion in the opposite direction. The task was presented only in instructor-signed ASL, yet both students moved into written algebraic notation to construct their equations. Student 1 signed using numerical choices, wrote an equation, gestured to cancellation, and counted on her fingers, demonstrating how the ASL prompt served as an entry point that necessitated a shift to written symbolic work to maintain the mathematical structure of the solution ($x = -3$).

Student 1's problem solving was characterized by repeated representational repair. Student 1's work was marked by repeated attempts to repair mismatches among representations. She often began with one written or signed transformation, evaluated it, marked it as uncertain, and then revised the representation. This pattern was most visible in Problem 1, where she repeatedly moved between the original equation, her written algebraic transformations, and signed self-monitoring.

After distributing and beginning to combine terms, Student 1 paused and signed "What next?" She then attempted to move terms, wrote without signing, paused again, and marked a mistake with "No my bad" and "My bad." Later, after another transformation, she signed "My no wrong," paused, then signed "I wrong," returned to the original equation, and began reconstructing the distributive step. These moments show representational repair: she noticed that her current written or signed form did not adequately preserve the relationship she was trying to represent, so she attempted to rebuild the representation from an earlier point.

A particularly clear repair sequence occurred when Student 1 rejected one of her own intermediate results. After producing an expression involving division, she stated that she did not think the answer was right because the division result did not make sense to her, then signed "So I start again" and returned to the original equation. The finding here is not that the repair led to a correct solution. Rather, the transcript shows that Student 1 used the written equation, signed commentary, and her sense of numerical reasonableness to evaluate and revise her representations.

These moments indicate that the student was monitoring her own Representational Coordination. When the current symbolic representation (written or signed) failed to align with her sense of mathematical reasonableness (e.g., when a division result "can't" work), she marked the breakdown. She attempted to rebuild the representation from an earlier point. In Problem 2, she repeated this repair, applying several numerical transformations ($200-100$, $200+100$, $25+25$) before revising her approach. The finding here is that Student 1's errors were not evidence of deficiency but rather moments when her process of coordinating representations became visible and open to revision.

Student 2's representational work was concentrated in written inscription and became only partially visible in signed explanation. Student 2 showed a different

representational profile. Whereas Student 1 externalized her revisions publicly through signing, gesture, and writing, Student 2 frequently worked silently for extended periods, suggesting that his representational activity was internalized through written inscription.

In Problem 1, after a three-minute silent period, he offered a condensed signed explanation. Similarly, in Problem 2 and Problem 3, his explanations followed long periods of silent work. The transcript captures his eventual explanation, but the video record suggests a significant amount of Representational Transformation occurred in his written work, which remained largely private until the final reporting phase. This finding distinguishes between “representational activity” and “observable externalization.” Student 2’s written work and silent periods suggest ongoing mathematical processing, but the signed transcript provides limited access to his intermediate revisions, unlike Student 1’s more continuous externalization.

The annotated table task created the greatest difficulty in representational coordination. Problem 2 required students to coordinate several representations at once: a written contextual statement, an annotated table, variable labels, numerical quantities, and an implied algebraic relationship. The table displayed monthly fee (m), cost per text (p), number of texts (t), and total cost (C), with two cases: \$25 for 100 texts and \$40 for 200 texts. Although the task included visual support, both students’ responses suggest difficulty coordinating the table representation with a stable algebraic model.

Student 1 appeared to recognize the task as visually distinct, calling it “picture math” and showing the paper to the camera. After sign-reading the prompt, she signed that the paper was hard and began testing relationships among the visible numbers. Her work moved through several numerical transformations: $(200 - 100)$, $(200 + 100)$, 300 texts, $(25 + 25)$, $(25 + 20)$, and $(200/40)$. These transformations show active engagement with the quantities. Still, they do not demonstrate stable coordination between the table’s two rows as simultaneous representations of a shared monthly fee and a shared per-text cost.

Later in the same task, Student 1 continued to revise her representation of the situation. She counted on her hands, wrote in the air, considered 300 texts, connected $(25 + 15)$ to 40, and eventually described “3-months per 300 texts” with a total of \$55, ending with “I hope I got right.” Her response suggests that she was trying to construct a numerical pattern from the table. Still, the relationship among monthly fee, per-text cost, number of texts, and total cost remained unstable.

Student 2’s response also suggests difficulty coordinating the table and algebraic relationship. After a long period of silence, he explained the problem by division and concluded that the cost was \$0.50 per text, then identified \$25 as the monthly fee. His explanation used numbers from the table, but it did not show coordination of both rows as constraints on the same equation.

The finding is therefore not that visual representations were unhelpful. Rather, this particular visual/table representation required students to coordinate multiple representational elements simultaneously. The students had to connect natural-language information, table structure, variable labels, two numerical cases, and the algebraic relationship ($C = m + pt$). Both students used numbers from the table, but neither transcript shows a stable transition from the table representation to the corresponding algebraic representation.

Summary of Findings

Across the six problem-solving episodes, the central finding is that D/HH students’ mathematical work involved ongoing coordination, transformation, and externalization of

representations. Student 1 made her activity highly visible through signing, writing, gesturing, and public repair. Student 2 appeared to rely heavily on silent written inscription, making his representational process less visible until the final explanation. The annotated table task posed the greatest challenge because it demanded simultaneous coordination of multiple representational elements into a single model. These findings reinforce the framework: students do not simply receive information; they construct mathematical meaning by navigating a complex landscape of representational tools.

Discussion

This study investigated how D/HH eighth-grade students navigate mathematical meaning through representational coordination. The findings demonstrate that D/HH students' mathematical work is not simply a matter of choosing the "right" modality (ASL, text, or diagram) but a dynamic process of constructing meaning across available registers. The following sections discuss the theoretical implications of these findings, connecting student activity to broader theories of representation, and then present practical implications for instruction, assessment, and curriculum.

Beyond Modality: Representation as Constructive Activity

The findings challenge the historical reliance on "modality-comparison" designs in D/HH mathematics education. By moving away from asking which modality "performs better," this study reveals that D/HH students do not simply receive information in one format and respond within that same register. As demonstrated in Problems 1 and 3, students actively transformed written English to ASL, and ASL into written algebraic notation. This aligns with Ainsworth's (2006) assertion that translation among systems is a demanding, high-level cognitive task.

These transformations were not merely "accommodations" but essential components of the students' mathematical thinking. When Student 1 converted written text to ASL, she was not just reading; she was constructing an internal representation to manipulate the equation. Similarly, when Student 2 used silent written inscription to process the phone-plan table, he was using the page as a primary workspace for representational transformation. These observations support the argument that for D/HH learners, ASL and writing are not parallel, isolated tools, but interconnected components of a single, multimodal representational system (Krause, 2019, 2023).

The Instability and Productivity of Representational Repair

A striking pattern in the data was the visible "representational repair" (i.e., moments where Student 1 identified a breakdown and reconstructed her reasoning). These moments are consistent with Duval's (2006) distinction between *treatment* (manipulating an expression within a register) and *conversion* (moving between registers). Student 1's errors, such as her misuse of "topic" or difficulty with distribution, were not merely procedural failures; they were visible sites of semiotic negotiation.

This finding suggests that the traditional view of errors as purely deficit-based obscures the *reasoning in progress*. When Student 1 signed "I wrong" and returned to her original equation, she was utilizing her own meta-representational awareness to monitor the stability of her coordinate system. This aligns with research in mathematical metacognition, which emphasizes that effective learners can monitor their own symbolic work and troubleshoot breakdowns in real time (Goldin & Shteingold, 2001). For D/HH students, these self-

corrective moments are particularly valuable sites of learning, as they force the student to align their internal concepts with the external sign vehicles being used.

Distributed Cognition and the Visibility of Reasoning

The divergence between Student 1 (who externalized her work continuously) and Student 2 (who utilized long periods of silent inscription) illustrates that “representational activity” and “externalized reasoning” are not the same. Student 2’s silent work periods align with research by diSessa (2004) regarding internal representation: some students may perform extensive representational transformations privately before committing them to public explanation.

This difference has significant implications for how we interpret D/HH mathematical performance. Relying solely on signed explanation may lead to an underestimation of the cognitive work performed by students who favor silent, written-first approaches. Teachers must be attuned to the fact that for many students, written inscription is not just a place to *record* an answer, but the primary site for *constructing* the mathematical relationship (Lesh, Post, & Behr, 1987).

The Coordination Demands of Visual Displays

The difficulty both students encountered with the annotated table (Problem 2) reinforces that visual displays are not inherently transparent. While research suggests visual-spatial supports can be beneficial for D/HH students (Blatto-Vallee et al., 2007), our findings demonstrate that without explicit coordination, these displays can become a source of representational overload. The students struggled to coordinate the table’s rows and columns with the underlying algebraic relationship ($C = m + pt$). This supports Schnotz and Bannert’s (2003) finding that pictorial representations can interfere with mental model construction if the learner lacks a convention for interpreting how the visual components map to the mathematical structure.

ASL and Gesture Functioned as Mathematical Representations, Not Only Explanations

The findings support attention to ASL, gesture, and embodied action as representational forms. Student 1’s air-drawing of a long-division structure, gesture for cancellation, and counting down on her fingers in Problem 3 show that some mathematical relationships were represented physically rather than only through written notation.

This matters because representation theory does not have to be limited to conventional written inscriptions. Mathematical representations can include diagrams, tables, symbols, gestures, spoken or signed language, and physical actions when these forms are used to stand for mathematical objects or relationships (Goldin & Shteingold, 2001; Lesh et al., 1987). In this study, ASL and gesture sometimes represented operations, groupings, quantities, or transformations. They were not merely after-the-fact explanations of work completed elsewhere.

At the same time, the findings show that embodied or signed representations were not automatically precise. Student 2’s Problem 3 explanation included ambiguous notation and signing, including “x 18,” possible confusion between 8 and 18, and movements that did not form stable signs. This suggests that signed and embodied representations, like written representations, can be partial, unstable, or difficult to interpret. Their value lies not in being

inherently clearer than written notation, but in being part of the representational system through which students construct and communicate mathematical relationships.

Correctness Alone Underrepresents Students' Representational Activity

The findings also caution against evaluating students' mathematical thinking only through final answers. A correctness-centered analysis would miss much of the representational work in the transcripts. Student 1's solutions were often unstable, but her transcript revealed extensive representational activity: converting written text into signed form, writing algebraic expressions, revising transformations, checking reasonableness, gesturing operations, and restarting. Student 2's explanations were often brief, but his long silent work periods suggest that written inscription played a substantial role in his problem solving.

This does not mean that correctness is unimportant. Mathematical accuracy remains a central concern. However, in a study focused on representation, correctness is only one dimension of analysis. A mathematically incorrect response can still reveal how a student interpreted the original representation, what transformations they attempted, where coordination failed, and how they tried to repair the mismatch. Conversely, a concise final answer may provide limited evidence of the representational pathway that produced it.

This distinction is especially important for D/HH students because mathematical understanding may be distributed across written work, ASL, gesture, spatial representation, and embodied action. The students' final answers alone would not show how they moved among those representations or where their representational coordination became unstable.

Task Format and Representational Demand Were Intertwined

The findings also clarify why the study should avoid treating the three tasks as a clean comparison of modalities. Each task differed not only in presentation format but also in representational demand. Problem 1 required students to evaluate and transform a symbolic equation embedded in written English. Problem 2 required students to convert a contextual table into an algebraic relationship. Problem 3 required students to generate an equation from an ASL prompt and then verify that it had a specified solution. The study procedures identify the three formats as written English, annotated diagram, and ASL, but the tasks also differed substantially in what kinds of representational work they required.

This point strengthens rather than weakens the study. The findings show that students' reasoning was shaped by the interaction between presentation format and representational demand. The annotated table task did not simply test whether visual support helped; it tested whether students could coordinate a table, variables, quantities, and an implied algebraic model. The ASL task did not simply test whether signed presentation helped; it required students to construct a symbolic representation from a signed prompt. The written-English task did not simply test reading; it required students to interpret and transform an existing equation.

Thus, the more defensible conclusion is that students' mathematical work depended on the representational demands of each task's ecology. Presentation format mattered, but it mattered as one part of a broader representational system.

Contribution of the Study

Framed through representation theory, this study contributes to research on D/HH mathematics learning by showing how two students constructed algebraic meaning through

representational conversion, coordination, inscription, and repair. The manuscript's literature review identifies a need for research on how Deaf students move among ASL, written English, visual representations, diagrams, sketches, and embodied forms while solving mathematical problems. The present findings address that need by showing the representational pathways students used in real time.

The study's strongest contribution is not a claim that one modality is superior. Rather, it shows that D/HH students' mathematical reasoning can be understood as work across representations. Student 1's reasoning became visible through repeated representational repair: signing, writing, pausing, revising, gesturing, and restarting. Student 2's reasoning appeared more concentrated in silent written inscription and became visible primarily through compressed signed explanation. The annotated table task showed that visual representations can still impose substantial coordination demands when students must connect contextual, tabular, numerical, variable, and algebraic forms.

These interpretations provide the basis for the following Implications section, which can consider what this representational analysis suggests for task design, instruction, assessment, and future research with D/HH mathematics learners.

Implications

The findings from this exploratory study have implications for mathematics instruction, assessment, teacher preparation, curriculum design, and future research involving D/HH learners. Because the study involved two students and three tasks, the implications should not be read as generalizable claims about all D/HH students or as evidence that one presentation modality is uniformly superior to another. Rather, the implications are analytic: they identify instructional and research considerations that become visible when students' mathematical reasoning is examined as representational coordination across ASL, written English, algebraic notation, diagrams, gesture, embodied action, and written work.

Implications for Mathematics Instruction

The first implication is that mathematics instruction for D/HH students should explicitly teach students how to coordinate representations rather than assuming that students will naturally move among written language, symbols, diagrams, and ASL. In the present study, both students often identified relevant numbers and operations, but they had difficulty preserving relationships among variables, constants, operations, and equality across representations. For example, the written equation task required students to move from printed English and algebraic notation into signed explanation and written symbolic work. In contrast, the diagram task required them to interpret a table as a model of a fixed-fee and rate relationship. These are not merely language tasks or computation tasks; they are representational coordination tasks.

Instruction should therefore include deliberate modeling of how one representation maps onto another. Teachers might explicitly ask students to show where a quantity appears in the written prompt, in a table, in an equation, and how it can be explained in ASL. For algebraic word problems, this could involve routine classroom questions such as "What stays the same?" What changes? Which quantity is the rate? Which quantity is fixed? Where is the unknown represented? How does this table become an equation? How would we sign the relationship before writing it symbolically? Such instruction would help students attend not only to individual numbers but also to the mathematical structure connecting those numbers.

A second instructional implication is that visual supports should not be treated as automatically accessible. The annotated table in the second task appeared to provide visual organization. Still, it also required students to understand variables, columns, rows, repeated quantities, and the relationship between a fixed monthly fee and a cost per text. The students' difficulty with this task suggests that visual representations may reduce some demands while introducing others. Teachers should therefore provide explicit instruction in how to read diagrams, tables, graphs, and annotated visuals as mathematical representations. A "picture" in a mathematics task is not simply an aid; it is often a dense representational object that must be interpreted.

A third implication is that ASL should be used not only to translate English prompts but also to develop mathematical meaning. ASL can support spatial organization, comparison, sequencing, and relational reasoning. However, signed access alone does not guarantee mathematical understanding. The ASL-only task reduced the need to decode written English, but it introduced the demand of creating an equation with a specified solution. Instruction should therefore use ASL as a language of mathematical reasoning, not merely as an accommodation after mathematics has already been framed in English. Students should have opportunities to generate, compare, revise, and justify mathematical representations directly in ASL.

Implications for Mathematical Discourse

The findings also suggest that teachers should create classroom routines in which students are expected to make their reasoning visible across multiple forms. Student 1's work was highly visible because she signed, paused, revised, counted, gestured, wrote, restarted, and commented on her own uncertainty. Student 2, by contrast, often worked silently for long periods and then produced a brief signed explanation. Both patterns are meaningful, but they create different opportunities for teachers to understand student thinking.

For instruction, this means teachers should not rely only on final answers or brief explanations. They should invite students to externalize reasoning in multiple ways: through ASL explanations, written work, diagrams, pointing, gesture, and reflection on errors. Questions such as "Show me where you changed your thinking," "What did you notice was wrong?" or "Can you explain that part in ASL and then write the equation?" may help students connect internal reasoning to public mathematical discourse.

The findings also support the value of metacognitive talk in ASL. Student 1's repeated self-corrections, including moments such as "my bad," "I wrong," and "What next," were not distractions from mathematics. They were evidence of monitoring, revision, and persistence. Mathematics classrooms should normalize this kind of visible uncertainty. For D/HH students, signed self-explanation and peer explanation may provide important opportunities to rehearse mathematical language, test relationships, and revise reasoning before finalizing an answer.

Implications for Assessment

The findings raise important concerns about how D/HH students' mathematical knowledge is assessed. If assessment relies primarily on final written answers, it may miss substantial evidence of mathematical reasoning. In this study, students' transcripts showed partial understandings, representational attempts, self-monitoring, and structural noticing even when final answers were incorrect. Conversely, a student who gives a brief final answer

may have reasoning that remains hidden unless written work, timing, gesture, and video evidence are also considered.

Assessment practices should therefore allow D/HH students to demonstrate mathematical understanding through multiple representational forms. This may include ASL explanations, written equations, diagrams, tables, gestures, and teacher-student conferences. Rubrics should distinguish among different dimensions of performance: comprehension of the problem, identification of relevant quantities, coordination of representations, use of mathematical structure, procedural accuracy, and clarity of explanation. Such rubrics would allow teachers and researchers to see whether a student's difficulty lies in reading the prompt, interpreting a diagram, translating into symbolic notation, maintaining equation balance, or carrying out arithmetic.

This implication is especially important for word problems. A D/HH student may struggle with English relational language while still demonstrating mathematical insight in ASL or through visual-spatial reasoning. Conversely, a student may sign-read a problem accurately but fail to preserve the mathematical relationships embedded in the prompt. Assessments should therefore be designed to separate, as much as possible, mathematical reasoning from unnecessary language barriers while still recognizing that language and mathematics are deeply connected.

Implications for Curriculum and Materials Design

Curriculum materials for D/HH students should be designed with representational coordination in mind. Problems should not simply be translated into ASL or supplemented with pictures. Instead, materials should help students see how mathematical relationships are represented across forms.

For example, a curriculum unit on linear relationships might present the same situation through a signed narrative, a written word problem, a table, a graph, an equation, and a diagram. Students could be asked to explain what each representation shows, what each representation hides, and how the representations connect. In the phone-plan problem, students would need explicit support to understand that the monthly fee remains constant, the cost per text is a rate, the number of texts changes, and the total cost depends on both the fixed fee and the variable text cost.

Materials should also include ASL-accessible mathematical vocabulary and conceptually precise signed explanations. This does not mean enforcing rigid one-to-one translations between English terms and signs. Rather, it means supporting shared classroom conventions for representing mathematical ideas such as parentheses, subtraction, negative values, equality, rate, variable, coefficient, and solution. The transcripts suggest that imprecision around terms such as "parenthesis," "negative," "minus," and "steps" can influence how students enter a problem. Curriculum materials should therefore support both mathematical vocabulary and the conceptual relationships those terms represent.

Implications for Teacher Preparation and Professional Development

Teachers of D/HH students need preparation that integrates mathematics content knowledge, ASL proficiency, and knowledge of mathematical representations. It is not enough for teachers to know mathematics or to know ASL separately. They need to understand how mathematical concepts can be expressed, transformed, and sometimes distorted as they move among English, ASL, symbolic notation, diagrams, and embodied action.

Professional development should help teachers analyze student work in multimodal ways. Teachers should learn to attend to signs, gestures, pauses, pointing, eye gaze, written marks, use of diagrams, and self-corrections as evidence of mathematical thinking. They should also learn to identify where representational coordination breaks down. For instance, when a student incorrectly solves an equation, the teacher might ask whether the difficulty arose from misunderstanding the written prompt, misrepresenting an operation in ASL, losing the balance of the equation, misusing inverse operations, or failing to connect a table to an equation.

There are also implications for collaboration among mathematics teachers, teachers of D/HH, interpreters, and ASL specialists. In settings where interpreters are involved, they need access to the lesson's mathematical goals, not only the English words being used. Mathematical interpretation requires preserving relationships among quantities, operations, comparisons, and symbolic structures. Collaboration can help establish consistent ways of representing mathematical concepts in ASL while remaining responsive to students' own signs, gestures, and reasoning strategies.

Implications for Research

For researchers, this study suggests the need for fine-grained qualitative analyses of D/HH students' mathematical reasoning across representations. Larger studies are needed, but scale should not come at the expense of detail. The richness of the present data lies in the word-for-sign transcripts, pauses, gestures, writing, and embodied actions. Future research should continue to use video-based methods that preserve the visual-spatial and embodied features of ASL mathematical discourse.

Future studies should also be designed to distinguish presentation modality from task demand. In the present study, the three problems differed not only in format but also in mathematical structure: evaluating a proposed solution, modeling a rate relationship, and generating an equation from a target solution. This is not a flaw, but it does limit causal claims about modality. Future research could use mathematically parallel tasks presented in different formats to examine how modality itself influences reasoning. Other studies could intentionally vary task structure while holding modality constant to better understand how D/HH students coordinate representations in different mathematical domains.

Researchers should also examine differences in students' externalization of reasoning. Student 1's reasoning was highly visible, while Student 2's reasoning was more compressed and often preceded by long silent work periods. Future studies should combine ASL transcripts, written work, video analysis, and stimulated recall interviews when possible. This would help researchers understand what students were thinking during silent work and how they decide what to make visible in a signed explanation.

Finally, future research should include attention to students' language histories, ASL proficiency, mathematical vocabulary, reading proficiency, and prior instructional experiences. These factors may influence how students interpret written prompts, use ASL to explain mathematics, read diagrams, and coordinate symbolic notation. However, such variables should be used to contextualize students' reasoning, not to frame D/HH learners through the lens of deficit assumptions.

Implications for Equity

The broader equity implication is that access in mathematics is not achieved simply by providing the same problem in a different format. Access requires attention to how students

make meaning with and across representations. A printed problem, a signed prompt, and a diagram are not equivalent containers for identical content. Each representation shapes what students notice, how they organize relationships, and where they may encounter difficulty.

For D/HH students, equitable mathematics instruction must therefore value ASL, visual-spatial reasoning, embodied action, and written mathematical notation as interconnected resources. Students should not be expected to demonstrate mathematical competence only through English-dominant forms of communication. At the same time, they need explicit support to navigate English-based mathematical texts, symbolic conventions, tables, diagrams, and formal notation, as these remain central to schooling and assessment.

The central implication of this study is that D/HH students' mathematical thinking becomes most visible when researchers and teachers look beyond correctness and attend to the representational pathways students use. Errors, pauses, gestures, revisions, and nonstandard signs can reveal how students are trying to coordinate mathematical meaning. An equitable approach to D/HH mathematics education should therefore ask not only whether students arrived at the correct answer, but how they represented the problem, what relationships they noticed, where coordination broke down, and what instructional supports could help them connect their reasoning across ASL, written English, visual representations, and algebraic notation.

Conclusion

This study investigated the real-time cognitive and communicative work of D/HH students as they navigated algebraic word problems. By shifting the analytic focus from "modality-comparison" to "representational coordination," we aimed to uncover the dynamic ways in which students construct mathematical meaning. The findings suggest that mathematical understanding for D/HH students is not a passive reception of information presented in a specific format, be it printed English, an annotated diagram, or signed ASL, but a constructive, active process of transformation.

The analysis of the three problem-solving tasks revealed three critical insights that reshape our understanding of D/HH mathematical competence. First, representational coordination is a turbulent, ongoing process; the "representational repair" demonstrated by Student 1 illustrates that identifying an error and reconstructing a relationship is a sophisticated site of mathematical learning, not merely a failure of procedure. Second, there is a divergence between observable externalization and internalized cognitive activity. Student 2's extensive periods of silent written work demonstrate that the absence of visible ASL explanation does not equate to the absence of representational transformation; rather, it suggests that for some learners, written inscription serves as the primary workspace for mathematical construction. Third, visual displays and annotated tables are not inherently "transparent" scaffolds. They introduce complex coordination demands that require explicit instructional attention to support, rather than interfere with, algebraic reasoning.

Ultimately, this study contributes to a shift in the field's approach to D/HH mathematics education. We have argued that access is not achieved simply by translating English-based curricula into ASL or by appending visual support to text. Rather, equitable instruction must treat ASL, visual-spatial reasoning, embodied action, and formal mathematical notation as a single, interconnected representational system. When we look beyond final answers to examine the representational pathways students construct, including their hesitations, gestures, restarts, and revisions, we gain a more precise and humane understanding of their mathematical reasoning.

For the D/HH community, this implies that our pedagogical goal should not be to enforce a “best” modality, but to foster representational competence. By supporting students in navigating the interplay between their linguistic and mathematical registers, we honor their full communicative landscape and provide them with the tools necessary to engage with mathematics as a dynamic, constructive discipline. Future research, curriculum design, and assessment practices that center this representational work will not only improve D/HH achievement but will also reflect a commitment to a more equitable vision of mathematical access.

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