Mathematics Teachers Attending and Responding to Students' Thinking: Diverse Paths across Diverse Assignments

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Abstract

Professional development (PD) programs often evaluate their impact on teachers' learning by assessing teachers either individually or in groups. The goal of this paper is to illustrate the variety of paths teachers might follow as a result of working in groups within online PD settings. Data are drawn from a PD program for grades 5-9 mathematics teachers. Participants took a series of three online graduate level semester-long courses focused on mathematical content knowledge and student mathematical thinking. During the final course, teachers were asked to complete a series of four group interview assignments that involved attending to student thinking, and an individual final project that involved designing, implementing, and analyzing a learning activity that responded to and built on student thinking. We present a detailed analysis of the work from a group of four teachers, whose learning paths were particularly diverse. We also analyze the feedback provided by the PD facilitators. The group made great strides throughout the four interview assignments in their attention to student thinking. However, the teachers' individual postings on an online forum showed that each teacher shifted in a different direction. Likewise, their individual final projects were dissimilar and demonstrated different approaches to responding to student thinking in the context of their classrooms. This case study shows that teachers' group work may not necessarily be indicative of their individual learning at course completion. Our findings suggest the need to examine teachers' learning from multiple perspectives and by means of varied types of assignments.

KEYWORDS

Professional Development • Student Thinking • Responsive Teaching • Interviews • Noticing • Mathematics Education

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INTRODUCTION

Rationale

This study was conducted in an online professional development (PD) program for grades 5-9 mathematics teachers. One of the goals of the program was to help teachers enhance their abilities to attend and respond to student thinking. With this goal in mind, we engaged teachers —organized into groups of 3-4 colleagues from the same school district— in numerous activities; some of them were to be completed in groups, whereas others were to be completed individually. This paper illustrates a variety of paths teachers' learning may follow as a result of working in groups within PD settings. Our main claim is that in order to have a full picture of teacher growth in PD settings, one must examine multiple levels of learning, including the work teachers produce both in groups and individually. The ultimate goal of this paper, therefore, is to raise PD facilitators' and researchers' awareness of the importance of providing teachers with different kinds of activities and of evaluating teacher learning from multiple angles.

The role of attending and responding to student thinking

Many countries have undergone significant reforms of their educational systems over the past decades. Without a doubt, the importance of adopting student-centered teaching approaches has been one of the most commonly emphasized themes in reform materials around the world. In mathematics education, reform materials in the United States highlight the teacher's central role in promoting student-centered teaching in the classroom. For example, the National Council of Teachers of Mathematics (NCTM) suggests that teachers should orchestrate the teaching of mathematics by "posing questions and tasks that elicit, engage, and challenge each student's thinking" and by "asking students to clarify and justify their ideas orally and in writing" (NCTM, 1991, p. 35). State curriculum materials[1], as in many other countries, also ask teachers to delve into and make use of students' thinking in instruction.

The idea of orchestrating teaching based on student thinking is increasingly present among researchers across content areas (Borko, 2004; Jacobs, Lamb, & Philipp, 2010; Levin, Grant, & Hammer, 2012; Sherin, Jacobs, & Phillip, 2011). Researchers in teacher education and professional development recommend that teachers systematically *explore* students' understandings of the content at hand (i.e., What are my students thinking while solving specific problems?), then *interpret* these understandings (i.e., Why are they thinking like that?), and finally *respond to* students' understandings through instruction (i.e., How can I use these ideas in the most productive way?). The answers to these three basic questions can inform teachers' decision-making practices, including deciding upon follow-up questions to ask students, new problems to pose, and further pedagogical moves (Mason, 2010). A considerable and growing body of research literature shows that this way of teaching —called "responsive teaching" by some authors (e.g., Hammer, Goldberg, & Fargason, 2012)— leads to enhanced student achievement and hence constitutes a major avenue to improve education (Carpenter, Fennema, Peterson, Chiang, & Loef, 1989).

In mathematics education, some researchers have used the term "noticing" to refer to the skills involved in attending and responding to student thinking. Jacobs, Lamb, Philipp, and Schappelle (2011) conceptualized the notion of professional noticing as an expertise with three

[1] Whereas many states in the United States acknowledge the importance of student thinking at the educational policy level, other states have different agendas. For instance, California adopted a language arts program that encourages and sometimes forces teachers to read from a script in class (Joseph, 2006).

central skills, "attending to children's strategies, interpreting children's understandings, and deciding how to respond on the basis of children's understandings" (p. 99). Van Es (2011) proposed a similar three-part definition of noticing that includes "attending to noteworthy events, reasoning about such events, and making informed teaching decisions on the basis of the analysis of these observations" (p. 135). The overall goal of the PD activities described in this study was to help teachers develop these three skills. For brevity, here we refer to them simply as *attending* (which for us encompasses both exploration and interpretation) and *responding* to student thinking. The focus of the PD activities was to help teachers attend and respond to how students reason while solving specific mathematical problems, with special attention to the content of the mathematical ideas they express (verbally, graphically, using gestures, etc.).

There is compelling evidence that knowledge of students' thinking and learning has the potential to positively influence teachers' instructional practices (e.g., Fennema et al., 1996) and student outcomes (e.g. Hill, Rowan, & Ball, 2005). However, several studies in mathematics education show that before participating in PD programs, teachers' knowledge about these matters tends to be partial. For example, in the study conducted by Carpenter et al. (1989), teachers were able to identify some of the primary strategies often used by students to solve certain arithmetic problems, and also recognize some distinctions among these problems. However, teachers were rarely able to relate critical dimensions of the problems at hand (such as the problem's context or the specific numbers used) to students' solutions. In addition, teachers' knowledge of children's ideas did not play a critical role in planning instruction. In a similar vein, Santagata's (2011) study showed important differences in the ways teachers made sense of classroom events depending on their level of experience with analyzing student thinking. Teachers with limited experience tended to be hesitant in their claims, and their descriptions of events were rather superficial and inaccurate. In contrast, teachers with more experience used their knowledge of instructional strategies to focus their attention on important elements, making multiple interpretations of these events and formulating compelling hypotheses.

Many PD programs in different content areas have focused on attending and responding to student thinking. Examples in the field of mathematics education are the Cognitively Guided Instruction (CGI) program (Carpenter et al., 1989), the Purdue Problem-Centered Mathematics program (Cobb, Wood, Yackel, & McNeal, 1993), and the Learning to Notice in Video-Clubs project (van Es & Sherin, 2008). These and other programs have shown that teachers' knowledge of student mathematical thinking is not static but rather dynamic, and that it can be developed in multiple ways (e.g., analyzing videos, conducting interviews and examining student written work, and interacting with students in class).

Different levels of foci in looking at teachers' learning in professional development

Research focusing on how teachers learn to attend and respond to student thinking in PD settings has traditionally adopted a single analytical level to assess teacher learning, assessing it either through their individual or group work. Note that in this section we are not referring to how PD was conducted (e.g., whether teachers engaged in group and/or individual activities) but how teachers' learning was examined at course completion. Many studies have focused on the individual teacher as the main unit of analysis. Overall, these studies show that teachers' learning varies greatly: whereas some teachers achieve meaningful changes in the direction intended by PD designers and facilitators, others achieve only part of the goal or change in ways that were not what the PD designers had intended, or do not change at all (Fennema et al., 1996).

There are some aspects of teachers' knowledge and teaching practices that may change more easily than others. For instance, as teachers go through PD, their observations and analyses of students' thinking become less evaluative and more descriptive and interpretative, and the claims they make about student thinking are increasingly backed up by evidence (Star, Lynch, & Perova, 2011). We observed similar changes in the interview assignments submitted by the group of teachers featured in this paper. Interestingly, however, some teachers did not show individual evidence of these changes in their final projects. Regarding classroom practices, teachers generally find it easy to incorporate strategies that elicit students' thinking (e.g., asking open-ended questions such as "How did you solve that problem?"). However, they seem to struggle with how to follow up on students' ideas and with how to use these ideas to make instructional decisions (Franke & Kazemi, 2001). In fact, not all teachers seem to develop the ability to use children's ideas to inform their teaching. The development of this ability tends to be very slow (Cohen, 2004).

In contrast to the studies described above, there is another community of PD researchers for whom collective teacher learning is the main focus of attention (e.g., Cobb, 2006; Sherin et al., 2011). For these researchers, learning constitutes a social process that occurs as individuals participate in communities of practice. Learning is thus conceived to be most effective when it involves a community of learners (e.g., teachers) that works together towards a shared goal. Activities in which teachers work in collaboration with other fellow teachers are therefore considered *essential* in this framework. Researchers who adopt this perspective have tended to assess teachers' learning in groups —for example, by looking at shifts in the topics discussed by teachers during their meetings, at how the norms and work dynamics of groups of teachers evolve, or at how the noticing skills of groups of teachers change over time (Goldsmith & Seago, 2011; van Es, 2011; van Es & Sherin, 2008). Consistent with the view of learning as a social process, these researchers oftentimes do not examine teachers' individual knowledge or classroom practices.

In this paper, we claim that examining teachers' learning only individually or only in groups cannot provide us with a full picture of how teachers evolve in PD settings. The original intent of our study was to examine the ways in which teachers' attention and response to students' thinking shifted over time. Through this investigation, by exploring changes both in individual teachers and across teachers, we found that teachers' group work throughout the course did not necessarily correspond to what the teachers did on their own at course completion. While we would not expect teachers to perform identically, as any group task reflects a combination of individual expertise and is strongly mediated by aspects such as collaboration, leadership, and the nature of task structure, the shifts in the group work were not mirrored by shifts in the individuals. Thus, this paper illustrates a variety of paths teachers' learning may follow as a result of working in groups within PD settings. As will be seen, the sample case we analyze is particularly interesting because, while all teachers made progress throughout the course, none of their individual changes were similar to the change demonstrated by the group. This sample case was purposefully selected to illustrate these tensions between the group and individual levels.

Analytic Framework

The analytic categories we used to examine teachers' work are based on van Es's (2011) and Goldsmith and Seago's (2011) frameworks for learning to notice students' mathematical thinking. Goldsmith and Seago (2011) highlight the importance of shifting "away from

evaluating student work to identifying and interpreting it for evidence of students' mathematical reasoning" (p. 170). In characterizing how teachers discussed student thinking, we focused on whether teachers explicitly considered *why* a student might have had a specific idea or performed a specific action. *Evaluating*, van Es (2011) explains, includes comments that are laden with values and judgments, when teachers do not explain why a student gives a particular answer. For instance, the group we analyzed in this study wrote: "once questioned, students seemed to have a good grasp of the concept, but there was a lack in independent thinking." Since this was written without further explanation as to why students had "a good grasp of the concept" or why "there was a lack in independent thinking," and instead as a judgment of student performance, this was considered as an example of an evaluative comment. In contrast, *interpreting* involves using evidence to reason through teaching and learning issues. That is, teachers explain why a student may have given a specific answer using specific evidence to substantiate their claims. For example, in one assignment, the group we focus on wrote how the student "has a good understanding of the difference between the rate the water is moving and the rate at which the container is filling. She says, it will seem like the water is filling in slower, just because it's wider." Because this reflection included a transcript and additional analysis to substantiate the claim made this quote was considered as an example of *interpreting*. Finally, in the middle of the two extremes we considered *exploring* or *describing*, which is when teachers describe the overall story of an event of interest, "similar to following the plot of a story" (van Es, 2011, p. 135).

We determined whether teachers evaluated, explored/described, or interpreted student thinking by also examining the kinds of questions they asked. As Mason (2010) notes, "The key to effective questioning lies in rarely using norming and controlling questions, in using focusing questions sparingly and reflectively, and using genuine enquiry-questions as much as possible. This means being genuinely interested in the answers you receive as insight into learners' thinking, and it means choosing the form and format of questions in order to assist learners to internalise them for their own use (using meta-questions reflectively)" (p.12). In alignment with Mason's (2010) distinctions, we considered that norming and controlling questions were associated with evaluating, whereas questions exhibiting genuine inquiry were associated with either exploring/describing or interpreting, depending on the specific formulation/intent of the question.

When examining teachers' work, we also attuned to whether their claims were *specific* or *general*. We made this distinction because general claims about student understanding do not demonstrate a focus on individual student thinking, and can oversimplify situations for learning and teaching (van Es, 2011). For instance, stating that a student "doesn't understand fractions" does not demonstrate a deep knowledge of what that student knows about particular aspects of fractions. When teachers make specific claims, it shows a greater attention to student thinking, which in turn may affect their teaching strategies (Goldsmith & Seago, 2011). Consistently, we attuned to teachers' use of evidence. We valued teachers' analyses that incorporated excerpts of transcripts and student work to substantiate claims, as we considered that providing actual evidence demonstrated closer attention to student thinking (Goldsmith & Seago, 2011).

We also analyzed whether teachers' claims were *positive* or *negative*. Students arrive in classrooms with a wealth of knowledge and intuitions. We can more productively help students learn by building off what they already understand, rather than focusing on what they do not understand (Hammer et al., 2012). In addition, we looked at the content of teachers' claims. Based on Star et al. (2011), we distinguished whether teachers were focusing on aspects related to vocabulary and symbol use *versus* understanding at a conceptual level. Finally, from Unit 3

onwards, teachers were asked to develop potential follow-up questions for their students and to connect their interviews to their teaching. In our analysis, we considered *open-ended* questions to be richer than right/wrong questions, and in alignment with our PD project's goals, hoped to see teachers connect what they were learning about student thinking to reflections about their everyday teaching (van Es, 2011).

Goals

The goal of this paper is to illustrate the variety of paths teachers' learning might follow as a result of working in groups within PD settings. We present a detailed analysis of the work of a group of four teachers (the group "K2S"). We explore teachers' learning across two types of assignments focused on attending and responding to student thinking: *interview assignments* on student thinking, and a *final project* that involved designing, implementing, and analyzing a learning activity that responded to and built on student thinking. In our analysis, we also look at teachers' postings on an online forum and analyze the feedback provided by the PD facilitators.

Our study differs in multiple ways from previous PD research in mathematics education. First, we look at teachers' work across several assignments. Second, instead of asking teachers to analyze someone else's work (e.g., classroom videos, learning activities), we asked teachers to analyze their own work. Finally, this study is different because we used an online environment to interact with the teachers while they completed the course assignments. Online interaction presents both advantages and disadvantages for teacher PD compared to in-person interaction. Among the advantages, discussion dynamics are likely to be more effective online because teachers can spend more time reflecting on their responses than in face-to-face settings, multiple participants can contribute at the same time, and the period available for teacher interaction is generally longer. However, there is generally a lack of opportunity to discuss the assignments face-to-face with facilitators, which would be helpful to clarify teachers' concerns, to make sure they accurately understand the main goal/s of each assignment, and to create an environment of trust. As will be seen in our study, the way assignments are presented, structured, and phrased is extremely important in online environments because teachers oftentimes interpret questions differently, or do not fully address in-depth questions or ideas that are essential to PD designers.

CONTEXT FOR THE RESEARCH

The professional development program

This study was conducted within "The Poincaré Institute" a PD program for grades 5-9 mathematics teachers in the northeastern United States (see http://sites.tufts.edu/poincare/; Teixidor-i-Bigas, Schliemann, & Carraher, 2013). The Poincaré Institute aims to help grades 5-9 mathematics teachers deepen and broaden their own understanding of both middle school mathematics and middle school student mathematical thinking and learning, with the final goal of enhancing students' learning. The first cohort of teacher participants (N=56), from whom the data for this study are drawn, took a series of three graduate level semester-long courses from January 2011 to June 2012. Three core mathematical ideas pervaded the content of these courses: algebra and functions, multiple representations, and modeling and applications. The courses covered numerous mathematics topics, including properties of numbers (fractions, rational numbers, integers), arithmetic (the basic operations of addition, subtraction, multiplication, and division), and algebra (functions, equations, slopes, and solutions of linear and polynomial equations).

Throughout the courses, we asked teachers to complete varied mathematical problems and activities related to mathematical thinking, learning, and teaching. For the assignments analyzed in this study, teachers were divided into groups of two to four participants from the same school district. Two facilitators (generally a mathematician and a mathematics educator) worked with each group. Facilitators provided constructive feedback, suggested new ideas and questions to prompt further reflection and discussion among teachers, and encouraged them to read other teachers' work. The first, second, and last authors of this paper were part of the team of facilitators. In addition, the second author was one of two facilitators assigned to work with this specific group of teachers. Both the teachers' assignments and the feedback from the facilitators were posted online.

Our work with the teachers was neither prescriptive nor directive. Regardless of the specific activity proposed, we did not provide teachers with rigid solutions nor with corrective feedback. Moreover, we never told teachers how they *should* teach their students. We instead fostered teachers' reflection and discussion —both of their own and their students' mathematical work— as a way to promote their learning. The goals of the feedback we gave to teachers were to help them to observe aspects of students' thinking that they had not explored, described, or interpreted in their analysis, and to raise new questions and ideas (for further details, see Teixidor-i-Bigas et al., 2013).

The structure of the course and the assignments

Teachers were requested to complete assignments every week during the three courses of our PD program. Assignments varied in content and form, as will be described below. The data analyzed here were collected during Course 3, titled *Invariance and Change*, which addressed the growth and behavior of different types of functions (e.g., linear, quadratic, exponential). The course was composed of four units, each involving three weeks of work. The four units had the same structure:

- Week 1 presented an introduction to a specific mathematical topic;
- Week 2 offered a more in-depth elaboration and applications of the topic;
- Week 3 asked each teacher in a group to interview students about that topic and video record the interview, to analyze the interviews with other teachers from their group, and to post a group reflection on the set of interviews.

The course ended with an individual final project. As mentioned above, this paper focuses on the group interview assignments carried out during Week 3 in each of the 4 units and on the individual final project. For clarity, the mathematical content of each unit will be described in the Results section.

In line with the literature on attending and responding to student thinking described earlier (e.g., Jacobs et al., 2010; Levin et al., 2012; Sherin et al., 2011), both the interview assignments and the final project were designed to emphasize the following themes: (a) Students have powerful ideas and representational competencies that enable them to learn mathematics; (b) In order for teachers to help students "mobilize" their resources, first it is important to know what these resources are; (c) Teachers therefore need to be able to "enter into students' minds" (Ginsburg, 1997) and "give students reason" (Duckworth, 2006); (d) Students should have many varied opportunities to talk about and represent their mathematical ideas and to solve problems in different ways; and last but not least, (e) Teachers should consider how students think of and learn specific topics in order to make instructional decisions when teaching these topics.

THE STUDY

Description and selection of participants

The case study presented in this paper features one of the 18 groups in the cohort, called group K2S, whose members were teachers Kyle, Laurel, Liz, and Sophia (all names are pseudonymous). Table 1 shows relevant information about these four teachers.

Table 1. Characteristics of the four members of the "K2S" group.

	Kyle	**Laurel**	**Liz**	**Sophia**
Gender	Male	Female	Female	Female
Educational background	B.S. Psychology (minor Education)	B.A. Mathematics (minor Secondary Education)	B.A Mathematics (minor Education) M.A. Mathematics	B.S. Mathematics (minor Education) M.A. Math Education
Years of teaching experience (total)	7	6	9	9
Years of math teaching experience	5	6	9	9
Grades taught	9-12	9-11	9-12	7-8

We decided to focus on K2S because we considered it to be a compelling case to show the diversity of ways in which teachers can evolve in their attention and response to students' thinking and to illustrate the tensions between the group and the individual analytical levels. In selecting this group, we used the rationale adopted by Nemirovsky, Kelton, and Rhodehamel (2013), whose goal was "not representativeness but rather the enrichment of the reader's own perception" (p. 385). We do not mean to generalize our observations, as we are aware that not all groups of teachers in our program progressed in the same way.

Data sources analyzed in this study

Table 2 shows the structure of the assignments we focus on in this paper (interview assignments and final project), as well as communications related to each assignment (feedback by PD facilitators and teachers' comments on the online forum). Items analyzed in this study are indicated with asterisks (*). As can be observed, some of these data sources were individual submissions of work; their "ownership" can therefore be attributed to an individual teacher. In contrast, others were explicitly requested as group submissions (e.g., joint analysis of student interviews). In these cases, it is impossible for us to know what each teacher contributed to each of the assignments.

Interview assignments. The main goal of the interview assignments was to help teachers attend to students' mathematical thinking. The assignment was introduced as follows: *Your biggest challenge will be to "enter the student's mind" and to understand their thinking without leading him/her in one direction or giving away the "right" answer. Some tips for avoiding this are to ask "Could you tell me more about that?" or "How do you know?"* Every teacher was asked to conduct and video record at least one interview (hence, each teacher interviewed different students) and to transcribe at least the section on which they wanted to focus in their group analysis. Then, we asked teachers to produce a collective analysis of the interviews carried out by all members of the group, focusing on the most interesting, surprising, and/or puzzling ideas and representations produced by the student/s.

Table 2. Structure of assignments and related items.

		Item	Generated by
Interview assignments (x 4)	Assignment	Video recorded interview with student(s)	Individual teachers
		Partial transcript of interview(s) *	Individual teachers
		Written analysis of interviews *	Group of teachers
	Related items	Feedback posted on the online forum *	PD facilitators
		Posts and reaction to feedback posted on the online forum *	Individual teachers
Final project	Assignment	Design of classroom activity *	Individual teachers
		Video of activity implementation *	Individual teachers
		Written analysis of activity implementation *	Individual teachers
	Related items	Feedback posted on the online forum	PD facilitators
		Posts and comments on work of other teachers posted on the online forum	Individual teachers

(*) Items analyzed in detail for this paper

The questions suggested to teachers for the analysis of the interviews evolved slightly as the course progressed. There were always questions asking teachers to discuss their students' thinking and initial ideas (e.g., *What did the set of your group's interviews show about students' ways of thinking about inequalities?*). We also consistently asked teachers the question, *What did students say or do that surprised you?*, and asked them to *Use evidence from the drawings and transcripts of the interviews to support your ideas.* In Units 1 through 3, teachers were asked how their students' approaches might help or hinder their understanding of the topic in the future (e.g., *How might the students' ways of approaching this problem help or hinder their understanding of equations and inequalities in future mathematics?*) These questions, however, were not included in the Unit 4 interview assignment and in the final project. Moreover, in Units 1 through 3 we asked teachers to think about possible follow-up questions to be asked to the interviewees (e.g., *What more would you like to be able to ask your students in order to better understand their thinking?*). In turn, the Unit 4 interview assignment and the final project asked teachers to relate their findings to their teaching (e.g., *What did your interviews reveal that may be relevant for your work as a teacher?*).

Final project. After the four group assignments in which teachers explored and interpreted students' thinking in an interview setting, teachers were asked to complete the final project individually, the main aim of which was to respond to student thinking in class. Specifically, the final project was composed of different activities: design a classroom activity on specific mathematical content and specific aspects of student mathematical thinking using their prior interview findings; implement (and document by video recording and collecting all written work produced during the activity by teacher and students) it by focusing on responding to student thinking in class; and analyze it from the point of view of making sense of student thinking. For their final project submission, teachers were asked to prepare a 10-minute video clip from the activity implementation and write an analysis of it, using examples from video transcripts or students' written work to support their claims.

Analysis

In this study, we analyzed the data sources detailed in Table 2. We first carried out multiple readings of all written documents and viewings of the 10-minute videoclips. To

conduct the analysis we used Glaser and Strauss' (1967) constant comparative method and Glaser's (1998) theoretical memoing. The constant comparison and refinement of descriptive codes continued until no new characteristics emerged from the data. We then created a detailed narrative account, or "thick description" (Geertz, 1973), to characterize the work of each teacher.

RESULTS: INTERVIEW ASSIGNMENTS

This section presents a qualitative description of the shifts observed in the K2S group in the four interview assignments. We describe the teachers' work chronologically (Units 1-4). Within each unit, each of the data sources is illustrated with characteristic examples. Our description includes excerpts from teachers' written work and from the feedback provided by the two PD facilitators assigned to the group, who were experts in mathematics and mathematics education, respectively. The facilitators consistently used two different methods to give feedback: 1) they gave specific comments inserted into the document submitted by the K2S group, and 2) wrote an overall summary to highlight their most reoccurring comments. We also present selected messages posted by the teachers and the facilitators on the online forum site, in which teachers responded to specific aspects of the feedback, commented on each others' work, and shared ideas for future submissions.

The headings for the sub-sections describing each unit summarize the main characteristics of the teachers' work for that unit. This brief statement condenses the most notable attributes of the work and leads into the description that follows. In turn, the qualitative description supports the characterizations in the headings.

Unit 1: Focusing on general (mainly negative) aspects of students' mathematical thinking

Unit 1 dealt with equations and inequalities, particularly with systems of equations and transformations of expressions. The unit also presented functions using both geometric and algebraic approaches. In the interview assignment, teachers were given three choices of mathematical tasks to pose to their students, all focusing on the inequality $a<12$:

Where could you place a on a number line if a is less than 12?
Where could you place a on a number line if a+3<12?
Where could you place a on the number line if |a-7|<12?

Teachers' written analysis of interviews

The primary focus of K2S's analysis was on vocabulary and symbolic issues (e.g., *None of the students used dots*), whereas students' conceptual understandings played a minor role. The few statements on conceptual understanding made by K2S were evaluative and judgmental (e.g., *None of the students wrote the [correct] expression a<12*). Rather than describing or interpreting students' thinking, the teachers assessed what students knew through a correct *versus* incorrect lens. In addition, most claims were general (e.g., *there was a lack of independent thinking*) and had a negative focus. In other words, the teachers described what students *could not* do instead of what they *could* do. However, they neither presented evidence to back up their claims nor explained further the approaches students took. Finally, when discussing potential questions the teachers would have liked to ask to follow up on students' responses, they wrote many questions with right or wrong answers (e.g., *Is ½ a solution? // Can you give me a solution that's not an integer? // Are there more numbers between 11 and 12?*). In general, the questions proposed tended to underestimate students' competencies.

Facilitators' feedback

The facilitators offered multiple observations in their feedback. Although they did not explicitly ask the group to be less evaluative or to provide more supporting evidence, their feedback targeted these issues by asking the teachers to consider the transcripts "more specifically." The facilitators suggested that the teachers should focus on how students explained their ideas and describe the students' thinking in greater detail. Another suggestion was to reflect on the impact the questions asked during their interviews might have had on the students and to look beyond simply evaluating what the students said and did in response to those questions. Finally, as illustrated in the following example, the facilitators suggested questions teachers could have asked to further explore students' conceptual understandings:

> *In your report, the students used letters instead of dots to mark points on the number line, and this surprised you. We wondered why this surprised you. At the beginning of your interview, you could ask: 'Give me a number that is less than 12.' Then you would know how the students mark points on the number line later* [since letters were explicitly mentioned in the task statement, see above].

Teachers' posts on the online forum

The excerpts below illustrate teachers' reactions after reading the feedback provided by the facilitators. Sophia and Laurel's posts addressed the idea of trying to be more aware when conducting the interview. Both acknowledged that they could learn more about their students' thinking by becoming more conscious of their moves during the interview, the questions asked, and how they were asked.

> **Sophia:** *I know that with Kim* [one of the students interviewed], *I should have questioned her more, but hopefully I will get better with practice. When I played the video back, I thought of many things that I should have said. I think next time as I am typing the transcripts, I will include those thoughts… watching the video, I can't believe I didn't ask the simple question, 'Why?' I think with that one word question I could have learned a lot more about James and Kim!*
>
> **Laurel:** *All of your comments have really helped me to be conscious of the things I do during my interview.*

In addition, Sophia expressed her interest in continuing to learn about student thinking and shared with us her excitement about the interview assignment. As will be seen throughout the paper, Sophia and Kyle demonstrated to be considerably more "aligned" —although not completely— with the goals of our project than Laurel and Liz.

> **Sophia:** *It was very interesting to interview students one on one and really focus on what each student understands about equations and inequalities. I am excited to continue with student interviews. I think the second time around, I will ask even more questions and not assume as much about what the students are telling me.*

Unit 2: Starting to explore students' conceptual understandings

Unit 2 explored linear and nonlinear functions, the quadratic formula, and what it means to "move everything to one side" when solving an equation. The problem provided for the interview assignment dealt with rates and how they relate to graphs:

> *Elizabeth, Patty, and Carly are cousins. Next year, they would like to send their grandmother on a vacation for her birthday, but it will cost $3,000. The girls decide that they have one year to raise $1,000 each. Elizabeth starts saving a lot of money on the very first day, but each day she puts less money into her bank account than the day before. Patty figures out exactly how much money she will need to save each day to reach $1,000 in one year and puts the same amount of money into her account each day. Carly begins by saving very little, but each day she puts more money into her account than the day before. Ask the student: "What do you think the graphs for each cousin will look like?" Ask the student to draw each graph on the worksheet and to explain to you what each graph shows and why.*

Teachers' written analysis of interviews

The following excerpt illustrates the characteristics of K2S's Unit 2 group submission. The excerpt features student Jem (grade 9), who was interviewed by one of the teachers:

> *Jem generally understood the idea of what was happening and she understood that there is a difference between the three savings plans. She understood that saving the same amount of money the whole time (Patty) would be a constant rate of change and* [she was] *able to represent this on the graph by drawing one line from (0, 0) to (1, 1000). Jem says, 'Oh… yea. So this represents Patty's [labels the center graph-line] because she put in the same amount everyday so it goes up at a steady interval.' There are other times that Jem uses the word interval incorrectly as a replacement for the word rate, at the beginning of the interview she says 'And Elizabeth [labels the top graph] because she started out saving more money than before and then she slowed it down so the graph got… whatever the word is.'*

As requested by the facilitators in their Unit 1 feedback, the teachers began to explore and describe how students "attempted" the problem, shifting from their evaluative and judgmental tone in the previous unit and using a more specific, descriptive voice (e.g., [Jem] *understood that saving the same amount of money the whole time (Patty) would be a constant rate of change*). The group provided evidence of Jem's understanding using a quote and a description of what Jem drew during the interview. However, the analysis of K2S also contained general claims. For instance, they stated that Jem said, *'the slope represents the diminishing amount of money that she is putting in the account...' Jem has an understanding that slope represents the rate of change*. This claim did not address specific ideas related to slope and rate of change, but instead broadly captured Jem's understanding. This Unit 2 submission was still somewhat focused on definitions and symbolic issues. For example, the teachers showed concern with Jem's use of the word 'interval.' They highlighted several moments in the interview when the student used this word in non-canonical ways, and used those moments to claim that Jem used the word incorrectly to stand for the word 'rate.' Finally, teachers made descriptions of students' thinking that focused on both positive and negative aspects, as can be observed in the above-presented excerpt.

Facilitators' feedback

The facilitators provided positive comments in response to teachers' increasing use of evidence (e.g., *This is a nice example to support your claim*) and appreciated teachers' shift towards the positive aspects of student thinking (e.g., *I really like that you are focused on what*

the students DO know). However, the facilitators continued to suggest that K2S should offer increased detail in their analysis of the interviews. As in Unit 1, the feedback cited several examples to call for more evidence (e.g., *We would have found it very helpful to have more evidence to support your claim and more insight into the student's mathematical understanding*). In addition, teachers were asked to provide further information about possible follow-up questions (e.g., *A couple of times you just indicated that you would ask Jem and Jojuan* [a second student interview the teachers focused on in their report] *more about the rate of change of the function. We would like to hear more about what kind of questions you have in mind!*).

Teachers' posts on the online forum

The message Laurel posted in response to Unit 2 feedback clearly illustrates how looking at teacher learning exclusively in groups can potentially obscure their individual shifts. In their Unit 2 submission, the K2S group had reflected on two different interviewing approaches they themselves had adopted, which are described below. The facilitators asked what the group members discovered from using these approaches. After reading the feedback, Laurel replied at length:

> ***Laurel:*** *In regards to your last comment, we had a lengthy discussion about the process Liz and I took when interviewing. I set up the problem to my student and told him what I expected. In my mind, there was a right and wrong answer. I guided Jojuan as he worked through the problem so he could understand the correct way to graph the three different situations. […] Liz, on the other hand, gave the problem to the students and had them graph the three graphs. After the fact, she questioned the student to see what their thought process was and how they obtained the graphs they did. I think she was able* [to] *ask some interesting questions because of this. […] I think both processes are good methods and it really depends on what you want to take from the overall interview. I think my student grew confident after the first graph and he knew that it was a good model of the information. This could affect the rest of the interview in a positive way. I am curious what the other teachers do as they go through their interviews??*

This excerpt highlights how at times different members of the same group might have different ideas. Laurel is still thinking of right or wrong answers and guided Jojuan to the correct way to graph the situations, whereas Liz seemed to be more attuned to the goals of our assignment (i.e., "entering the student's mind" and "giving students' reason"). These discrepancies between the members of the group, which were not evident in the teachers' group submission, are discernible here and will become apparent again when we look across their final projects.

Unit 3: More is less. Describing many students, providing little evidence

Unit 3 elaborated on the ideas of change, covariation, and slope. The interview assignment asked teachers to devise their own interview situation relating to these concepts. The problem K2S chose involved matching water containers with their corresponding graphs of height of water as a function of time, as shown in Figure 1.

Teachers' written analysis of interviews

In this unit, K2S described all five interviews that the members conducted, which led to a rather superficial analysis. For example, the following paragraph summarized Nina's (grade 9) interview:

Nina chose graph (b) for container A because she misunderstood what the y-axis was measuring. She thought it was measuring the rate of the water vs. time, not the depth of the water in the container vs. time. Moving onto container B, she immediately eliminates graph (f) because she understands that this graph is showing a decrease, then an increase, which does not correspond to the container filling up. She also eliminated graph (i) for the same reason. She correctly chooses graph (c), but has difficulty verbalizing why and winds up changing her answer to graph (e). It would have been nice to ask where on graph (e) she sees it 'really getting started' and where 'it's getting slow at the top.' For container C, Nina chooses graph (g) correctly. Again, it would have been helpful to ask where on the graph it is shows when the rate is slow and when the rate is fast.

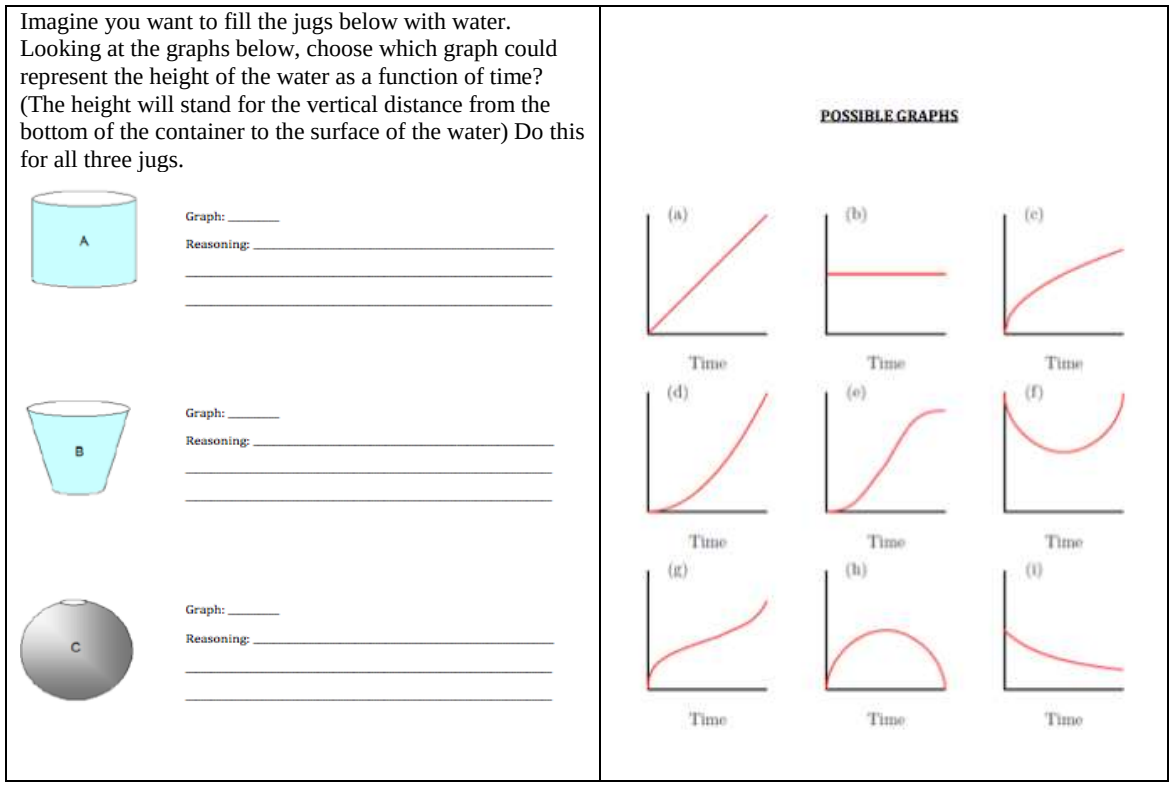

Figure I. Interview situation designed by the K2S group for Unit 3 assignment

The teachers primarily described Nina's *actions* (that is, which graphs she chose) and secondarily described and interpreted the *understandings* behind her actions. Although they included some interpretive commentary as to why Nina gave specific responses, such as "She thought it was measuring the rate of the water vs. time," their interpretations were rather cursory. The group explained what Nina understood about several specific situations without making broad statements, which constitutes a departure from previous units. It therefore seems that K2S was responding to the facilitators' calls for greater specificity regarding student thinking. However, despite the facilitators' prior requests for more evidence, the group did not provide any explicit evidence to support their claims. This lack of evidence could relate to the sheer number of interviews the group decided to analyze in their report.

K2S's claims were again both positive and negative, and still had an emphasis on correctness (e.g., *Nina chose graph (b) for container A because she misunderstood what the y-axis was measuring… She correctly chooses graph (c), but has difficulty verbalizing why and winds up changing her answer to graph (e)*). However, the teachers did propose several follow-up questions that were grounded in Nina's thinking; they wanted to understand what she meant at certain moments in the interview (e.g., *It would have been nice to ask where on graph (e) she sees it 'really getting started' and where 'it's getting slower at the top'*). This demonstrates that K2S was focused on Nina's conceptual understanding, rather than symbolic or vocabulary issues. The questions proposed in Unit 3 also marked a departure from previous units, given the open-ended nature of the questions and their focus on student thinking. Finally, K2S reflected on how they would modify the interview task if they were to conduct it again. The proposed modifications aimed at responding to the challenges shown by students (e.g., *If we were to use this problem again, we might consider labeling the y-axis as depth of container to avoid misconceptions-misunderstandings*).

Facilitators' feedback

The facilitators began their response by praising the group for including questions that would have helped the teachers understand their students' thinking. They then requested, again, for more probing questions during the interview (e.g., *We could have found it very helpful to ask 'why' questions to get more insights into your students' understanding*). They also requested more details in the analysis and challenged the teachers to look beyond 'correctness' (e.g., *at some points it seems that you were hoping that your students would arrive at the 'correct' answer… We feel like it would have been helpful to get deeper into your students' thinking rather than merely making sure that they provide a correct answer*). Additionally, the facilitators asked the teachers to consider several questions related to teaching and asked teachers to make connections to their teaching practices (e.g., *Have you learned anything that might impact the way in which you teach this content? What's the value of interviewing related to teaching?*). The goal of this request was to start to prepare teachers for their final projects.

Teachers' posts on the online forum

Along with discussing the content of the activity used in the interview, K2S teachers used the online forum to write about how they questioned students. Laurel's interview was still focused on getting students to find the correct answer. As in the previous unit, her comments on the online forum suggest that she was becoming increasingly aware of this aspect and how it differed from the project's goal of attending to student thinking. This self-critique represents an important step forward in this aspect.

> **Laurel:** *It is such an interesting thing to read the questioning I used and wonder why I worded things a certain way. I definitely think my mind was narrow and focused on the answer. I have to work on that and jump deeper into the questions I asked. […] I never press further if they are correct and I always assume what they are thinking but never directly ask them. I am even worse when they get the wrong answer because my line of questioning guides them to the answer instead of guiding me to their line of thinking.*

Kyle wrote about student ownership of the problem, raising interesting issues about how the way the problem was presented influenced students' responses.

> **Kyle:** *I think it is easier to start dialog and ask good questions when the student draws the graph on their own. They also ask questions that give some insight into their thinking when they have to graph on their own.*

Finally, Sophia's reflection demonstrates how she was immersed in student thinking and how interested she was in discussing how her questions affected students' responses.

> **Sophia:** *I need to make a note for myself to* [ask] *more probing questions when the students get the answer right. I think when students answer incorrectly, I am more likely to ask question*[s] *to find out what they are thinking. Then when a student get*[s] *the answer right, I make too many assumptions about their understanding and I need to question them more.*

While these three members of the K2S group commented on their approaches to interviewing (Liz did not comment on this issue), they were still in different places in terms of attending to student thinking. Laurel did not focus her interview on student thinking but on correctness. Thus, on the online forum she elaborated on the need for her to start looking beyond a correct answer. In contrast, Kyle and Sophia's interviews did focus on student thinking. However, Kyle elaborated on how the features of the task itself affected students' responses, whereas Sophia reflected on how students' answers affected her own line of questioning.

Unit 4: Interpreting specific (positive) aspects of students' conceptual understandings

Unit 4 included a discussion of functions and their role in the world. It involved primarily modeling and the structure of word problems. In the interview assignment, teachers were given two potential problems to choose from, one involving distances and the other involving painting a wall. The K2S group picked the latter problem:

> *(a) Joe can paint a wall by himself in 2 hours and Sam can paint the same wall by himself in 4 hours. How long will it take them to paint the wall if they work on the wall together? (b) Joe can paint 1/2 of a wall in an hour and Sam can paint 1/4 of the same wall in an hour. How long will it take them to paint the wall if they work on the wall together?*

Teachers' written analysis of interviews

Similar to Unit 3, the group focused on several students, briefly describing four students' approaches to the problem. As the following excerpt shows, they primarily described how students approached the problem and made interpretations of their students' understandings.

> *This student* [grade 9 student] *uses diagrams to solve. They begin by determining how much of the wall each person paints in 1 full hour. They realize ¾ of the wall is painted and that ¼ of the wall remains unpainted. He keeps dividing the unpainted sections in half and adding the time elapsed to the 1 hour. Somewhere, this student makes a mistake and obtains 1 hour and 26 minutes but we thought the process was a very interesting process! This student was an algebra student so we think that might have been a contributing factor.*

In this quote, the group begins descriptively, listing the tasks that the student did when approaching the problem. However, the group shifted to interpretation, noting that the student had an "interesting process" and used their description to explore why the student gave a specific

answer. Indeed, the group did not make any broad claims about student understandings. This constitutes a major shift from Unit 1, when the group made broad and evaluative claims exclusively. K2S's claims about students in Unit 4 were generally positive. Even when describing students' errors they noted that the student's approach was "a very interesting process!" This radically differs from their Unit 1 analysis, which was entirely focused on what students did not know. Unit 4 was very much attuned to students' conceptual understandings and representations. A focus on correctness is notably absent from this analysis, which demonstrates that K2S reacted to the facilitators' feedback.

K2S not only provided summaries of four students but also provided extensive commentary on one of the interviews by inserting comments into the transcript, which is analogous to providing quotes in the analysis itself. This kind of presentation for their analysis mimicked the way in which facilitators had in prior units inserted comments within the interview transcripts. For instance, the student gave the following response: *No, the three hours would be, let's say Joe paints first for 1 hour and then Sam paints for 2 hours, it would be three hours.* The group commented as follows within the interview transcript: *Tyler* [the student interviewed] *is understanding that these are individual completions of the wall. He is able to go from 2 hours and 1 wall to 1 hour and ½ a wall. Similarly, he is able to go from 4 hours and 1 wall for Sam to 2 hours and ½ a wall.* The group's analysis here was deeper than their prior summative analyses. They provided their perception of the student's understanding by highlighting a quote, and then describing how it reflected his understanding. Within the interview transcript, they provided a total of fourteen comments of varying depth.

Facilitators' feedback

The facilitators expressed that K2S asked interesting questions and praised the group for highlighting when they might have asked different questions. The feedback acknowledged the difficulties intrinsic to interviewing and praised teachers for their progress (e.g., *We also acknowledge how hard it is to carry out these interviews! They require a lot of experience, practice, and attention, and it's hard to strike a balance between being focused and being open ended*). Overall, the tone of the feedback was very positive (e.g., *You've made great progress during the semester*).

The suggestions primarily concerned connections to teaching, which were rarely addressed in K2S's Unit 4 analysis, even though this was one of the specific requests made in this unit. The goal of the facilitators was to help the teachers connect the interviews to classroom practice and to prepare them for the final project, in which they had to design, implement, and analyze a classroom activity "in response" to student thinking. Thus, the facilitators suggested that teachers could think about other possible clarifying questions (e.g., *think about what kinds of questions are most helpful for students to have these 'aha' moments*), as well as about next pedagogical steps (e.g., *I would be interested in hearing what you would like to do next in order to clarify students' common misconceptions for the painting problem*). Moreover, some comments asked teachers to provide further details, such as why the students' participation in Algebra I was a "contributing factor" (see above, Unit 4 excerpt from teachers' analysis).

Teachers' posts on the online forum

The group did not make any connections to teaching or reflect on their interviewing approaches, even though the facilitators had provided positive feedback on that front. However, Laurel did provide a brief reflection on the forum:

> **Laurel:** *I was surprised at how many of my* [Algebra I] *honors students answered the question incorrectly. We talked about it afterwards and I asked them if it made sense to take 3 hours? I asked how long it would take if each person was by themselves. I asked, using those numbers, what were unreasonable answers? The students, at this time, realized it HAD to be less than 2 hours! Yay! After that, we worked through it using a couple of methods and they were able to understand how to solve it. I bet if I gave them another problem with rates and combined work, they would be able to figure it out.*

This post reflects that Laurel was still quite focused on correctness, despite the group's shift away from correct-or-incorrect. Laurel did discuss how her line of questioning affected her students' responses and proposed questions she could ask in the future to see how her students would perform, which shows that she was slightly more attuned than before to the goals of the course.

RESULTS: FINAL PROJECTS

Overall, by Unit 4, the K2S group was able to both describe and interpret the understandings demonstrated by students in the interviews, generally adopting a positive perspective when looking at students' ideas. Increasingly, the group also used evidence to support their claims, which were more and more specific and detailed. Despite the facilitators' requests, however, the group was less focused on the implications of the interview findings for their teaching practice. As will be seen below, the final projects of all four teachers were different than the group work in Unit 4. Each teacher only demonstrated *some* of the achievements demonstrated by the group.

In the following, we present a snapshot for the final project of each teacher. We first briefly describe the content of the activity designed, then summarize the content of the 10-minute video clip submitted, and finally detail the main features of the analysis teachers wrote about their activity implementation. Excerpts are provided to illustrate our descriptions.

Liz: Exploring student thinking slightly, but not responding to it

The activity Liz designed for her final project was procedural in nature. It required solving problems using systems of equations. Because this topic had not been tackled in the interview assignments, we argue that this activity was not designed in response to students' thinking. In her 10-minute video clip, Liz tried to capture the entire lesson. She showed episodes of herself lecturing and asking students questions about how they found their answers (e.g., *What did x and y stand for in the word problems?*). In addition, Liz showed examples of students' written work.

Her analysis of the activity implementation was fairly brief. Liz focused on describing the activity and made general claims about what students learned, which were evaluative and descriptive in nature, using little evidence to back up her claims. She only wrote a few sentences about how her students' understandings changed as a result of the activity:

> *Students FINALLY began to realize that substitution AND elimination are two methods that can be used to solve the same type of problem. This is seen in the video when Mattias asks this at the end of problem #2. Students also showed more evidence of understanding the solution they get from a system. At the end of problem #2, Danielle asks if she can check her answer by plugging in the values for x and y into their solutions. The variables x and y had meaning attached to them*. [Emphasis in original.]

Liz maintained an emphasis on correctness, apparent from her comment that students "FINALLY" began to understand the two methods that would help them find the correct answer. She was, however, open to having students use multiple solution methods (e.g., *I encouraged students to try different methods- ones that they felt were 'easier' given the problem and the types of equations that were formed from the word problem*). She briefly referred to the questions asked by Danielle and Mattias, although without interpreting their understandings. Finally, Liz wrote little about implications of her final project for her teaching practice. The implications she referred to were rather vague (e.g., *The next time I teach systems word problems, I think I will try and look at a variety of examples together so students realize that not all problems get set up the same way*).

Laurel: Implementing an activity based on interview findings, but backgrounding students' thinking

The activity designed by Laurel was thoughtfully planned and accounted for aspects identified in previous interview assignments. It involved students analyzing the correspondence between graphs and equations of linear and quadratic functions. The use of mathematical concepts and representations was emphasized in the activity design (e.g., *I chose to teach this lesson because I had never made a connection between graphs and solving equations before… I am taking many things away from this course, but the one thing I value the most is the strengthening of visual models*). Laurel's reflection demonstrates that she was aware of the importance of making connections *across* representations to promote students' learning, which is progress from the group's focus on symbols (dots *or* letters) and vocabulary in Unit 1.

The 10-minute video clip Laurel submitted did not contain questions aimed at attending to students' thinking. Instead, she showed herself talking through the material and answering the questions posed by two students. She used a very traditional, teacher-centered approach. Laurel did include a clip where she was working one-on-one with a student. However, she did not ask any questions then either. In her brief analysis of the activity implementation, she tended to be evaluative and descriptive, making primarily general, correction-focused claims. She focused on whether her students were correct or not, saying generally that students "struggled with the entire concept" instead of specifically citing what students knew and did not know. When she did give specific student examples she tended to be descriptive but not especially detailed. For instance, she mentioned that a student "was able to see the connection" and "talk(ing) through the first few problems" was helpful, but did not provide any more details. Although Laurel's claims were primarily general, she also made a few specific claims. For instance, she mentioned that a student "was able to identify that the right side of the equation and his graph was the same." However, Laurel's ultimate goal was still correctness, which is apparent in the way she discussed her students' work. Although she did briefly mention what the students' knew, it was to describe how they eventually achieved the correct answer.

Kyle: Responding to student thinking in class, but still making general claims

The activity Kyle designed asked students to make connections between graphing and writing equations in the context of word problems. As with Laurel, he seemed particularly attuned to the use of multiple representations (written sentences, tables, graphs, and equations), consistent with the emphasis of the PD courses. The 10-minute video clip Kyle submitted featured, using his own words, an "interactive lecture." The camera was focused on him and never showed any students or student work. Kyle posted the problems on the board at the front of

the class and started solving them for the students. For instance, he himself drew a table on the board and students shouted out values that corresponded with the table. When students shouted out answers, Kyle did not ask follow-up questions as to how students found their answers. He did ask for student answers, rather than delivering a straightforward lecture with no interaction.

Even though Kyle's video did not show evidence of students' thinking or written work, his analysis was primarily focused on students. The kinds of claims he made were positive but rather general, as well as descriptive and (to a lesser extent) interpretative. In the following excerpt, Kyle vividly described a moment when his students "oohed" and "ahhed," describing the classical image of student understanding.

> *My students began to make connections and build their understanding throughout the video. You will hear many voices in the background, as well as the multitude of oohs, and ahhs, as the students begin their understanding. At the 2:49 mark, students are making sense of the word problem with a table. They are decontextualizing the problem and converting the word*[s] *to a number table.*

This is a rather superficial description, as Kyle did not actually describe what students understood at that particular moment. Yet this description highlights Kyle's attention to the importance of students' understanding. In addition, Kyle presented many broad reflections on the implications of the final project for his teaching practice. His reflections were interspersed with observations from the classroom, often to justify his actions as a teacher. After being surprised by his students' actions during the activity, he wrote that, "this was a chance for me to allow them to direct the learning and I'm thankful I remained flexible because students typically stay more engaged when they are commanding their own learning." This ability to critically examine the activity he himself designed and to adapt it to better respond to students' needs demonstrates his attunement to the goal of attending and responding to student thinking.

Sophia: Fascinated by student thinking, but backgrounding her teacher moves

Sophia implemented the "painting the wall" problem used in the Unit 4 interview assignment. The 10-minute video clip she submitted clearly reflected her focus on student thinking. Her camera was directed at the students at all times, and she showed many selected pieces of student work to explain how they thought about the problem. Her students appeared to be leading the discussion. Given that there were disagreements about how to solve the problem, Sophia invited her students to write their ideas on the board. In the 10-minute video she submitted, Sophia does not intervene at any moment.

Her written analysis was primarily specific and descriptive, and interpretative to a lesser extent. She focused on students' conceptual understandings and on how representations can help students' learning. Unlike the other teachers, she chose to look at two students in detail. This decision reflected her desire to explore students' thinking in greater depth than might be possible through a classroom-wide analysis. This is part of what Sophia wrote about Mia, one of the students chosen:

> *Mia had a limited understanding of the problem. Her original answer for the painting problem was 3 hours. She explains her thinking in the first part of the video. At 0:16 she says, 'If they are painting the wall together, Joe paints half and that takes him 1 hour. Sam painted the other half and that takes 2 hours, so that's obviously 3 because 2 plus 1 is 3.' I then had 3 students show me multiple ways to represent what was happening in the problem. One student showed looking at the fraction of the wall each person painted*

> *and trying to get that to be 1 wall and also the same time for both people. This is the method that Mia builds her understanding on.*

In this excerpt, Sophia described Mia's process in detail, giving quotes and describing the exact methods that Mia and her group used. The information she provided was very specific. In addition, Sophia also offered detailed interpretations of student thinking. Some of her initial claims about Mia's thinking were rather general (e.g., *Mia had a limited understanding of the problem*), although she justified her interpretations with a wealth of detail (e.g., *Mia was adding how long it would take each painter to paint the wall, rather than finding out how long it would take for them to paint the wall together*). Sophia's claims were grounded in the problem, rather than generalizing Mia's knowledge. This was reflective of Sophia's attention to giving a complete description of student thinking.

Another important point that Sophia focused on in her analysis revolved around the role of representations. All four teachers discussed using multiple representations in their activities. However, Sophia was the only one who incorporated student-created representations, such as proportions and drawings. Her choice demonstrates that she was truly interested in describing students' conceptual understandings and the process of solving, rather than on having students find the right answer or follow a prescribed process. Finally, Sophia did not make explicit connections between this activity and her general teaching practice. However, she did briefly discuss how previous interviews had shaped her activity design.

DISCUSSION

The PD literature currently emphasizes the need for engaging teachers in multiple kinds of activities and for looking at their learning across different tasks and contexts (Kazemi & Hubbard, 2008). In this study, conducted within an online PD program for grades 5-9 mathematics teachers, we explore the interplay between the learning teachers demonstrate collectively and individually when engaging in activities focused on attending and responding to student thinking (Jacobs et al., 2010; Levin et al., 2012; Sherin et al., 2011). More specifically, we illustrate the broad range of understandings that teachers can potentially achieve as a result of working in groups in online PD settings (An & Kulm, 2010), and show how at times, teachers' individual work differs radically from that generated by the group. We purposefully selected the K2S group to serve an illustrative function. In the Results section, we have described the learning of the four group members across a series of four *interview assignments* on student thinking, and a *final project* that involved designing, implementing, and analyzing a learning activity. In our analysis, we have also looked at teachers' postings on an online forum and the feedback provided by the PD facilitators.

Table 3 summarizes the most important shifts identified in K2S's group analyses across the four interview assignments (Units 1-4). Unit 1 analysis was primarily evaluative and emphasized the negative aspects of students' understandings. The claims made were general and focused on correction of the vocabulary and symbols used by students (i.e., using letters *vs.* dots). The teachers rarely used evidence to substantiate their claims, and the follow-up questions they proposed were right-or-wrong in nature. They then shifted towards being more descriptive (Units 2-4) and eventually interpretative as well (especially in Unit 4). They also increased the use of evidence and open-ended follow-up questions, and began to contemplate the role of interviewing and questioning within classroom situations. The group's final analysis (Unit 4) focused on students' conceptual understandings and representational competencies, and the claims presented were specific and primarily positive.

There are multiple factors that might have fostered these shifts in the group's ability to attend to student thinking. One of the factors is the feedback provided by the course facilitators, which challenged the teachers' work and offered constructive criticism, but was neither negative nor prescriptive. The facilitators often asked teachers to provide more specific information about students' thinking, requested more evidence for the claims made, and proposed new ideas and insights aimed at triggering further reflection and discussion. Another potential contributing factor is the interaction among the teachers themselves. They discussed the facilitators' feedback using the online forum, which became a powerful tool to follow up on teachers' individual progress. In addition, the teachers held face-to-face meetings (monthly with representatives of the PD program, weekly on their own), where they worked on the assignments together and discussed their different views. Other factors that might have helped K2S change were reading other groups' work (the facilitators consistently encouraged K2S to look at the analyses submitted by other groups) and the nature of the assignments themselves.

While the K2S group seemed to improve their abilities to attend to student thinking throughout the units, the individual online forum postings revealed a more complex picture. The group showed positive shifts in several areas (e.g., from general to specific claims, from negative to positive claims, towards increasing use of evidence). However, Laurel and Liz did not change their personal views regarding some of these areas. For example, the group moved away from correctness in Unit 2, but the online posts show that Laurel was still concerned with correctness in Unit 4. Similarly, the individual posts in Unit 3 illustrate how Sophia, Kyle, and Laurel were "on different pages" regarding interview approaches, and more generally, regarding the role of attending to student thinking.

Table 3. – Summary of results throughout the four interview assignments.

	Unit 1	**Unit 2**	**Unit 3**	**Unit 4**
EMPHASIS OF THE ANALYSIS				
Evaluating	Emphasis	-	-	-
Exploring / describing	-	Emphasis	Emphasis	Emphasis
Interpreting	-	Little emphasis	Little emphasis	Emphasis
CLAIMS				
Specific – General	General	Both	Both	Specific
Positive – Negative	Negative	Both	Both	Mainly Positive
Focus of the claims made	Vocabulary and symbols used	Vocabulary and symbols used	Conceptual understandings	Conceptual understandings
AMOUNT OF EVIDENCE PROVIDED	Low	High	Medium	High
TEACHERS' QUESTIONS				
Type of follow-up questions proposed	Right/Wrong	Open-Ended	Open-Ended	Open-Ended
Do teachers reflect on how the questions asked might have had an effect on student thinking?	No	Yes	Yes	Yes

Differences among the teachers were even more striking when their final projects were considered. Following the guidelines provided, Laurel, Kyle, and Sophia designed activities on mathematical topics related to the ones addressed in previous interviews. Liz, instead, focused on a topic that had not been explored during the course. Thus, there is no evidence that Liz designed her activity in response to her students' thinking since there had been no assignment in the course related to the topic she focused on. Her activity was procedural in nature and emphasized the development of computational skills. In contrast, the other three teachers' activities aimed at helping students develop conceptual understandings. Whereas Laurel and Kyle asked students to establish links among multiple conventional representations, Sophia encouraged students to produce their own idiosyncratic representations. Moreover, the ways teachers interacted with students during the 10-minute videos were substantially different: Laurel did not ask students any questions; Kyle just asked them to share the answers they were getting; Liz asked them to explain how they got their answers; and finally, Sophia's lesson was led by the students themselves, who held a discussion on how to solve the problem at hand.

Table 4. – Summary of results in the individual final projects.

	Liz	**Laurel**	**Kyle**	**Sophia**
ACTIVITY Does the teacher make explicit how the activity responds to specific aspects of students' thinking, based on the findings of prior interviews?	No	No	Yes	Yes
EMPHASIS OF THE ANALYSIS				
Evaluating	Emphasis	Emphasis	-	-
Exploring / describing	Emphasis	Emphasis	Emphasis	Emphasis
Interpreting	-	-	Little emphasis	Little emphasis
CLAIMS				
Specific - General	General	Both	General	Specific
Positive - Negative	Positive	Positive	Positive	Positive
Focus of the claims made	Procedures and Computations	Conceptual Understandings	Conceptual Understandings	Conceptual Understandings
AMOUNT OF EVIDENCE PROVIDED	Low	Low	High	High
TEACHERS' QUESTIONS Do teachers reflect on how the questions asked might have had an effect on student thinking?	Very little	Very little	Yes	Yes

Likewise, the written analyses of the activity implementations demonstrated entirely different approaches. Table 4 shows the main characteristics of each analysis. Overall, it is notable that the individual analyses of all four teachers were different from the final analysis for Unit 4 that they had produced as a group (see Table 3). Recall that in Unit 4, the K2S group not only explored students' thinking but also offered compelling interpretations. The claims made, which dealt with specific aspects of students' conceptual understandings, were mainly positive

and supported by evidence. Furthermore, the group reflected on potential teaching implications and proposed numerous follow-up questions. Individually in their final projects, however, the teachers only demonstrated *some* of these achievements.

Laurel and Liz emphasized "correctness" in their analysis. They rarely used evidence and did not reflect on the connections between their interventions and students' thinking. In contrast, Kyle and Sophia focused primarily on exploring and secondarily on interpreting students' conceptual understandings. They both provided significant evidence to support their claims. Kyle made both specific and general claims, and explained ways in which his own interventions might have constrained and elicited specific student responses. Sophia's claims were much more specific. However, she did not establish connections between her final project and her teaching practice, but with what she had learned about students' thinking in the Unit 4 interview assignment.

CONCLUSIONS

This sample case illustrates how different our conclusions about teachers' learning could be depending on the analytical level adopted; in particular, when we look at teachers' work in groups or individually. The four teachers featured here seemed to follow an "ideal" path when working as a group on the interview assignments. Their attention to students' thinking shifted in the direction intended by the PD designers and facilitators (Teixidor-i-Bigas et al., 2013). However, the individual online forum postings show that each teacher was actually following a different path, and that teachers were shifting in different paths. This was confirmed through our analysis of their individual final projects, where teachers demonstrated a wide variety of approaches to responding to student thinking. Whereas all teachers made progress throughout the course, none of their individual approaches was identical to the approach demonstrated by the group.

Based on this evidence, some readers might question the importance of the group assignments and wonder, "Why bother using group assignments when teachers' shifts are so different? Why not just work with teachers and assess them individually?" It seems clear that the collective *zone of proximal development* (Vygotsky, 1978) of the K2S group was greater than the individual zones of proximal development of each of its four members. It may be the four teachers' shared understandings that allowed the group to make progress from Unit 1 to 4 even though none of the teachers was able to show the same characteristics in their individual final projects. Following a similar logic, the changes observed in the individual teachers might not have been possible without the work previously done by the group. Based on this argument, we propose that the individual level and the group level maintain a dialectical relationship. Both levels seem to influence and be influenced by one another.

Many plausible hypotheses could be proposed to try to explain the differences observed in this case between the group and the individual levels:

- For example, some could argue that the four teachers might have started the course at different points. Based on the evidence gathered and our interaction with the teachers, we endorse this interpretation. Indeed, whereas Sophia and Kyle seemed attuned to student thinking from the outset, Laurel and Liz seemed to hold more traditional views on teaching and learning (teacher-centered). We acknowledge that our PD courses probably did not fit the initial needs, expectations, and interests of Laurel and Liz. In fact, we were able to observe a similar "mismatch" among other teachers, which allowed us to change the content and design of our program over subsequent years. Yet, both Laurel and Liz

did seem to become aware of the importance of attending to student thinking, which in our view constitutes an important accomplishment.

- Similarly, others might think that the teachers did not contribute equally to the group work. In particular, one could speculate that Laurel and Liz might have contributed less than Kyle and Sophia. Alternatively, one might think that the collective analyses produced by the teachers could have been dependent on what each teacher "brought to the table" for discussion, as teachers interviewed different students. We do not have evidence as to who wrote the different parts of the group reports. However, we do have evidence that all teachers took the work seriously, conducted the interviews, and read and responded to the facilitators' feedback.
- Finally, we could argue that perhaps teachers were not able to transfer the learning they achieved in the interview assignments to the final project because these assignments involved different mathematical tasks, and the nature of the work was different. In other words, increased capability to notice students' thinking in one-on-one interviews (as captured in the group assignments) might not necessarily translate immediately to changes in teachers' practice, as characterized by responsive pedagogies. Most likely, all four teachers needed more time and support to use their attention to student thinking skills to guide their instructional practices. According to numerous studies, this is not an easy and automatic process but rather a slow and unpredictable one (e.g., Carpenter et al., 1989; Franke & Kazemi, 2001).

We would be remiss if we did not acknowledge this study's limitations, which include the fact that we have focused on a single case study. Our intent was to illustrate an interesting phenomenon that might occur among certain groups of teachers while working in PD programs. We by no means intend to suggest that the same differences between the group and the individual levels occur across all teachers. Further studies should be conducted to explore to what extent the observations presented here appear among other groups of teachers enrolled in PD programs, both in mathematics and other content areas.

The case presented in this study has several implications for PD designers and facilitators. First, using both group and individual activities is important to promote teachers' learning, in particular their ability to attend and respond to student thinking. Doing group work is productive and has the potential to trigger individual shifts, as described above. But at the end of the day, teachers are on their own in their classrooms with students. It is therefore essential to also examine their learning individually, and observe how they design, implement, and critically analyze learning activities. Indeed, implementing the student-centered teaching approaches referred to in the Introduction requires much more than noticing student thinking. If change is to be widespread, initiatives for teacher PD should incorporate more practice-based activities that support teachers to adopt key practices associated with responsive teaching. Second, our study highlights the importance of providing teachers with "spaces" where they can interact and discuss their ideas, like the online forum utilized in this study. The online forum can also be beneficial for course facilitators' attention to teachers' learning paths; for instance, without the online space PD facilitators might not have known that Laurel was still focused on correctness even though the group had shifted away from that perspective in their assignments. Finally, this study shows the importance of providing teachers with systematic and constructive feedback as well as follow-up support over extended periods of time (Borko, 2004). Helping teachers develop the ability to attend and respond to student thinking is demanding and challenging, and it requires plenty of time and effort on the part of both teachers and PD facilitators. In our program,

teachers took three semester-long courses but still did not change in the ways the PD designers intended (see Teixidor-i-Bigas et al., 2013). This suggests that shorter PD experiences might not be appropriate to help teachers meet the challenge of attending and responding to student thinking, and ultimately to enhance students' learning.

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