STRAND A: Computation

Unit 1 *Decimals and Fractions*

Student Text

Contents

1 Decimals and Fractions

1.1 Decimals

The use of numbers involving decimal points is very important. Recall that:

$\frac{1}{10} = 0.1$ "one tenth"

$\frac{2}{10} = 0.2$ "two tenths"

$\frac{1}{100} = 0.01$ "one hundredth"

$\frac{7}{100} = 0.07$ "seven hundredths"

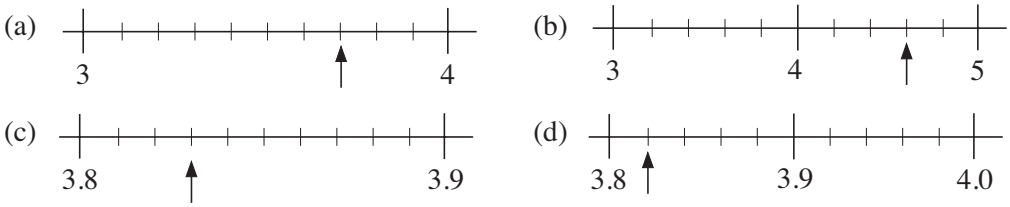
$\frac{42}{100} = 0.42$ "4 tenths and 2 hundredths"

$\frac{82}{100} = 0.82$ "8 tenths and 2 hundredths"

$\frac{7}{1000} = 0.007$ "7 thousandths"

Worked Example 1

Read the value indicated by each pointer.



Solution

(a) The marks on the scale are $\frac{(4-3)}{10} = 0.1$ units apart, so the arrow points to 3.7.

(b) The marks on the scale are $\frac{(5-4)}{5} = 0.2$ units apart, so the arrow points to 4.6.

(c) The marks on the scale are 0.01 units apart, so the arrow points to 3.83.

(d) The marks on the scale are 0.02 units apart, so the arrow points to 3.82.

Worked Example 2

Find

(a) $0.17 + 0.7$ (b) $0.624 + 0.41$ (c) $0.12 + 0.742$

Solution

(a)
$$\begin{array}{r} 0.17 \\ +\ 0.7 \\ \hline 0.87 \end{array}$$

(b)
$$\begin{array}{r} 0.624 \\ +\ 0.41 \\ \hline 1.034 \end{array}$$

(c)
$$\begin{array}{r} 0.12 \\ +\ 0.742 \\ \hline 0.862 \end{array}$$

Note how the decimal points are lined up above each other.

Worked Example 3

A boy spent J$15.48 on football stickers, J$3.33 on sweets and J$105.95 on a comic. Find the total he spent in J$.

Solution

$$\begin{array}{r} 15.48 \\ 3.33 \\ +\ 105.95 \\ \hline 124.76 \end{array}$$

He spent J$124.76.

Exercises

1. Write each of these as a decimal.

(a) $\frac{7}{10}$ (b) $\frac{8}{10}$ (c) $\frac{3}{10}$

(d) $\frac{5}{100}$ (e) $\frac{21}{100}$ (f) $\frac{42}{100}$

(g) $\frac{5}{1000}$ (h) $\frac{151}{1000}$ (i) $\frac{22}{1000}$

(j) $\frac{8}{100}$ (k) $\frac{13}{100}$ (l) $\frac{16}{1000}$

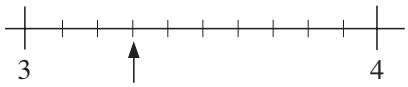
(m) $\frac{5}{10}$ (n) $\frac{4}{100}$ (o) $\frac{321}{1000}$

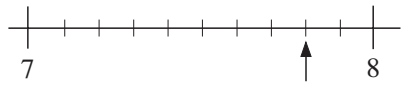
2. Write each of these as a fraction.

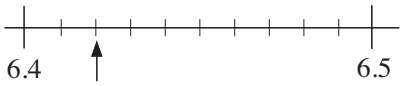
(a)	0.4	(b)	0.3	(c)	0.04
(d)	0.32	(e)	0.45	(f)	0.06
(g)	0.08	(h)	0.14	(i)	0.008
(j)	0.147	(k)	0.036	(l)	0.04
(m)	0.1	(n)	0.009	(o)	0.107

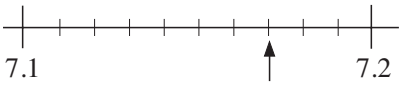
3. Read the value indicated by each pointer.

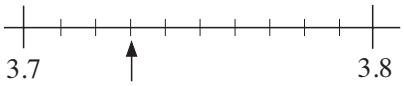
(a) 

(b) 

(c) 

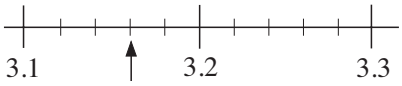
(d) 

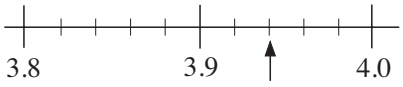
(e) 

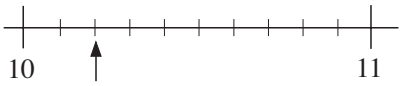
(f) 

(g) 

(h) 

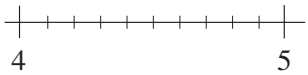
(i) 

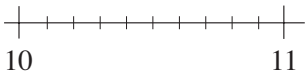
(j) 

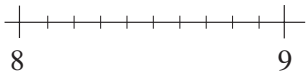
(k) 

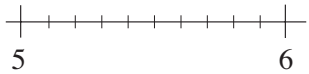
(l) 

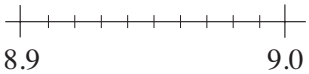
4. On a copy of the scale, mark as accurately as possible the given value.

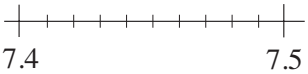
(a) 4.6 

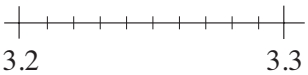
(b) 10.4 

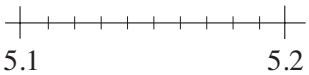
(c) 8.7 

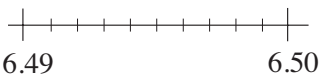
(d) 5.45 

(e) 8.91 

(f) 7.47 

(g) 3.245 

(h) 5.175 

(i) 6.495 

5. Find

(a)	0.7 + 0.81 =	(b)	0.004 + 0.42 =	(c)	0.1 + 0.182 =
(d)	0.863 – 0.024 =	(e)	0.802 + 0.3 =	(f)	0.321 – 0.04 =
(g)	0.86 – 0.002 =	(h)	0.85 – 0.112 =	(i)	0.386 – 0.014 =
(j)	8.67 + 3.2 =	(k)	8.571 + 3.72 =	(l)	4.8 + 12.68 =
(m)	18.2 – 9.47 =	(n)	28.6 + 102.05 =	(o)	82.01 – 32.004 =

6. In each number below, does the 5 represent

5 tenths, 5 hundredths or 5 thousandths?

(a)	0.152	(b)	0.522	(c)	0.05
(d)	3.572	(e)	1.475	(f)	3.115

7. The table below shows how four American friends spent their pocket money in one week.

Name	Sweets	Stickers	Toys	Magazines/Comics
Peter	84 cents	24 cents	70 cents	80 cents
Katie	25 cents	–	80 cents	49 cents
Lora	16 cents	48 cents	99 cents	65 cents
Michael	86 cents	48 cents	42 cents	99 cents

(a) Find the total that each child spent in 2 weeks.

(b) Find the amount that the four children spent altogether in one week.

(c) How much did they spend on sweets in one week?

8. Convert the following amounts in cents to $s.

(a)	328 cents	(b)	152 cents	(c)	842 cents
(d)	1121 cents	(e)	48 cents	(f)	127 cents
(g)	64 cents	(h)	32 011 cents	(i)	8421 cents

9. In the USA, Mr Krishnan buys five 20 cent stamps and three 26 cent stamps.

(a) How much does he spend in $s?

(b) How much change would he get from a $5 note?

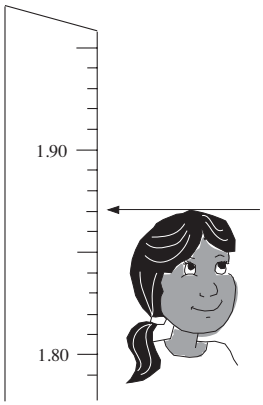
10. The cost of a ticket to enter a fair in Miami is $1.10 for adults and 65 cents for children.

(a) Find the cost if Mrs Barnes goes to the fair with her two children.

(b) Find the cost if Mr Barnes also goes.

11. Janice measures her height, in metres.

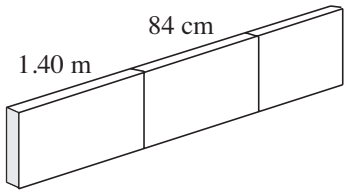
How tall is she?



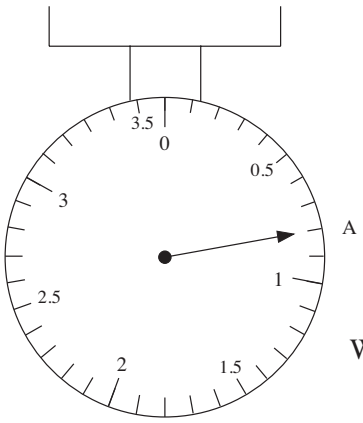
12. Chippy the carpenter marks a 3 metre length of wood into three pieces.

One piece is 1.40 metres long.
Another piece is 84 centimetres long.

How long is the third piece of wood?



13.



These kitchen scales weigh in kilograms.

Write down the weight when the pointer is at A.

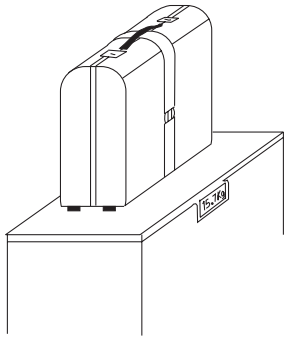
14. John goes into a shop to buy some 19 cent stickers.

(a) What is the greatest number of stickers he can buy for $1?

(b) How much change should John get from $1?

15. Ann flies from London to Montego Bay.

At London Airport her suitcase is weighed and the scales show 15.7 kg.



In Jamaica she buys four presents for her family.

They weigh:

4 kg, 2.50 kg, 0.75 kg, 3.60 kg.

(a) What is the total weight of these presents in kilograms?

(b) Ann puts the presents in her suitcase when she packs it to fly home. What does it weigh now?

(c) If her suitcase now weighs more than 20 kg, there is an extra charge. She has to pay 15 dollars for every kg or part kg over 20 kg.

How much does Ann have to pay?

1.2 Multiplying and Dividing With Decimals

When multiplying or dividing by 10, 100, 1000, etc. the decimal point can simply be moved to the left or the right, although it should be noted that it is really the *digits* that are moving rather than the decimal point. When numbers such as 20, 200 or 300 are involved, the numbers can be multiplied by 2 or 3 and then the decimal point can be moved the correct number of places.

Worked Example 1

Find

(a) 362×100 (b) 4.73×10 (c) $576 \div 10$ (d) $4.2 \div 1000$

Solution

(a) To multiply by 100 move the decimal point 2 places to the right.

To do this it is necessary to add two zeros to the number. So

$$362 \times 100 = 362.00 \times 100$$

$$= 36\ 200$$

(b) To multiply by 10, move the decimal point one place to the right. So

$$4.73 \times 10 = 47.3$$

(c) To divide by 10, move the decimal point one place to the left. So

$$576 \div 10 = 57.6$$

(d) To divide by 1000 move the decimal point three places to the left. To do this it is necessary to put some extra zeros in front of the number.

$$4.2 \div 1000 = 0.0042$$

Worked Example 2

Find:

(a) 3.4×20 (b) $\dfrac{14.8}{20}$ (c) $\dfrac{42}{0.7}$

Solution

(a) First multiply the 3.4 by 2 to give 6.8. Then multiply the 6.8 by 10 to give 68; so

$$3.4 \times 20 = 3.4 \times 2 \times 10$$
$$= 6.8 \times 10$$
$$= 68$$

(b) First divide 14.8 by 2 to give 7.4. Then divide by 10 to give 0.74.

$$\frac{14.8}{20} = \frac{7.4}{10}$$
$$= 0.74$$

(c) First multiply both numbers by 10 so that the 0.7 becomes a 7. This will make the calculation easier. Then divide 420 by 7 to give 60.

$$\frac{42}{0.7} = \frac{420}{7}$$
$$= 60$$

Exercise

1. Find.

(a)	4.74×10	(b)	6.32×100	(c)	$41.6 \div 10$
(d)	12.74×100	(e)	$16.58 \div 100$	(f)	$32.4 \div 10$
(g)	6.3×100	(h)	4.7×1000	(i)	$3.2 \times 10\,000$
(j)	47×1000	(k)	$6.8 \div 1000$	(l)	$82 \div 100$
(m)	$192 \div 1000$	(n)	$14 \div 1000$	(o)	0.18×1000

2. Find.

(a)	1.8×20	(b)	4.7×300	(c)	15×700
(d)	66×2000	(e)	15×400	(f)	1.3×8000
(g)	$66 \div 20$	(h)	$74 \div 200$	(i)	$21 \div 3000$
(j)	$35 \div 5000$	(k)	$3.42 \div 20$	(l)	$52 \div 400$
(m)	18.1×600	(n)	47.2×500	(o)	$4.95 \div 50$
(p)	3×0.02	(q)	15×0.04	(r)	$5 \times 0.000\,7$

3. Find.

(a)	$\frac{16}{0.4}$	(b)	$\frac{500}{0.2}$	(c)	$\frac{64}{0.8}$
(d)	$\frac{24}{0.04}$	(e)	$\frac{264}{0.02}$	(f)	$\frac{465}{0.15}$
(g)	$\frac{156}{0.03}$	(h)	$\frac{48}{0.012}$	(i)	$\frac{56}{0.08}$

4. A factory produces screws which it sells at 1.2 cents each.

   (a) Find the income in cents from the sale of:

   (i) 300 screws (ii) 50000 screws (iii) 4000 screws

   (b) Convert your answers to (a) from cents to dollars.

   (c) $3600 was paid for a batch of screws. How many screws were in this batch?

5. A company in the USA made a large profit one year and decided to give a bonus to each department. The bonus was divided equally among all the staff in each department.

Department	**Total Bonus**	**Number of staff**
Production	$12 487	100
Sales	$8 260	20
Delivery	$5 350	50
Finance	$4 896	40

   Find the amount of the bonus that would be paid to staff in each department.

6. A snail moves at a speed of 0.008 miles per hour.

   (a) How far would the snail travel in 1.5 hours?

   (b) How long would it take the snail to travel:

   (i) 40 miles (ii) 0.72 miles?

7. The cost of making a chocolate bar is J$5.27.

   (a) What is the cost of producing:

   (i) 4000 (ii) 17 000 (iii) 30 000 chocolate bars?

   (b) A consultant says that he can reduce the production costs by 40 cents per bar. How much would this save on the production of:

   (i) 5000 (ii) 22 000 (iii) 30 000 chocolate bars?

8. A new pop group is trying to produce their first CD.

   (a) Their manager finds that it will cost $1.20 to make each CD. If they can afford to spend $1800 on making CDs, how many can they make?

   (b) Another CD manufacturer will make the CDs for 90 cents each. How many more can they make at this price?

9. It is established that a truck can carry 64 000 cans of soft drinks. Each can contains 0.33 litres of drink.

   Find the total volume of the drink carried by the truck.

10. For a major sporting event, a stadium is expected to hold its limit of 70 000 spectators.

    (a) How much money is taken in ticket sales if the price of the tickets were:

    (i) \$5 (ii) \$8 (iii) \$11?

    (b) If \$432 000 is taken in ticket sales when the ticket price is \$6, how many spectators will not be able to get into the ground?

11. (a) $\boxed{900 \times 0.6}$

    Work out the answer to this sum in your head. *Do not use a calculator.* Explain clearly the method you used.

    (b) $\boxed{40 \div 0.8}$

    Work out the answer to this sum in your head. *Do not use a calculator.* Explain clearly the method you used.

12. Write the value of

$$(11.2)^2 - (0.375 \div 3)$$

    (a) exactly

    (b) to two significant figures.

13. Using a calculator, or otherwise, determine the exact value of

    (a) $2.3^2 + 4.1^2$ (b) $\dfrac{0.18}{0.6} - 0.003$

14. Using a calculator, or otherwise determine the value of $3.48 + \dfrac{3.335}{2.3}$ and write the answer

    (a) exactly

    (b) correct to one decimal place

    (c) correct to one significant figure.

Information

The sides of the Great Pyramid of Giza in Egypt are about 230.5 m long. Although it was built thousands of years ago by thousands of slaves, the lengths of its sides vary by no more than 11.5 cm!

Challenge!

Without moving 6 adjacent numbers of the face of a clock, rearrange the other six so that the sum of every pair of adjacent numbers is a prime number.

1.3 Fractions and Decimals

Some fractions can be written as decimals with a fixed number of decimal places, for example:

$$\frac{1}{4} = 0.25$$

These are called *terminating* decimals. Others have an infinite number of decimal places, for example:

$$\frac{1}{3} = 0.333\,333\ldots$$

Numbers that contain an infinite number of decimal places are usually rounded to a specified number of significant figures or decimal places.

Remember that s*ignificant figures* are counted from left to right, starting from the first non-zero digit; *decimal places* are counted after the decimal point.

Worked Example 1

Round each number in the list below to:

(i) 3 significant figures (ii) 3 decimal places.

(a) 4 732.165 (b) 4.736 1 (c) 417.923 5

(d) 0.056 234 (e) 0.004 721

Solution

(a) (i) $4\,732.165 = 4\,730$ to 3 significant figures.
Note that only the first 3 figures are considered.

(ii) $4\,732.165 = 4\,732.165$ to 3 decimal places. There is no charge as there are exactly 3 figures behind the decimal point.

(b) (i) $4.736\,1 = 4.74$ to 3 significant figures. The first three figures are considered and the 3 is rounded up to a 4, because it is followed by a 6.

(ii) $4.736\,1 = 4.736$ to 3 decimal places. The 6 is not rounded up because it is followed by a 1.

(c) (i) $417.923\,5 = 418$ to 3 significant figures. The first 3 figures are used and the 7 is rounded up to 8 because it is followed by a 9.

(ii) $417.923\,5 = 417.924$ to 3 decimal places. There are three figures behind the decimal point and the 3 is rounded up to a 4 because it is followed by a 5.

(d) (i) $0.056\,234 = 0.056\,2$ to 3 significant figures. Note that the zeros at the start of this number are not counted.

(ii) $0.056\,234 = 0.056$ to 3 decimal places.

(e) (i) $0.004\,721 = 0.004\,72$ to 3 significant figures. Note that the zeros in front of the 4 are not counted.

(ii) $0.004\,721 = 0.005$ to 3 decimal places. The 4 is rounded up to a 5 because it is followed by a 7.

Worked Example 2

Convert each of the following fractions to decimals,

(a) $\frac{1}{4}$ (b) $\frac{2}{3}$ (c) $\frac{4}{5}$ (d) $\frac{3}{7}$

Solution

In each case the bottom number should be divided into the top number. This will require long division.

(a) To convert $\frac{1}{4}$, divide 4 into 1.

$$\begin{array}{r} 0.25 \\ 4\overline{)\,1.00} \\ \underline{8} \\ 20 \\ \underline{20} \\ 0 \end{array}$$

So $\frac{1}{4} = 0.25$.

(b) To convert $\frac{2}{3}$, divide 3 into 2.

$$\begin{array}{r} 0.666\ldots \\ 3\overline{)\,2.000} \\ \underline{18} \\ 20 \\ \underline{18} \\ 2 \end{array}$$

So $\frac{2}{3} = 0.666\,6\ldots = 0.667$ to 3 decimal places.

(c) To convert $\frac{4}{5}$, divide 5 into 4.

$$\begin{array}{r} 0.8 \\ 5\overline{)\,4.0} \\ \underline{40} \\ 0 \end{array}$$

So $\frac{4}{5}$ is exactly 0.8.

(d) To convert $\frac{3}{7}$ into a decimal, divide 7 into 3.

$$
\begin{array}{r}
0.428571\ldots \\
7\overline{)\,3.000000} \\
28 \\
\hline
20 \\
14 \\
\hline
60 \\
56 \\
\hline
40 \\
35 \\
\hline
50 \\
49 \\
\hline
1
\end{array}
$$

There will be an infinite number of decimal places in this case, but

$$\frac{3}{7} = 0.428\,6$$

correct to 4 decimal places.

Exercises

1. Write each of the following numbers correct to:

   (i) 2 decimal places (ii) 2 significant figures.

(a)	18.643	(b)	1 024.837	(c)	16.04
(d)	181.435	(e)	16.824	(f)	0.083 741
(g)	0.009 562	(h)	4.837 5	(i)	3.864 9

2. Write the number 48 637.012 45 correct to

(a)	3 significant figures	(b)	2 decimal places
(c)	4 decimal places	(d)	4 significant figures
(e)	3 decimal places	(f)	2 significant figures.

3. Write each number correct to the number of decimal places or significant figures specified.

(a)	0.00472 (2 s.f.)	(b)	48.234 (3 s.f.)	(c)	15.83 (1 s.f..)
(d)	4.862 (2 d.p.)	(e)	18.415 (2 d.p.)	(f)	21.804 (2 d.p.)
(g)	14862 (2 s.f.)	(h)	0.00463 (3 d.p.)	(i)	0.004178 (3 s.f.)
(j)	15682 (3 s.f.)	(k)	54631 (2 s.f.)	(l)	31.432 (3 s.f.)
(m)	14.176 (4 s.f.)	(n)	0.815 (2 s.f.)	(o)	1.84149 (3 d.p.)
(p)	15.013 (3 s.f.)	(q)	14.1704 (3 d.p.)	(r)	201.04 (3 s.f.)

4. The number of spectators that enter a football ground for a big match is 44 851.
   (a) Write this number correct to 1, 2, 3 and 4 significant figures.
   (b) Which of your answers to (a) makes the number of spectators appear
      (i) the largest (ii) the smallest?

5. Each of the fractions below can be written as a terminating decimal. Write each fraction as a decimal.

   (a) $\frac{1}{2}$ (b) $\frac{3}{4}$ (c) $\frac{2}{5}$

   (d) $\frac{3}{5}$ (e) $\frac{1}{8}$ (f) $\frac{5}{8}$

   (g) $\frac{3}{8}$ (h) $\frac{7}{8}$ (i) $\frac{1}{5}$

6. Write each of the following fractions as a decimal correct to 4 decimal places.

   (a) $\frac{1}{3}$ (b) $\frac{1}{6}$ (c) $\frac{4}{7}$

   (d) $\frac{1}{7}$ (e) $\frac{5}{7}$ (f) $\frac{5}{6}$

7. (a) Write $\frac{1}{9}$, $\frac{2}{9}$, $\frac{4}{9}$ and $\frac{5}{9}$ as decimals correct to 5 decimal places.
   (b) Describe any patterns that you notice in these decimals before they are rounded.
   (c) How would you expect $\frac{7}{9}$ and $\frac{8}{9}$ to be written as decimals? Check your answers.

8. (a) Write $\frac{1}{11}$, $\frac{2}{11}$, $\frac{3}{11}$ and $\frac{4}{11}$ as decimals correct to 5 decimal places.
   (b) By looking at any patterns that you observe, write down
   $$\frac{5}{11},\ \frac{6}{11},\ \frac{7}{11},\ \frac{8}{11},\ \frac{9}{11} \text{ and } \frac{10}{11}$$
   as decimals.
   (c) Check your answers for $\frac{7}{11}$ and $\frac{10}{11}$ by division.

9. Write down two different numbers that are the same when rounded to:
   (a) 2 decimal places and 2 significant figures,
   (b) 3 decimal places and 5 significant figures,
   (c) 1 decimal place and 8 significant figures,
   (d) 4 decimal places and 2 significant figures.

10. (a) Change $\frac{4}{5}$ to a decimal.
   (b) Write these numbers in order of size. Start with the smallest.
   $$0.805, \quad 0.85, \quad \frac{4}{5}, \quad 0.096$$

1.4 Negative Numbers

A number line like this is very helpful when dealing with negative numbers.

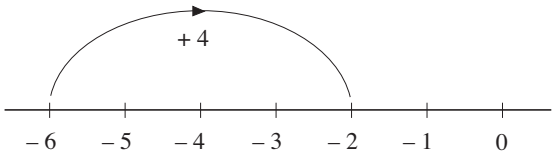
When comparing numbers on the number line, the number on the right is always greater; the number on the left is always less.

Worked Example 1

An ice cube taken from a deep freeze is at a temperature of $-6°\text{C}$. Its temperature rises by 4 degrees. What is its new temperature?

Solution

The number line can be used as shown.



$$-6 + 4 = -2$$

Worked Example 2

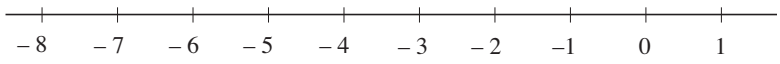
Write down numbers which are:

(a) less than -4

(b) greater than -5 but less than -2

(c) less than -3 but greater than -8.

Solution



(a) Any number less than -4 must be to the left of -4 on the number line, for example -5, -6, -7, etc.

(b) A number greater than -5 must be to the right of -5, and to be less than -2 must also be to the left of -2, for example -4 and -3.

(c) To be less than -3 the number must be to the left of -3 but to be greater than -8 the number must be to the right of -8, for example -7, -6, -5, -4.

Exercises

1. Copy and complete the following table of temperature changes.

Original Temperature	*Temperature Change*	*Final Temperature*
6° C	-10° C	
-4° C	$+2^\circ$ C	
-5° C	-3° C	
-10° C	-4° C	
2° C	-10° C	
-6° C	$+1^\circ$ C	
-5° C	-2° C	

2. Place each set of numbers in increasing order.

   (a) $-10, -4, -6, 2, -5, 3$ (b) $3, -1, 0, -4, 7$

   (c) $7, -2, -4, -6, 3, 0$ (d) $-4, 0, -7, 7, 5, -2$

   (e) $-1, -4, 0, -5, 6, -3$

3. Write down all the integers which lie between:

   (a) -6 and -10 (b) -4 and 2 (c) -3 and 0

   (d) -1 and -3 (e) -9 and 0.

4. Write down a whole number which is:

   (a) greater than -10 but less than -6 (b) less than -6

   (c) greater than -5 but less than 0 (d) greater than -10

   (e) less than -6 but greater than -9 (f) less than -4.

5. Insert the correct $<$ or $>$ sign between each pair of numbers.

   (a) $3 \quad 2$ (b) $-5 \quad -6$ (c) $0 \quad -1$

   (d) $-7 \quad -10$ (e) $-2 \quad -4$ (f) $-1 \quad -6$

   (g) $-5 \quad 0$ (h) $-9 \quad -6$ (i) $-8 \quad -2$

Challenge!

If ABCDE $\times 4 =$ *EDCBA, find A, B, C, D and E if none of them is zero.*

1.5 Operations with Negative Numbers

The following rules for working with negative numbers are important to remember.

Multiplication	*Division*
$+ \times + \Rightarrow +$	$+ \div + \Rightarrow +$
$+ \times - \Rightarrow -$	$+ \div - \Rightarrow -$
$- \times + \Rightarrow -$	$- \div + \Rightarrow -$
$- \times - \Rightarrow +$	$- \div - \Rightarrow +$

When multiplying and dividing negative numbers, if the signs are the same the result will be a positive number; if the signs are different the result will be a negative number.

You can see how to use these rules in the following Worked Examples.

Worked Example 1

(a) $6 \times (-4) = ?$ (b) $-6 \times (-7) = ?$

(c) $42 \div (-2) = ?$ (d) $-33 \div (-11) = ?$

Solution

(a) $6 \times (-4) = -24$ (b) $-6 \times (-7) = 42$

(c) $42 \div (-2) = -21$ (d) $-33 \div (-11) = 3$

Worked Example 2

(a) $-6 + 7 = ?$ (b) $-6 - 8 = ?$

(c) $-6 - (-4) = ?$ (d) $-7 + (-3) = ?$

Solution

(a) $-6 + 7 = 1$ (b) $-6 - 8 = -14$

(c) $-6 - (-4) = -6 + 4$
$= -2$

(d) $-7 + (-3) = -7 - 3$
$= -10$

Exercises

1. (a) $-6 \times -2 = ?$ (b) $-7 \times 2 = ?$ (c) $-8 + -4 = ?$
   (d) $-8 - 12 = ?$ (e) $-9 + 2 = ?$ (f) $-4 + (-2) = ?$
   (g) $-6 - (-5) = ?$ (h) $-7 \times (-4) = ?$ (i) $-3 - (-4) = ?$
   (j) $-64 \div (-2) = ?$ (k) $-18 \div 9 = ?$ (l) $-21 \div (-3) = ?$
   (m) $-4 + (-6) = ?$ (n) $-2 - (-4) = ?$ (o) $-6 - 7 = ?$
   (p) $-24 \div 2 = ?$ (q) $-7 \times (-8) = ?$ (r) $-7 - (-8) = ?$
   (s) $-14 + 8 = ?$ (t) $-12 - 3 = ?$ (u) $-16 - (-8) = ?$
   (v) $4 - 12 = ?$ (w) $3 - (-2) = ?$ (x) $8 \div (-4) = ?$

1.6 Estimating Answers

If you do a calculation such as

$$\frac{4.1721 \times 3.846}{18.21 + 5.73}$$

you need to use a calculator to find the answer. This section looks at ways of estimating the answers to calculations such as this, and giving answers to a specified accuracy.

Worked Example 1

Give each of the following numbers to (i) 2 significant figures (2 s.f.)
(ii) 1 decimal place (1 d.p.)

(a) 17.47 (b) 0.0784 (c) 4.96

Solution

(i) (a) $17.47 \approx 17$ to 2 s.f.
(b) $0.0784 \approx 0.078$ to 2 s.f.
(c) $4.96 \approx 5.0$ to 2 s.f.

(ii) (a) $17.47 \approx 17.5$ to 1 d.p.
(b) $0.0784 \approx 0.1$ to 1 d.p.
(c) $4.96 \approx 5.0$ to 1 d.p.

Worked Example 2

Estimate the answers to each of the following problems.

(a) 18.42×3.76 (b) $\dfrac{47.932}{4.071}$ (c) $\dfrac{18.51 + 11.23}{3.0712}$

Solution

Estimates can be obtained by using each number correct to 1 or 2 significant figures (s.f.).

(a) $18.42 \times 3.76 \approx 20 \times 4$
≈ 80

(b) $\dfrac{47.932}{4.071} \approx \dfrac{48}{4}$
≈ 12

(c) $\dfrac{18.51 + 11.23}{3.0712} \approx \dfrac{20 + 10}{3}$
$\approx \dfrac{30}{3}$
≈ 10

Exercises

1. Write each of the following numbers correct to 1 significant figure.

(a)	47.316	(b)	18.45	(c)	27.65
(d)	9.632	(e)	15.01	(f)	149.32
(g)	62.84	(h)	0.176	(i)	0.039 4
(j)	1.964	(k)	21.87	(l)	1.849

2. Estimate the answers to the following problems:

(a)	6.74×8.31	(b)	4.35×12.46	(c)	236×4.321
(d)	16.67×3.21	(e)	5.92×105.3	(f)	16.78×32.51
(g)	$\dfrac{192.7}{17.35}$	(h)	$\dfrac{284}{37.2}$	(i)	$\dfrac{963}{51.8}$
(j)	$\dfrac{47.63}{0.4185}$	(k)	$\dfrac{36.72}{8.26}$	(l)	$\dfrac{17.24}{0.374}$

Now, using a calculator, find the answer to each problem in Question 2, giving your answer to 4 significant figures. In each case compare your answers and estimates.

3. Estimate the answers to each of the following calculations.

(a)	$\dfrac{6.6 \times 9.5}{32.4}$	(b)	$\dfrac{0.32 \times 8.43}{6.21}$	(c)	$\dfrac{12.8 + 45.3}{17.3}$
(d)	$\dfrac{33.6 + 77.9}{15.72}$	(e)	$\dfrac{888 + 723}{38.4}$	(f)	$\dfrac{560 + 2.01}{29.47}$
(g)	$\dfrac{16.5 \times 3.82}{4.162}$	(h)	$\dfrac{82.4 + 91.9}{1.04 + 1.43}$	(i)	$\dfrac{82.6 \times 19.41}{0.024 \times 405}$

4. When cars leave a factory they are parked in lines until they are delivered. The length of each car is 4.32 m. A line contains 54 cars.
   (a) Estimate the length of a line, if there are no gaps between the cars.
   (b) Find the length of a line if there are no gaps between the cars.
   (c) If there is a gap of 0.57 m between each car, estimate the line length and find the actual length.

5. A cross-country runner has an average speed of $6.43\ \text{m s}^{-1}$.

   $\left(\text{Average speed} = \dfrac{\text{Distance}}{\text{Time}}\right)$

   (a) Estimate and find the distance run in 200 seconds, if he runs at his average speed.
   (b) Estimate and find, to 3 significant figures, the time it takes him to run 1473 m.

6. Drivers at a motor racing circuit complete practice laps in times of 130.21, 131.43 and 133.62 seconds. The length of the circuit is 5214 metres.

   (a) Estimate the average speed of the drivers. $\left(\text{Average speed} = \frac{\text{Distance}}{\text{Time}}\right)$

   (b) Find their speeds correct to 2 decimal places.

7. A car travels 12.43 km on 1.12 litres of gasoline.

   (a) Estimate and then calculate the distance that the car would travel on 1 litre of gasoline.

   (b) Estimate the distances that the car would travel on

   (i) 41.1 litres and (ii) 33.8 litres

   of gasoline.

8. A factory in the USA produces 108 CD players every day. The cost of producing the CD players is made up of \$4125 for labour costs and \$2685 for parts.

   Estimate and then calculate:

   (a) the total cost of producing a CD player,

   (b) the cost of the parts to make a CD player,

   (c) the cost of the labour to make a CD player.

9. Vinyl floor tiles are made so that they are square with sides of length 48 cm.

   Estimate and then calculate the number of tiles needed for rooms with sizes:

   (a) 6.41 m by 3.28 m (b) 3.84 m by 2.91 m (c) 4.29 m by 4.6 m.

10. (a) Write down the numbers you could use to get an approximate answer to

$$59 \times 32$$

   (b) Write down your approximate answer.

   (c) Using a calculator find the difference between your approximate answer and the exact answer.

11. Flour costs J\$96.5 per kilogram. Ryan bought 205 kg and shared it equally among 14 people. He calculated that each person should pay J\$141.30.

   Without using a calculator, use a rough estimate to check whether this answer is about the right size.

   You must show all your working.

Challenge!

The growth rate of the human hair varies from person to person. On average, a human hair grows at a rate of 0.35 mm per day. If the length of a hair is 6 cm, how long will it take the hair to grow to a length of 26 cm?

1.7 Using Brackets and Memory on a Calculator

By using the bracket and memory keys on a calculator it is possible to carry out tasks fairly quickly and easily.

Some of the standard memory keys which are found on a calculator are:

[Min] Places the current number into the memory, replacing any previous number.

[MC] Clears the memory.

[M+] Adds the number displayed to the memory.

[MR] Recalls the number that is currently in the memory.

Brackets can be used to tell the calculator the order in which to do calculations.

For example, to find:

$$\frac{3.62 + 4.78}{3.9 - 1.4}$$

use

[(] [3] [.] [6] [2] [+] [4] [.] [7] [8] [)] [÷] [(] [3] [.] [9] [−] [1] [.] [4] [)] [=]

Worked Example 1

Find:

(a) $\dfrac{3}{3.2 + 1.8}$ (b) $\sqrt{\left(\dfrac{5.2 - 3.6}{4.7}\right)}$

Solution

(a) Use the brackets as shown below

[3] [÷] [(] [3] [.] [2] [+] [1] [.] [8] [)] [=]

to obtain 0.6.

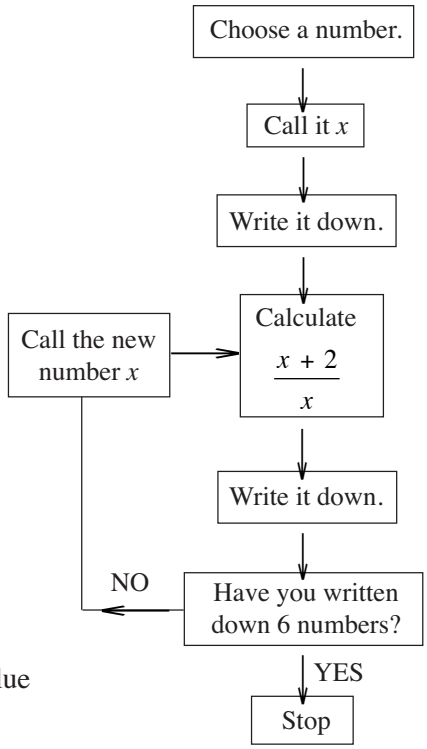
(b) Use brackets to enclose the top part of the fraction ,as shown below,

[(] [5] [.] [2] [−] [3] [.] [6] [)] [÷] [4] [.] [7] [=] [√]

to obtain 0.5835 correct to 4 decimal places.

Worked Example 2

Follow the instructions given in the flow chart for a student who chooses the number 20 as a starting point.



Solution

Starting with 20 leads to the calculation

$$\frac{20 + 2}{20} = 1.1$$

To perform the remaining calculations, follow the steps below.

1. Press (Min) to place the value displayed in the memory.
2. Press (+) (2) (=) which adds 2 to the value of x.
3. Press (÷) (MR) (=) which divides the displayed value by the number in the memory.
4. Go back to Step 1.

Worked Example 3

A factory produces plastic tanks in 4 different sizes.

The table opposite shows the orders placed one day.

Find the value of the orders, using the memory keys on your calculator.

Tank Size	Price	Number Ordered
Giant	\$126	5
Large	\$ 87	16
Medium	\$ 56	44
Small	\$ 33	31

Solution

1. First press (MC) to clear the memory.
2. For the *Giant* tanks, the value of the order is given by 126×5. Find this on your calculator and press the (M+) key.
3. For the *Large* tanks, find 87×16 and press (M+) again.
4. For the *Medium* tanks, find 56×44 and press (M+) again.
5. For the *Small* tanks, find 33×31 and press (M+) again.
6. Finally press (MR) to obtain the total, which is \$5509.

Exercises

1. Carry out the following calculations, using the bracket keys on your calculator.
Give all answers to 3 significant figures.

(a) $4 \times (8.1 + 16.2) =$ (b) $(5.6 - 3.2) \times 11.4 =$ (c) $\dfrac{15.6 + 3.2}{5.3} =$

(d) $\dfrac{19 + 24}{16} =$ (e) $\dfrac{33}{127 - 84} =$ (f) $\dfrac{19 + 61}{20 + 32} =$

(g) $\sqrt{\dfrac{4}{9 + 24}} =$ (h) $\dfrac{14.1 \times 2}{18 + 4} =$ (i) $\sqrt{\dfrac{16 + 22}{18 - 4}} =$

(j) $\left(\dfrac{8.2 + 4}{13 + 7}\right)^2 =$ (k) $\dfrac{3 + 4.9}{7.32 \times 18.4} =$ (l) $\left(\dfrac{4.7 - 3.2}{8 \times 0.22}\right)^2 =$

2. Work through the flow chart of *Worked Example 2,* starting with a number of your own choice.

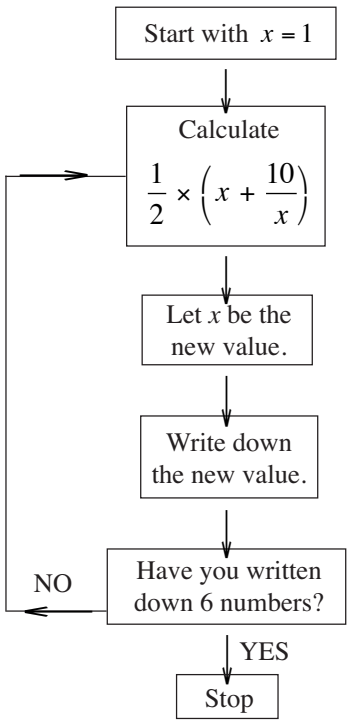
3. Find the mean of each set of numbers, using the brackets on your calculator.

(a) 15, 16, 17.5, 18, 20.

(b) 22, 21, 32, 28.

(c) 112, 114, 140, 130, 132, 126, 128, 110.

4. Use the flow chart shown in the diagram, giving your final answers to 5 significant figures.



(a) Work through the flow chart as shown.

(b) Follow the flow chart but start with $x = 2$ instead of $x = 1$.

(c) How does this affect your final answer?

5. (a) Carry out the following calculations on your calculator inserting brackets where shown.

(i) $(24 \times 2) + (12 \times 4) + (3 \times 15) =$

$24 \times 2 + 12 \times 4 + 3 \times 15 =$

(ii) $(24 + 2) \times (15 + 3) =$

$24 + 2 \times 15 + 3 =$

(iii) $(24 \times 2) \div (5 \times 3) =$

$24 \times 2 \div 5 \times 3 =$

$24 \times 2 \div (5 \times 3) =$

(b) In each of the following, decide which brackets, if any, could be missed out without changing the answer that would be obtained.

Check your answers with your calculator.

(i) $(3 \times 6) + (5 \times 51) + (15 \times 2) =$

(ii) $(3 + 6) \times (5 \times 2) =$

(iii) $(3 - 4) \times (8 - 2) =$

(iv) $(3 + 4) \div (5 \times 2) =$

(v) $(3 \times 4) \div (5 + 2) =$

(vi) $(3 \times 2) \div (4 \times 6) =$

6. The formula

$$A = 2\pi r(r + h)$$

is used to calculate the surface area of a drinks can.

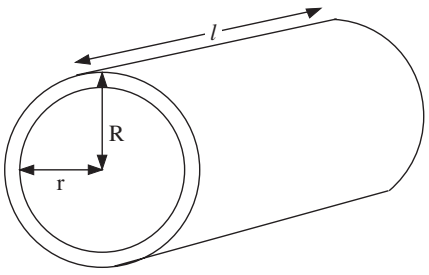
Using π on your calculator, find A to 1 decimal place, if

(a) $r = 6$ cm and $h = 10$ cm,

(b) $r = 3.7$ cm and $h = 7.4$ cm.

7. The volume of plastic used to make a pipe is given by the formula

$$V = \pi l(R^2 - r^2)$$



(a) Using π on your calculator, find V to the nearest whole number, if

(i) $R = 25$ mm, $r = 20$ mm and $l = 3000$ mm,

(ii) $R = 3$ cm, $r = 2.4$ cm and $l = 500$ cm.

The formula can be rearranged as

$$l = \frac{V}{\pi(R^2 - r^2)}$$

(b) Find l if:

(i) $V = 800\ \text{cm}^3$, $R = 5\ \text{cm}$ and $r = 4.5\ \text{cm}$.

(ii) $V = 100\ \text{cm}^3$, $R = 1\ \text{cm}$ and $r = 0.8\ \text{cm}$.

8. Find the value of f using the formula

$$\frac{1}{f} = \frac{1}{u} + \frac{1}{v}$$

if $u = 28.2$ and $v = 18.4$. Give your answer correct to 3 significant figures.

9. The acceleration due to gravity, g, on any planet can be found using the formula

$$g = \frac{Gm}{d^2}$$

Find g if $G = 6.67 \times 10^{-11}$, $m = 7.4 \times 10^{30}$ and $d = 8.4 \times 10^{9}$.
Give your answer correct to 2 decimal places.

10. Use a calculator to find the value of

(a) $\dfrac{3.86 + 17.59}{5}$ (b) $\dfrac{9.76 + 1.87}{18.3 - 15.8}$

(c) $\dfrac{330}{1.2 \times 5.5}$ (d) $\dfrac{1}{\sqrt{(0.16)}}$

11. Use your calculator to evaluate

(a) $(2.37 - 8.42)^2$ (b) $\sqrt{(2.37 - 8.42)^2 + 17.42}$

12. Maria uses the formula

$$a = \frac{v - u}{t}$$

She has to calculate the value of a when $v = 118.07$, $u = 17.76$ and $t = 4.8$.
Maria estimates the value of a without using her calculator.

(a) (i) Write down numbers Maria could use to estimate the value of a.

(ii) Write down the estimate these values would give for the value of a.

Maria then uses her calculator to find the value of a.

(b) Here is the sequence of keys that she presses.

[1] [1] [8] [.] [0] [7] [−] [1] [7] [.] [7] [6] [÷] [4] [.] [8] [=]

This gives an answer of 114.37, which is not the correct answer.

Why does she get the wrong answer?

13. William uses his calculator to work out

$$\frac{4.2 \times 86}{3.2 \times 0.47}$$

He is told to do this in one sequence, writing down only the answer. He presses the keys as follows:

This gives him the wrong answer. Explain what is wrong with William's method.

Investigation

Within 4 consecutive years, Mrs Morton gave birth to four lovely children. Today, x years later, Mr and Mrs Morton find out that the product of their four children's ages is 3024.

How old is each child now, assuming that all the children are of different ages?

Investigation

In his will, a man left 23 cows to his three children. The eldest child was to have half of the herd, the second child should have one third and the youngest one eighth. The children could not decide how to divide up the cows without it being necessary to kill any of them.

A wise man came to the scene. He brought along his only cow and put it with the other 23 cows to give a total of 24 cows. He gave half of the 24 cows (12) to the eldest child, one third of the 24 cows (8) to the second child and one eighth of the 24 cows to the youngest child. He then took his own cow back. What is wrong with this solution?

Investigation

In country X, only 5 cent and 8 cent stamps are available. You have to post letters which cost 23 cents, 27 cents, 77 cents and $19.51 respectively. Which of these amounts can you make exactly?

Make a complete list of the amounts between 1 cent and 99 cents which ***cannot*** *be made exactly.*