Curriculum Opportunities for Number Sense Development:

A Comparison of First-Grade Textbooks in China and the United States

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Abstract

As a key concept in early mathematics curriculum, number sense is crucial for children's learning of other mathematics concepts. An earlier curriculum with a strong focus on number sense development presumably helps children perform better in mathematics later on. Chinese students outperformed their United States (U.S.) peers on number sense at lower grade levels, and on many other mathematics areas at both lower and higher-grade levels. This study analyzed the representation of number sense and its connection to other mathematics concepts in both traditional and reformed first-grade textbooks in China and the United States, and explored the learning opportunities that the textbooks in each country provide for their children in developing number sense. It found that Chinese textbooks focused more on the meaning and representation of number, place value, base-ten concepts, and on the connection of number sense to number operation while U.S. textbooks focused more on number counting, patterns, and the connection of number sense to data analysis. Thus, the textbooks in the two countries offered different learning opportunities for their students to develop number sense and its connection to other mathematics content.

Descriptors: Comparative analysis, elementary mathematics curriculum, mathematics learning, and number sense

Curriculum Opportunities for Number Sense Development: A Comparison of First-Grade Textbooks in China and the United States[i]

U.S. mathematics education reform has been encouraging students to understand big mathematics ideas, their relationships, and use these ideas to solve problems; number sense is one of these big ideas about which students need to develop a deep understanding in early schooling (National Council of Teachers of Mathematics, 1989, 1991, 2000). Number sense can be defined as one's conceptual understanding of numbers and numerical relationships, one's earlier understanding of number sense is seen useful to extend and deepen their understanding of number (Soder, 1990) and other relevant mathematics concepts later (Gersten & Chard, 1999). This understanding of number sense includes the meaning and representation of numbers, different ways to count numbers, identification of number patterns through comparison, decomposition, and re-compositions of numbers, judgment of different quantities, magnitudes, and measurements, understanding of the value of a number based on the place or position of its digits, especially in the decimal numerical system, and mental estimation of various quantities using numbers (Faulkner, 2009; Ghazali, Rahman, Ismail, Idros, & Salleh, 2003; Reys et al., 1999; Sowder & Schappelle, 1994; Yang & Cobb, 1995).

However, U.S. students are weak in such an understanding based on the series of data from the Trend in International Mathematics and Science Study (TIMSS) (Mullis, et al., 1997; Mullis, Martin, & Foy, 2005; Mullis, et al., 2000). Chinese students show strong performance in understanding numbers and its properties at various grade levels in the international comparisons

(Aunio, et al., 2006; Ee, Wong, & Aunio, 2006; Miura, Okamoto, Vlahovic-Stetic, Kim, & Han, 1999) and the direct comparison between Chinese and U.S. students (Benjamin, 2006; Huntsinger, Jose, Liaw, & Ching, 1997; Miller, Smith, Zhu, & Zhang, 1995; Zhou, Peverly, & Lin, 2005).

Although student mathematics performance in a particular country can be influenced by various factors such as students' cultural and racial backgrounds, language, school structure in which teaching practice is situated, teaching culture, and teachers knowledge in the two countries (Wang & Lin, 2005), mathematics curriculum in the country is considered as one of the important factors influencing students' learning (Hood & Parker, 1994; Schmidt, McKnight, Cogan, Jakwerth, & Houang, 1999). Mathematics textbooks, an important component of intended curriculum, are assumed to influence the enacted curriculum that teachers developed and the modes of students' mathematics learning, which in turn can shape student performance differences between the two countries (Li, Ding, Capraro, & Capraro, 2008; Li, 2000; Ma, 1999; Wang, 2001). Studies show that three-quarters of the eighth grade teachers in the US used one textbook all or most of the time and covered at least three-quarters of the textbook in a given year, which suggests that teachers may study their mathematics curriculum intensely for teaching (National Research Council, 2001), while Chinese teachers relied more heavily on their mathematics curricular knowledge and relevant examination for planning and teaching mathematics lessons (Cai & Wang, 2010).

The existing curriculum comparisons across different countries based on the curriculum study of the Third International Mathematics and Science Study (Schmidt, Houang, & Cogan, 2002) revealed that U.S. mathematics curriculum is “a mile wide, an inch deep” as it covers too many topics that are often incoherent and repeated year after year with weak intellectual challenge for the students (p. 3). However, Chinese textbooks were found to cover fewer topics and show a strong logical connection between different concepts based on the comparison of Chinese and U.S. mathematics textbooks (Li, 2000; Li, et al., 2008; Zhou & Peverly, 2005).

Other research (Sood & Jitendra, 2007) compared three traditional and one reformed textbooks in the US with each other to see the extent to which they reflect the ideas of effective mathematics teaching. It found that the U.S. traditional textbooks provided more opportunities for students to learn number relationships, more direct and explicit instructions, and more common feedback while the reformed textbook is moving towards focusing more on real-world connections, relational understanding, various models to develop number sense concepts, and hands-on activities to enhance children engagement.

However, the existing studies on mathematics curriculum comparison focused either on general patterns of curriculum differences or paid attention to the similarities and differences of curriculum change patterns within a country. Few studies paid attention to the representation of big mathematics ideas specifically in curriculum material across different countries; moreover, almost no studies have identified the change of these representations and patterns from a comparative perspective, especially the change of some important concepts, such as number sense and number understanding, the learning of which starts at an early grade level and extends to later schooling and has a strong relationship with the learning of other mathematics concepts. This study is designed to compare the Chinese and U.S. first-grade traditional and reformed mathematics textbooks within each country and across the two countries for their similarities and differences in representing number sense properties, relationship of number sense to other mathematics ideas, and to the real world and collaboration.

We expect that this comparison will provide useful empirical data for further examination of the following assumption embedded in the U.S. mathematics reform: The quality of students' early number sense development influences their subsequent understanding of number (Soder, 1990) and other mathematics content (Gersten & Chard, 1999), and helps build a strong mathematical foundation for their school success later (Sood & Jitendra, 2007); while earlier curriculum material with a strong focus on number sense can provide important opportunity in shaping student learning in the above area (Howe & Epp, 2006).

In particular, this study explores specifically the following two research questions. First, what are the similarities and differences between Chinese and U.S. first-grade textbooks in representing various properties of number sense, different relationships of number sense to other mathematics content and to real world and collaborations? And second, what are trends of change between the traditional and reformed textbooks within each country and across the two countries in representing the above properties and connections of number sense?

Theoretical Framework

Three theoretical assumptions guided the inquiry of this study and the design of a textbook coding system. The first assumption is that number sense encompasses a person's conceptual understanding of numbers and numerical relationships and his or her ability to solve daily life problems using this understanding, and it has various properties (Howden, 1989; Reys, et al., 1999; Sowder & Schappelle, 1994; Yang & Cobb, 1995). These properties include the following.

1. The meaning and representation of numbers with different levels of abstraction that children need to master (Faulkner, 2009; Reys, et al., 1999; Sowder & Schappelle, 1994).
2. Sequential, skip, and reverse counting of numbers that children are able to conduct (Ghazali, Rahman, Ismail, Idros, & Salleh, 2003; Jordan, 2007).
3. Various number patterns and relationships that children can identify through comparison, decomposition, and re-compositions of numbers (Reys, et al., 1999; Sowder & Schappelle, 1994; Tsao, 2005; Yang & Cobb, 1995).
4. Their understanding of place value, the value of a number based on the place or position of its digits, including their understanding of based-ten, the value of any number in the decimal numerical system in which the position of each digit denotes power of ten multiplied with that digit and each position has a value ten times that of the position

to its right starting from zero (Faulkner, 2009; Ghazali, et al., 2003; Reys, et al., 1999; Sowder & Schappelle, 1994).

5. Different number quantities, magnitudes, and measurements that children are able to compare and judge (Faulkner, 2009; Reys, et al., 1999; Sowder & Schappelle, 1994; Yang & Cobb, 1995).
6. Their mental estimation of various quantities using numbers (Sowder & Schappelle, 1994; Yang & Cobb, 1995). These assumption of number sense and its properties becomes the conceptual base for us to code the properties of number sense in the textbooks of the two countries.

The second conceptual assumption is that children's number sense development is closely related to their learning of other mathematics concepts (Greeno, 1991). Not only do such relationships help children develop a stronger conceptual understanding about number, but it also supports their learning of other mathematics concepts (Reys, et al., 1999; Sood & Jitendra, 2007). Number sense is presumably related to such mathematics content as measurement, data representation, and number patterns (Sood & Jitendra, 2007), and operations in addition, subtraction, multiplication, and division (Fennell & Landis, 1994; Ghazali, et al., 2003; Reys, et al., 1999; Sood & Jitendra, 2007). Thus, it is important to understand the extent to which the textbooks in each country represent these relationships of number sense to other mathematics concepts. Such an assumption of the connections of number sense to other mathematics concepts becomes the conceptual base for us to design the coding system to code the connections of number sense to other relevant mathematics concepts in the textbooks of the two countries.

The third assumption is that the understanding of number sense and its relationships is a developmental process in which the following needs of children in mathematics learning should be carefully considered (Bruer, 1997; Ginsburg, 1997; Lock, 1996; Louange, 2007). First, children need to make personal connections of number sense to real-world situations, tools, and objects with which they are familiar (Jordan, 2007; McIntosh, Reys, & Reys, 1992). Second, earlier number sense development requires children's informal and formal interactions with peers (Robinson, Menchetti, & Torgesen, 2002; Sowder & Schappelle, 1994; Yang, 2000) and in rich play activities (Griffin, 2004; Louange, 2007). Thus, the analysis of textbooks from the two countries needs to focus on whether and to what extent these textbooks are adapted to children's needs in connecting number sense to children's real-world experience and their interaction with their peers. Such an assumption is the conceptual base for us to design the coding system to code the relationship of learning number sense to the real-world and peer interaction in the textbooks of the two countries.

Research Methods

Textbook Selection and Data Sources

Among the mathematics textbooks used in the elementary schools in the two countries, we chose the following two Chinese and two U.S. first-grade mathematics textbooks for this study. Of the two Chinese textbooks, one has been popularly used in Chinese schools for many years (Liu, Zhang, Lu, Yang, & Tao, 2001), which is chosen as the Chinese traditional textbook;

while the other is newly developed based on the ideas of mathematics education reform in China (Research Group of National Mathematics Curriculum Standards for Compulsory Education, 2001), which is used as Chinese reformed textbook. The two U.S. textbooks are selected in the same way as the Chinese textbooks, with one popular textbook used widely for many years in many schools (Charles, et al., 2004) and the other new textbook designed by following the ideas of mathematics education reform in the US (Technical Education Research Centers, 2004). We chose these textbooks for the consideration that the textbooks for this study need to have a wide impact on student learning in the country and demonstrate the direction towards which the mathematics education in each country is moving.

To select data from each selected textbook for our analysis, we read each textbook carefully for individual lessons relevant to number sense by following our theoretical framework. These lessons from each of the four textbooks in the two countries are used as the major sources of data for the analysis in the study.

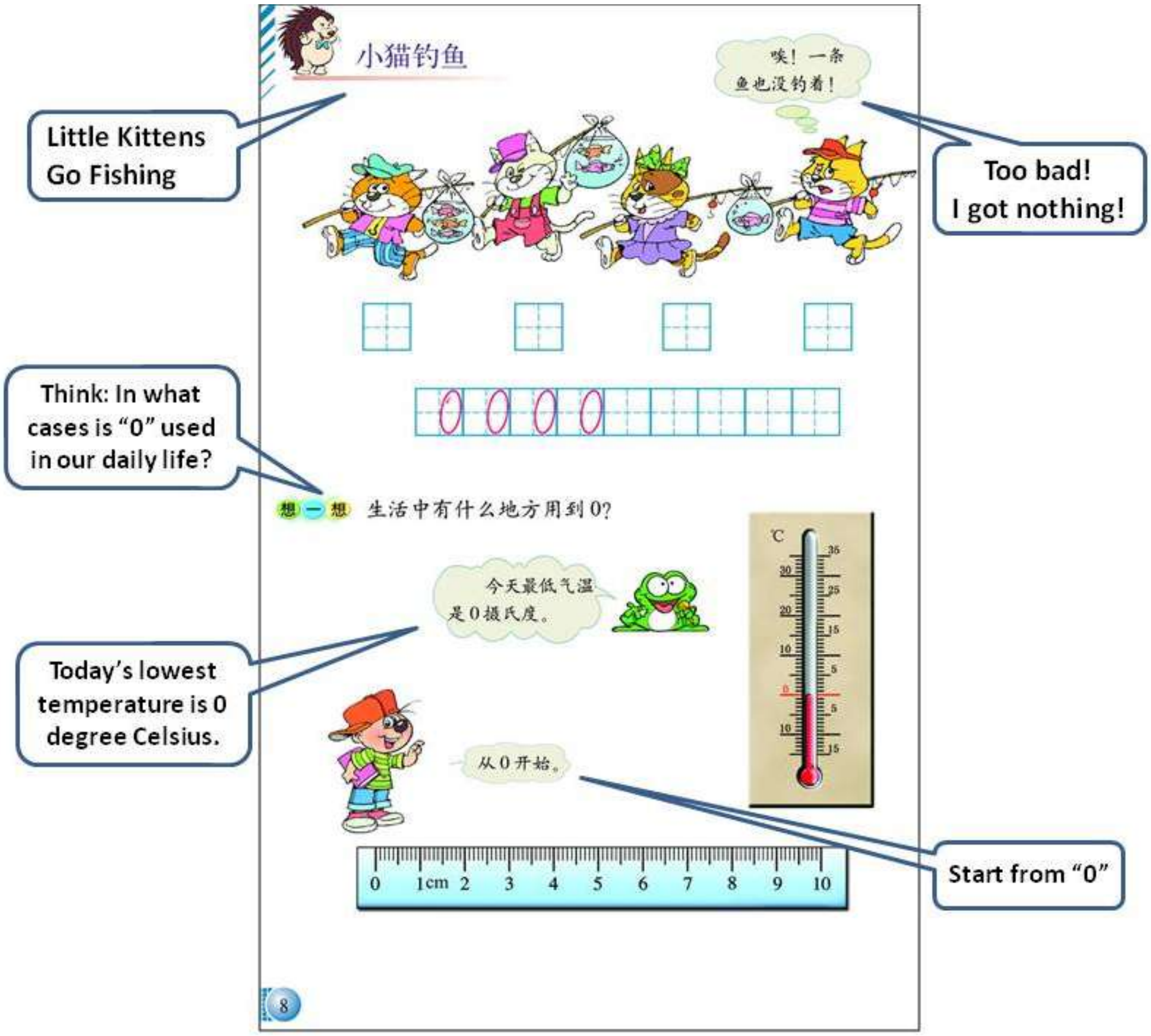
Data Analysis, Coding System, and Procedures

To answer our research questions, we conducted the following phases of analyses. First, we read each lesson relevant to number sense from each textbook carefully for individual meaning chunks. The meaning chunk here indicates a specific portion of the page in the textbook, such as a description, an example, a picture, a problem, or a series of questions, that serves a particular purpose in the lesson where it comes. Each meaning chunk identified in each lesson of number sense was then coded and recoded following the coding system.

This coding system includes the following three dimensions consistent with each of the three theoretical assumptions for this study. The first dimension is directly related to different number sense properties as developed under the first theoretical assumption about what number sense involves (Faulkner, 2009; Ghazali et al., 2003; Reys et al., 1999; Sowder & Schappelle, 1994; Yang & Cobb, 1995), which is used to capture the range and depth of particular number sense properties in each meaning chunk. Under this dimension are six sub-categories:

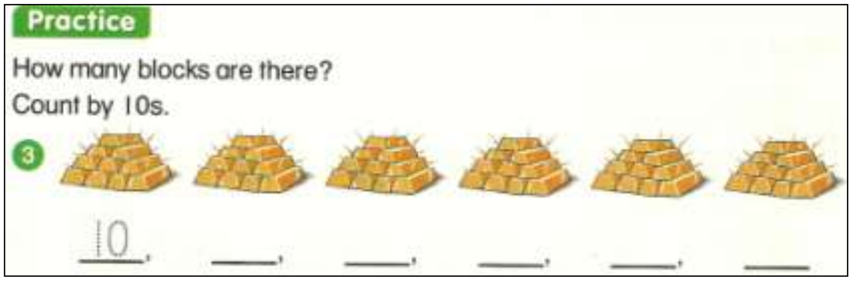
1) *Number meaning and representation* including zero, ordinal, and cardinal numbers (Faulkner, 2009; Reys, et al., 1999; Sowder & Schappelle, 1994). The meaning chunk example shown in Figure 1 below from the Chinese reformed textbook is coded as number meaning and representation as it helps students understand three different meanings of number zero as quantity, dividing point, and starting point.

Figure 1: Meaning Chunk Example of Number Sense Property-Meaning of Zero



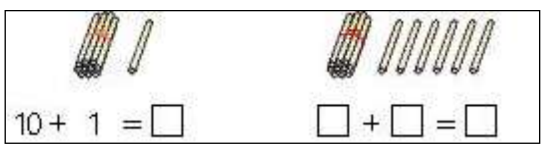
2) *Various ways of counting* including sequential, reverse, and/or skip counting as suggested in the relevant literature (Ghazali et al., 2003; Jordan, 2007). Figure 2 is a meaning chunk example from the U.S. traditional textbook coded as skip counting since it helps student learn how to count by 10s.

Figure 2: Meaning Chunk Example of Number Sense Property-Skip Counting



3) *Understanding of place value and/or base ten concept* (Faulkner, 2009; Ghazali et al., 2003; Reys, et al., 1999; Sowder & Schappelle, 1994). In Figure 3, the meaning chunk from U.S. traditional textbook is coded as base-ten concept as students are asked to demonstrate their understanding about the relationship between 10 and 1 here.

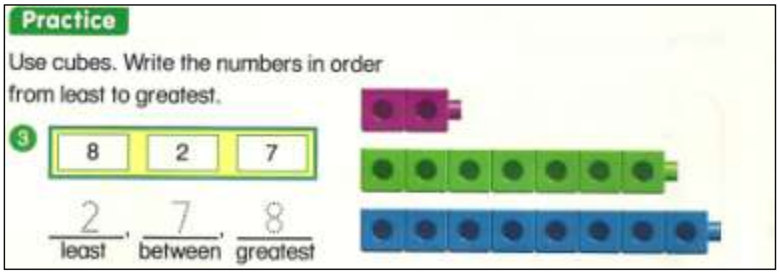
Figure 3: Meaning Chunk Example of Number Sense Property-Represent Base Ten Concept



4) *Number composition* including decomposing and/or recomposing numbers at different levels (Reys, et al., 1999; Sowder & Schappelle, 1994; Yang & Cobb, 1995). For example, when students are asked to provide all the possible ways to make number 8 using addition in a meaning chunk, the meaning chunk would be coded as number composition.
5) *Number relationship* in quantities, magnitudes, and measurements at greater than,

less than, and/or equivalence levels (Faulkner, 2009; Reys, et al., 1999; Sowder & Schappelle, 1994; Yang & Cobb, 1995). In Figure 4 below, the meaning chunk example from the U.S. traditional textbook is assigned a number relationship code because it asks students to compare which number is bigger or smaller than the other two.

Figure 4: Meaning Chunk Example of Number Sense Property-Number Relationships



6) *Estimation of quantity* at certain, likely, less likely, and impossible levels (Sowder & Schappelle, 1994; Yang & Cobb, 1995). When students are asked to tell the total number of cubes in a bowl without actually counting them in a meaning chunk, this chunk is coded as estimation of quantity as students are asked to estimate the likely number of cubes.

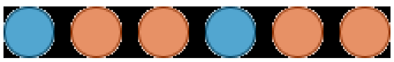
The second dimension is about the number sense connections to other mathematics content and concepts (Greeno, 1991). This dimension of coding is developed to check the extent and complexity that each meaning chunk connects number sense to other mathematics concepts. Within this dimension are four subcategories as suggested:

1) *Connection to measurements* at length, height, weight, capacity, and/or size levels

(Sood & Jitendra, 2007). For example, when students are asked to use number measures to represent the length of an object in a meaning chunk, the chunk would be coded as number sense connection to measurements.

2) *Connection to patterns* in identifying, extending, creating, and/or translating patterns (Sood & Jitendra, 2007). For example, the meaning chunk in Figure 5 below from U.S. traditional textbook is coded as the connection of number sense to pattern because students need to use number sense to identify what colored ball will follow the two orange balls based on the patterns shown.

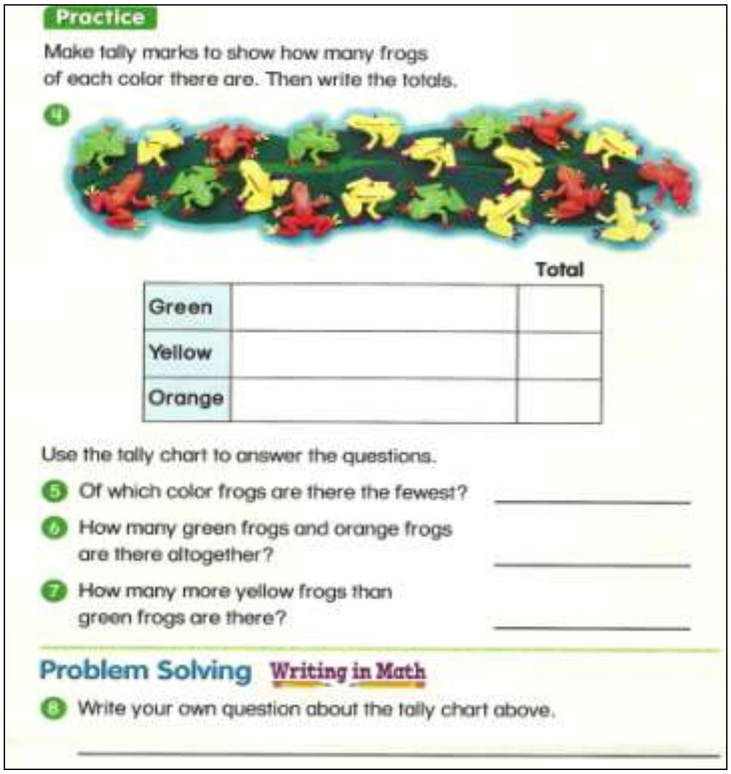
Figure 5: Meaning Chunk Example of Number Sense Connection to Patterns



3) *Connection to operations* at addition, subtraction, and mixed operation levels (Fennell & Landis, 1994; Ghazali, et al., 2003; Reys, et al., 1999). The example shown in Figure 3 is also coded as connection to operation as students are asked to calculate addition using number sense property, base-ten concept.

4) *Connection to data analysis* including translating numbers to graph, comparing, judging, and raising question about data as suggested (Sood & Jitendra, 2007). The meaning chunk example in Figure 6 from U.S. traditional textbook is assigned a code of the number sense connection to data analysis because it asks students to translate numbers from the picture to a tally graph.

Figure 6: Meaning Chunk Example of Number Sense Connection to Data Analysis



Practice

Make tally marks to show how many frogs of each color there are. Then write the totals.

4

		Total
Green		
Yellow		
Orange		

Use the tally chart to answer the questions.

5 Of which color frogs are there the fewest? ______

6 How many green frogs and orange frogs are there altogether? ______

7 How many more yellow frogs than green frogs are there? ______

Problem Solving Writing in Math

8 Write your own question about the tally chart above.

______

The third dimension is number sense connections to the real world (Jordan, 2007; McIntosh et al., 1992) and collaboration (Robinson et al., 2002; Sowder & Schappelle, 1994; Yang, 2000). This dimension is developed to examine whether each meaning chunk is presented in relation to the real-world objects, tools, situations, and/or ask students to work with their peers. Under this dimension are four sub-categories of codes:

1) *Connection of number sense to real-world items* as suggested (Jordan, 2007). Whenever a concrete, nonverbal object, picture, and/or cartoon is used in a meaning chunk to represent numbers or amount, the code of connection of number sense to real-world items is coded.

2) *Connection of number sense to read world situations* (Jordan, 2007; McIntosh et al., 1992). Whenever a real-world situation or event, for example, a story and a context, is used to situate the concepts related to number sense, the code of connection of number sense to real-world situation is assigned.
3) *Connection of number sense to real-world tools* as suggested in the literature (McIntosh et al., 1992). This category of coding is developed to capture the relationship between number sense and real-world tools, such as clock, calendar, thermometer, currency, and other tools in any meaning chunk.
4) *Connection of number sense to collaborations* (Robinson et al., 2002; Sowder & Schappelle, 1994). Whenever students are asked to play or work with other peers in a meaning chunk, the meaning chunk is coded as the number sense connection to collaboration.

Reliability of coding is checked between the two researchers and an external, independent coder, who is literate in both Chinese and English. The coder was trained to gain a thorough understanding of the coding system that is developed on the basis of the theoretical framework of this study. She independently coded 15% of each textbook before embarking on the coding process as the researchers did. The coding results by the independent coder were compared to that obtained by the researchers. The inter-rater agreement between the different coders reaches 95%. Any inconsistent coding emerged in the whole coding process was discussed among the researchers and the coder in order to ensure the consistent reliability of all codes assigned.

After coding and recoding of all the meaning chunks in all the lessons related to number sense from each textbook, the frequency of each code is calculated for each textbook. The frequencies of each code from textbooks are compared within each country and across the two countries to see their similarities and differences.

To test whether the differences in frequencies from the above comparisons are statistically significant, Chi Square statistics is applied in the following steps as suggested since the data of this study are nonparametric (Shavelson, 1996). First, we subtract dimension or category codes from their corresponding total codes to obtain the necessary data for each comparison group. Then, the data in each comparison group are weighted since the total codes for each group are different. Lastly, the Chi Square test statistics are calculated for each group through the cross tabulations procedure. In the above steps, SPSS software was used.

Results of Study

This comparative study yielded many interesting findings. In this section, first of all, we present the similarities and differences among the four textbooks in light of the ratio of their total meaning chunks to coding counts within each country and across the two countries. In the second place, we show how codes are distributed among the three dimensions of the textbooks within each country and across the two countries. Lastly, we demonstrate similarities and differences within each of the three dimensions of number sense among the textbooks within each country and across the two countries.

Ratio of Meaning Chunks to Coding Counts

The comparison of the ratios of the meaning chunks to coding counts among the four textbooks allows us to see whether and to what extent a particular country's textbook is fragmented or concentrated in designing and ranging its content. Our analysis along this line shows that the U.S. textbooks have a substantial change from the traditional to reformed textbook in representing number sense and relevant content. In contrast, the Chinese textbooks have much smaller changes in this area. Based on Table 1, U.S. traditional and reformed textbooks have the greatest and smallest ratios of their numbers of meaning chunks to coding counts, which are 1:2.98 and 1:7.28 respectively. The ratios of meaning chunks to coding counts in the Chinese traditional and reformed texts are very close, which are 1:4.18 and 1:5.23 correspondingly. This finding suggests that the U.S. reformed textbook moves towards the direction of having fewer topics that contain more content in terms of number sense properties, the connection of number sense to other mathematics concepts, and to real world and collaboration. This feature of having fewer topics but more content in each topic in U.S. textbook design resembles that of its Chinese counterparts.

Table 1
Ratios of Meaning Chunks to Coding Counts in Four Textbooks

	Meaning Chunks	Coding Counts	Chunks & Counts Ratio
US Traditional Textbook	2185	6519	1:2.98
US Reformed Textbook	89	648	1:7.28
Chinese Traditional Textbook	610	2552	1:4.18
Chinese Reformed Textbook	534	2795	1:5.23

Distribution of Codes in Three Dimensions

Our comparative analysis of the code distributions of the four textbooks in three dimensions also reveals several findings. First, Chinese traditional and reformed textbooks pay more attention to the connection of number sense to other mathematics concepts with 50% and 63% of its total codes respectively than its U.S. counterparts, which have correspondingly 36% and 47% based on Table 2. The results of Chi-square tests on the differences between Chinese traditional and U.S. traditional, and between Chinese reformed and U.S. reformed textbooks are all statistically significant at 99% confidence level.

Table 2

Distributions of Different Categories of Codes in Four Textbooks

Category	Textbook	Category Codes	Total codes	%	US tradition		US reform		Chinese tradition		Chinese reform	
					χ^2	ρ	χ^2	ρ	χ^2	ρ	χ^2	ρ
Connection to other	US tradition	2,338	6,519	36%	N/A	N/A	30.92	.000**	159.64	.000**	589.36	.000**
math content	US reform	304	648	47%	30.92	.000**	N/A	N/A	2.39	.122	57.55	.000**
	Chinese tradition	1,284	2,552	50%	159.64	.000**	2.39	.122	N/A	N/A	89.16	.000**
	Chinese reform	1,764	2,795	63%	589.36	.000**	57.55	.000**	89.16	.000**	N/A	N/A
Number sense	US tradition	2,767	6,519	42%	N/A	N/A	11.19	.001**	57.41	.000**	356.04	.000**
Properties	US reform	231	648	36%	11.19	.001**	N/A	N/A	.80	.370	53.48	.000**
	Chinese tradition	862	2,552	34%	57.41	.000**	.80	.370	N/A	N/A	93.70	.000**
	Chinese reform	613	2,795	22%	356.04	.000**	53.48	.000**	93.70	.000**	N/A	N/A
Connection to world	US tradition	1,414	6,519	22%	N/A	N/A	6.36	.012*	38.22	.000**	56.16	.000**
& collaboration	US reform	113	648	17%	6.36	.012*	N/A	N/A	.89	.346	2.49	.115
	Chinese tradition	406	2,552	16%	38.22	.000**	.89	.346	N/A	N/A	0.93	.335
	Chinese reform	418	2,795	15%	56.16	.000**	2.49	.115	0.93	.335	N/A	N/A

Note. *p<.05; **p<.01.

Additionally, the reformed textbooks become more focused on the connection of number sense to other mathematics content than their traditional counterpart in each county. This finding is clearly demonstrated in Table 2 in that Chinese and U.S. reformed textbooks respectively have about a 13% and a 11% increase than their traditional textbooks in this dimension. The Chi-square test results suggest that the differences between the two Chinese textbooks and between the two U.S. textbooks are both statistically significant at the 99% confidence level.

Furthermore, U.S. traditional and reformed textbooks pay much more attention to number sense properties than the Chinese traditional and reformed textbooks. U.S. traditional and reformed have about 42% and 36% of its total code counts respectively while Chinese traditional and reformed textbooks have 34% and 22% of the codes correspondingly for number sense properties as shown in Table 2. The Chi-square tests of the differences between U.S. and Chinese traditional textbooks and between U.S. and Chinese reformed textbooks both show statistically significant results at the 99% confidence level. However, within-country comparisons demonstrated that textbooks in each country decrease their attention to number sense properties with a 12% decrease for China and a 6% decrease for the US from its traditional to reformed textbooks. The results of testing differences between two U.S. textbooks and between two Chinese textbooks both are statistically significant at the 99% confidence level.

Lastly, Chinese traditional and reformed textbooks have more equal attention to the connection of number sense learning to the real world and collaboration with 16% and 15% of its total codes respectively. In comparison, the U.S. textbooks decrease their attention to this dimension from 22% in the traditional textbook to 17% in the reformed textbook. The Chi-square tests on the differences between the two Chinese textbooks and between U.S. and Chinese reformed textbooks are not statistically significant but the differences between the two U.S. textbooks and between U.S. and Chinese traditional textbooks are significant at the 95% and 99% confidence level respectively. This finding suggests that the U.S. textbooks are moving closer towards the direction of Chinese textbooks regarding the connection of number sense to the real world and collaboration.

Distribution of Number Sense Property Codes

Our analyses of code distribution about number sense within each country and across the two countries lead to several findings. First of all, both U.S. textbooks pay substantially more attention to the counting properties of number sense than their Chinese counterpart but the textbooks in each country reduce such a focus. Table 3 indicates that U.S. traditional and reformed textbooks have about 61% and 44.6% of their total codes in the category respectively, which is greater than the 47.9% and 35.2% of Chinese traditional and reform textbooks respectively. The results of Chi-square tests on the differences between Chinese and U.S. traditional textbooks and between Chinese and U.S. reformed textbooks are both statistically significant at the 99% confidence level. It is also clear that Chinese textbooks reduce 12.7% of their total codes from its traditional to reformed textbooks in the counting property of number sense while U.S. textbooks reduce 16.4%. The test on the difference between two Chinese textbooks is significant at the 99% confidence level and the difference between the two U.S. textbooks is significant at the 95% confidence level.

Table 3
Distributions of Different Codes within Number Sense Properties

Category	Textbook	Category Codes	Total Codes	%	US tradition		US reform		Chinese tradition		Chinese reform	
					χ^2	ρ	χ^2	ρ	χ^2	ρ	χ^2	ρ
Various ways of counting	US tradition	1,687	2,767	61.0%	N/A	N/A	23.78	.000**	45.96	.000**	135.06	.000**
	US reform	103	231	44.6%	23.78	.000**	N/A	N/A	.81	.369	6.24	.012*
	Chinese tradition	413	862	47.9%	45.96	.000**	.81	.369	N/A	N/A	23.53	.000**
	Chinese reform	216	613	35.2%	135.06	.000**	6.24	.012*	23.53	.000**	N/A	N/A
Number meaning & representation	US tradition	32	2,767	1.2%	N/A	N/A	70.39	.000**	238.53	.000**	272.45	.000**
	US reform	20	231	8.7%	70.39	.000**	N/A	N/A	3.07	.080	5.88	.015*
	Chinese tradition	111	862	12.9%	238.53	.000**	3.07	.080	N/A	N/A	1.37	.242
	Chinese reform	92	613	15.0%	272.45	.000**	5.88	.015*	1.37	.242	N/A	N/A
Place value & base ten concept	US tradition	174	2,767	6.3%	N/A	N/A	11.35	.001**	55.71	.000**	104.46	.000**
	US reform	2	231	0.9%	11.35	.001**	N/A	N/A	32.31	.000**	45.99	.000**
	Chinese tradition	123	862	14.3%	55.71	.000**	32.31	.000**	N/A	N/A	6.10	.014*
	Chinese reform	117	613	19.1%	104.46	.000**	45.99	.000**	6.10	.014*	N/A	N/A
Number relationship	US tradition	436	2,767	15.8%	N/A	N/A	4.68	.031*	.16	.693	14.03	.000**
	US reform	49	231	21.2%	4.68	.031*	N/A	N/A	4.79	.029*	.07	.799
	Chinese tradition	131	862	15.2%	.16	.693	4.79	.029*	N/A	N/A	11.29	.001**
	Chinese reform	135	613	22.0%	14.03	.000**	.07	.799	11.29	.001**	N/A	N/A
Different number composition	US tradition	207	2,767	7.5%	N/A	N/A	57.76	.000**	2.57	.109	2.37	.124
	US reform	51	231	22.1%	57.76	.000**	N/A	N/A	28.99	.000**	49.12	.000**
	Chinese tradition	79	862	9.2%	2.57	.109	28.99	.000**	N/A	N/A	6.00	.014*
	Chinese reform	35	613	5.7%	2.37	.124	49.12	.000**	6.00	.014*	N/A	N/A
Number estimation	US tradition	231	2,767	8.3%	N/A	N/A	9.69	.002**	65.23	.000**	21.54	.000**
	US reform	6	231	2.6%	9.69	.002**	N/A	N/A	7.44	.006**	.07	.792
	Chinese tradition	5	862	0.6%	65.23	.000**	7.44	.006**	N/A	N/A	12.96	.000**
	Chinese reform	18	613	2.9%	21.54	.000**	.07	.792	12.96	.000**	N/A	N/A

Note. *p<.05; **p<.01.

In addition, Chinese textbooks pay substantially more attention than their U.S. counterparts to the properties of number meaning and representations. According to Table 3, Chinese traditional and reformed textbooks devote 12.9% and 15% of the total number sense codes respectively to the property of meaning and representations while U.S. traditional and reformed texts have only 1.2% and 8.7% correspondingly. The Chi-square tests show that the difference between Chinese and U.S. reformed textbooks is statistically significant at the 95% confidence level and the difference between Chinese and U.S. traditional textbooks is significant at the 99% confidence level.

Moreover, the two Chinese textbooks devote more attention to the number sense property of place-value and base-ten concept than their U.S. counterparts. Moreover, Chinese textbooks increase while U.S. textbooks decrease their attention to this number sense property from their traditional to reformed textbooks in each country. Based on Table 3, Chinese traditional and reformed textbooks have 14.3% and 19.1% of their total codes of number sense respectively focusing on the property of place-value and based-ten concept while their U.S. counterparts have only 6.3% and 0.9% respectively. The Chi-square tests show that the differences between U.S. and Chinese traditional textbooks and between U.S. and Chinese reformed textbooks are both statistically significant at the 99% confidence level. The difference between the two Chinese textbooks is significant at the 95% confidence level and the difference between the two U.S. textbooks is significant at the 99% confidence level.

Furthermore, both Chinese and U.S. textbooks attribute substantial and increased attention to number relationship property relevant to understanding of quantities, magnitudes, and measurements at greater than, less than, and/or equivalency levels. As shown in Table 3, Chinese traditional and reformed textbooks have 15.2% and 22.0% of their total codes in number sense respectively for number relationship property with 6.8% increase while U.S. traditional and reformed textbooks have 15.8% and 21.2% correspondently with 5.4% code increase. The Chi-square test on the difference between two U.S. textbooks is statistically significant at the 95% confidence level and the difference between two Chinese textbooks is at the 99% confidence level. However, there are no statistically significant differences between the Chinese and U.S. traditional textbooks and between the Chinese and the U.S. reformed textbooks.

Furthermore, U.S. textbooks increase their attention while Chinese textbooks decrease their already low attention on the number composition property. Based on Table 3, the U.S. traditional textbook has 7.5% while its reformed textbook has 22.1% of its total number sense codes for the number composition property. In contrast, Chinese traditional and reformed textbooks have 9.2% and 5.7% in the category respectively. The Chi-square test on the difference between the two U.S. textbooks show statistically significant result at the 99% confidence level and the difference between the two Chinese textbooks is statistically significant at the 95% confidence level. The difference between the two traditional textbooks across the two countries is not statistically significant while the difference between the two reformed textbooks across the two countries is significant at the 99% confidence level.

Lastly, both Chinese and U.S. textbooks pay less attention to the estimation property of number sense. Based on Table 3, each textbook attributes less than 9% of its total number sense codes to this property.

Distribution of Codes in Number Sense Connection to Other Mathematics Concepts

Our analyses of the attention of the textbooks in two countries to the connection of number sense to other mathematics concepts reveal the following findings. In the first place, Chinese textbooks pay more attention to the connection of number sense to operations than their

U.S. counterparts. As shown in Table 4 below, Chinese traditional and reformed textbooks attribute 84.0% and 90.5% of their total codes in this dimension to the number sense connection to operation. In contrast, U.S. traditional and reformed textbooks have 75.4% and 53.3% of their total codes respectively for the connection to operations. The Chi-square tests of the differences between Chinese and U.S. reformed textbooks and between their traditional textbooks both show statistically significant results at the 99% confidence level.

Table 4

Distributions of Different Codes within Connections to Other Mathematics Concepts

Category	Textbook	Category Codes	Total Codes	%	US tradition		US reform		Chinese tradition		Chinese reform	
					χ^2	ρ	χ^2	ρ	χ^2	ρ	χ^2	ρ
Connection to operations	US tradition	1,754	2,338	75.0%	N/A	N/A	63.76	.000**	38.80	.000**	160.40	.000**
	US reform	162	304	53.3%	63.76	.000**	N/A	N/A	135.09	.000**	281.40	.000**
	Chinese tradition	1,078	1,284	84.0%	38.80	.000**	135.09	.000**	N/A	N/A	29.34	.000**
	Chinese reform	1,596	1,764	90.5%	160.40	.000**	281.40	.000**	29.34	.000**	N/A	N/A
Connection to measurements	US tradition	296	2,338	12.7%	N/A	N/A	.06	.807	72.23	.000**	59.53	.000**
	US reform	40	304	13.2%	.06	.807	N/A	N/A	38.40	.000**	24.59	.000**
	Chinese tradition	51	1,284	4.0%	72.23	.000**	38.40	.000**	N/A	N/A	3.75	.053
	Chinese reform	97	1,764	5.5%	59.53	.000**	24.59	.000**	3.75	.053	N/A	N/A
Connection to patterns	US tradition	127	2,338	5.4%	N/A	N/A	31.58	.000**	8.33	.004**	78.30	.000**
	US reform	42	304	13.8%	31.58	.000**	N/A	N/A	10.62	.001**	196.24	.000**
	Chinese tradition	101	1,284	7.9%	8.33	.004**	10.62	.001**	N/A	N/A	118.41	.000**
	Chinese reform	8	1,764	0.5%	78.30	.000**	196.24	.000**	118.41	.000**	N/A	N/A
Connection to data analysis	US tradition	161	2,338	6.9%	N/A	N/A	57.96	.000**	10.67	.001**	21.40	.000**
	US reform	60	304	19.7%	57.96	.000**	N/A	N/A	88.98	.000**	121.14	.000**
	Chinese tradition	54	1,284	4.2%	10.67	.001**	88.98	.000**	N/A	N/A	.81	.368
	Chinese reform	63	1,764	3.6%	21.40	.000**	121.14	.000**	.81	.368	N/A	N/A

Note. *p<.05; **p<.01.

Secondly, Chinese textbooks increase while U.S. textbooks decrease their attention to the connection of number sense to operations from its traditional to reformed textbooks in each country. As shown in Table 4, Chinese textbooks have about 6.5% increase of its total codes while U.S. textbooks have about 21.7% decrease of its total codes in this connection. The Chi-square tests show that the differences between the two Chinese textbooks and between the two U.S. textbooks are both statistically significant at the 99% confidence level. This finding suggests that Chinese and U.S. textbooks moved in opposite directions in their number sense connection to operation.

Additionally, U.S. textbooks devote substantially more attention to the connection of number sense to measurements than their Chinese counterparts and such attention holds consistent within each country. The U.S. textbooks have 12.7% and 13.2% of their total codes in this dimension for the connection to measurements while Chinese traditional and reformed textbooks have 4.0% and 5.5% respectively as shown in Table 4. The Chi-square tests on the difference between Chinese and U.S. traditional textbooks and the difference between Chinese and U.S. reformed textbooks both demonstrate statistically significant results at the 99% confidence level. However, the differences between the two U.S. textbooks and between the two Chinese textbooks are both non-significant.

Furthermore, Chinese textbooks decrease while U.S. textbooks increase their attention to the connection of number sense to patterns, which suggest that textbooks in both countries move in opposite directions in this area. Based on Table 4, U.S. traditional and reformed textbooks have 5.4% and 13.8% of its total codes in this dimension for the connection of number sense to patterns respectively, which counts as 8.4% of increase. In contrast, Chinese traditional and reformed textbooks have 7.9% and 0.5% with 7.4% decrease. The Chi-square tests on the differences between two US textbooks and between two Chinese textbooks both show statistically significant results at the 99% confidence level. The Chi-square tests on differences between Chinese and U.S. traditional textbooks and between Chinese and U.S. reformed textbooks both indicate statistically significant results at the 99% confidence level.

Lastly, U.S. textbooks pay more attention to the connection of number sense to data analysis than their Chinese counterparts and increase their attention to this connection from its traditional textbook to reformed textbook. As demonstrated in Table 4, U.S. traditional textbook has 6.9% while its reformed textbook devotes 19.7% of its total codes in this dimension to the connection of number sense to data analysis, which counted as 12.8% increase. However, both Chinese textbooks have only less than 5% of its total topics in this aspect. The Chi-square tests on the differences between Chinese and U.S. traditional textbooks and between Chinese and U.S. reformed textbooks both show significant results at the 99% confidence level. The difference between the two U.S. textbooks is also statistically significant at the 99% confidence level while the difference between the two Chinese textbooks is not significant.

Distribution of Codes in Number Sense Connection to Real World and Collaboration

The attention of the four textbooks to the number sense connection to real world and collaboration is also analyzed, which revealed the following similarities and differences. First of all, U.S. textbooks decrease their attention from its traditional to reformed textbooks while Chinese textbooks pay consistent and substantial attention to the connection of number sense to real-world items. As shown in Table 5, U.S. traditional and reformed textbooks have 51.6% and 30.1% of their total codes in this dimension for the connection of number sense to real-world objects respectively and there is a 21.5% decrease from its traditional to its reformed textbook in this aspect, while Chinese traditional and reformed textbooks devote 46.8% and 52.2 % of the total codes to this connection correspondingly. The Chi-square test on the difference between the two U.S. textbooks shows statistically significant results at the 99% confidence level while the difference between the two Chinese textbooks is not significant. The difference between Chinese and U.S. traditional textbooks is non-significant but the difference between Chinese and U.S. reformed textbooks is significant at the 95% confidence level. This finding suggests that the difference between U.S. textbooks becomes increasingly greater regarding the connection of number sense to real-world objects.

Table 5

Distributions of Different Codes within Connections to Real World and Collaboration

Category	Textbook	Category Codes	Total Codes	%	US tradition		US reform		Chinese tradition		Chinese reform	
					χ^2	ρ	χ^2	ρ	χ^2	ρ	χ^2	ρ
Connection to real life objects	US tradition	730	1,414	51.6%	N/A	N/A	19.42	.000**	2.94	.086	.04	.850
	US reform	34	113	30.1%	19.42	.000**	N/A	N/A	10.06	.002**	17.37	.000*
	Chinese tradition	190	406	46.8%	2.94	.086	10.06	.002**	N/A	N/A	2.36	.124
	Chinese reform	218	418	52.2%	.04	.850	17.37	.000*	2.36	.124	N/A	N/A
Connection to real world tools	US tradition	541	1,414	38.3%	N/A	N/A	13.01	.000**	12.84	.000**	61.14	.000**
	US reform	24	113	21.2%	13.01	.000**	N/A	N/A	2.41	.120	.74	.390
	Chinese tradition	116	406	28.6%	12.84	.000**	2.41	.120	N/A	N/A	13.71	.000**
	Chinese reform	74	418	17.7%	61.14	.000**	.74	.390	13.71	.000**	N/A	N/A
Connection to real-world situations	US tradition	101	1,414	7.1%	N/A	N/A	42.08	.000**	18.92	.000**	57.72	.000**
	US reform	28	113	24.8%	42.08	.000**	N/A	N/A	7.44	.006**	1.30	.254
	Chinese tradition	57	406	14.0%	18.92	.000**	7.44	.006**	N/A	N/A	4.94	.026*
	Chinese reform	83	418	19.9%	57.72	.000**	1.30	.254	4.94	.026*	N/A	N/A
Connection to collaboration	US tradition	42	1,414	3.0%	N/A	N/A	106.18	.000**	41.15	.000**	39.04	.000**
	US reform	27	113	23.9%	106.18	.000**	N/A	N/A	13.41	.000**	14.39	.000**
	Chinese tradition	43	406	10.6%	41.15	.000**	13.41	.000**	N/A	N/A	.02	.887
	Chinese reform	43	418	10.3%	39.04	.000**	14.39	.000**	.02	.887	N/A	N/A

Note. *p<.05; **p<.01.

Curriculum opportunities

Additionally, U.S. textbooks pay more attention than their Chinese counterparts to the connection of number sense to real-world tools but both countries decrease their attention in this area. Table 5 shows that Chinese traditional and reformed textbooks have 28% and 17.7% of their total codes in this dimension for the number sense connection to real-world tools respectively with 11.1% decrease while U.S. traditional and reformed textbooks have 38.3% and 21.2% for this category with a 17.1% decrease. The Chi-square tests on the differences between the two U.S. textbooks and between the two Chinese textbooks both indicate statistically significant results at the 99% confidence level. The difference between Chinese and U.S. traditional textbooks is statistically significant at the 99% confidence level while there is no significant difference between the two reformed textbooks across the two countries, which suggests that the textbooks in both countries move close to each other in the number sense connection to real-world tools.

Furthermore, all textbooks in the US and China pay some of their attention to the connection of number sense to real-world situations and increase their attention to this connection in their reformed textbooks. As shown in Table 5, Chinese traditional and reformed textbooks attribute 14.0% and 19.9% of their total codes in this dimension to the number sense connection to real-world situations respectively, which suggests 5.9% increase. U.S. traditional and reformed textbooks have correspondingly 7.1% to 24.8% in this connection, which counts as a 17.7% increase. The Chi-square test on the differences between the two U.S. textbooks shows statistically significant results at the 99% confidence level and the differences between the two Chinese traditional textbooks is also significant at the 95% confidence level. However, the difference between Chinese and U.S. traditional textbooks are statistically significant at the 99%

confidence level while there is no significant difference between Chinese and U.S. reformed textbooks. This finding suggests that the difference between the textbooks in China and US become smaller in light of the connection to a real-world situation.

Lastly, Chinese textbooks maintain their substantial attention to engaging students in working with each other in developing number sense and related topics while U.S. reformed textbooks substantially increase their attention to the collaboration in the dimension. In Table 5, Chinese traditional and reformed textbooks devote 10.6% and 10.3% of their total codes in this dimension respectively to the collaboration among students. In contrast, the U.S. traditional textbook only has 3.0% of its total codes devoted to the collaboration while the U.S. reformed textbook has 23.9% devoted to the collaboration with a 20.9% increase. The Chi-square test on the differences between the two Chinese textbooks show non-significant results while the difference between the two U.S. textbooks is statistically significant at the 99% confidence level. The difference between Chinese and US traditional textbooks is significant at the 99% confidence level in favor of the Chinese traditional textbook while the difference between Chinese and U.S. reformed textbooks is also significant at the 99% confidence level but in favor of the U.S. reformed textbook. This finding suggests that US textbooks have a dramatic change in terms of attention to collaboration among students than their Chinese counterparts.

Discussion and Implications

This study provided some valuable findings regarding the similarities and differences in number sense and relevant representation among the reformed and traditional textbooks used in China and the US, which adds to the knowledge bases upon which mathematics teaching and learning differences between the two countries can be better understood (Schmidt et al., 2002).

Curriculum opportunities

These findings also help us understand the learning opportunities that these textbooks offer for young students in each country to develop their number sense and relevant ideas as well as the directions in which the textbooks in each country are moving in light of improving their representations and learning opportunities.

Our study shows that the two textbooks from each country have paid most of their attention to the connection of number sense to other mathematics content and concepts and increased their attention to this connection in their reformed textbooks. This finding suggests a similar approach that the two countries use to improve students' number sense development through their textbooks that stress the connection of number sense to different mathematics concepts. Such a similarity seems not surprising since the connection of different mathematics concepts in curriculum has been one of the major foci in the current U.S. standards-based mathematics education reform that has been developed to improve mathematics teaching and student mathematics learning (Romberg, 1992). This is also part of the tradition to which Chinese mathematics textbooks are designed and developed (Ma, 1999; Zhou & Peverly, 2005).

However, as for how to develop the connection of number sense to other mathematics content and concepts, the two countries show some clearly different foci. First, the two Chinese textbooks pay more attention to the connection of number sense to operation than the two U.S. textbooks. In addition, they have increased this attention substantially while the U.S. textbooks have decreased their attention in this connection. This finding suggests that engaging students in calculation processes is and continues to be an important characteristic of Chinese textbooks as a way to improve their number sense, which again reflects the finding that the logical connections of mathematics concepts to calculations is a major characteristic of Chinese textbooks for engaging students in learning mathematics (Li, 2000)

Second, the two U.S. textbooks pay substantially more attention than the two Chinese textbooks to the connection of number sense to measurement and data analysis. This finding mirrors the research finding on mathematics achievement differences between the two countries, in which Chinese students outperformed U.S. students generally, but they are not necessarily better performers on measurements in earlier grade levels (Wang & Lin, 2009) and often underperformed in understanding data graphs in comparison with U.S. students (Brenner, Herman, Ho, & Zimmer, 1999; Cai, 1997; Stevenson, et al., 1990).

In developing learning opportunities for students to understand the properties of number sense, U.S. and Chinese textbooks show some similarities and some substantial differences. First, although the textbooks in both countries stress the counting property of number sense most and have paid increasingly more attention to the number relationship property of number sense, the Chinese textbooks pay more attention to the number sense properties, such as number meaning and representation, place value, and base ten concepts than their U.S. counterparts. In addition, the Chinese textbooks have increased their attention while the U.S. textbooks have decreased their attention to place-value and base-ten property, which are seen as important for developing students' conceptual understanding of number sense (Faulkner, 2009; Reys, et al., 1999; Sowder & Schappelle, 1994). The higher performance of Chinese students in number understanding than U.S. students as they progress through various grade levels might be explained by the more intense focus on the number sense properties of number meaning and representation, place value, and based ten concept in Chinese mathematics curriculum, which provide Chinese students more opportunities in developing conceptual understanding of number (Wang & Lin, 2009).

Second, although this composition property of number sense has been highly valued by the research community for students to develop number sense as well as understand other relevant mathematics concepts (Reys, et al., 1999; Sowder & Schappelle, 1994; Tsao, 2005; Yang & Cobb, 1995), the U.S. reformed textbook increases its attention substantially to such a property while both Chinese textbooks pay less attention to it. This finding seems to be in contrast with the research finding (Ma, 1999) that Chinese teachers are reported to have paid substantial attention to the composition property of number sense in their teaching practice. The difference between the weak focus on the composition property in textbooks and the strong focus on it in teaching practice in China suggests that, within the Chinese centralized curriculum context, Chinese teachers are not always practicing their teaching in a lockstep manner with their textbooks (Wang & Paine, 2003). However, the lockstep relationship between teaching and curriculum materials has been a constant worry among the scholars in the west about the side-effects of standards-based curriculum reform (Cochran-Smith, 2001; Hargreaves & Dawe, 1990; Helsby & McCulloch, 1996). Therefore, it is important for the research community to develop a better understanding why and how such differences between textbooks and teaching practice occur in a centralized curriculum context.

This study further shows that both U.S. and Chinese textbooks pay similar attention to the connection of number sense to real-world objects and tools. This characteristic of first-grade textbooks in the two countries reflects that the tradition and reform of mathematics curriculum and teaching are influenced to a certain extent by the suggestions of child development psychology that strongly encourages teachers to use concrete materials and manipulatives to engage students in learning mathematics (Putnam, Lampert, & Peterson, 1990).

However, the two Chinese textbooks pay consistent attention to connecting number sense to real life situations and to collaborations while their U.S. counterparts provide inconsistent attention to this connection. This result mirrors the findings that Chinese students are good performers in solving real-world problems (Brenner, et al., 1999; Cai & Silver, 1995; Chen & Stevenson, 1995; Stevenson, et al., 1990) in the collectivist Chinese cultural tradition. The substantial increased attention to these connections in the U.S. reformed textbook echoes the call of U.S. mathematics education reform for the development of students' mathematics understanding in real life contexts (Borko, 2004; Leinhardt, 1992; Resnick, 1987) and through their collaboration (National Council of Teachers of Mathematics, 1989, 1991, 2000).

Our analysis in the study further suggests that the textbooks in both countries have changed substantially in some similar directions. One of these directions is that the reformed textbooks in both countries contain fewer topics, each of which encompasses more content and concepts. This characteristic is clearly reflected in the finding that meaning chunks are reduced from Chinese and U.S. traditional textbooks to their reformed textbooks while average coding counts for each meaning chunk are increased. This change in the textbooks of the two countries reflects the mathematics curriculum reforms in both countries that call for reducing the amount of content and number of topics while increasing the depth and connections among those topics and content (Liu, 1995; Liu, et al., 2001; Romberg, 1992; Schmidt, McKnight, & Raizen, 1996).

Another similarity in change is that the textbooks in both countries decrease their attention to the connection of number sense to real-world tools and pay relatively little attention to estimation of quantity within the dimension of number sense property. Such a change is not consistent with the call of mathematics reform in both countries in which the use of real-world tools and the inclusion of estimation in mathematics teaching and learning are strongly encouraged by the mathematics education reform policies in both countries (Liu, 1995; National Council of Teachers of Mathematics, 1989, 2000). Thus, it will be interesting to find out why these changes occurred in each country against their reform calls.

The last similarity is that both Chinese and U.S. reformed textbooks increase their attention to the connection of number sense to real-life situations. While there is a much bigger leap in this connection from U.S. traditional textbooks to its reformed one, both Chinese and U.S. reformed textbooks become more similar in encouraging students to collaborate with each other in learning about number sense and relevant topics. Again, this change in the textbooks of the two countries reflects the call of the mathematics curriculum reform in both countries for situating student mathematics learning in the real-life contexts and through collaborations (Liu, 1995; Liu et al., 2001; Romberg, 1992; Schmidt et al., 1996)

In spite of these similarities, the textbooks in both countries also show some differences in their changes. First, the U.S. textbooks have increased their attention to the connection of number sense to patterns and made substantial leap in their attention to the composition properties of number sense, and the numbers sense connection to collaboration among students. In contrast, Chinese textbooks decrease their focus on the connection of number sense to patterns and pay less attention to composition property of number sense. The influences of these opposite changes in the textbooks of the two countries on mathematics and teaching and learning are worth further exploration.

Curriculum opportunities

Limitations of the Study

This study has several obvious limitations. First of all, the coding system in this study is developed according to the relevant assumptions about number sense and its relationship to other mathematics content, real world, and collaboration, the coding is conducted in a top-down approach to capture the important patterns and characteristics of the textbooks. Such a coding approach is clearly limited in identifying the patterns and characteristics of the textbooks developed and intended by the authors of the textbooks, which are more likely to be revealed through a ground up approach to coding as suggested (Strauss & Corbin, 1990).

In addition, the study used Chi-square to test the significance level about the frequency differences between two groups of codes in different categories among the four textbooks as suggested (Shavelson, 1996). Although Chi-square is a useful statistical tool in capturing the differences in such a context, especially when sample sizes of the study are large enough (Conover, 1999), the results of the Chi-square tests cannot provide useful information about the relationship among different patterns and characteristics of a particular textbook. Thus, the results of this study cannot be used to indicate any relationships between these patterns and characteristics of a particular sampled textbook.

Furthermore, this study only analyzed two first-grade mathematics textbooks from each country. In spite of their popularity in the two countries, it is not clear whether and to what extent the patterns and characteristics of number sense and relevant topics emerging from these textbooks reflect in the textbooks of the later grades in the two countries and in other first grade mathematics textbooks in the two countries. This is especially the case for the US where the decentralized curriculum control and market-driven curriculum production and distribution may lead to different patterns and characteristics of number sense and relevant topic representation in the textbooks of other grade levels and in other first grade mathematics textbooks (Cohen & Spillane, 1992). Thus, some cautions should be exercised when generalizing the findings from this study to the textbooks of other grade levels or the other first grade textbooks in the two countries.

Finally, although being able to identify some patterns and characteristics of number sense and relevant topic representation among the four textbooks from two countries, this study is not able to identify the direct influence of these patterns and characteristics on teachers' teaching and student learning of mathematics in the two countries. This is because teaching practice can be influenced by various other factors, such as students' cultural and racial backgrounds, school structure in which teaching practice is situated, teaching culture, and teacher knowledge, etc., which can be different in the two countries (Wang & Lin, 2005). It is also because the development and implementation of curriculum in China is centralized while in the US, such development and implementation is decentralized (Wang, 2001). Thus, the potential influences of the patterns and characteristics of these textbooks on teaching and learning of mathematics in each country can be mediated and interactively shaped by the other factors or different curriculum system unique for each country.

Conclusion

In spite of the above limitations, the current study has provided important understanding about the similarities and differences in representing number sense by the reformed and traditional textbooks in both countries. As was identified in this study, the changes in the textbook design of both countries are substantial and reflected in the reduction and increase of specific topics and content as well as in the organization and the structure of the textbooks. While the effects of these changes on teaching practice and student learning are not yet carefully documented, the challenges that these changes have created for teachers to adapt to are daunting as teaching is a culturally scripted practice and its changes often follow a gradual, evolutionary path rather than a sudden revolutionary transformation (Cuban, 1993; Kennedy, 2010; Stigler & Hiebert, 1999). In adapting to these changes of textbooks, our analysis suggests that U.S. teachers might face bigger challenges since there are greater changes in U.S. textbooks than their Chinese counterparts in light of topic-content ratio, number composition property, and connection of number sense to real-world situations and collaboration among students (Little, 1990; Stigler & Hiebert, 1999).

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