STUDENTS' PERCEPTIONS ABOUT THE SYMBOLS, LETTERS AND SIGNS IN ALGEBRA AND HOW DO THESE AFFECT THEIR LEARNING OF ALGEBRA: A CASE STUDY IN A GOVERNMENT GIRLS SECONDARY SCHOOL KARACHI

Abstract

Algebra uses symbols for generalizing arithmetic. These symbols have different meanings and interpretations in different situations. Students have different perceptions about these symbols, letters and signs. Despite the vast research by on the students' difficulties in understanding letters in Algebra, the overall image that emerges from the literature is that students have misconceptions of the use of letters and signs in Algebra.

My empirical research done through this study has revealed that the students have many misconceptions in the use of symbols in Algebra which have bearings on their learning of Algebra. It appears that the problems encountered by the students appeared to have connection with their lack of conceptual knowledge and might have been result of teaching they experience in learning Algebra at the secondary schooling level. Some of the findings also suggest that teachers appeared to have difficulties with their own content knowledge. Here one can also see that textbooks are also not presenting content in such an elaborate way that these could have provided sufficient room for students to develop their relational knowledge and conceptual understanding of Algebra.

Moreover, this study investigates students' difficulty in translating word problems in algebraic and symbolic form. They usually follow phrase- to- phrase strategy in translating word problem from English to Urdu. This process of translating the word problem from English to their own language appears to have hindered in the correct use of symbols in Algebra. The findings have some important implications for the teaching of Algebra that might help to develop symbol sense in both students and teachers. By the help of symbol sense, they can use symbols properly; understand the nature of symbols in different situations, like, in functions, in variables and in relationships between algebraic representations. This study will contribute to future research on similar topics.

INTRODUCTION

Mathematics is known as one of the gate keepers for success in all fields of life. It is a common saying that Mathematics is mother of all subjects. That's why it is considered to be more than a subject and is conceived as a key for solving the problem. The first question which arises in our mind as teachers that why should we teach Mathematics to our students? One of the main objectives of teaching and learning Mathematics is to prepare students for practical life. Students can develop their knowledge, skills; logical and analytical thinking while learning Mathematics and all these can lead them for enhancing their curiosity and to develop their ability to solve problems in almost all fields of life. This problem solving nature of Mathematics can be found in sub-disciplines of Mathematics such as in geometry, calculus, arithmetic and Algebra. Algebra is an important area of Mathematics. Algebra is a generalized form of arithmetic and for the purpose of generalization of arithmetic; the letters and signs are used. No doubt, the use of letters and signs make it an abstract subject. Because of nature of generalization and abstraction, Algebra is considered to be a difficult area of Mathematics.

This study has explored students' perceptions about the use of symbols and signs in Algebra. Here, this chapter discusses background of the study with some significance of this study for research. This chapter also presents the research question and concludes with some definitions related to research focus.

FRAME WORK AND PURPOSE OF THE STUDY

For learning of Algebra, learners should have a conceptual understanding about the use of the symbols and the context in which it is used. In other words, they should know the situation in which the algebraic statements are made. Hiebert et. al. (1997) cited in Foster (2007), says that, "when we memorize rules for moving symbols around on paper we may be learning some thing but we are not learning Mathematics" (p.164). Moreover, the use of symbols without an understanding cannot develop students' relational understanding of Algebra. Foster (2007) highlighted that if students are taught abstract ideas without meaning, this might not develop their understanding. He suggested that if teachers want students to know Algebra then they must be given a deeper understanding of the use of symbols.

Arcavi (1994) introduces the notion of *symbol sense* as a ‘desired goal for Mathematics education’. Symbol sense incorporates the ability to appreciate the power of symbols, to know when the use of symbols is appropriate and an ability to manipulate and make sense of symbols in a range of contexts. Symbol sense actually develops skills of the use of symbols and understanding of the situation. Making the sense of terms (letters) is one of the fundamental problems in learning of Algebra.

In most of the cases the letter is regarded by the learners as shorthand or abbreviation for any object or as an object in its own right (Collis, 1975). It is also a common misconception among the students. Early experiences with Algebra often lead students to develop this misconception where letters stands for abbreviations of objects. Kuchemann (1981) investigated in one of his research where a group of students’ response to the following problem:

> *Shirts cost s dollars each and pants cost p dollars a pair. If I buy 3 shirts and 2 pairs of pants, what do 3s + 2p represent?*

Response of the most of the students suggested *3 shirts and 2 pairs of pants.* This shows that they perceive ***s*** as shirts and ***p*** as paints rather then **s** for the number of shirts and ***p*** for the number of paints.

Furthermore, the research findings of the Kuchemann (1981) suggested that all students who participated in his research were asked another question:

> *Blue pencils cost 5 pence each and red pencils cost 6 pence each. I buy some blue and some red pencils and altogether it costs me 90 pence. If b is the number of blue pencils bought, and r is the number of red pencils bought, what can you write down about b and r?*

The most common response was $b + r = 90$. This response suggests a students’ strong tendency to conceive letters as labels denoting specific sets, which seems to be a result of the students’ attempt to accommodate their previous arithmetic experience with letters to the new meanings assigned to letters within an algebraic context. Perhaps this problem arose due to the use of symbols in other disciplines like in Chemistry they use symbols like O for oxygen and P for phosphorus. MacGregor and Stacey (1997) found that many eleven-year-olds who had never been taught Algebra thought that the letters were abbreviations for words such as h for height or for specific numbers. Further, he found that students have a misconception that these numbers were the "alphabetical value" of the letter such as h=8 because it was the eighth letter of the alphabet. Another interpretation stems from Roman numerals. For example, 10h would be interpreted as "ten less than h" because IV means "one less than five."

is Students regarded the letters as a specific but unknown number and can be operated on directly (Collis, 1975). In response to the problem given by Kuchemann (1981),

What can you say about p if p + q = 12 and p is a natural number greater than q?

Most of the students replied p = 7**.** The results highlighted that learners have no idea or they were not able to use correct interpretation of the letters that the letters may be more than one value. It is also highlighted that the learners have a belief that the letter should not have only a specific value but it should have been in whole numbers.

Collis (1975) indicated a problem of students' understanding and stated that the letter is seen as representing, or at least being able to take on, several values rather than just one. A study by Kuchemann (1981) in the *Concept of Secondary Mathematics and Science* (CSMS) project investigated the performance of school students aged 11-16 years old on test items concerning the use of algebraic letters in generalized arithmetic. The results showed that most of the students were unable to cope with items which require interpreting the letters as generalize numbers or specific unknowns. He also found the interpretation issue of pertaining letters in Algebra. The study highlighted that students misunderstanding of the letters seem to be reflected in their approach to the relevant relationship in problem situation.

As Schoenfeld and Arcavi (1988) and Leitzel (1989) cited in Bergeson, et.al.(2000) stated that the concept of variable is more sophisticated than teachers' expectation and it frequently becomes a barrier to a students' understanding of algebraic ideas. In this case the letter is seen as representing a range of unspecified values, and a systematic relationship is seen to exist between two such sets of values (Collis, 1975). Kuchemann (1981) found that, even though the interpretation that students choose to use depended in part on the nature and complexity of the question, most students could not cope consistently with items that required the use of a letter as a specific unknown. Schoenfeld and Arcavi (1988) cited in Bergeson, et.al.(2000) argue that "understanding the concept of [variable] provides the basis from transition from arithmetic to Algebra and is necessary for the meaning full use of all advance Mathematics." (p. 421) For many students letters are considered as potential numbers, or index or a sign indicating the place that an actual number will occupy in a process (Redford 2003 cited in Bardini, Radford, and Sabena n.d.).

Clement (1982) and Kuchemann (1981) have investigated that the majority of 15 year's old students were unable to interpret algebraic letters as generalized or even as specific unknown numbers. The study of Kuchemann (1981) shows that many students ignore the letters, replace them by numerical values or regard them as shorthand of names or measurement labels. Clement (1982) and Kieran, & Louise, (1993) indicated children's arithmetic experiences in elementary schools which lead them to different alternative frame works in Algebra. For instance, in arithmetic children have experience that letters denote measurements, for example 10 m to denote 10 meters, but in Algebra it may denote ten times unspecified number.

Traditionally children have limited experience with letters in elementary schools such as for finding area students use the formula A= *l* x *w* which shows the use of letters as labels in arithmetic. Children's such experience of using letters as measurement labels in arithmetic lead them to make alternative frameworks to treat numerical variables as if they stood for the objects rather than numbers.

Same letter can be used in different contexts with different meanings. The different meanings of the same letter or symbol in different contexts create problems in conceptual understating of the concepts of Algebra and in solving the algebraic problems (Zahid, 1998). Moreover, these letters and symbols are highly abstract in nature and can be predicted by understanding the context in which the symbols are used. Collis (1975) argued that the difficulties children have in Algebra relate to the abstract nature of the elements in Algebra. After knowing the use of letters it is important to review the literature about students' perceptions about the use of letters in algebraic expressions and equations.

Students' Concepts about Algebraic Expressions

A number of research studies have shown that Students' interpretation of symbols in algebra is not proper because some of the difficulties faced by the students are specific to algebraic expressions (Kuchemann, 1981& Clement 1982). For instance, a difficulty in algebraic understanding of expression was identified by Davis (1975). He called the "name-process" dilemma by which an expression such as 6x is interpreted in algebra as an indication of a process "What you get when you multiply 6 by x" and a "name for the answer". Sfard and Linchevski (1993) cited in Herscovics and Linchevski, (1994) have suggested that the term "process-product dilemma" better describes this problem. Collis' theory of the student's Acceptance of the Lack of Closure (ALC) is a little bit different which describes the level of closure at which the pupil is able to work with operations (Collis, 1975). He observed that at the age of seven, children require that two elements connected by an operation (e.g. 3 + 2) be actually replaced by a third element; from the age of 10 onwards, they do not find it necessary to make the actual replacement and can also use two operations (e.g. 6+4 +5); twelve year-olds can refrain from actual closure and are capable of working with formulas such as Volume = L x B x H; between the ages of 13 - 15, although students are not yet able to handle variables, they have no difficulty with symbolization as long as the concept symbolized is underpinned by a particular concrete generalization. Collis' ALC theory is particularly relevant to the teaching of algebraic expressions since the operations performed on the pro-numerals cannot be closed as in arithmetic. For example in the response of a question in a research most of the students could not accept 8 **x a** as the area of an indicated rectangle unless it was inserted in the formula "*Area of rectangle= 8 x a*".

Use Of Equal Sign

The misconceptions about the equal sign are common in the learners of Algebra (Carpenter et. al., 2003). The concept of equality is an important idea for developing algebraic concepts among the learners of Algebra. NCTM (2000) showed importance of the concept of equal sign (=) and suggested that more emphasis should be placed on students' interpretation of equal sign to ensure a foundation for learning Algebra. Much of elementary school arithmetic is answered oriented which reflects in students' algebraic solutions. Students who interpret the equal sign as a signal to compute the left side and then to write the result of this computation immediately after the equal sign might be able to correctly interpret algebraic equations such as 2x + 3 = 7 but not equations such as 2x + 3 = x + 4 (Carpenter et al., 2003). Researches highlighted that students tend to misunderstand the equal sign as an operator, that is, a signal for "doing something" rather than a relational symbol of equivalence or quantity sameness (NCTM, 2000). Students interpret this sign as an operator. Students who immediately place an answer following the equal sign without considering the relationship of the numbers on both sides of the equal sign is a counter indication of a relational interpretation for instance, 8 + 4 = 12 + 5 = 17. Falkner, Levi and Carpenter (1999) asked 145 American grade 6 students to solve the following problem:

$$8 + 4 = \square + 5$$

All the students thought that either 12 or 17 should go into the box. The equal sign meant "carry out the operation". They had not learned that the equal sign expresses a relationship between the numbers on each side of the equal sign." This is usually attributed to the fact that in the students' experience, the equal sign always "comes at the end of an equation and only one number comes after it" (Falkner et. al., 1999, p. 3). Another possible origin of this misconception is the "=" button on many calculators, which always returns an answer.

A major focus of recent research into the teaching and learning of Algebra has been the transition from arithmetic and Algebra. Difficulties with the transition from arithmetic to Algebra have been found to stem from problems relating to operational laws, the equals sign, and operations on and the meaning of the variable (Cooper & William, 2001).

RESEARCH DESIGN

I used qualitative research design for exploring students' perceptions about the use of symbols, letters and sings in Algebra. I preferred qualitative design because in this design the natural setting is the direct source of the data (Fraenkel & Wallen, 2003). In this study the researcher goes to observe research participants and to collect data in their natural setting without controlling any aspect of the research situation. As this research study was intending to find out students' perceptions, the affect of that perception on their learning and exploring the reasons of their perceptions. These questions, which are concerned with the process of phenomenon, are best answered through qualitative paradigm. As Creswell, (2003) supports this idea by saying, "This study is "concerned with the process rather than outcomes or product" (p.145).

I selected case study as the research method. This method allowed me to get in-depth understanding of the perceptions of students about the use of symbols in Algebra and in exploring the factors which affect students' perceptions. A case study is particularistic because it focuses on a specific phenomenon such as a program, event, process, person, institution, or group.

RESEARCH SETTING

Sample and Sampling Procedure

"A sample in a research study is a group on which information is obtained" (Fraenkel & Wallen 2006, p.92). I wanted the participants to be from government school Karachi Pakistan and to be the students of Science group. Moreover, they should have experience of learning Algebra in previous classes. These boundaries led me to follow Maxwell's (1996) suggestion of using purposeful sampling when persons are "selected deliberately in order to provide important information that [cannot] be gotten as well from other choices" (p. 70).

Students were the primary sample of the study to explore their perceptions and the influence of learning opportunities on their understanding of the Algebra. I conducted this study with the students belonging to the same age group. Teacher

I conducted study with one teacher. She was my secondary participant because this research is intending to find about learning opportunities inside the classroom and teacher has an important role in this regard. I selected a Mathematics teacher. She was teaching Mathematics since last fourteen years in secondary school.

Procedure

I conducted eight focused group interviews with students and two interviews with teacher of about 40 or 45 minutes. The time and place of interview was according to the choice of research participants. Before each interview the students were given a task which they were supposed to solve in 10 – 15 minutes. After participants' completion of Task and the subsequent discussion of their strategies, I then shared two or three work samples of the students' strategies that could enhance discussion. These alternative students' work samples were used for investigating students' perceptions about symbol sense, algebraic thinking and their perception about the use of letters with their additional justifications.

RESULTS AND ANALYSIS

Students' Perceptions about Mathematics

Before exploring students' perceptions about the use of symbols in Algebra, I preferred to elicit their perceptions about Mathematics and Algebra in general. This elicitation helped me to find out the root causes of different issues in learning Algebra which I will discuss later on.

On a probing question about Mathematics, a participant (students) replied.
"Sir I like Mathematics because when I do sums [mathematical problems] I enjoy." (In: January 29, 2008) Another student replied "I like Mathematics because my elder brother is Mathematics student in college [studying in grades 11 - 12] and he helps me in solving different problems" (In: January 29, 2008).
Another student shared that, "I also like Mathematics because when I do Mathematics sums I like it and enjoy doing them but when I do not get the correct answers I dislike Mathematics" (In: January 29, 2008).

These responses show that students like Mathematics because of their achievements in solving problems and getting correct answers. The students who enjoy doing Mathematics could solve problems like doing puzzles and riddles, to get amused of it. Also it suggested that the students gave up their efforts of solving problems when they got stuck and when they could not find solution to the problems. On the other side the students who did not like Mathematics had different feelings towards Mathematics, as one student thought, "I do not like Mathematics because it is very difficult, mostly each problem has different solutions and it is difficult to remember all these solutions." (In: January 29, 2008). One more student shared, "Sir for me trigonometry and theorems are difficult in Mathematics" (In: January 29, 2008). Likewise, the next quote also showed a problem, "For me Algebra is difficult because it has very big formulas and we could not understand how to use them" (In: January 29, 2008).

The above data suggested that students had difficulties in different areas within Mathematics like some students highlighted trigonometry, some highlighted theorems and some had highlighted Algebra as difficult area, therefore, they appeared to conceive of Mathematics as a difficult subject.

As evident from above quotations it appeared that most of the students liked Mathematics. Data also highlighted that the students who liked Mathematics appeared to have support from their siblings, parents or teachers. Furthermore, their interest in Mathematics could be associated with their feelings of success in solving problems. For instance, data showed that they enjoy doing sums when they get correct answers. On the other hand, the students who disliked Mathematics showed difficulties in understanding the mathematical problem and did not get correct answers. Moreover, the data also showed that many students were either afraid of doing Mathematics which could be associated with socially constructed fear towards Mathematics prevailing in society or students feel boredom of sustaining their engagement with Mathematics.

Students' Perceptions about Algebra

As this study focuses on the area of Algebra so I investigated students' insights about the Algebra. In the response of the question about Algebra, a student shared that, "I like Mathematics but I do not like Algebra. Algebra is difficult subject because we *don't know the value of x or y"* (In: January 29, 2008). Another student said

> I also like Mathematics because I like to solve sums and getting answers. I enjoy solving the exercises given in Mathematics. But in Mathematics the part of Algebra is a difficult subject because usually in Algebra the *values* are not given and we have to find the answer so it is difficult to get answer without any given value. (In: January 29, 2008)

Students shared that

> Sir I do not like Algebra because of big and difficult formulae. I am difficult to remember these formulae and I could not understand where I should use these formulae. For example in Factors I feel complexity that which formula I suppose to use to solve it." (In: January 29, 2008)

The above quote highlighted that *very big formulas in Algebra make* it difficult for her because she could not remember them. Students, who had previously learned algebraic formulae in one context, found difficulty in applying these formulas in other/unfamiliar contexts. Skemp (1986) attributed this difficulty of students' ability to use formula in different contexts as instrumental understanding rather than relational understanding of the formula. Relational understanding suggests that students become able to apply their knowledge in solving problems in different situations.

The data also revealed that students had some strong rationales for their disliking. They highlighted the problem of interpreting letters and variables and use of letters in Algebra. Moreover, they also indicated that they had some concerns about the methods of solving the algebraic problems which they indicated by saying like, formulae are difficult and when and where to use them.

Students' Perceptions about The Use Of Letters In Algebra

I used different tasks to identify students' perceptions about the use of symbols in Algebra. These tasks were based on the concepts underlying understanding of the symbols in terms, in expressions and in equation. Symbols are considered as driving force of algebraic thinking. This research study results have revealed the evidence that students' difficulties in Algebra could be related to their difficulties and misinterpretation of symbolic notations. According to Kieran (1992), misconceptions and common errors are rooted generally from the meaning of symbols. This study highlighted students' perceptions which were rooted in the multiple meanings or roles that same symbol assume in different contexts. This study also investigated that students were having difficulties in using, analyzing, or understanding symbols in different situations. It is worthwhile here to closely examine and discuss students' perceptions about the use of symbols in Algebra and discus findings.

I used the following task for exploring students' perceptions about letters and their skills to use symbols in algebraic expressions:

> *A piece of rope 3 meters long is cut into two pieces. One piece is x meters long. How long is other piece?*

Students' work sample

Six students out of ten solved that problem as under

A piece of rope = 3m

One piece is = x m

Other piece =?

Solve

3/2 = x

Other piece = 1.5 m

The students used different strategies to solve the task. I selected a strategy which six out of ten students used. The selected strategy was discussed (In: Feb 20, 2008)

> T: How do you feel about the task?
>
> S: It is not difficult task.
>
> S: It is slightly difficult. (an other student)
>
> T: Why you feel that it is difficult?
>
> S: Sir, it is difficult because of x.
>
> T: Why you think that putting x make it difficult?
>
> S: Because the value of x is not given. How can we solve such type of problems in Algebra without given values.

It is evident from the above data that students were confused because of the use of the variable x. They were expecting that the value of x with any numbers should be given in this task. The data revealed that students had limited concepts of the letters in Algebra most of them had misconception that all letters used in algebraic tasks should have any one and fixed numerical value. On further discussion over participants' own solutions of the given problem, students shared that they had experienced in using x as unknown. For instance, a student stated that

> Sir, the problem says us to find the value of other piece of the rope which is x. I suppose x to be found in this problem and divided by 2 because the problem say that cut in two parts. So for getting two parts I divide it by two. (In: Feb 20, 2008)

In the above response a student who could not solve the problem asked a counter question, "But how do you think that both parts are equal. It is not given in the problem (to cut in equal parts)" (In: Feb 20, 2008). The student replied to her by saying, "I get idea of two from the given problem" (In: Feb 20, 2008). This discussion influenced other students so most of them suggested that the value should be given in the problem. At that point students asked about any value. For knowing the reasons behind their demand I asked

> T: But why I (Teacher) should give you the word equal in the problem.
>
> S: Because it will help us in the getting same answers. Other wise the answers may be changed like one can get 1 meter and 2 meters or 1.5 m and 1.5 meters. (In: Feb 20, 2008)

Another reason possibly might be their single answer approach experience in arithmetic which made them to come up with a single answer. For example, Kuchemann (1981) identified as particular numerical value to cause closure.

In the solutions most of the students divided given length of the rope in two equal measurements. In the discussion on their solution they all agreed that in the given problem it is possible that both pieces may not be equal. It might be 1 meter and 2 meters. As one of the students suggested that in the problem, word equal parts should be included. The findings of the study suggest that student's arithmetic experience of getting a single answer is highly influenced in their perceptions which mold them to put any value (number) and get the answer. The above dialogues showed that students got confused in word problems where the letter was given, specially the letters which they had already used in their pervious exercises for totally different purposes. In pervious classes of arithmetic they did lot of problems by unitary method. They used to use x for knowing the value of the unknown in unitary method. Thus in such type of the problems in Algebra they also used the same method and used x for same purpose.

After some discussion on their responses I felt that they were feeling difficulty in solving the task, and not come to use variable in their answer so I chose MCQ (multiple choice questions) approach. I gave them three choices. Like

(1) x-3 (2) 3- x (3) 3x

I was astonished to see that students were not ready to accept these answers. As, one of them argued, "How can we cut x-3 or 3-x pair? And how you can measure it (x-3)? We cannot measure it." (In: Feb 20, 2008)

It was evident from this quote that the students were thinking to get single answer in numbers because they wanted to answer in this problem which could be possible in real situation and could be measured. For instance, they try to get answers like 1.5 or 2 m, etc because the given problem was asking about the measurement of other piece. As a student argued, “how can we measure x-3 in real situation?” (In: Feb 20, 2008). The finding from this task highlighted an important aspect of Algebra which is algebraic thinking. The situation above indicates students’ lack of algebraic thinking. This indicates that students had no conceptual understanding of the generalization nature of Algebra and the use of letters for generalization which is the basic concept of Algebra. Their lacking in the concept of generalization is also evident from some other tasks where they were accepting 2x+3y as answer but in the word problem they were not accepting the answer with x as a variable or specific unknown. On further probing I found that the confusion was with the wording and real life situation. Probably it seems difficult to cut a rope in 3-x meter piece in a real situation. Students could accept such results in the condition when students have algebraic thinking and the concept of generalization nature of Algebra. Stacey and MacGregor (1997) said, “*Algebraic thinking* is about generalizing arithmetic operations and operating on unknown quantities. It involves recognising and analysing patterns and developing generalisations about these patterns. In Algebra, symbols can be used to represent generalisations” (p.12).

Some students also tried to solve the task by putting zero at the end of the expression. For example, a student stated that “we can solve it by making it equation.” (In: Feb 20, 2008) She put zero after this expression which also indicated that they want a single answer. Even she could not solve the problem but making equations by putting zero is indicating that they try to solve the problems by putting them in comfort zone. Literature also highlighted the issue as Wagner & Parker (1993) argued that students often force algebraic expression into equation by adding ‘= 0’ when asked to simplify or solve.

These all responses and the methods they used are due to their arithmetic experiences. Moreover, it shows that they probably did not have clear understanding of Algebra and its nature of generalization. Because such type of response clearly indicated that they do not expect any letter in the final answer especially in the word problems.

I used another task for my further investigation. I found that the second task was little bit difficult for the participants.

There are 24 hours in a day. How many hours in y days?

Students solved the task by supposing different values like two students suppose the value of y is 2, three supposed 7, and other have 365, and 30 days in their solution. In the discussion students argued on their responses. The students who supposed the value of y is 2 have no argument as she had an idea that any number could be supposed, the students who supposed 7, 30 and 365 argued that as days are indicated that it may ask about week, month or year so they supposed the days in a week, month or year. But one thing was common that all students were agreeing that all responses are correct but the in the given task the value of y should be given.

The views about the task identified that they were feeling it a difficult task. According to them third value was not given in this task. A student inquired that, "How we solve the problem with only one value." (In: Feb 22, 2008) One participant replied, "By putting the value of y. suppose any value of y and solve that task" (In: Feb 22, 2008). The data of the study suggests that they believe in a value for solution of the task for any variable. Even they also indicated that the answer should be same as students shared, "But how it is possible because in this case we all may have different answer" (In: Feb 22, 2008). Their single and common answer approach needed a common value so one of the students said that, "Sir, please give us any value of y. other wise, we could not solve it" (In: Feb 22, 2008).

Students' comments show that in this problem they were also feeling difficulty in accepting the answer in the presence of y. Even they used different values of y and tried to get answer but they were not able to use y as a number or unknown variable. Additionally, it was also found that they might have procedural knowledge through which they used a correct operation to solve the task, but the lack of conceptual understanding of letters for making an algebraic expression was found. The discussion on students' responses confirms and strengthens the pervious findings of this study about their concept of generalization.

It is evident from the data that all groups solved the problem by putting different values of y and multiplied it with 24. The answers were different; they all were confused about their answer because of the variation in their answers. Their method of solving the problem shows that they have clear understanding of mathematical process of solving the problem as they all multiplied both given and evaluated values.

From the task based interviews and classroom observations the study explored that students have misconceptions about the letters that all letters presented in Algebra have a number or value. Their responses and work samples show that they perceive letters to have a fixed value and they named it as hidden value or unknown value and named the letter as variable. For instance, in the response of the question why should we use letters in algebraic expressions? They all have same point of view that "the letters show any values which we do not know or we have to find out." (In: Feb 18, 2008) Additionally, some students replied that when we don't know the value or we want to know the value of any unknown then we put x, y or any other letter. Here they also shared an example from their real life experience that, "For example we use xyz in our common language when we do not want to disclose the name of any person" (In: Feb 18, 2008). Their replies indicated that they have a limited understanding of the use of letters in Algebra. Their analogy reflects that they think the letters are only representation of or for the hidden things or letters are used for discovering some unknown or hidden value. From analyzing of the textbooks of class sixth to class eighth I investigated that the textbooks also prefer the fixed natural numbers for variables, and make Algebra more figurative. At the initial stage it is acceptable as Kuchemann (1981) also highlighted in his research that for the students of age 11-13, Algebra should t be more figurative. But at high school level (age group of 14-16) the students should be introduced to abstract and generalized nature of Algebra. At this stage they should know that symbols are not only having some values but they represent the relationships. Collis (1975) and Kuchemann (1981) argued that supposing a value in any algebraic expression lead learners to incorrect responses where an unknown is given a particular numerical value to cause closure.

Students' Perceptions about the Use of Letters as Short Form of the Objects

The data of the study reveals that in some tasks students perceive letters as short forms and abbreviations of some objects especially in the word problems. For instance, I used the Students and Professors task (adopted from Kuchemann 1981)

Q: At a University there are six times as many students as professors.
What would be the equation?

a) P = S/6 **b)** 6S = P **c)** S > P d) 6S > P

The responses of the students revealed that only one participant responded P = S/6. In the discussion she could not justify her strategy or solution. As she used trial and error method so she solved it by chance. It was evident from her responses. She could not explain her answer and could not reply to why she divided S with 6.

For knowing students perceptions about the letter used in the task I probed about the letters S and P. Their responses indicated that all students agreed that S stands for students and P stands for professors (In: Feb 25, 2008). They all have idea that the letters used in this problem are the abbreviations. The study is indicating the issue of students' perceptions about the letters in which they used the letters as objects. For instance, in the response to differentiating 3m in Algebra and in Physics the students replied that both 'm' are unknowns. They replied that in Algebra it is known as variable while in physics it is used for showing meter but in both cases it indicates that it has a value. One of the students shared an example "For example we say how many meters are in one Km?" then we have to find the value of meter as we do not know the value and we have to find out so it is also an unknown" (In: Feb 18, 2008).Another participant shared her arguments by saying that, "We use m in physics as in Algebra , because it is common saying that *Mathematics is mother of all subjects*" (In: Feb 18, 2008).

Students' perceptions about the letters are not different form the teachers' perceptions. For exploring teacher's view about the letter she shared that

> Letters are used in Algebra show the variables. But in many cases these letters also indicated as abbreviation or short hand names of any object. For example in Algebra we use f for functions. In word problems I usually use short hands like for father f and son s etc. I think other words could create difficulties for students (In: January 8, 2008).

The above quote indicates that teacher also has the same perception that in word problems the abbreviations should be written. The same thinking was reflected in students' perceptions.

This study highlighted that in the word problems like the problem of students and professors where students used S for students and P for professors but in other algebraic expressions like x + 3y or 3a – 5b when the students were asked that what a, b or x, y indicate for? They replied that these all letters are variables. Many of them were accepting it as an answer in the result of different operations of algebraic expressions. The above data suggests that only in word problems many students perceive letters are abbreviations of some objects. It shows that in abstract type of problems like in x + 3y they were accepting it as product while in real problems they are not ready to accept it as answer. Kuchemann (1981) studied that students have perception that letters are abbreviations of some thing. , but in my case that students have perception that all abbreviations are abbreviations as well as unknowns and variables. Another study on the student-professor problem showed that students often altered the meaning of the literal symbols in problem situations, changing them so that they were used as labels rather than as varying quantities (Philipp, 1992).

Language Issues

In the response of the task of students and professors five students wrote 6S= P. On discussing their responses the students shared, "Sir, as problem says that six times as many students which indicate that students are many in quantities so we multiplied S with 6" (In: Feb 25, 2008). It is appear from the above quote that participant got the wrong idea from the language of the word problem. It shows that the interpretation of the words given in the task plays a vital role in problem solving

> T: (asked to the participant who choose S>P) Why you select S>P?
>
> S: Sir, the given problem indicated that "are six times as many" which indicate that students are greater then professors. So I have written S>P. (In: Feb 25, 2008).

Another student pointed out that why you have ignored 6. But the respondent could not answer, "The problem says that six times as many so I have written 6S>P" (In: Feb 25, 2008). Five students wrote 6S = P results of the other researches who used the same problem like Clement et. al. (1981) in their research investigated in the responses of the same word problem that 68% students of secondary school replied the same answer 6S = P. This situation shows students' perceptions and misconception in learning Algebra is not different from any other context because my study also confirms the findings of the pervious researches.

The findings of this study suggested that in word problems students developed algebraic equations or expressions by phrase to phrase translation of the given word problem. On probing of their answers, they all indicated the same issue. Clement (1982) shared two reasons for such type of misunderstandings the first is the word order matching and second is static compression. Such type of understanding led them to make wrong algebraic expressions or equations. As in interview a student shared,

> Sir as we have given 6 times Students which means 6 multiplied by S because a stand for students, and then it says as professors and P shows for professors and it given to make an equation so we make equation (In: Feb 25, 2008).

From their written responses as well as from their discussion I found students in my case study were misinterpreting the language given in the task. For example, three students selected S > P and 6S > P. On discussion on their responses they indicated that as the word '*as many*' is another word for greater than so we used the symbol of greater then. Some times the word ratio or times also make puzzle because in the case of the P and S problem we used the word times. It indicates that in word problems usually language become a problem and makes students puzzled. Because in our daily language we usually use words which could not be used in Algebra or arithmetic as these words can be used in other contexts. For instance, in this situation we used the words *as many* in the above problem that create confusion because the words as many can be translated in Urdu as '***ziyadah'*** which means greater than.

Students Understanding of Arithmetic

It also appears from the responses of the students that their poor understating of arithmetic concepts affect their learning of Algebra and interpreting variables. For example, in the task of students and professors students could not solve the task because of their poor understating of the concept of ratio. As many of them had written 6S = P which indicates the problem with the concept of ratio. Many students come to the study of early Algebra with poor understandings of arithmetic. However, it is likely that failure to understand the structures of arithmetic (e.g., commutative law, distributive law, fractions, integers and operations) will place an added cognitive load on students when it comes to the study of Algebra.

Students' Perception about Variables and Specific Unknowns

This study explored students' perception about the variables and specific unknowns. For exploring students' perception about the variable I used flowing task.

"Which is greater, 2n or n + 2? Explain?" (Adopted from NCTM)

The majority of the students in the group solved it by putting only one supposed value and decided that wither 2n or n+2 is greater. Some students solved it by putting different (supposed) values of n. I selected four strategies used by the students to prove the task.

Solution I		**Solution II**		**Solution III**		**Solution IV**	
Suppose n = 1		Suppose n = 2		Suppose n = 3		Suppose n=4	
2n =2(1) =2	n+2 1 + 2 = 3	2n = 2(2) = 4	n=2 2+2 = 4	2n = 2 (3) = 6	n+2 2+ 3 =5	2n = 2 (4) = 8	n+2 4+2 = 6
2n < n+2		2n = n+1		2n > n+2		2n > n+2	

In discussion on the strategies and solutions students tried to justify their solutions. Due to the different responses of the task they insisted for a fixed value to get a common response. For example, a participant shared that, Sir we could not decide that which is greater and which is less some times it becomes greater and for some value it becomes less while in one case it become equal (In: March 05, 2008) .

A student insisted by saying, "For getting a fixed answer we should given the value of n. then we can say accurately that which is greater" (In: March 05, 2008).

Additionally one more student claimed that

> Sir I think 2n is greater because in two cases in gave greater value. For instance I put 4 and 5 and found that 2n is greater. I feel that may be in other cases it will give us greater value. (In: March 05, 2008)

Another, participant stressed that

> Yes sir I agreed with her because I think that in multiply we get greater values rather than in addition. For example if we add 2 and 3 we get 5 but if we multiply it we will get 6. (In: March 05, 2008)

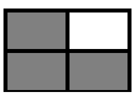
It is evident from participants' comments that they had a partial concept of variables in Algebra. Their responses indicate that they were not accepting different values of variable n which shows that they have a misconception that letters in Algebra have fixed value and they could not show more than one value. This problem involves the comparison of two expressions, both using the same variable. There is a need to think of the variable as taking on a range of values while making this comparison. Their responses show that most of the students used two or three numerical examples to support their responses. Although these answers were technically correct, they indicated a tendency toward arithmetic thinking unlike more general algebraic thinking. Such as, a student shared that "2n" is greater; she argued that "multiplication makes numbers larger". This persistent a view of some students that multiplication makes numbers larger than addition. Moreover, the data indicated that all students used only natural numbers as the referents of the variable in this task. It shows that they thought the variable as being only natural numbers. It is possible that they have less experience of using negative or other integers to prove their expressions.

The data of my study indicated that after putting different values students experienced that greater than or less than or equal to depend on the given values. So, most of the students asked for giving them any value fro n. As they argued that "how can we say which is greater without any given value" (In: March 05, 2008). In such type of situations, students follow their arithmetical thinking rather then algebraic thinking. In addition, they used natural numbers for solving the tasks. It highlighted their perception that in such type of real life problems only natural numbers should be appropriate. It also indicated the problem with their perception about negative integers. For conceptual understanding of the Algebra students should have a clear understating of the concept of negative numbers (Dickson, Brown, & Gibson, 1984). Textbook analysis highlighted that in their pervious classes (seventh and eighth) the exercises and examples of finding relationships or putting values in algebraic expression given in the textbooks gave only natural numbers. So their pervious experience of working with such type of problems with natural numbers is reflected in different tasks given in this study. Kuchemann's (1981) study highlights the ease with which beginning Algebra students could associate letters as representing particular values versus letters as representing relationships.

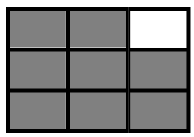
Students' Perception about the Use of Variable for Generalization

For identifying students' perceptions about variable I used a pattern seeking activity. In an interview with the students, they claimed that they are did not used such type of activity before. Because it was their first experience to solve such type of pattern so it made this task more challenging to them. In addition, the ways in which they attend to certain perceptual aspects of the pattern like trial and error method and arithmetical solutions, made it difficult for them to express generality, either verbally or symbolically. In the result not a single student was able to get any generalized pattern.

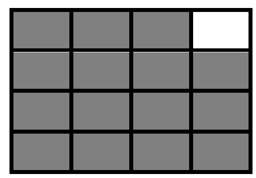
Q: How many gray tiles and white tile will there be in 10th explain how you figure out.



Pattern #1



Pattern #2



Pattern #3

The students solved the pattern by different ways; here I am, sharing one way of solution which I also used for discussion

Pattern # 4 White = 1 Gray = 24	**Pattern # 7** White = 1 Gray = 63	**Pattern # 10** White = 1 Gray = no reply
Pattern # 5 White = 1 Gray = 35	**Pattern # 8** White = 1 Gray = 80	
Pattern # 6 White = 1 Gray = 48	**Pattern # 9** White = 1 Gray = 99	

In this task students seemed to be in difficulty as I already shared that it was students' first experience with such type of task. Only three students out of ten solved it by manually doing arithmetic operations as shown in table 1. Another strategy used by a student in which she drew different squares and made patterns. Some used trial and error strategy. Not a single students could use any letter (as variable) to generalize the pattern. I observed that many students started to solve the problem with numerical strategies but they could not continue it. I think their lack of understanding of the concepts of generalization results the lack of flexibility to try other approaches. In addition they were unable to see possible connections between different forms of representations and generalizations like the use of letters or variables. Literature highlights that it is crucial for students' success in Algebra that they make sense of these concepts and be able to use symbols to express generality. The use of patterning activities to develop meaning for algebraic expressions suggests that hard work is needed by students in order for them to express the observed numerical and geometric patterns in a letter-symbolic form.

Furthermore, in the task of the pattern I observed that some students noticed that the terms of the sequence increased by squaring the pattern number and that this common increment applies to all terms. In other words, they did generalize something, but they continued doing it with arithmetic rule as shown in the table # 1. They generalized some figures, but could not able to use this information to make an expression for the 10th, or for whatever, term of the sequence. It also indicated from their response on the question that how can we find the 100^{th} or 1000^{th} term. A student remarked that, "Sir it needs months to solve the problem" (In: March 08, 2008).

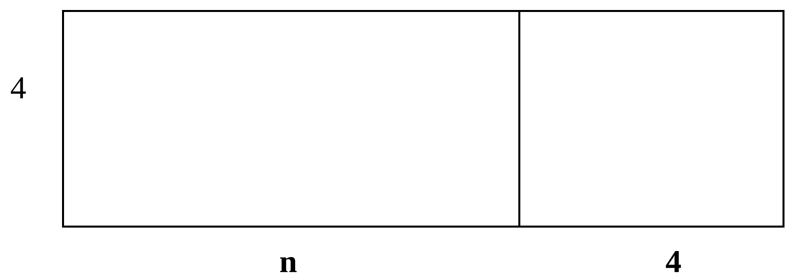
Clement (1982) stated that Arithmetic generalizations are those that do not involve a rule that provides one with an expression of "whatever term" of the sequence. For example, in the task of pattern the students were supposed to find out the 10^{th} pattern. For tenth or any other pattern seeking, it is important that students should have skills of transforming expressions from arithmetic to Algebra. As Clement (1982) claims that, moving from arithmetic to algebraic generalizations is a process that has been found to take time. Teaching process as well as curriculum and textbooks also play a vital role in improving students understanding about the generalization and in helping students in developing their algebraic thinking in the result they can develop algebraic expressions and equations.

Over all, students' strategies in solving the pattern were predominantly numerical. The students' solved the task by trial and error method. But it is clear that they have no idea to generalize the patron. This indicates their less algebraic thinking which restricted them to use symbols to solve the problems.

As I already discussed that Algebra is form of generalization of arithmetic. The example of pattern and how students solved it results in that students have no concept of generalization. For instance, in the task of pattern the learners should have to think more deeply and logically of the solution and use variable for generalizing purpose.

For further investigating students understanding and perception tot use a variable, I used another task.

Q: Write down the area of the rectangular? Write attest three different ways?



Students solved this task in different ways here I am presenting the solution which students tried to get the answer.

Method I	**Method II**	**Method III**	**Method IV**
Length of rectangular = n,4 Breath of rectangular = 4 A= bxl A= 4 x 4 A= 16	Length of rectangular = n,4 Breath of rectangular = 4 A= bxl A= 4 x n A= 4n	Area = Length x Breath Area = 4 (n, 4) Area = 16n	Area = 4n x 4 Area = 16 n

In discussion on their solution a student who solved the task with method I and method II reported that

> Sir, in method I, I took length 4 m and breathe 4 m and multiplied both, while in method II I select 4 as length and n as breath so I get is 4n. I think there is not one rectangle there are two rectangles so I solved two different rectangles (In: March 25, 2008) .

The above quote shows that the student seemed to be confused about two things one was her interpretation of the task that asked participant for different ways of solutions so she thought that there were two different rectangles, and she was supposed to give two solutions for these two different rectangles. Second is the figure it self which is separated in two rectangle. On further probing she argued that, “I identified that there are two rectangles by a separation line between both of them and different size of the length which are 4 and n” (In: March 25, 2008) .

The above statement highlighted that student had less skills in combining and representing length by using n and 4 to represent 4+n. In the discussion on method III a student stated that, “Sir I solved with method III. I took n and 4 as breath and 4 as length, and multiplied all and get 16n.” (In: March 25, 2008) Another student said,

> I think problems of the area we must multiply the given terms. As in this problem we have given 4, 4 and n so for multiplying them we should use the formula which is ***Area = Length x Breath***, by putting length 4 m and breathe n and 4 meters (In: March 25, 2008)

It is evident from their responses and solutions that some students accepted that the length of the rectangle is n and 4. But it is also looking like a dilemma with students' perception of the concept of variable that they could not express any number with any variable like 4+n. On probing question “if the length of the rectangle is 4 and 8 rather then 4 and n then what will be the length” (In: March 25, 2008)? All students agreed that then length would be 12 meters. They also agreed on the process of addition yet they could not write 4+n. This shows that they have no clear idea of how they can get length or breath in the presence of any unknown letter or variable. The data suggested that they have a clear understanding that for getting length both numbers should be added. But they were not able to write 4+ n. For instance in method III, students have clear idea that in bracket they had written (n, 4) they verbally say n and 4 but conceptually they were not adding it. Here the data suggest that they did not have relational understanding however they had instrumental understating of the use of + sign. It further highlighted that their conceptual understanding of the process some times does not help them in symbolic representation.

Method 1 was much more frequent where students totally ignored n. I did not found ignoring the symbol in any other case but in this task most of the students ignored it. Collis (1975) also found the same situation in his research and found that novice learners of Algebra may have such difficulty. But at the level of secondary school where learners had already three years experience with Algebra could not be expected of it.

Of course, these all methods suggest that the common issue within students which is already discussed is a limited conceptual understanding of the use of symbols as variable in Algebra. Furthermore, the data also identified that students were feeling difficulty in transition from arithmetic to Algebra.

. The concept of variable is a complex concept in Algebra because it is used with diverse meanings in different situations. Variables depend precisely from the particular way to use them in the problem-solving. The notion of variable could take on a plurality of conceptions some of them are general number, unknown and functional relation. This research investigated that students meet many difficulties in the use of variables in Algebra. It is possible that they derive from the inadequate construction of the concept of variable in their Algebra classes. Kuchemann (1981) in his research investigated that most of the pupils between 13 and 15 years treat the letters in expressions or in equations like specific unknowns before as generalized numbers or variables in a functional relation.

Students' Perception about Terms and Expressions

In my study I used some tasks in which students were supposed to simplify the expressions. Here I am sharing one of them, in this task students were asked to solve 3(x+ 2y). Students solved it in three different ways.

Way I	**Way II**	**Way III**
3(x+2y) = 3x+2y = 5xy	3(x+2y) = 3x+6y = 5xy	3(x+2y) = 3x+2y = 3x+6y

In the discussion on the strategies and products students shared

T: (Student who solved way I) how did you come with solution? (In: March 26, 2008)

S: Sir I first multiply 3 with x and then add 2y.

T: why you add them (In: March 26, 2008)

S: There is a + sign which shows that we should add the terms given in the expression.

It appears from the above data that students' experiences of arithmetic of getting single answer influenced the algebraic solutions. It is common in arithmetic that the operation signs could not come in the final answer. The same experience is reflected here in both ways, way I and way II. On the response of a participant's argument, another participant commented that," But how can you add two different variables" (In: March 26, 2008. On further probing that why different letters cannot be added? The student replied that, "It is common rule that we cannot add two different letters" (In: March 26, 2008). Most of the students were agreed on the concept that the different letters could not add, so they agreed that way III is correct. This result shows that to these students 3x+6y is not acceptable as a solution to them the solution should be a single answer. They were not accepting the dual nature of the expression in which expression look process and product at same time. They are not looking the expression as a process and a product. Some students shared about their final answer, "There is a + sign which shows that we should add the terms given in the expression" (In: March 26, 2008)

The data suggest that the plus sign (+) led them to do some calculation to produce an answer. On further discussion on different answers students argued that we cannot add x and y because both are different and different letters could not add or subtract. Furthermore, they all agreed that the solution is not final it would be final by putting any values in the given x and y variables. This also indicating the issue which I already discussed that students' experience of arithmetic led them to a single answer that's why the students were not accepting even 3x+6y as product. The data of my study reveals that students' perception about use of symbols affects their perceptions of algebraic expressions because algebraic expressions are a blend of letters or variables and signs.

Students' Perception about Equation

The data of this study indicated that students had different perceptions about the equations. On probing about the equation a student replied, "In equation we have two sides left hand side and write hand side. We say it equation when the answers of both the sides should be equal" (In: March 22, 2008). Another student replied that,

> In equations we have two equal quantities like three apples cost 6 rupees can be written by putting equal sign in between. 3 apples = 6 rupees. This equal sign shows that both sides are equal if we simplify it we can get the answer that one apple costs 2 rupees. (In: March 22, 2008)

These responses suggest that students had perception that equation shows that answer of both the sides should be same. Their responses identified that the sign of equality is used for showing that both sides of the sign are equal. They also said that it can be used for getting answers. Their responses also highlighted that the sign of equality is used for sum up or final answer.

This study explored students misconceptions in solving the word problems of equations. Students prefer to solve word problems by arithmetic reasoning rather than first representing the problem by an algebraic equation and then applying algebraic transformations to that equation. For example, the findings of my study show that many students relied on arithmetic approaches even in problems where they were specifically encouraged to use algebraic methods, as in the following:

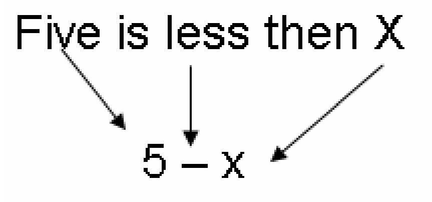
Q: A friend gives you some money. Can you tell which is larger, the amount of money your friend gives you plus six more rupees OR Three times the amount of money your friend gives you? Please explain your answer

Students often used arithmetic process in solving this problem. They supposed a value and then tried to solve the problem. I felt that it was a difficult task for the students to develop an equation. Developing an equation form word problems is a difficult task as Kieran and Chalouh (1993) emphasis that setting up the equation requires an analytic mode of thinking that is exactly opposite to that used when solving a problem arithmetically In fact, when permitted to choose their own solving methods, students find word problems presented in verbal form easier to solve than equations, or "word-equations. However, students' persistent difficulties with the framing of equations to represent word-problem situations lead to questions regarding the feasibility of such approaches for developing algebraic competence.

Kieran (1992) argued that pupils are usually not able to make sense of the algebraic equations as they do not really understand the structure of relations in the equation. Students with insufficient conceptual knowledge about algebraic terms and expressions could not interpret or write the symbolic form of the equation. In the above task they felt difficulty in the arrangement of different terms and in using an appropriate operation and relation between both terms.

The data revealed that among the students the greatest difficulty is of reproducing or forming equations for written or verbal problems. Translating form written or verbal statement to symbolic equation or from English to Math causes a great deal of confusion

(Rosnick 1981). For instance, in the task in which students were suppose to write an expression *"Five is less than x"* most of the students perceive it as subtraction and wrote 5-x. As literally the word less than indicates the subtraction so the answer was given like above.



In the response of this statement students tried to solve the problem according to the situation being described in the problem because in their culture specially in Urdu common language we use "*teen kum pachaas*" which is perceived as 50 – 3. Such language issue developed students' wrong perception in the use of symbols. As in the response students wrote 5- x that indicated that they could not able to represent their understanding by using appropriate sign. The main reason is translating the word problems in mathematical or algebraic expressions. Besides daily language one of the main reasons is teaching process. It is our common experience that teachers use to encourage students to look key words. In given problem less then is supposed as a key word by the participants. The meaning of less than in Urdu it '*Kam'* (کم) so students used minus sign rather then sign of less then. Wagner and Parker (1993) stated, "Though looking for key words can be a useful problem-solving heuristic, it may encourage over-reliance on a direct, rather than analytical, mode for translating word problems into equations" (p. 128).

On probing about the uses of equality sign a student shared, "the equal sign in Algebra is used for showing both sides equal or it is also used for continuation of the problem solution by putting it against the expression" (In: March 22, 2008). Another student shared that "after equal sign the figure shows the answer, and we use equal sign for getting answer in the calculators" (In: March 22, 2008) . These quotes indicate that many students failed to correctly interpret the equal sign (=) as a symbol to denote the relationship between two equal quantities in an equation. For them this sign is interpreted as a command to carry out the calculation. Literature also highlighted the same perceptions about the use of equal sign (Falkner et al., 1999; Cooper & William, 2001). Much of elementary school arithmetic is answer oriented. Students who interpret the equal sign as a signal to compute the left side and then to write the result of this computation immediately after the equal sign might be able to correctly interpret algebraic equations such as 2x + 3 = 7. These researches suggested that the proper interpretation of the equal sign helps students to algebraic manipulation.

Effect of Teaching Process on students learning of Algebra

Students shared that they began to learn Algebra from class sixth. They identified that they learned only some basic rules in pervious class. Mathematics teacher in her interview stated that,

> From sixth class the students stared Algebra, but usually teachers do not pay attention to the chapter so students base remain weak in Algebra. In seventh class they focus on only solving some selected exercises which could not help them in understanding Algebra (In: Feb 17, 2008).

On probing about the teaching process the teacher shared that

> It is dilemma of our school that we have not single teacher including myself with Mathematics back ground. In lower classes teachers use guides books to solve the problem of Mathematics and only copy and paste on the black board. A big problem is with the course which is lengthy so teachers could not complete it. They usually ignore Algebra and geometry. In the result when they come into the tenth class they do not have basic information about Algebra so feel difficulties in understating Algebra. (In: Feb 18, 2008)

It appears from the quote that a major problem in students' difficulties in learning algebraic concepts is pedagogy. As it is a highly abstract in nature and naturally such type of abstract subject cannot be taught through abstract way. Moreover, it is a common practice in my context which I also experienced as learner and teacher of Mathematics that teachers directly start Algebra by giving learners the idea about the rule of addition, subtraction or multiplication of the letters. They usually start exercises given in the textbooks without any prior discussion or doing activity in the classroom. In the result, they could not get the conceptual understating of algebraic concepts.

It is evident from the interviews that Mathematics teacher illustrate some difficulties in the concepts of Algebra. As in the reply of what is Algebra? Mathematics teacher responded that “Algebra is a subject in which we use letters for solving expressions and equations by following some rules and methods” (In: Feb 17, 2008). Likewise, the same beliefs were reflected in the responses of the students.

CONCLUSION

Algebra is a language used to express mathematical relationships. Students need to understand how quantities are related to one another and how Algebra can be used to concisely express and analyze those relationships. The aim of the study was to explore students’ perceptions about the use of symbols, letters and signs and the effect of their perceptions on their learning of Algebra. The study revealed that the students have many misconceptions in the use of symbols in Algebra, which affect their learning of Algebra. It is vital that students recognize that the symbols that are used to represent an unknown quantity or variable have different meanings in different contexts. Algebra is so significant as a part of Mathematics that its foundation must begin to be built in the very early grades. It must be a part of an entire curriculum which involves creating, representing, and using symbols for relationships. But getting desired objectives teachers’ content knowledge and content provided by textbooks also play a significant role for promoting students relational knowledge and conceptual understanding of Algebra. For relational understanding the concepts of Algebra and use of Algebra as a tool to use it in real world situations it is important that the teachers should develop students’ algebraic thinking and symbol sense. To assure that all children have conceptual understating of the use of symbols in Algebra, these concepts must be incorporated throughout the entire Mathematics curriculum. So, that all students could know and apply Algebra in solving their real problems confidently regardless of their ultimate career.

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