UNIT 1 *Decimals and Fractions* Introduction to SIM

Learning objectives

This unit revises the key topics of *decimals* and *fractions*. After completing this unit of STUDENT INSTRUCTIONAL MATERIAL (SIM) you should

- understand the equivalence of decimals and fractions
- be able to convert between decimals and fractions
- be able to multiply and divide decimals in contexts
- be able to undertake, confidently, calculations with negative numbers
- understand why estimating answers is an important mathematical skill
- be able to use a calculator efficiently to make complex numerical calculations.

Key points and principles

- Decimals and fractions are two ways of expressing numbers.

  For example, $\frac{1}{2} \equiv 0.5$ (the sign '$\equiv$' is sometimes used to mean 'equivalent to')

- You can easily convert from one form to another.

  For example, $\frac{1}{4} = 0.25$, $\frac{1}{10} = 0.1$, $0.45 = \frac{45}{100} \left(= \frac{9}{20}\right)$

- Always write fractions in their simplest form.

  For example, $\frac{4}{20} = \frac{1}{5}$, $\frac{7}{56} = \frac{1}{8}$

- Note that the decimals 3.4, 3.40, 3.400, etc. are all the same number, that is, $\frac{17}{5}$ or $3\frac{2}{5}$ and that 3 is the same as 3.0 and 3.00, etc.

- The rules for calculations involving negative numbers.

- Appreciate the difference between answers given to an accuracy involving significant figures and decimal places.

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Facts to remember

- Equivalence of fractions and decimals; see examples shown in the table opposite.

Fraction	*Decimal*
$\frac{1}{10}$	0.1
$\frac{1}{8}$	0.125
$\frac{1}{5}$	0.2
$\frac{1}{4}$	0.25
$\frac{1}{3}$	$0.\dot{3}$
$\frac{1}{2}$	0.5
$\frac{2}{3}$	$0.\dot{6}$
$\frac{3}{4}$	0.75
1	1.0

- Equivalent fractions, for example, $\frac{1}{3} = \frac{2}{6} = \frac{3}{9} = \frac{4}{12}$, etc.

- $\frac{a}{b} \times \frac{c}{d} = \frac{a \times c}{b \times d}$ i.e. you multiply both numerator and denominator

- $\frac{a}{b} \div \frac{c}{d} = \frac{a}{b} \times \frac{d}{c}$ i.e. dividing is the same as multiplying by the reciprocal.

  For example, $4 \div 2 = 4 \times \frac{1}{2}$

- $+ \times + \Rightarrow +$
  $+ \times - \Rightarrow -$
  $- \times + \Rightarrow -$
  $- \times - \Rightarrow +$

- $+ \div + \Rightarrow +$
  $+ \div - \Rightarrow -$
  $- \div + \Rightarrow -$
  $- \div - \Rightarrow +$

We can summarise these results using the rule that when multiplying or dividing by a negative number,

if the signs are the same, the result will be positive

if the signs are different, the result will be negative.

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Glossary of terms

Fractions are rational numbers; that is, numbers of the form $\frac{a}{b}$ where a and b are integers (whole numbers) and $b \neq 0$

Decimals are the extension of whole numbers into tenths, hundredths, thousandths, etc.

For example, 5.4 means $5\frac{4}{10}$ $\left(\text{or } 5\frac{2}{5}\right)$; 0.23 means $\frac{23}{100}$, etc.

Equivalent fractions are different forms of the same fraction, obtained by multiplying or dividing the numerator and denominator by the same factor.

For example, $\frac{1}{2} = \frac{5}{10} = \frac{23}{46} = \frac{250}{500}$, etc.

The *simplest form* is where there are no common factors in the numerator and denominator.

For example, $\frac{1}{2}$ or $\frac{2}{7}$ or $\frac{3}{20}$

Estimation

This is when we estimate the answer to a calculation by making appropriate approximations.

For example, $\frac{21.5 \times 9.7}{41.4} \approx \frac{20 \times 10}{40}$

$= 5$

Significant figures

This is when a number is approximated to a specified number of important (significant) figures. The significance of a digit is dependent on the place value of that digit; hence we count significant figures from left to right (from greater to lesser place values) starting from the first non-zero digit.

For example, 11.747 to 1 significant figure is 10

2 significant figures is 12

3 significant figures is 11.7

4 significant figures is 11.75

Decimal places

This is when a number is given to a specified number of decimal places; decimal places are counted after (to the right of) the decimal point.

For example, 11.747 is 11.7 to 1 decimal place

11.75 to 2 decimal places

0.0749 is 0.1 to 1 decimal place

0.07 to 2 decimal places

0.075 to 3 decimal places