STRAND E: Measurement

Unit 11 *Units of Measurement*

Student Text

Contents

denotes that the topic is not on the current CXC/CSEC Mathematics syllabus and therefore not examined, but is of relevance to the content of the Unit.

11 Units of Measurement

11.1 Units and Measuring

Different units can be used to measure the same quantities. It is important to use sensible units. Some important units are listed below.

1 km = 1000 m
1 m = 100 cm
1 m = 1000 mm
1 cm = 10 mm

1 tonne = 1000 kg
1 kg = 1000 g

1 litre = 1000 ml
1 m^3 = 1000 litres
1 cm^3 = 1 ml

Worked Example 1

What would be the best units to use when measuring,

(a) the distance between Negril and Kingston,

(b) the width of a watchface,

(c) the mass of a person,

(d) the mass of a letter,

(e) the mass of a truck,

(f) the volume of medicine in a spoon,

(g) the volume of water in a swimming pool?

Solution

(a) Use km (or miles).

(b) Use mm or cm.

(c) Use kg.

(d) Use grams.

(e) Use tonnes.

(f) Use ml.

(g) Use m^3.

Shading denotes that the topic is not on the current CXC/CSEC Mathematics syllabus and therefore not examined, but is of relevance to the content of the Unit.

Worked Example 2

(a) How many mm are there in 3.72 m?

(b) How many cm are there in 4.23 m?

(c) How many m are there in 102.5 km?

(d) How many kg are there in 4.32 tonnes?

Solution

(a) 1 m = 1000 mm

So

$3.72\text{ m} = 3.72 \times 1000$

$= 3720\text{ mm}$

(b) 1 m = 100 cm

So

$4.23\text{ m} = 4.23 \times 100$

$= 423\text{ cm}$

(c) 1 km = 1000 m

So

$102.5\text{ km} = 102.5 \times 1000$

$= 102\,500\text{ m}$

(d) 1 tonne = 1000 kg

So

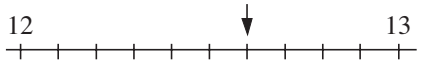
$4.32\text{ km} = 4.32 \times 1000$

$= 4320\text{ kg}$

Worked Example 3

What value does each arrow point to?

(a)



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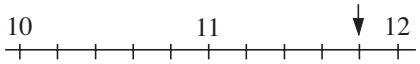

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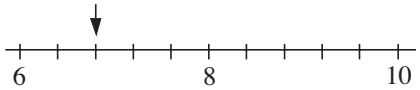


Solution

(a) Here the marks are 0.1 units apart.
So the arrow points to 12.6.

(b) Here the marks are 0.2 units apart.
So the arrow points to 11.8.

(c) Here the marks are 0.4 units apart.
So the arrow points to 6.8.

Exercises

1. Measure each line below. Give the length to the nearest mm.

   (a) ———————————————————————————

   (b) ——————

   (c) ——————————————————

   (d) ———————————————————————————

   (e) ——

2. Which units do you think would be the most suitable to use when measuring:

   (a) the distance between two towns,
   (b) the length of a sheet of paper,
   (c) the mass of a sheet of paper,
   (d) the mass of a sack of cement,
   (e) the volume of a water in a cup,
   (f) the volume of water in a large tank?

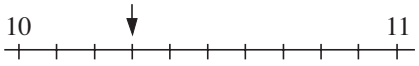
3. (a) How many grams are there in 12.3 kg?
   (b) How many mm are there in 4.7 m?
   (c) How many mm are there in 16.4 cm?
   (d) How many m are there in 3.4 km?
   (e) How many cm are there in 3.7 m?
   (f) How many ml are there in 6 litres?

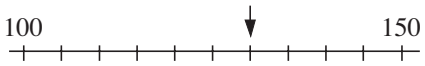
4. Copy and complete the table below.

Length in m	*Length in cm*	*Length in mm*
4		
	311	
		1500
	374	
8.62		

5. Read off the value shown by the arrow on each scale.

(a) 

(b) 

(c) 

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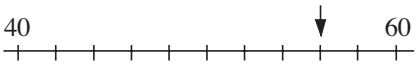
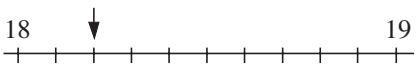
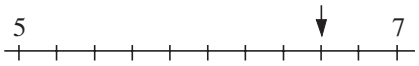
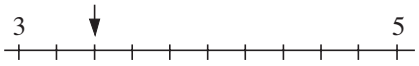
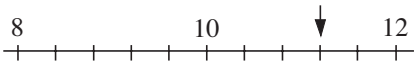
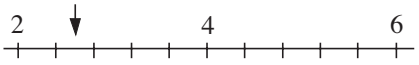
(e) 

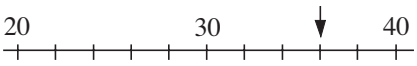
(f) 

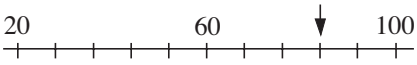
(g) 

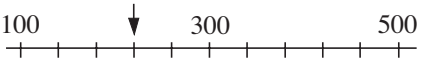
(h) 

(i) 

(j) 

(k) 

(l) 

(m) 

6. A jug contains 1 litre of water.

   (a) If 150 ml is poured out, how much water is left?

   (b) A glass holds 200 ml of water. How many glasses could be filled from a full jug?

7. State whether the following lengths would be best measured to the nearest m, cm or mm.

   (a) Your height.

   (b) The length of a ship.

   (c) The height of a hill.

   (d) The thickness of a book.

   (e) The height of a building.

   (f) The length of a pencil.

   (g) The width of a nail.

11.2 Upper and Lower Bounds

When measurements are made, they can only be obtained to a limited degree of accuracy. For example if the length of a line is given as 11 mm, this means that it is 11 mm to the nearest mm. In fact, if l is the length then it lies in the range

$$10.5 \leq l < 11.5$$

Worked Example 1

For each length below state the range of values within which the length must be.

(a) 18 cm (b) 12.7 cm (c) 11.06 m

Solution

If l represents the given length then:

(a) $17.5 \leq l < 18.5$ (b) $12.65 \leq l < 12.75$ (c) $11.055 \leq l < 11.065$

Note

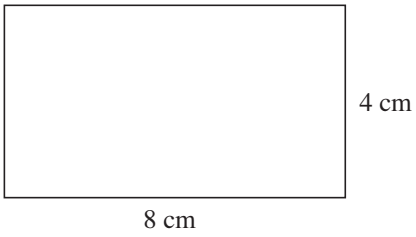
The *lower bound* is the smallest number which will round to the given number, while the *upper bound* is the smallest number which will *not* round to the given number.

Worked Example 2

If the length of the sides of the rectangle are given to the nearest cm, find:

(a) the minimum possible perimeter

(b) the range of possible areas.



Solution

First note that the length, l, of the rectangle is 8 cm to the nearest cm, so $7.5 \leq l < 8.5$.
Similarly the width, w, of the rectangle lies in the range $3.5 \leq w < 4.5$.

(a) To find the minimum possible perimeter, use the minimum possible length,

$$\begin{aligned} \text{Minimum perimeter} &= 7.5 + 3.5 + 7.5 + 3.5 \\ &= 22 \text{ cm} \end{aligned}$$

(b) The range of values for the area can be calculated using the maximum and minimum lengths of each side.

$$\begin{aligned} \text{Minimum area} &= 3.5 \times 7.5 \\ &= 26.25. \end{aligned}$$

$$\begin{aligned} \text{Maximum area} &= 4.5 \times 8.5 \\ &= 38.25 \end{aligned}$$

So

$$26.25 \leq \text{area} < 38.25$$

Exercises

1. For each quantity below, state the range of values within which it must lie.

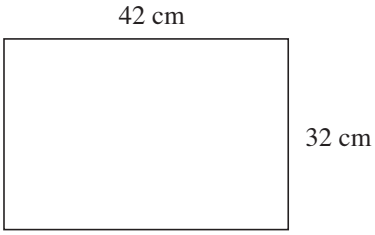
   (a) $x = 4.7$ (b) $l = 42$ (c) $A = 15.62$

   (d) $d = 16.2$ (e) $r = 11.68$ (f) $m = 14.24$

   (g) $w = 218$ (h) $l = 15.20$ (i) $w = 5.00$

   (j) $v = 20.0$ (k) $A = 18.09$ (l) $r = 31.451$

2. The lengths of the rectangle are given to the nearest cm.



   (a) State the range of values within which the length, l, and the width, w, must lie.

   (b) Find the range of values within which,

   (i) the perimeter (ii) the area

   of the rectangle must lie.

3. The radius of a circle is 12 mm to the nearest mm.

   (a) State the range of possible values for the radius.

   (b) Find the range of possible values for the:

   (i) diameter (ii) circumference (iii) area.

4. The quantities x and y are given to 1 significant figure as $x = 20$ and $y = 40$. Find the minimum possible value of each expression below.

   (a) $x + y$ (b) $y - x$ (c) xy (d) $\frac{x}{y}$ (e) $\frac{y}{x}$

5. A student runs 100 m in 12.8 seconds. Find the possible speeds of the student if:

   (a) the distance is to the nearest metre and the time is to the nearest 0.1 s,

   (b) the distance is to the nearest cm and the time is to the nearest 0.1 s.

6. (a) The mass of a plant on kitchen scales is 2.474 kg. What is the possible mass of the plant?

   (b) The mass of another plant on scientific scales is 1.6280 kg. What are the upper and lower bounds of the mass of the plant?

7. Twenty computer monitors are packed in a single container. Each monitor weighs 12.6 kg, to the nearest 0.1 kg.

   (a) Calculate the lower bound for the total weight of the monitors.

   (b) Calculate the difference between the upper and lower bounds for the total weight of the monitors.

8. Jafar has a piece of wood that has a length of 30 cm, correct to the nearest centimetre.

   (a) Write down the minimum length of the piece of wood.

   Fatima has a different piece of wood that has a length of 18.4 cm, correct to the nearest millimetre.

   (b) Write down the maximum and minimum lengths between which the length of the piece of wood must lie.

9. A full jar of coffee weighs 750 g. The empty jar weighs 545 g. Both weights are accurate to the nearest 5 g.

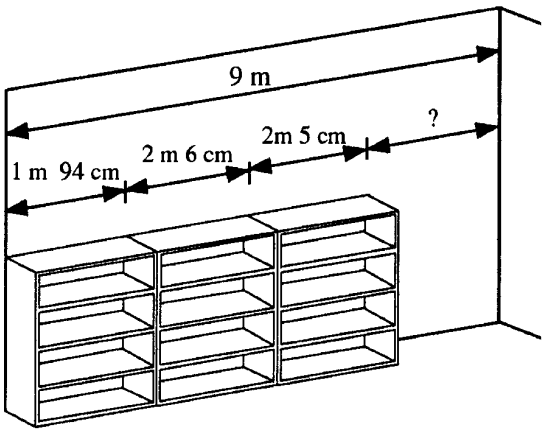
   Calculate the maximum and minimum possible values of the weight of coffee in the jar.

10. Four shelf units are to be fitted along a library wall.

    The wall is 9 m long.

    All measurements are to the nearest cm.

    What is the maximum length of the fourth shelf unit?



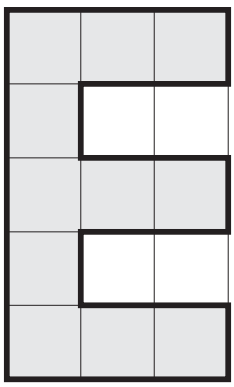
11.3 Estimating Areas

A square with sides of 1 cm has an area of 1 cm^2.

Area = 1 cm^2

Worked Example 1

Find the area of the shaded shape.

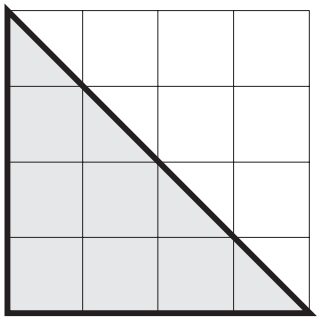


Solution

The shape covers 11 squares, so its area is 11 cm^2.

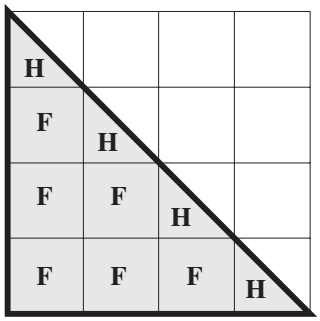
Worked Example 2

Find the area of the shaded triangle.



Solution

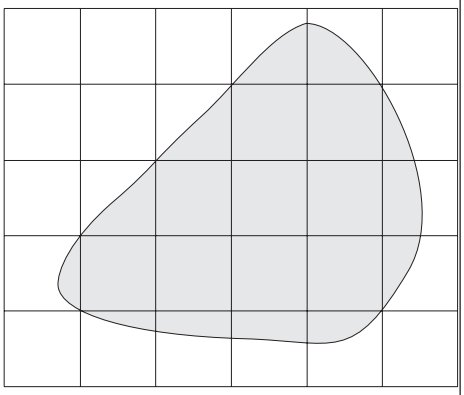
The triangle covers 6 full squares marked **F**, and 4 half squares marked **H**.



$$\text{Area} = 6 + 2$$
$$= 8\,\text{cm}^2$$

Worked Example 3

Estimate the area of the shape shaded in the diagram.



Solution

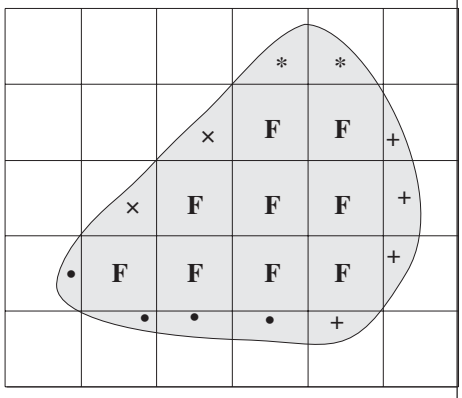
This is a much more complicated problem as there are only 9 full squares marked **F**, but many other part squares. You need to combine part squares that approximately make a whole square. For example,

the squares marked * make about 1 full square;

the squares marked × make about 1 full square;

the squares marked + make about 1 full square;

the squares marked • make about 1 full square.



Thus the total area is approximately

$$9 + 4 = 13\ \text{cm}^2$$

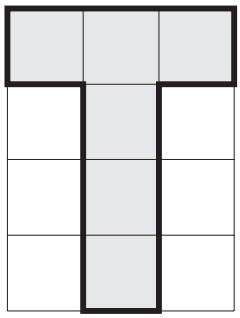
Challenge!

Use 12 matchsticks to form 6 equilateral triangles, all of the same area. Move only 4 matchticks from your figure so as to get 3 equilateral triangles, 2 of which are of the same area.

Exercises

1. Find the area of each of the following shapes.

(a)



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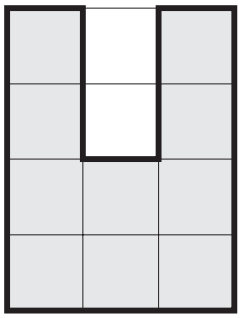
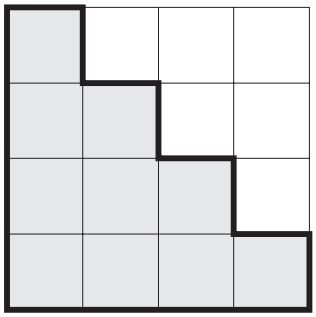
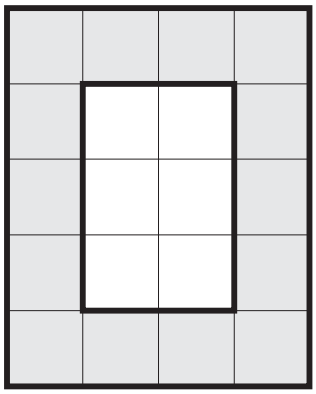


(c)



(d)



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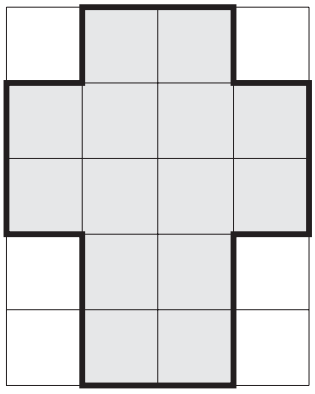

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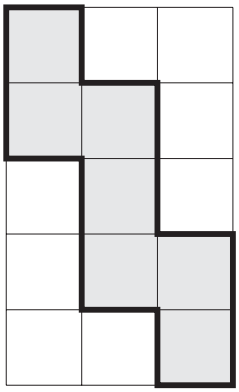
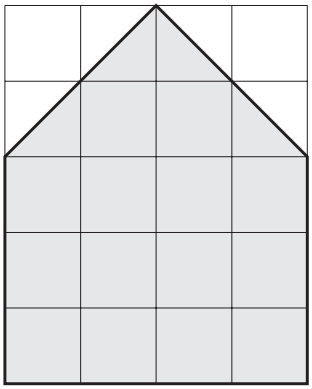


2. By counting the number of whole squares and half squares, find the area of each of the following shapes.

(a)



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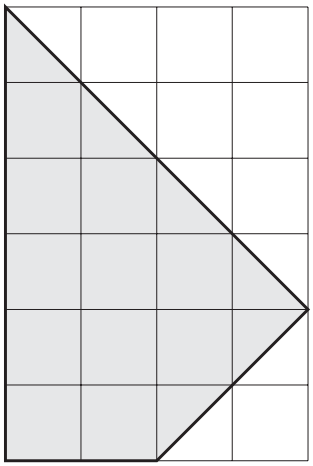

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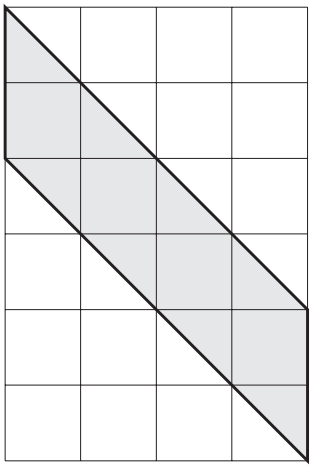

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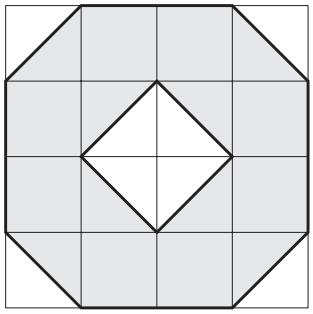

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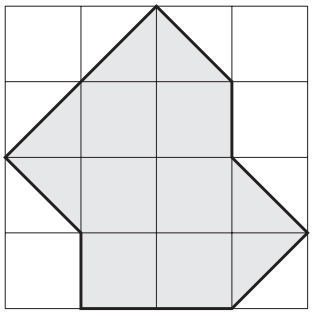

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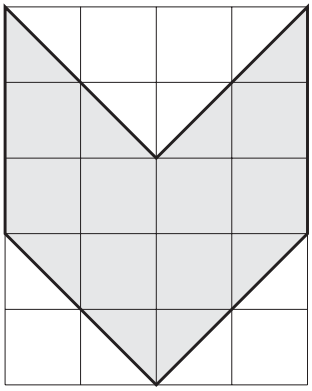
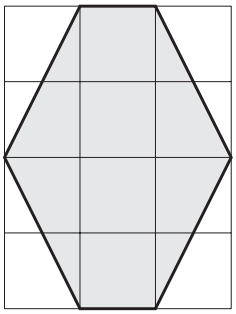


3. Estimate the area of each of the following shapes.

(a)



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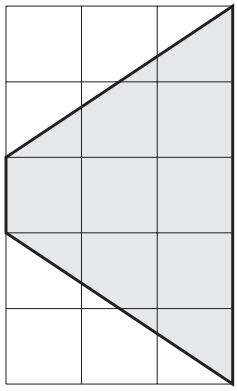

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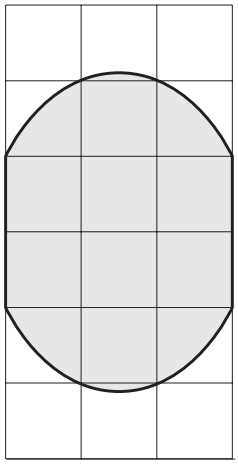

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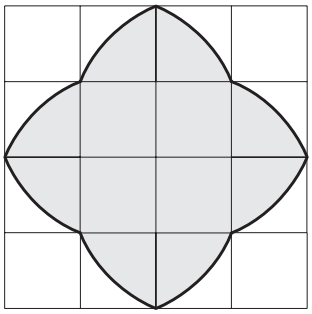

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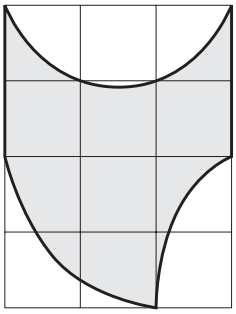
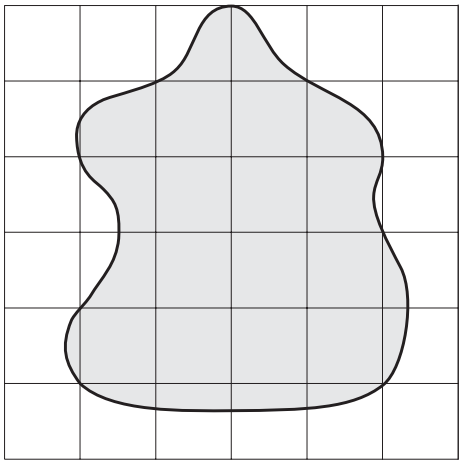
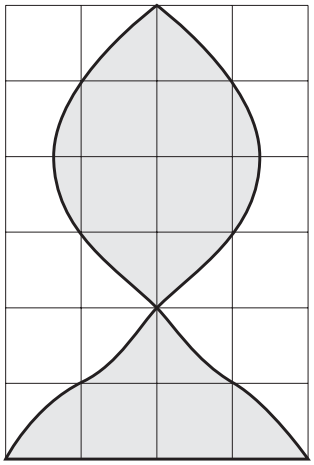


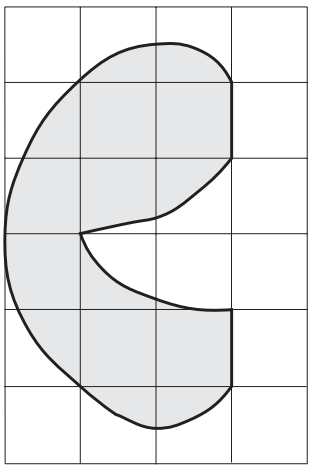
(f)



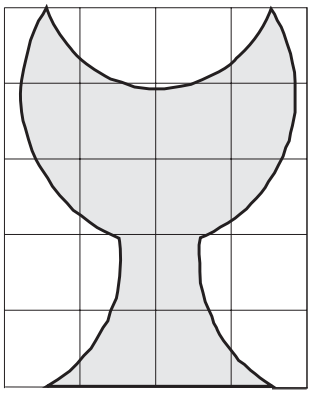
(g)



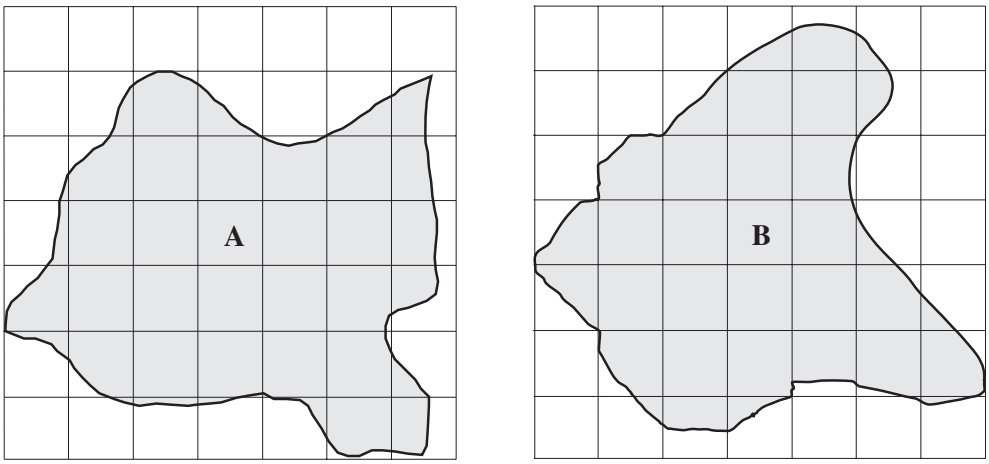
(h)



(i)

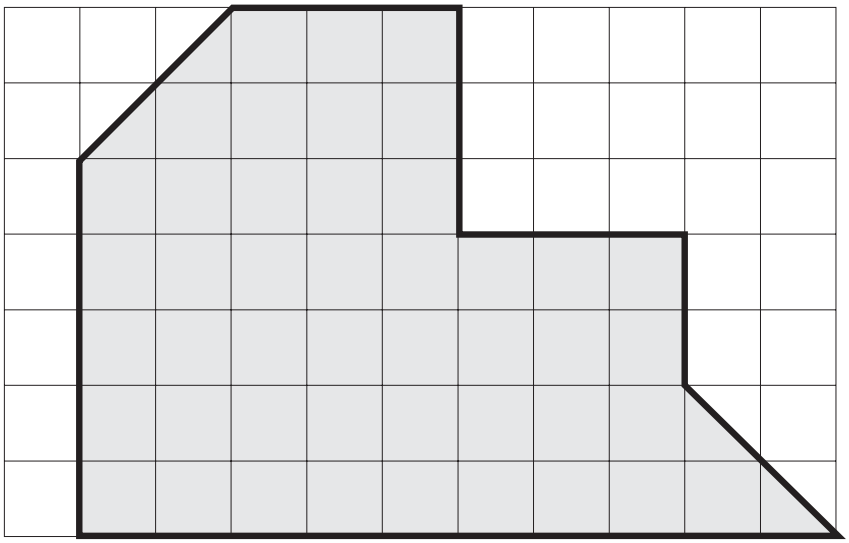


4. The diagrams below shows the outlines of two islands, A and B. The grid squares have sides of length 1 km. Find the approximate area of each island.



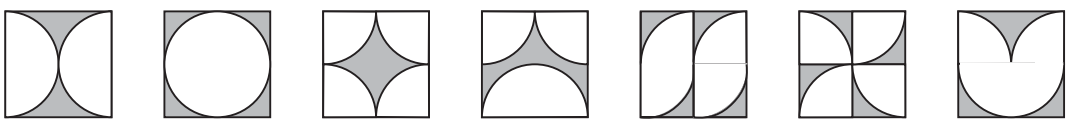
5. Each of the squares in this grid has an area of 1 square centimetre.

Work out the area of the shaded shape.



Investigation

Which of the following shaded figures has the greatest area? The squares are of the same length and the curved lines are all arcs of circles.



11.4 Conversion of Units

It is useful to be aware of both metric and Imperial units and to be able to convert between them.

Imperial Units

1 foot	=	12 inches
1 yard	=	3 feet
1 mile	=	1760 yards
1 pound (lb)	=	16 ounces
1 stone	=	14 pounds
1 gallon	=	8 pints

Conversion Facts

1 kg is about 2.2 lbs.

1 gallon is about 4.5 litres.
1 litre is about 1.75 pints.

5 miles is about 8 km.
1 inch is about 2.5 cm.
1 foot is about 30 cm.

Worked Example 1

John is measured. His height is 5 feet and 8 inches.

Find his height in:

(a) inches, (b) centimetres, (c) metres.

Solution

(a) There are 12 inches in one foot, so

$$\text{John's height} = 5 \times 12 + 8$$
$$= 60 + 8$$
$$= 68 \text{ inches}$$

(b) 1 inch is about 2.5 cm, so

$$\text{John's height} = 68 \times 2.5$$
$$= 170 \text{ cm}$$

(c) 1 metre = 100 cm, so

$$\text{John's height} = 1.7 \text{ m}$$

Worked Example 2

A family travels 65 miles on holiday. Convert this distance to km.

Solution

As 5 miles is approximately equal to 8 km, first divide by 5 and then multiply by 8.

$$65 \div 5 = 13$$
$$13 \times 8 = 104$$

So 65 miles is approximately the same as 104 km.

Worked Example 3

John weighs 8 stone and 5 pounds. Find John's weight in:

(a) pounds,

(b) kg.

Solution

(a) There are 14 pounds in 1 stone, so

$$\text{John's weight} = 8 \times 14 + 5$$
$$= 112 + 5$$
$$= 117 \text{ lbs}$$

(b) As 1 pound is about 0.45 kg,

$$\text{John's weight} = 117 \times 0.45$$
$$= 53 \text{ kg} \quad \text{(to the nearest kg)}$$

Worked Example 4

A line is 80 cm long. Convert this length to inches.

Solution

$$1 \text{ inch} = 2.5 \text{ cm}$$

$\frac{80}{2.5} = 32$, so the line is about 32 inches long.

Exercises

1. Convert each quantity to the units given.

   (a) 3 inches to cm
   (b) 18 stone to pounds
   (c) 6 lbs to ounces
   (d) 6 feet 3 inches to inches
   (e) 15 kg to lbs
   (f) 3 yards to inches
   (g) 3 feet to cm
   (h) 5 gallons to litres
   (i) 120 inches to cm
   (j) 45 kg to lbs
   (k) 9 litres to pints
   (l) 45 gallons to litres
   (m) 8 litres to pints
   (n) 6 gallons to pints

2. Convert each quantity to the units given. Give answers to 1 decimal place.

   (a) 8 lbs to kg
   (b) 3 lbs to kg
   (c) 16 pints to litres
   (d) 10 cm to inches
   (e) 400 cm to feet
   (f) 80 ounces to pounds
   (g) 182 lbs to stones
   (h) 50 litres to gallons
   (i) 84 inches to feet
   (j) 52 cm to inches
   (k) 16 litres to gallons
   (l) 3 pints to litres
   (m) 6 lbs to kg
   (n) 212 cm to feet

3. A car travels on average 10 km for every litre of gasoline. The car is driven from Montego Bay to Black River, a distance of 41 miles.

   (a) How far does the car travel in km?
   (b) How many litres of gasoline are used?
   (c) How many gallons of gasoline are used?

4. Jimar is 6 feet 2 inches tall and weighs 11 stone 5 pounds.

   Sam is 180 cm tall and weighs 68 kg.

   Who is the taller and who is the heavier?

5. Jelan and Calvin go fruit picking. Jelan picks 8 kg and Calvin picks 15 lbs. Who has picked the greater weight of fruit?

6. A customer asks for a sheet of glass 15 inches by 24 inches. What would be the area of the glass in cm^2?

7. Here is a rule to change miles into kilometres.

Multiply the number of miles by 8 Divide by 5

(a) Use this rule to change 30 miles into kilometres.

(b) Write down an equation connecting kilometres (K) and miles (M).

(c) Use your equation to find the value of M when $K = 100$.

8. (a) Convert 48 kg to grams.

A box contains 280 hockey balls. The hockey balls weigh 48 kg in total.

(b) Calculate the weight of one hockey ball to the nearest gram.

One kilogram is approximately 2.2 pounds.

(c) Estimate the weight of the box of hockey balls in pounds.

9. The same quantity can sometimes be measured in different units.

(a) Write out the statement below, filling in the missing unit.

Choose the unit from this list:

millimetres, centimetres, metres, kilometres

1 inch = 2.54 . . . . . .

(b) Write out the statement below, filling in the missing unit.

Choose the unit from this list:

millimetres, litres, gallons, cubic metres

4 pints = 2.27 . . . . . .

10. Mary is 5 feet 3 inches tall.

1 cm = 0.394 inches

12 inches = 1 foot

(a) Calculate Mary's height in centimetres. Give your answer to an appropriate degree of accuracy.

(b) An electronic weighing scale gives Mary's weight as 63.4792 kg.

Give her weight correct to an appropriate degree of accuracy.