# SUPERITEM TEST: AN ALTERNATIVE ASSESSMENT TOOL TO ASSESS STUDENTS' ALGEBRAIC SOLVING ABILITY

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## Abstract

Superitem test based on the SOLO model (Structure of the Observing Learning Outcome) has become a powerful alternative assessment tool for monitoring the growth of students' cognitive ability in solving mathematics problems. This article focused on developing superitem test to assess students' algebraic solving ability through interview method. The interview data analysis found that the high ability students seemed to be more able to seek the recurring linear pattern and identify the linear relationship between variables. They were able to co-ordinate all the information given in the question to form the algebraic expression and linear equation. Whereas the low ability students showed their abilities more on counting and drawing method. They lacked understanding of algebraic concepts to express the relationship between the variables. The findings provided evidence on the significance of superitem test in assessing algebraic solving ability.



**Keywords**: *SOLO model, superitem, interview, algebraic solving ability.*


## Background

Recent advance in the understanding of how students learn should lead to the important role in assessment. According to Webb, Norman and Briars (1990), assessment must be an interaction between teacher and students; with the teacher continually seeking to understand what a student can do and how a student is able to do it and then using this information to guide the instruction. Thus, assessment procedures and practices must be revised and improved in order to provide the useful information for the current curricular and instructional reform. As a consequence, the movement toward alternative assessment is motivated. Alternative assessments are thought better at providing teachers and administrators with a more complete picture of what each student might know and understand about mathematics – related skills or concepts rather than comparing the knowledge of individuals to a standard norm. Proponents of alternative assessment claim that changing to alternative-based will encourage instructional techniques that promote critical thinking and problem solving skills amongst the students. The general belief is that alternative assessments are likely to be more authentic or real in nature than traditional assessments and, therefore, more closely aligned with the true goals that teachers have for their students' learning (Raboijane, 2005). In this study, researchers explored alternative types of assessment to gain a better understanding about what students had learnt, and how their conceptions or misconceptions were identified. To directly measure student gains from interaction with the test, a superitem test had been developed. The intention was to go beyond traditional questions to see if students could demonstrate their solving abilities in the topic of algebra, especially the linear pattern.

### *What Is Superitem Test?*

Collis, Romberg and Jurdak (1986), Lam & Foong (1998), Wilson & Iventosh (1988) developed the possible use of superitem based on the SOLO taxonomy as an alternative assessment tool for monitoring the growth of students' cognitive ability in solving mathematics problems. Superitem consists of a problem situation and four different complexity levels of items related to it. The problem situation is often represented by text, diagram or graphic. While the items represent four levels of reasoning defined by SOLO model which include unistructural, multistructural, relational and extended abstract. Thus, within any superitem, a correct response to an item would indicate the cognitive ability to respond to the information in the stem at the certain level reflected in the SOLO structure.

SOLO model assumes a latent hierarchical and cumulative cognitive dimension. It found that when students answer the given tasks, their responses to the task can be summarized in terms of five levels ( Biggs & Collis, 1982; Biggs & Collis, 1989; Chick, 1988; Chick, 1998; Wilson & Iventosch, 1988; Wongyai & Kamol, 2004; Wilson & Chavarria, 1993), ranging from prestructural to extended abstract. These are described as follow:

a. Prestructural - the learner does not understand the point of question. He/she gives an answer without even dealing with the problem. In other words, the learner fails to engage in the problem.
b. Unistructural - the learner focuses on one or a few relevant information given to provide a response to the direct concrete reality involved in the problem. The information is obtainable from either the stem or from the given diagram. In other words, a quick closure (answer) is achieved on the basis of minimal use of given information. For example, the learner uses and refers to the concrete object (picture) given in the stem to find the next term of the given pattern.
c. Multistructural - the learner picks up more relevant information given to obtain the solution but does not integrate them. The information given may use as a recipe where a set of instructions are followed in sequence to solve the problem. For instance, the learner begins to identify the relationship between the variables of the pattern and able to describe how to move from one term in a sequence to next term. He is able to see the pattern given as successive process.
d. Relational - the learner integrates all aspects of given information with each other into a coherent structure. In other words, the given information are insufficient to solve the problem immediately. It must be carefully interrelated to produce a satisfactory solution. For example, the learner is able to generalize the relationship of the pattern symbolically based on the all the given information and pattern.
e. Extended abstract - the learner generalizes the structure into a new and more abstract situation. This may allows generalization to a new topic or area. For instance, the learner is able to extract the abstract general principle from the information given and use of deductive reasoning to form an alternative solution for the new pattern.

The five responses listed above represented both an increase in the use of the information available and an increase in the complexity of structure response. For example, at the unistructural and multistructural levels, responses may only involve one or more relevant aspects of given information in the stem and thus there is little relationship between the question and the given information. Further, in these levels, a student needs only to encode the given information and use it directly to give a response. Whereas, at the relational level or extended abstract level, the student need to make a generalization within the given information and an abstract principle which are not given directly in the stem. Furthermore, at the relational level or extended abstract level, the student need to understand the task in a way that is personally meaningful and links up with the existing knowledge.

***Using Superitem Test For Assessing Algebraic Solving Ability***

There is currently a general agreement among mathematics researchers that algebra is fundamentally the study of pattern and relationship. They have challenged the conventional view of algebra as a series of abstract rules regarding $x$'s and $y$'s, formal structure, manipulation of symbols and rote skills; arguing that algebra as a tool for problem solving, a method of expressing relationship, describing, analyzing and representing patterns, and exploring mathematical properties in a variety of problem situation. (Day &Jones, 1997; Fernandez & Anhalt, 2001; NCTM, 1988; Iman & Sirine, 2007). Many mathematics researchers and educators have focused on the investigation into the introduction and development of algebraic solving ability which can be viewed from different approaches, namely generalization, problem solving, modeling and functional. The nature of algebraic solving ability inherits in each approach is sufficient to generate powerful algebraic solving ability in classroom. However, as the instructional approaches change, so must the assessment techniques used to evaluate progress toward the new goals, both for evaluation of student performance and for research studies on the effectiveness of new instructional approaches. Nevertheless, the question of how to assess the new era of algebraic solving ability may still be problematic for many teachers. Macgregor and Stacey (1999) and Tricia (1999) stated that at the present time, the current assessment in algebra is too narrow in scope and still focusing on getting a correct answer, symbol manipulation, rote skill and little or no application. This scenario and stereotypical image of assessment in algebra still happens in Malaysia. TIMSS report (International Association for the Evaluation of Educational Achievement, 2007) has indicated that Malaysian students performed better (i.e., 24% of the students answered correctly) in the straightforward questions. For example: In Zedland, total shipping charges to ship an item are given by the equation $y = 4x + 30$, where $x$ is the weight in grams and $y$ is the cost in zeds. If you have 150 zeds, how many grams can you ship? However, in the questions that requiring solving the word problem that can be expressed as two linear equation with two variables, only 14% of Malaysian students answered correctly compare with Singapore (59%) and Korea (68%). Malaysia is far behind these two Asian countries. Research studies (such as Mashooque, 2009; Teng, 2002; Tall and Razali, 1993) have noted that symbol manipulation and procedural skills practice in algebra class among the secondary school students might serve to prolong the interpretation that algebra is a 'menagerie' of disconnected rules to deal with different contexts ('collect together like terms', ' turn upside down and multiply', 'do the same thing to both sides', 'change side, change sign' etc). It therefore exhibits the poor understanding of the basic concept and cognitive obstacles among the students as this practice to algebra relies almost exclusively on written symbolic forms as the tool to make representation, generalization and interpretation to the applied problem.

Fortunately, SOLO model has provided insight into an alternative assessment of cognitive ability and learning pointing the way to new approaches to classroom. The content and the structure of items can be designed based on this model to train the students in the well-developed thinking skill. Specially, the four level items involve investigating, formulating and analyzing students' thinking.

## Purpose of the Study

This study aimed to assess the Form Four (Grade 9) students' levels and processes of algebraic solving ability in using linear equation. The SOLO model was used as the framework to construct the test items. In order to capture the manifold nature of algebraic solving ability in using linear equation to solve the problem situations, the framework of this study incorporated four content domains of linear equation, namely linear pattern (pictorial), direct variation, concept of function and arithmetic sequence. However, this paper only focused on discussing the findings of linear pattern (pictorial) domain.

## Methodology

This study used qualitative approach to assess the students' algebraic solving ability based on SOLO model. Interview assessment was conducted to collect the data. It is one of the most direct methods to examine the depth of ideas that respondents hold about algebra concepts. Although this method of assessment is difficult to manage, the returns on the teacher's investment are great for the students as well as teacher. Students realize that their ideas have the teacher's undivided attention. The teacher has a clearer understanding of students' algebraic conceptual strengths, problem solving ability and cognitive ability.

There were eight questions which were designed according to the SOLO superitem format. Each superitem consisted of a situation or story (the stem) and four items related to it. The items represented four levels of reasoning defined by SOLO (unistructural, multistructural, relational, extended abstract).

### *Participants*

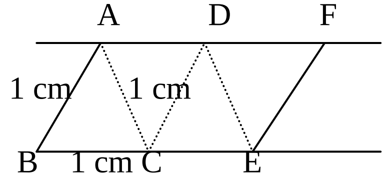
In Malaysia, basic topic of algebra is taught during lower secondary school and the topic of linear equation is taught in Form Two and Form Three (Grade 7 and Grade 8). Thus, the construction of an instrument to assess algebraic solving ability amongst Form Four students was important as the teachers could gain a greater awareness of students' algebraic solving abilities about this topic before they learnt the more complex topics which were required the basic algebra knowledge. The participants of this study consisted of nine Form Four students from a secondary school. They were selected based on their mathematics results in school mid-year examination, namely three high achievers, three moderate achievers and three low achievers.

### *Instrumentation*

In this study, the instrument of data collection consisted of eight superitems. All the superitems were open-ended questions. Two superitems were constructed for each content domain to be assessed. The following is an example of a superitem (superitem 2: linear pattern pictorial) designed for this study.

Problem: Triangle Train

Look at the triangle train below. The length of triangle train is determined by the number of equilateral triangles with the side 1 cm. The perimeter of the triangle train is 5 cm if the length is 3.




<u>*Unistructural*</u>

a. What is the perimeter of the triangle train if the length is 4? (interior lines don't count as part of perimeter)

**Answer:** ***6 cm***

*Descriptors:*

The item requires the response based on the concrete information (given terms in the diagram) to find the next term for the given sequence. The task can be answered most simply by counting.

<u>*Multistructural*</u>

**b.** What is the perimeter of the triangle train for the length (number of train) of 6 and 15?
**Answer:** ***8*** **cm and** ***17*** **cm**

*Descriptors:*
The item requires the given information being handled serially. That is, identify the recursive relationship between the terms in the sequence in order to compute some specific cases.

<u>*Relational*</u>

c. i What is the perimeter of the triangle train for the length (number of train) of ***h***?
**Answer*:* *h + 2***

ii. Try to write a linear equation to find the perimeter of the triangle train for any length of the triangle train. Let ***r*** represents perimeter of the train and ***s*** represents the length of the train.
**Answer:** ***r = s + 2***

iii. If the triangle train has a perimeter of 50 cm, what is its length? Try to apply the linear equation to solve this problem.
**Answer:**
***50 = s + 2***
***s = 48***

*Descriptors:*
The item requires the integration of all given information to make generalization and apply the rule to solve the related situation. If a student provides this response, it would demonstrate his/her algebraic solving ability in: i) identifying the linear relationship between the variables. ii) constructing a rule by using algebraic symbol. iii) applying the rule in related situation.

<u>*Extended Abstract*</u>

d. Can you try to suggest a new pattern of the train and form a linear equation to represent the perimeter (***r***) of the train for any length (***s***)?

*Answer:*
**1.** ***r = 2s + 2*** **(square train) or**
**2.** ***r = 2s + 4*** **(hexagon train)**

*Descriptors:*

This level represents the highest level of algebraic solving ability. The response shows an ability to extend the application of the given information in the new situation (creating new pattern) and recognize an alternative approach which is formed by the abstract concept (linear relationship).

## Data analysis

The information from the interview session was transcribed into writing form. Each interview was audiotaped and videotaped. Each session lasted between 30 minutes to an hour.

## The Findings Of Using Superitem Test In Assessing Form Four Students' Algebraic Solving Ability

The discussion of the findings was focusing on three out of nine Form Four students involved, namely Zul, Sri and Seal. The four processes of algebraic solving ability in solving superitem linear pattern pictorial (example of superitem above) were investigated as follow:

*a) Investigating the numerical pattern that comes next to it*

All the three students were able to solve and respond correctly by seeking out the sequence of pattern which comes next to it or extend the sequence by referring directly to the given diagram and information. The following dialogues depicted how the students responded to item a (based on the example of the above superitem). In the following dialogues, R denotes researcher, Zul, Sri and Seal denotes students.

**R:** Please tell me how did you get the answer for item a?
**Sri:** (counting the side of triangles in the given diagram). I count each side here to get the answer for the perimeter of the train.

**R:** How did you solve the item a?
**Seal:** (pointing to the diagram). I count from here. AB, AD, DF, EF, CE and BC.

*b) Investigate the pattern by computing specific cases*

When the students were confronted with various number tasks as a way to assess and refine their understanding of the pattern, subjects began to notice the pattern and understand the linear relationship involving an arithmetic operation. Sri and Seal did not use the manipulative to get the solution instead of substituting the specific values into the arithmetic expression that they formed. These were shown in the extracts below:

**R:** How did you solve the item b?
**Sri:** (Recalling) The length of train plus two…because the first and last side of the train. So, two. The length plus two.
**R:** How did you find the perimeter of the train for the length of 15?
**Seal:** (Pointing to the solution in finding the perimeter for the length of 15). 2 + 15 = 17. '2' means the first part and last part of the train and 15 means the number of triangle, interior lines not counted. So, increasing of every 1 triangle will increase 1 cm of perimeter.

Zu1 was unable to notice the linear pattern in the diagram. He solved the items by counting method. He drew the triangle to count the perimeter. He sought the pattern by working with manipulative to get the answers. He relied on the drawing and counting. His explanation is shown in the following dialogue:

**R:** How about item b? How did you solve it?

**Zu1:** I draw the number of triangle …then count the perimeter.

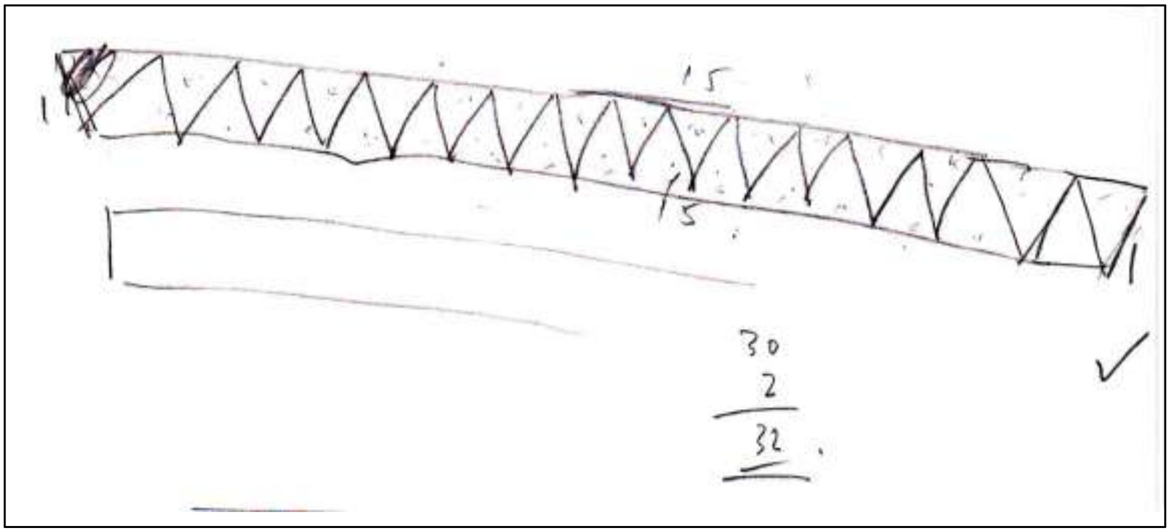



*Figure 1* Zul's solution for item b

*C i) Representing the unknown value with the use of letter*

Sri and Seal were able to gather the information and findings at the concrete level into more abstract level. In other words, they were able to transfer the meaning from arithmetic expression to an abstract conjecture. They linked their interpretation and mathematical findings to connect the counting action with an accurate symbolic representation in the form of algebraic expression. The following dialogues show how the algebraic expression was formulated:

**R:** What is the perimeter if the length is *h*? Try to explain your answer.

**Sri:** (*h*) plus 2 (referring to the specific cases that had been counted). I substitute the figure with *h.*

**R:** What is *h*?

**Sri:** *h* is an unknown… can be a certain number.

**R:** If the length is *h*, what is the perimeter?

**Seal**: *h* plus 2. I refer this (pointing to arithmetic expression)

The lack understanding of algebraic concepts such as unknown as opposed to an ability to make transition from arithmetic method to the application of algebraic symbols. For example, Zu1 was unable to use algebraic concept such as algebra expression to express or describe the numerical relationship that existed in the pattern into the abstract situation. The problem about lack understanding of unknown can be seen in the extract below:

**R:** For item ci, if the length is *h*, what is the perimeter? How did you get 20?

**Zu1:** I simply write the answers.

**R:** What does '*h*' mean?

**Zu1:** …. I don't understand actually.

*C ii) Writing linear equation to make generalization of pattern*

Sri and Seal were able to generate the linear equation for the problem situation based on the information given and the data that they collected. Sri's explanation is shown in the following extract:

**R:** Can you try to explain the linear equation that you formed to represent the pattern? How did you get it?

**Sri:** *r* equals s plus 2. I got it from the findings in question b (pointing to the arithmetic expression that he formed).

However, Zu1 was unable to use linear equation to express or describe the linear relationship that existed in the pattern. The lack understanding of concept variable can be seen in the extract below:

**R:** Can you describe and generalize the pattern in linear equation form?

**Zul**: *r* equals *5 cm* and *s* equals the length of train, also 5 cm.

**R:** Why?

**Zul**: *r* here I assume is a number…let's say 5 and *s* also same, 5….? I don't know how to represent it.

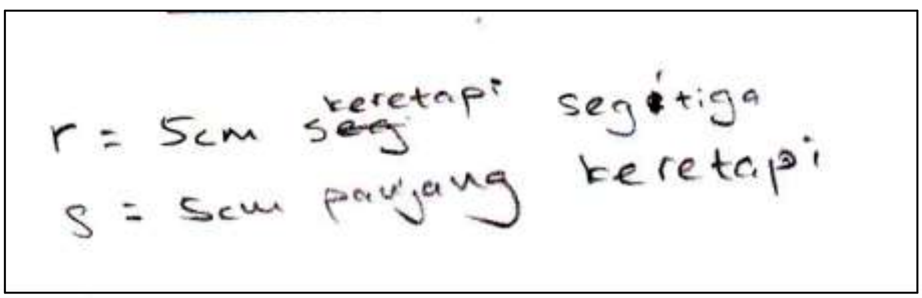



*Figure 2*: Zul's solution for item C (ii)

*C iii) Application of the rule to solve the related problem*

The equation was applied to represent the relationship between dependent variable and independent variable of the problem. Sri and Seal were able to analyze the problem apply the rule that they constructed. In the following extract, Sea1 attempted to explain the application of linear equation:

**R:** If the perimeter is 50 cm, what is the length of the train? How did you solve the problem?

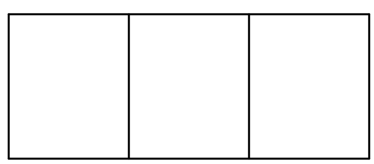
**Seal:** I use equation to solve it.
**R:** How did you solve it?
**Seal:** 50 equals *s* plus 2, *s* equals 48.
**R:** Any other method?
**Sea1:** I think this is the fastest.

However, Sri was unable to apply the rule due to the misunderstanding of the use of reverse operation between multiplication and division. These weaknesses had obstructed the progress of mapping the steps appropriately to the solution.

*d) Making generalization for the new pattern or new situation by forming the alternative solution*

Only Seal was able to generalize the new pattern by forming the alternative solution. In solving the item d, Seal was able to extract the abstract concept (linear pattern) from the information given to form the linear equation in order to generalize and represent the new situation that he created. He sought out the pattern by searching the given information. Then he tried to solve some additional cases to test and refine his understanding of pattern. In the final process, he developed a generalization method for figuring out the relationship between the length of square train and perimeter. He articulated a rule for the pattern by using equation. In the following extract, Sea1 tried to explain the solution:

**R**: How did you solve the item d?
**Seal:** I draw the square train to identify the sequence. From here, I get the equation.



1 - 4
2 - 6
3 - 8
So, the equation is $r = 2s + 2$.

The three students had provided the detailed responses in explaining their solving abilities within the four processes. Based on their responses, Zul was categorized in mulistructural level (low ability level). He was unable to notice the linear pattern. Sri was categorized in relational level as his inability to apply the rule due to the misunderstanding of the use of reverse operation while Seal was categorized in the highest ability level (extended abstract). He was able to generalize the new pattern by forming an alternative solution.

## Discussion and Conclusion

Superitem test is claimed to be applicable in assessing cognitive learning outcomes among different levels of students in mathematical problem solving (Collis, Romberg dan Jurdak, 1986; Lam dan Yeen, 1998; Wilson dan Iventosch, 1988). The findings of this study suggest that the high ability students (relational and extended abstract level) seemed to be more able to seek out the recurring linear pattern and identify the linear relationship between the variables. They were able to co-ordinate all the information given to generalize the pattern algebraically such as forming an algebraic expression and linear equation, used the linear pattern concept in more abstract situation such as forming a rule for the new linear pattern that they created. Also, they used their methods more consistently to find the solutions. This finding is in concurrence with the finding of previous study (Levins, 1999; Moooney, 2002) which found that students who exhibited relational level thinking generalized data displays and constructed complete, and sometimes unique, display that were representative of and appropriate for the data given. The students who exhibited highest level thinking (extended abstract level) were able to integrate and generalize all the relevent concepts and ideas which are not given in the task. They also showed their reasoning abilities and making hypotheses for the given situation.

Nevertheless, the low ability (unistructural and multistructural levels) students showed their abilities more on counting method where the given task is performed and understood serially. They failed to inter-relate the features of linear pattern given in the question due to the lack understanding of algebraic concepts especially unknown and linear equation. This finding is consistent with the finding of previous study (Mooney, 2002) regarding the characterization of middle school students' statistics thinking. The finding found that although the low ability students consistently use both visual and quantitative aspects of data, they made little or no connections to the contextual aspect of the data.

Besides, the finding of this study suggests that Zul and Sri had misconceptions as following:

i) Letters have no meaning or tend to link letter with numbers.
ii) The misunderstanding of the use of reverse operation between multiplication and division in solving the equation.

This findings was in accordance with the findings of previous studies (Cheah and Malone, 1996; Teng, 2002) found that majority of Form Three students and Form Four students had the similar misconceptions when simplifying algebraic expression or solving equation.

As noted above, the use of SOLO model not only suggest an item writing with the format of superitem, it also can be used to score the item, allowing for crediting partial knowledge. Thus, it provides teachers an indication of some of the levels of algebraic solving ability they can expect to encounter in their classroom. Notwithstanding, this test has the potential to contribute to both instruction and assessment. From the instructional perspective, it would seem prudent for teachers to use solving ability level descriptors as broad guidelines for organizing instruction and building problem task. From an assessment perspective, it appears to be valuable in providing teacher with useful background on students' initial solving ability, and in enabling them to monitor general growth in algebraic solving ability. In other words, the test provides a valuable tool for the teacher in planning learning goals and assessment tasks. As example, task like those of this study could be used in instruction and also as problem to assess students' solving abilities at various stages.

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