STRAND F: Statistics

Unit 16 *Data Presentation*

Student Text

Contents

denotes that the topic is not on the current CXC/CSEC Mathematics syllabus and therefore not examined, but is of relevance to the content of the Unit.

16 Data Presentation

16.1 Pie Charts

Pie charts, which represent quantities as sectors of a circle, can be used to illustrate data. They are particularly effective if there is only a small number of items to illustrate. In total a complete circle, i.e. $360°$, must always be used.

Worked Example 1

Tracey uses her pocket money of J$1800 per month in the following way.

Books	J$400
Sweets	J$300
Sport	J$600
Transport	J$400
Savings	J$100

Draw a pie chart to show how Tracey uses her pocket money.

Solution

Tracey has a total of J$1800.

$$\frac{360°}{1800} = 0.2°$$

So $0.2°$ should be used for each J$1. The angles needed are given in this list.

Books	$400 \times 0.2° = 80°$
Sweets	$300 \times 0.2° = 60°$
Sport	$600 \times 0.2° = 120°$
Transport	$400 \times 0.2° = 80°$
Savings	$100 \times 0.2° = 20°$

Alternatively, you could calculate each angle by taking the appropriate fraction of $360°$.

For example, for Books, we have

$$\frac{400}{1800} \times 360° = 80°$$

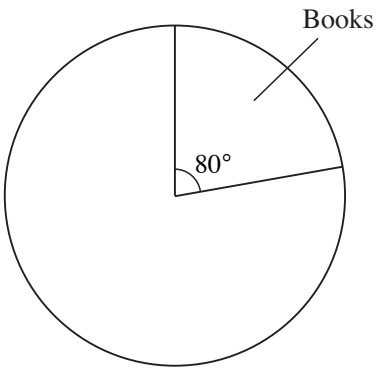
and for Sweets,

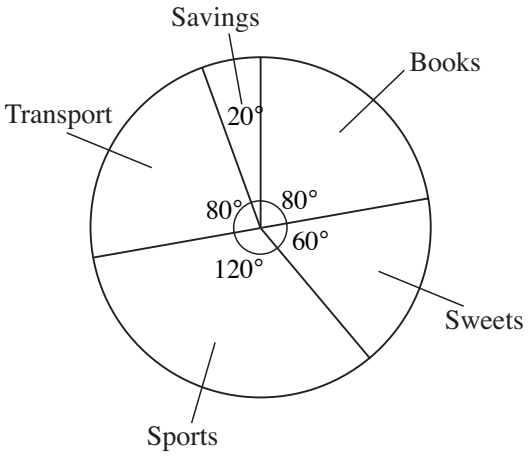
$$\frac{300}{1800} \times 360° = 60°, \text{ etc.}$$

The pie chart can now be drawn.

Shading denotes that the topic is not on the current CXC/CSEC Mathematics syllabus and therefore not examined, but is of relevance to the content of the Unit.

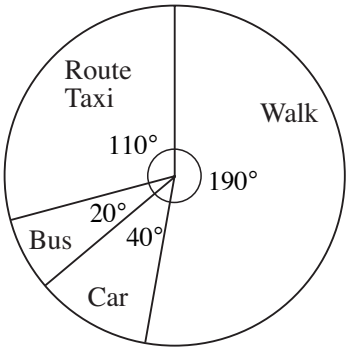
The diagram below, on the left, shows the first section for 'Books'.
The completed pie chart is on the right.





Worked Example 2

The pie chart was constructed after asking 72 students how they travel to school.



(a) How many of these students travel to school by:

(i) car, (ii) bus, (iii) route taxi?

(b) What percentage walk to school?

Solution

(a) There are 72 students so

$$\frac{360°}{72} = 5° \text{ per student}$$

(i) The angle for travelling by car is $40°$.

$$\frac{40°}{5°} = 8$$

So 8 students travel by car.

(ii) The angle for travelling by bus is $20°$.

$$\frac{20°}{5°} = 4$$

So 4 students travel by bus.

(iii) The angle for travelling by route taxi is $110°$.

$$\frac{110°}{5°} = 22$$

So 22 students travel by route taxi.

(b) The number who walk to school is given by

$$\frac{190^\circ}{5^\circ} = 38$$

so the percentage who walk is

$$\frac{38}{72} \times 100 \approx 52.8\%$$

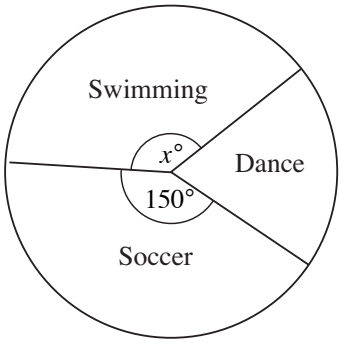
Note

Alternatively, you could just use the angles in the pie chart to give

$$\frac{190}{360} \times 100 \approx 52.8\%$$

Worked Example 3

The pie chart below, **not drawn to scale**, shows the Saturday morning activities of a group of 120 children.



(i) The sector for soccer is represented by an angle of 150°. Determine the number of children who play soccer on Saturday mornings.

(ii) Given that 46 children swim on Saturday mornings, calculate the value of x.

(iii) Determine the probability that a child chosen at random, dances on Saturday mornings. *(CXC)*

Solution

(i) Number playing soccer $= \frac{150}{360} \times 120$

$= 50$ children

(ii) Angle for swimming $= \frac{46}{120} \times 360^\circ$

$= 138^\circ$

(iii) Number dancing $= 120 - (50 + 46)$

$= 120 - 96$

$= 24$

So probability of dancing $= \frac{24}{120} = \frac{1}{5}$

(Probability is covered in detail in Unit 19.)

Exercises

1. In an opinion poll 360 people were asked who they would vote for in the next election. Their responses are:

Jamaica Labour Party	160
People's National Party	170
National Democratic Movement	20
Other	10

Draw a pie chart to show this.

2. Bethany recorded how she spent the last 24 hours. Her results are shown below.

Sleeping	9 hours
School	7 hours
Homework	2 hours
Watching TV	3 hours
Eating	1 hour
Travelling	2 hours

Draw a pie chart to show this information.

3. A student estimated that he had 30 hours available each week for home study and for sports. The table below shows the percentage of time he spent on each activity.

Activities	*%*
Languages	30
Mathematics	20
Computer Studies	15
Sciences	15
Sports	x

(a) Calculate the number of hours he spent on sports.

(b) Calculate the angles in a pie chart that would be used to represent the hours spent on

(i) Mathematics

(ii) Languages.

(c) Draw a pie chart to represent the distribution of hours in the week, which the student spends on the activities indicated in the table above.

(d) One hour in the 30 hours is chosen at random. Calculate the probability that the student is

(i) playing sports

(ii) studying Mathematics or Languages.

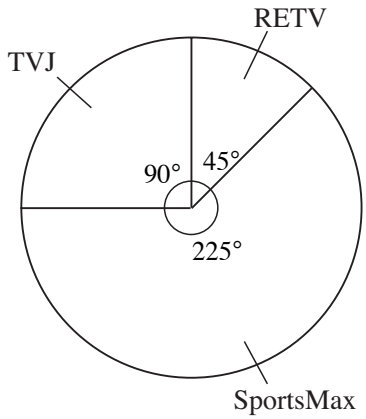
(CXC)

4. The 30 students in a class state their favourite sport. Their results are listed below.

Basketball	3
Cricket	9
Athletics	6
Netball	2
Football	10

Draw a pie chart to show this information.

5. The pie chart shows how the time Robert spends watching television is split between different channels, for one day.

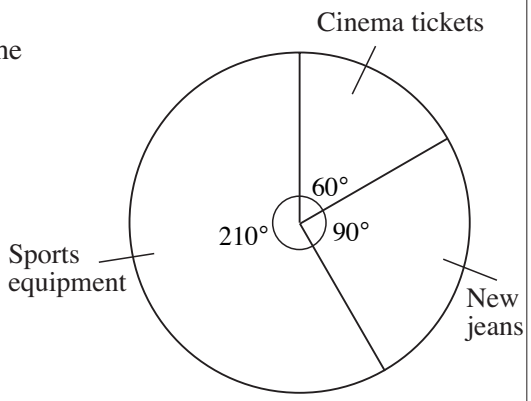


Ron spends 1 hour watching RETV

(a) How long does he spend watching TVJ?

(b) How long does he spend watching SportsMax?

6. Andrew was given $60 on his birthday. The pie chart shows how he spent this money.



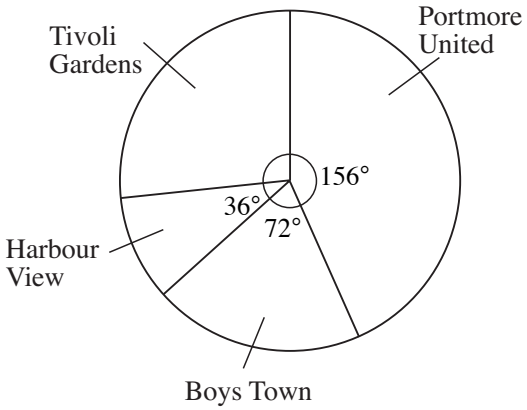
How much did he spend on:

(a) cinema tickets,

(b) his new jeans,

(c) sports equipment?

7. The pie chart shows the football teams supported by a class of students. There are 3 students who support Harbour View.



(a) What is the angle representing Tivoli Gardens?

(b) How many students support Boys Town?

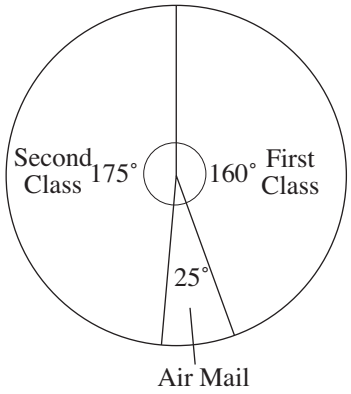
(c) How many students are there in the class?

(d) How many students support Portmore United?

8. A Post Office in a Jamaican town dealt with 720 letters in one week. They were sorted into First Class, Second Class and Air Mail.

The pie chart shows the different number of each type.

How many letters of each type were handled?



9. Kelly spent $J9000.
The table shows what she spent it on.

Items	Amount spent
Bus fares	J$1200
Going out	J$2500
Clothes	J$3000
CDs	J$1500
Other	J$800
Total Spending	J$9000

Kelly is asked to construct a pie chart to show her spending.

(a) Work out the angle of each sector in the pie chart.

Items	Angle of sector
Bus fares	
Going out	
Clothes	
CDs	
Other	
Total of angles	360°

(b) Construct the pie chart to illustrate the data.

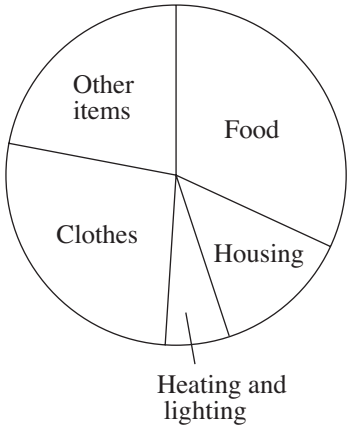
(c) What fraction of Kelly's spending was on clothes?

10. Andrew spends \$180 per week.
The way in which he spends his money is shown in the table.

Items	Spending (\$)
Food	30
Heating and lighting	12
Clothes	20
Other items	20
Housing	98
Total Spending	180

(a) Draw a pie chart to show how Andrew spends his money.

This pie chart shows how the average person spends money.



(b) Describe **one** way in which Andrew's spending differs from the average person's spending.

11. This question is about the way water is used in two Mozambique villages.

(a) In village *A*, 324 litres of water are used each day.
The pie chart shows how the water is used.

(i) How much water (in litres) is used each day for cooking?

(ii) What fraction of the water used is given to animals?

(b) In village *B*, the water is used as follows:

Cooking	20%
Washing themselves	50%
Washing clothes	20%
Washing pots	10%

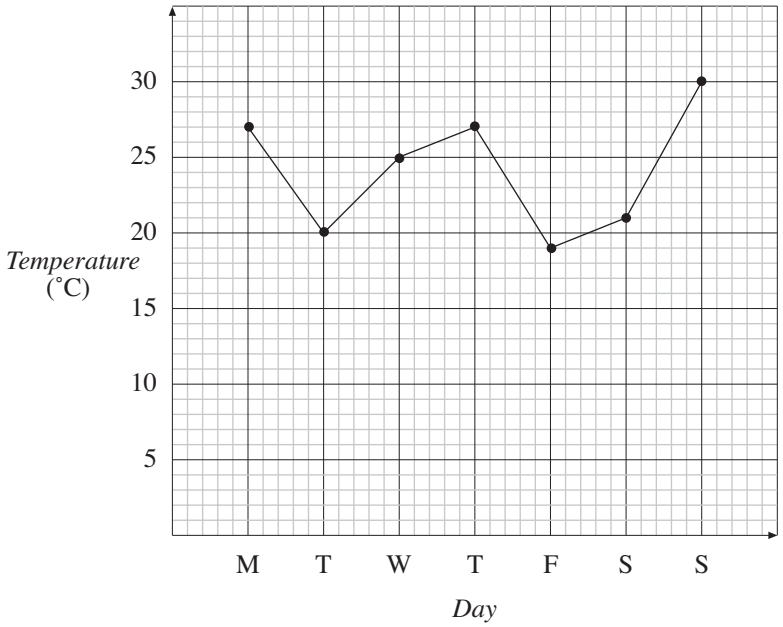
Represent this information in a pie chart.

16.2 Line Graphs

A *line graph* is drawn by plotting data points and joining them with *straight* lines. It is really only the actual data points that count, but by drawing the lines you get a better impression of the trend in the data points. This method of representation is particularly useful when illustrating trends over time.

Worked Example 1

Samuel recorded the temperature in his shed at 6 am each day for a week. His records are shown on this line graph.



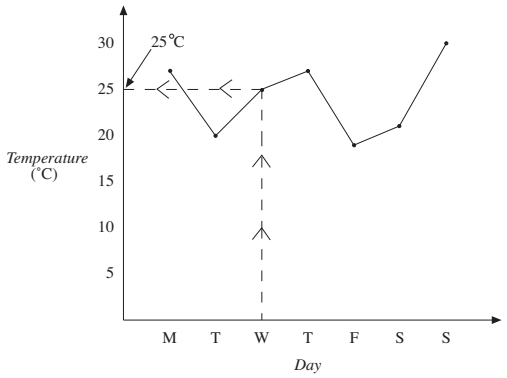
(a) What was the temperature on Wednesday?

(b) What was the lowest temperature recorded?

(c) What was the highest temperature recorded?

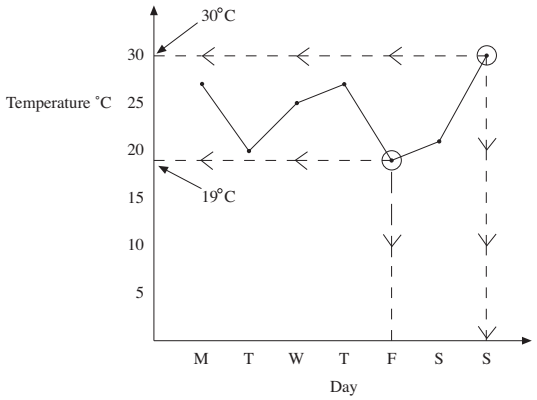
Solution

(a) For Wednesday the temperature can be read as $25°C$.



(b) The lowest temperature occurred on Friday and was $19°C$.

(c) The highest temperature occurred on Sunday and was $30°C$.



Worked Example 2

As part of a science project Evan records the height of a plant every week. His results are shown in this table.

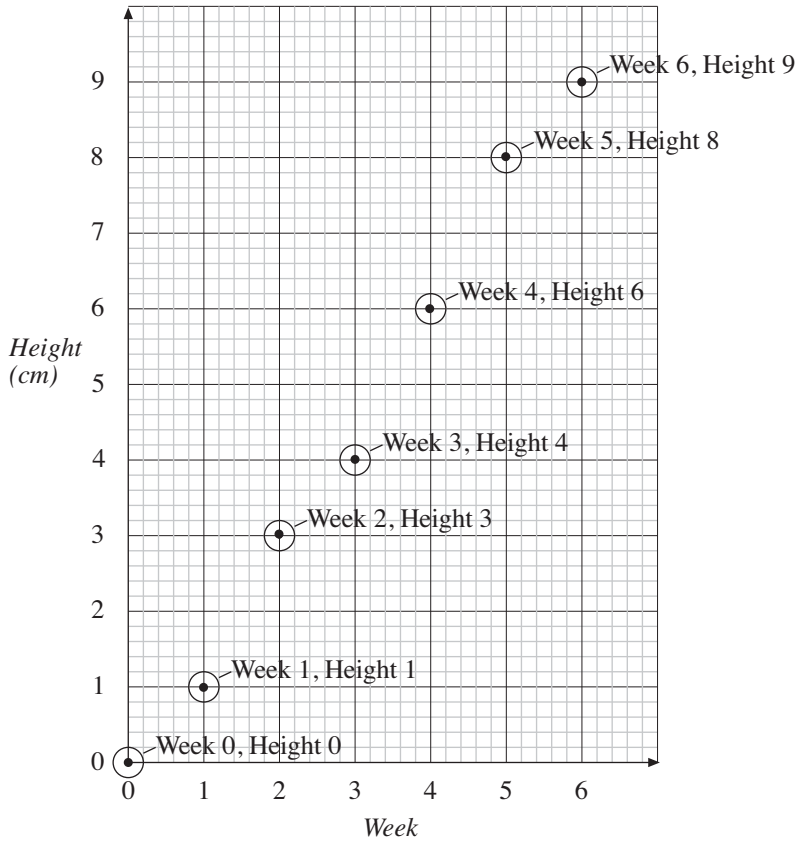
Week	0	1	2	3	4	5	6
Height (cm)	0	1	3	4	6	8	9

Draw a line graph to show the data.

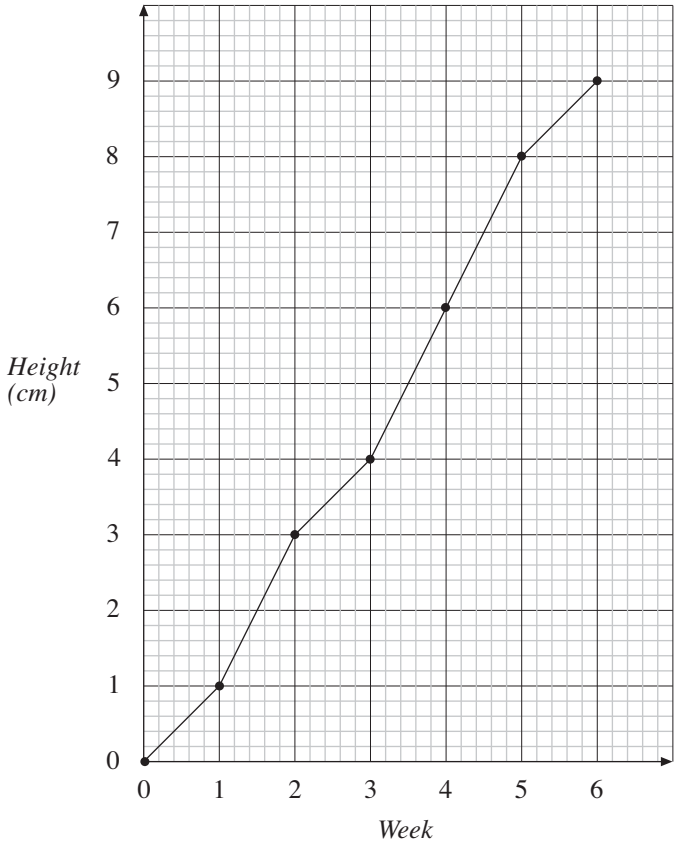
Solution

First draw a suitable set of axes.

Then plot a point for each measurement as shown on the following graph.

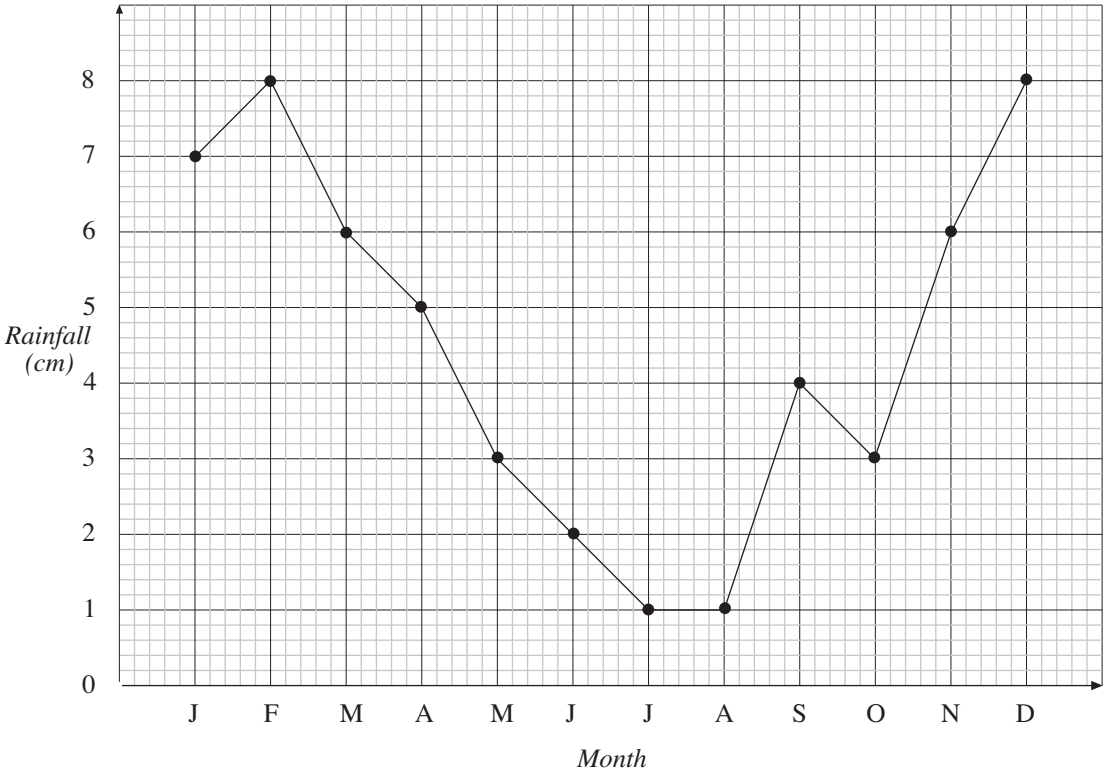


The points can then be joined with straight lines as shown in the next graph.



Exercises

1. The line graph shows the monthly rainfall for a town.



(a) How much rain was there in September?

(b) In which month was the rainfall 5 cm?

(c) Which months were the wettest?

(d) Which months were the driest?

2. Paul recorded the temperature outside his house in New York at 8.00 am every day. His results are in the table.

Day	M	T	W	T	F	S	S
Temperature (°C)	8	5	4	6	7	5	3

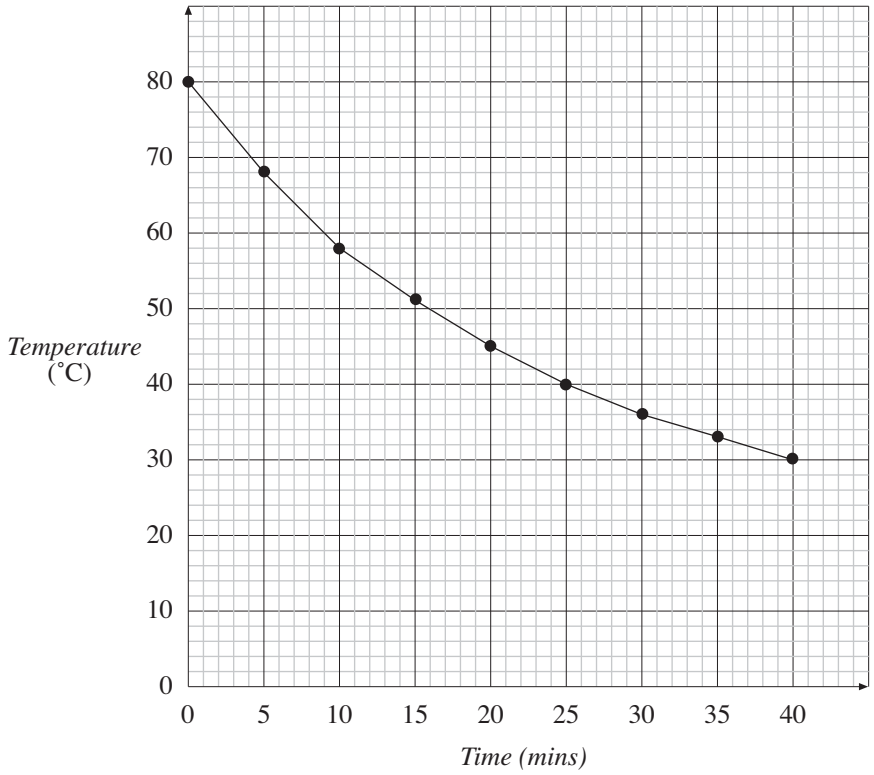
Draw a line graph for this data.

3. Annie counted the number of cars that drove past her while she was waiting at the bus stop each morning on her way to work.

Day	M	T	W	T	F	S
Number of cars	18	12	22	36	4	10

Draw a line graph for this data.

4. A mug was filled with hot water and the temperature was recorded every 5 minutes. The graph below shows the results.



(a) What was the temperature after 25 minutes?

(b) What was the temperature at the start of the experiment?

(c) When was the temperature 45°C?

(d) How long did it take for the temperature to drop from 68°C to 36°C?

5. Anna recorded the time it took her to walk to school every day for a week.

Day	M	T	W	T	F
Time taken (mins)	8	9	15	12	7

Draw a line graph for this data.

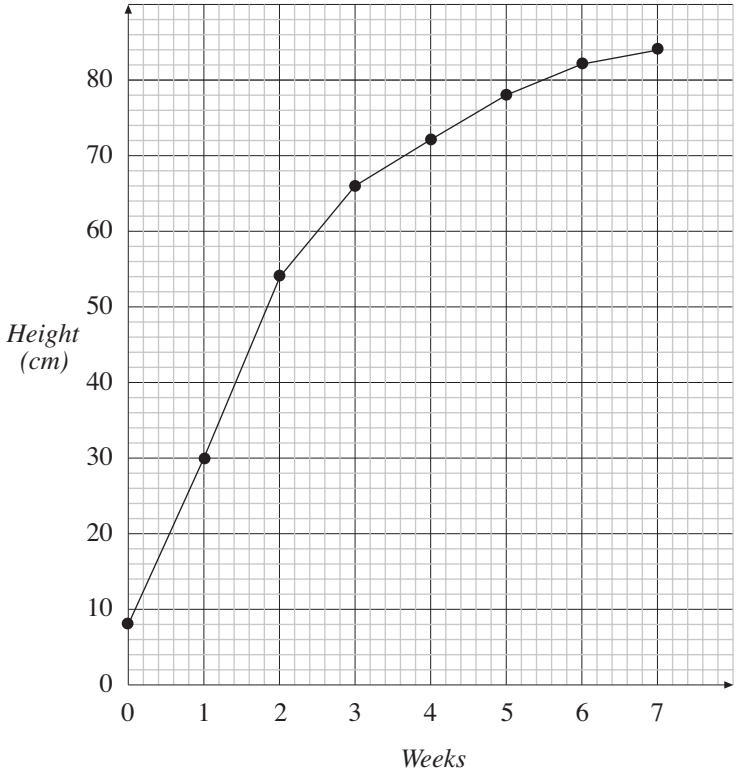
6. The following graph shows how the height of a sunflower plant changed in the weeks after it was planted in a garden.

(a) What was the height of the plant when it was planted in the garden?

(b) How much did the plant grow in the first week?

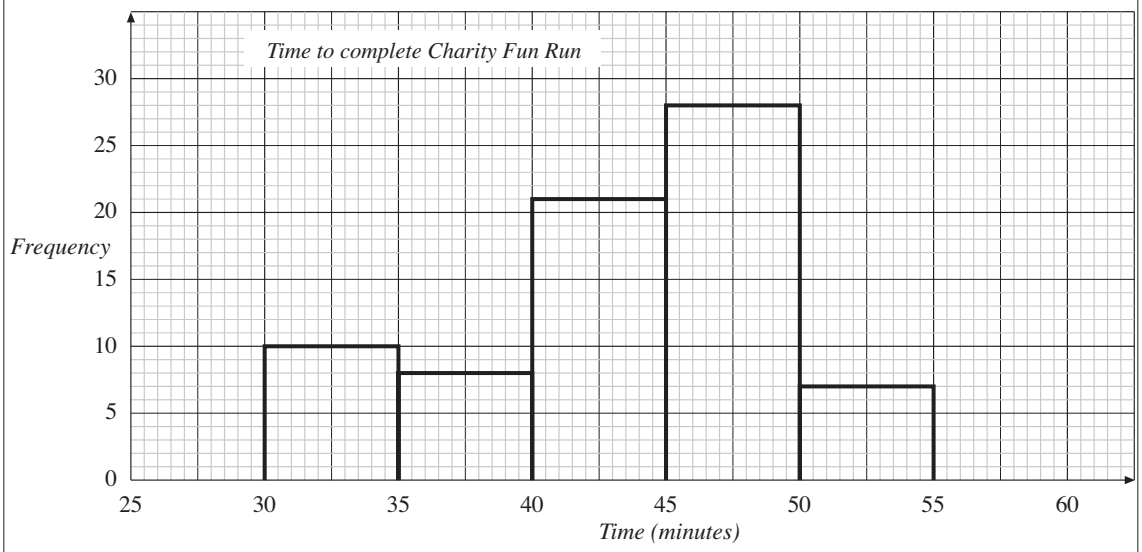
(c) What is the greatest height that the graph shows?

(d) How long did it take for the height to increase from 54 cm to 78 cm?



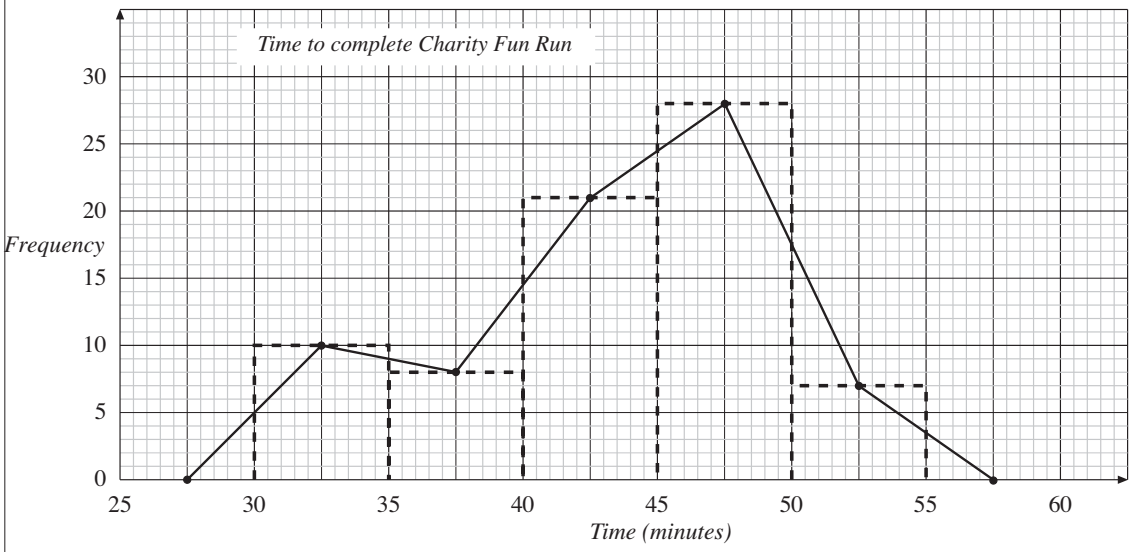
16.3 Frequency Graphs: Histograms

For *continuous data*, when any value over a range of values is possible, a *frequency graph* like the one below should be used, rather than a bar chart which is used for discrete data.



A graph like this is often called a *histogram,* and is characterised by having a continuous scale along the horizontal axis. Note that in this case the widths of the bars are all the same, but this is not always the case, as you will see in the next section. Care though must be taken about the end points. For example, the first *class interval* (in minutes) would normally be $30 \leq \text{time} < 35$, so that a time of 35 minutes would be in the second class interval.

A *frequency polygon* could also be used to show the same data, as on the following graph. Note how it is related to the histogram.



Worked Example 1

Use the data shown on the graphs above to answer these questions.

(a) How many people completed the Fun Run in between 40 and 45 minutes?

(b) How many people completed the Fun Run in less than 40 minutes?

(c) How many people completed the Fun Run in less than 1 hour?

Solution

(a) The 40-45 minute interval contains 21 people.

(b) The 30-35 and 35-40 minute intervals must be considered.
There are 10 people in the 30-35 minute interval.
There are 8 people in the 35-40 minute interval.
So there are $10 + 8 = 18$ people who complete the run in less than 40 minutes.

(c) The number in each interval is needed.
So the number of people is:

$$10 + 8 + 21 + 28 + 7 = 74$$

Worked Example 2

A group of students measured the reaction times of 50 other students. The times are given below correct to nearest hundredth of a second.

0.44	0.32	0.31	0.47	0.27	0.31	0.40	0.28	0.16	0.26
0.33	0.46	0.41	0.33	0.31	0.28	0.38	0.29	0.17	0.26
0.29	0.40	0.29	0.24	0.41	0.22	0.25	0.47	0.31	0.36
0.49	0.21	0.42	0.43	0.28	0.36	0.24	0.37	0.34	0.27
0.49	0.16	0.29	0.30	0.41	0.27	0.29	0.28	0.40	0.42

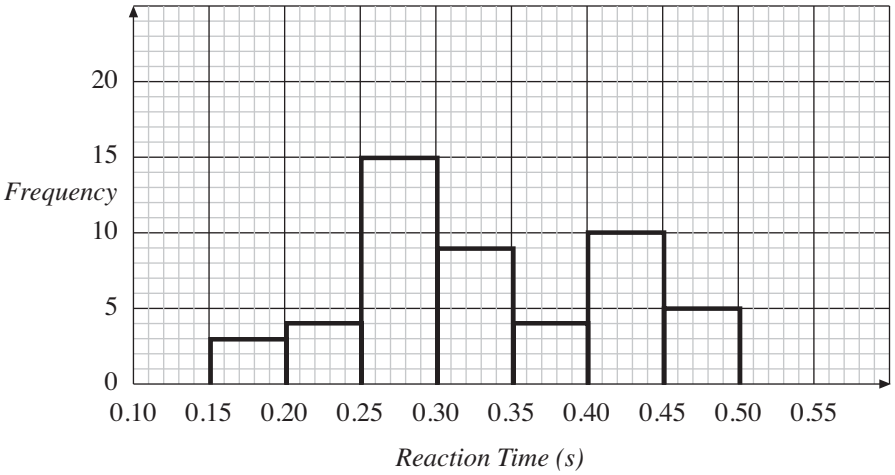
Draw a histogram for this data.

Solution

First the data must be collected into groups, using a tally chart.

Reaction Time(s)	*Tally*	*Frequency*
$0.15 \le t < 0.20$	𝍸	3
$0.20 \le t < 0.25$	𝍸	4
$0.25 \le t < 0.30$	𝍸 𝍸 𝍸	15
$0.30 \le t < 0.35$	𝍸 𝍸	9
$0.35 \le t < 0.40$	𝍸	4
$0.40 \le t < 0.45$	𝍸 𝍸	10
$0.45 \le t < 0.50$	𝍸	5

Now that the data has been collected in this way, the histogram below can be drawn.



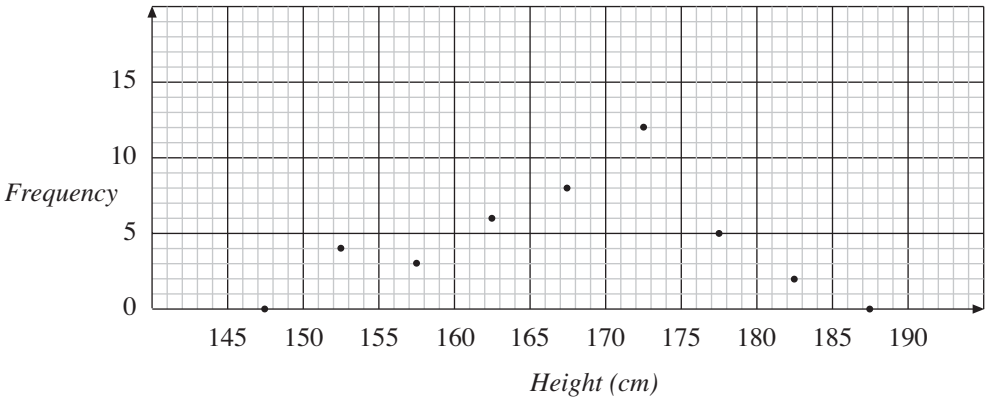
Worked Example 3

Draw a frequency polygon for the data on the height of children, given in cm, in the table below.

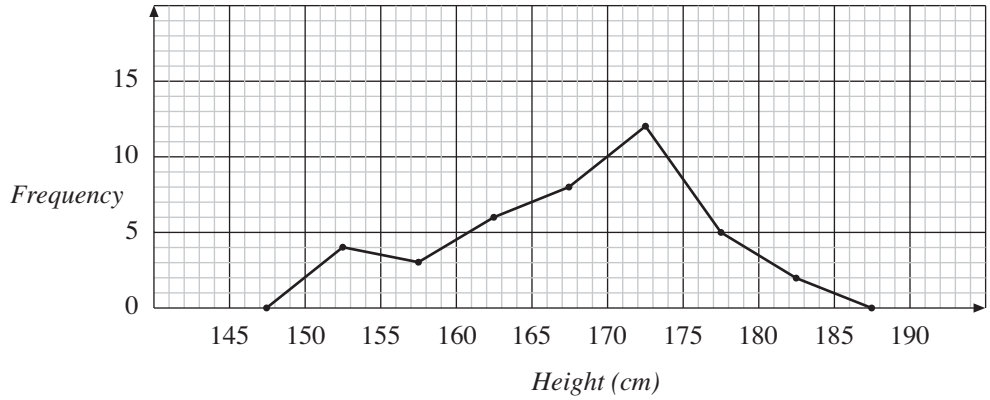
Height (cm)	Frequency
$150 \le h < 155$	4
$155 \le h < 160$	3
$160 \le h < 165$	6
$165 \le h < 170$	8
$170 \le h < 175$	12
$175 \le h < 180$	5
$180 \le h < 185$	2

Solution

Points should be placed above the centre of each interval. The height is given by the frequency. The following graph shows these points.

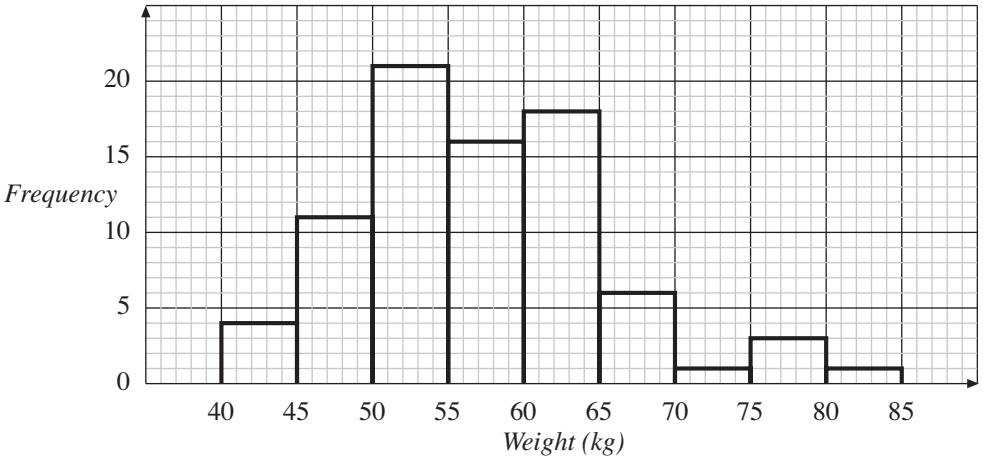


Note that points have been placed on the horizontal axis in the intervals that have frequencies of 0. The points can then be joined to give the frequency polygon below.



Exercises

1. The histogram below shows how the weights of students in a school class were distributed.



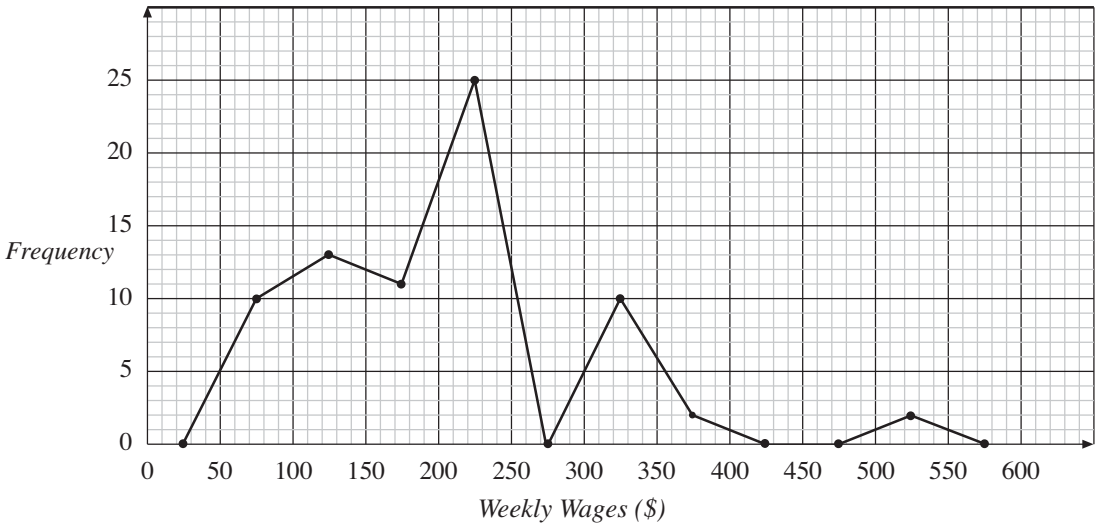
(a) How many students had a weight greater than 70 kg?

(b) How many students had a weight between 50 and 65 kg?

(c) How many students had a weight less than 50 kg?

(d) How many students were there in that class?

2. The frequency polygon shows the weekly wages of a large firm.



(a) How many people earn between \$300 and \$350 per week?

(b) How many people earn between \$100 and \$300 per week?

(c) How many people are employed by the firm?

(d) What are the largest and smallest possible weekly wages that the graph shows could be paid?

3. An orchard contains 100 apple trees. The weight of apples produced by each tree in one year was recorded. The results are given in the table .

Mass of apples (kg)	*Frequency*
$50 < m \leq 60$	5
$60 < m \leq 70$	7
$70 < m \leq 80$	13
$80 < m \leq 90$	10
$90 < m \leq 100$	20
$100 < m \leq 110$	22
$110 < m \leq 120$	18
$120 < m \leq 130$	5

Draw a histogram for the data.

4. A psychologist uses a test in which people have to solve a puzzle. He records the time it took people to solve the puzzle.

Time taken (mins)	$0 \leq t < 1$	$1 \leq t < 2$	$2 \leq t < 3$	$3 \leq t < 4$	$4 \leq t < 5$
Frequency	5	32	18	7	12

Draw a histogram for the data.

5. The finishing times for a cross country race were recorded to the nearest minute. Draw a suitable histogram for the data.

23	38	43	47
27	39	43	48
31	39	43	48
32	40	43	48
32	40	44	50
32	40	44	50
33	41	46	51
34	41	46	51
35	42	46	52
37	42	47	53

6. At the end of a football season a newspaper reported the average number of goals scored per match for 100 top footballers.

2.7	1.4	2.9	1.3	2.1	2.1	2.9	2.1	1.7	3.1
1.2	2.0	0.6	2.2	2.2	2.2	2.7	1.6	1.3	1.9
1.3	1.4	1.9	2.1	1.8	0.9	2.1	2.3	1.9	1.3
1.3	0.3	1.2	2.0	1.4	1.7	2.4	1.9	1.7	1.5
2.7	2.2	0.7	1.5	1.6	1.4	2.7	2.0	1.9	2.7
2.1	1.6	0.7	1.5	2.0	0.9	1.6	1.6	1.5	2.6
1.1	2.2	1.8	1.5	1.8	2.6	0.2	1.2	1.4	1.9
1.7	1.4	2.1	2.2	1.5	2.1	2.4	0.5	0.9	1.4
1.8	0.6	1.9	2.6	1.6	2.1	2.0	1.8	1.3	2.0
1.2	2.2	1.4	2.1	1.2	0.4	2.6	1.9	0.9	2.1

Use the data given to draw a suitable histogram and then draw a frequency polygon on top of the histogram.

7. The marks gained by a group of students in a mathematics test are shown below.

11	20	24	27
29	34	13	22
26	27	31	36
17	23	26	28
32	38	19	23
27	28	33	39

(a) Copy and complete the following frequency table to show the distribution of the marks.

Marks	Frequency
10 - 14	2
15 - 19	
20 - 24	
25 - 29	
30 - 34	
35 - 39	3

(b) Draw a histogram to represent the information in the completed frequency table from (a) above.

(c) Calculate the probability that a student chosen at random from those who wrote the test scored LESS THAN 25 marks.

(CXC)

8. The age distribution in a town is given in the table.

Draw a histogram to show the data.

Age	*Frequency*
$0 \le a < 10$	1800
$10 \le a < 20$	1500
$20 \le a < 30$	1450
$30 \le a < 40$	1600
$40 \le a < 50$	1250
$50 \le a < 60$	1150
$60 \le a < 70$	800
$70 \le a < 80$	500
$80 \le a < 90$	150

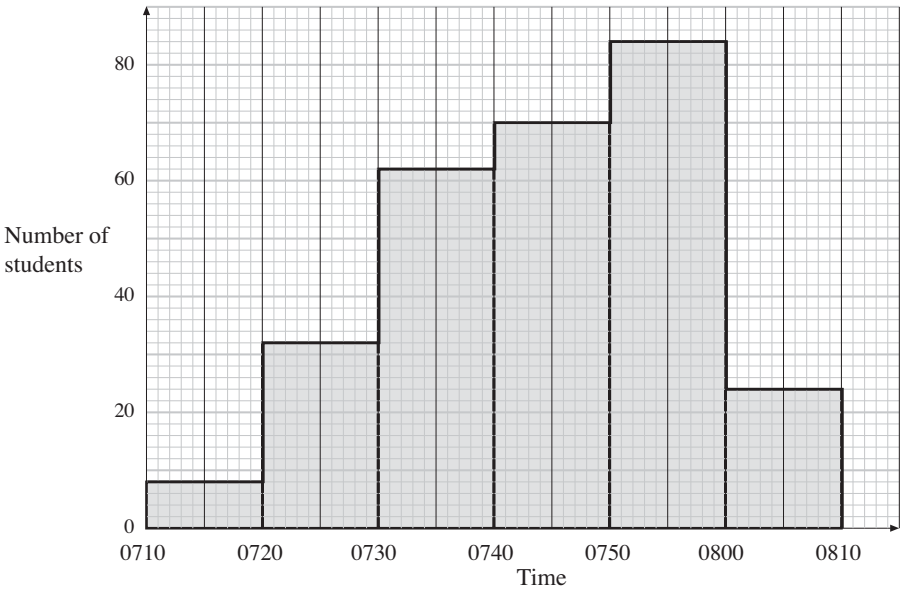
9. A vehicle hire company owns three types of car which are classified as small, medium and large. The distance travelled by each car is always recorded.

Distance (km)	$0 < t \le 100$	$100 < t \le 200$	$200 < t \le 300$	$300 < t \le 400$	$400 < t \le 500$
Small Cars	80	50	30	2	1
Medium Cars	30	45	67	70	10
Large Cars	5	12	16	24	12

(a) On the same set of axes draw frequency polygons for each type of car.

(b) Comment on the graphs you have drawn.

10. The graph shows the result of a survey of the times at which students arrived at school one day.



How many students arrived for school between 0730 and 0750?

11. The table below gives information about the expected lifetimes, in hours, of 200 light bulbs.

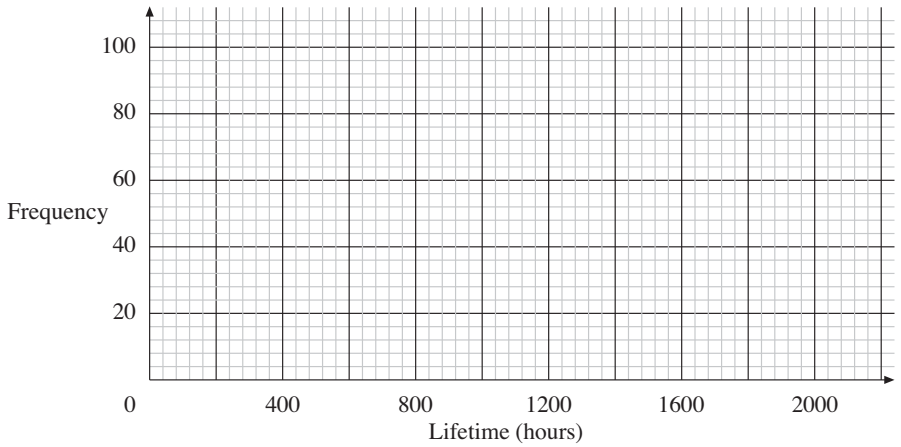
Lifetime (t)	$0 < t \leq 400$	$400 < t \leq 800$	$800 < t \leq 1200$	$1200 < t \leq 1600$	$1600 < t \leq 2000$
Frequency	32	56	90	16	6

(a) Mr Jones buys one of the light bulbs.

(i) What is the probability that it will not last more than 400 hours?

(ii) What is the probability that it will last at least 800 hours but not more than 1600 hours?

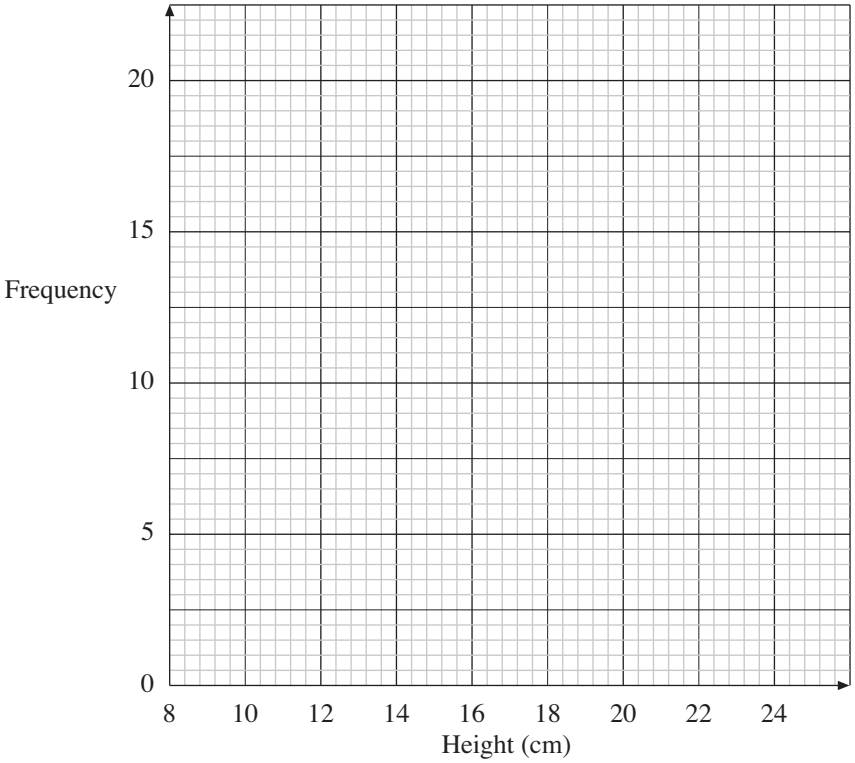
(b) Using axes similar to those below, draw a frequency polygon to illustrate the information in the table.



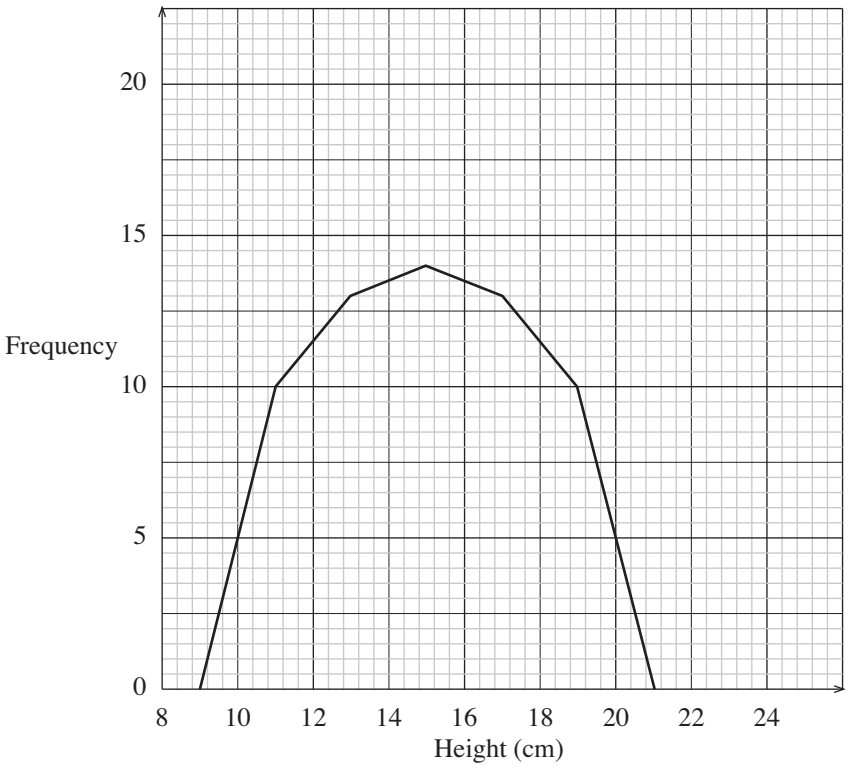
12. The height of each of 60 plants of type *A* was measured and recorded.

Height of plant (cm)	8 - 10	10 - 12	12 - 14	14 - 16	16 - 18	18 - 20	20 - 22
Number of plants	0	2	3	18	19	18	0

(a) Draw the frequency polygon of these results on a grid like the one below.



The following graph shows a frequency polygon of 60 plants of type *B*.



(b) Write down **two** differences between the two types of plant shown by the frequency polygons.

16.4 Histograms with Unequal Class Intervals

When drawing histograms it is possible that the intervals will not have the same width.

Consider the data given in the table below.

Weight (in grams)	*Frequency*
$0 \le w < 40$	5
$40 \le w < 50$	6
$50 \le w < 60$	8
$60 \le w < 70$	4
$70 \le w < 100$	2

The way the data have been presented makes it impossible to draw a histogram with equal class intervals.

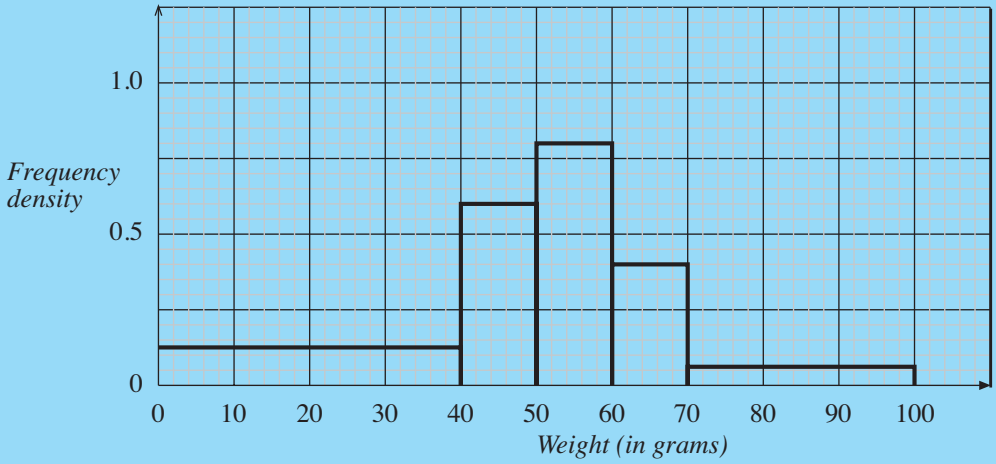
In order to keep the histogram fair, the *area* of the bars, rather than the height, must be proportional to the frequency. So on the vertical scale we plot *frequency density* instead of frequency, where

$$\text{Frequency Density} = \frac{\text{Frequency}}{\text{Class Width}}$$

Rewriting the table with an extra column for frequency density, gives

Weight (in grams)	*Frequency*	*Frequency Density*
$0 \le w < 40$	5	0.125
$40 \le w < 50$	6	0.6
$50 \le w < 60$	8	0.8
$60 \le w < 70$	4	0.4
$70 \le w < 100$	2	0.066...

and you can draw the histogram with frequency density on the vertical axis.



Note

You can see that it is the area that is proportional to the frequency – in fact, a frequency of 1 is represented by 10 little squares.

Worked Example 1

Traffic police recorded the speeds of vehicles passing a speed camera on an open road. Draw a histogram for this data.

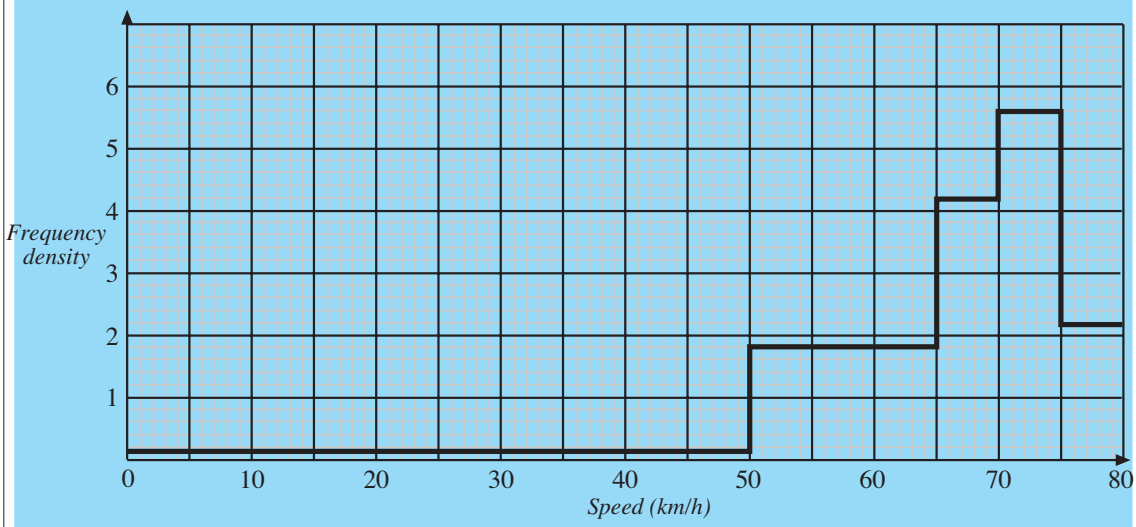
Speed (km/h)	*Frequency*
$0 \le v < 50$	5
$50 \le v < 65$	27
$65 \le v < 70$	21
$70 \le v < 75$	28
$75 \le v < 80$	11

Solution

The following table shows how the frequency density can be calculated.

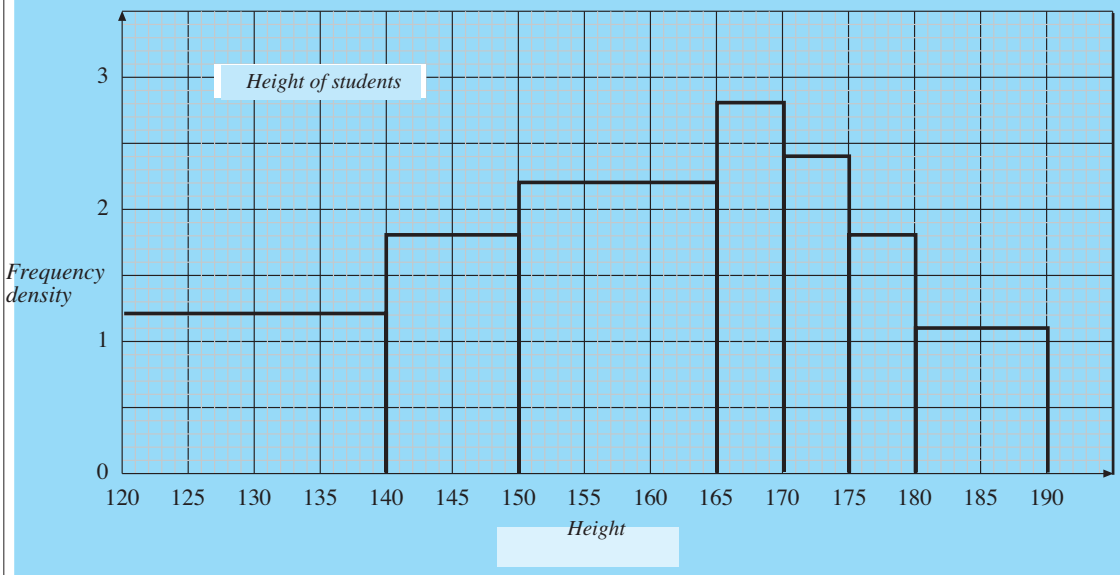
Speed (km/h)	*Class width*	*Frequency*	*Frequency Density*
$0 \le v < 50$	50	5	$\frac{5}{50} = 0.1$
$50 \le v < 65$	15	27	$\frac{27}{15} = 1.8$
$65 \le v < 70$	5	21	$\frac{21}{5} = 4.2$
$70 \le v < 75$	5	28	$\frac{28}{5} = 5.6$
$75 \le v < 80$	5	11	$\frac{11}{5} = 2.2$

The histogram is shown below.



Worked Example 2

The histogram below shows the results of a survey into the height of students in a school,



(a) find the number of students with heights between:

(i) 120 and 140 cm, (ii) 170 and 175 cm.

(b) find the total number of students measured.

Solution

(a) (i) For the 120 to 140 cm interval:

$$\text{Frequency Density} = 1.2$$
$$\text{Class Width} = 20$$

$$\text{Frequency} = 20 \times 1.2$$
$$= 24 \text{ students}$$

(ii) For the 170 to 175 cm interval:

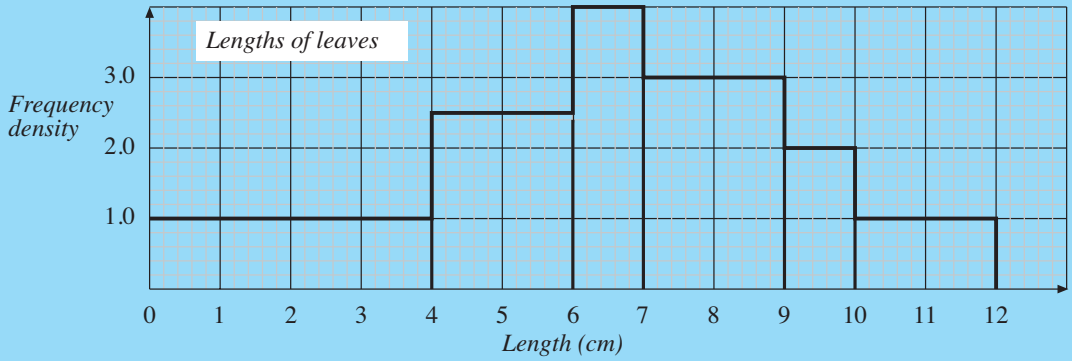
$$\text{Frequency Density} = 2.4$$
$$\text{Class Width} = 5$$
$$\text{Frequency} = 5 \times 2.4$$
$$= 12 \text{ students}$$

(b) To find the total, the numbers in each class interval must be found and added together.

$$\text{Total} = 20 \times 1.2 + 10 \times 1.8 + 15 \times 2.2 + 5 \times 2.8 + 5 \times 2.4 + 5 \times 1.8 + 10 \times 1.1$$
$$= 24 + 18 + 33 + 14 + 12 + 9 + 11$$
$$= 121$$

Exercises

1. For a project in Biology, Sharma gathered data on the length of leaves from a tree and drew the histogram below.

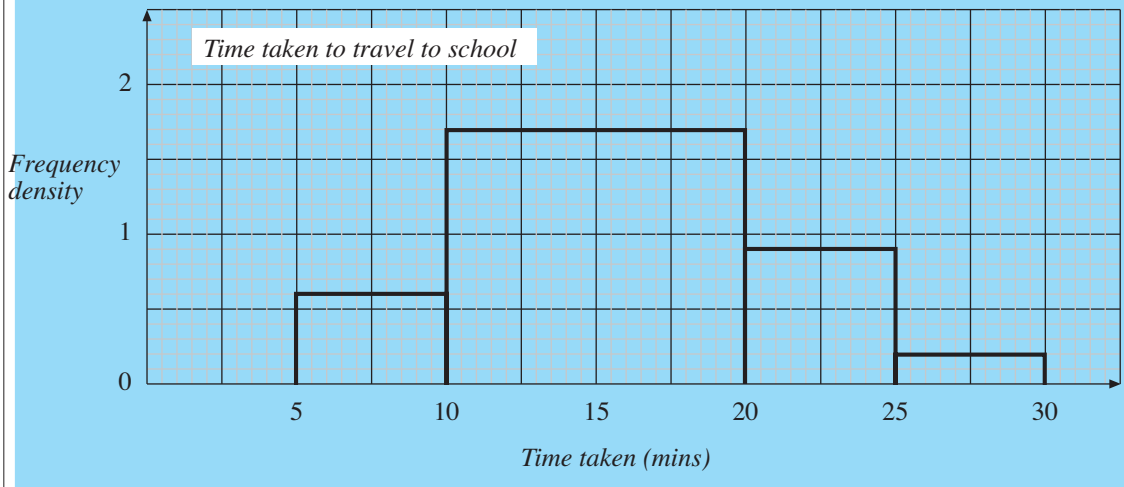


(a) How many leaves had a length less than 6 cm?

(b) How many leaves had a length greater than 9 cm?

(c) How many leaves did she measure?

2. Jennifer collected data on the length of time it took her to travel to school. She drew the histogram below.



This histogram contains an error. What is it?

3. A teacher recorded all the scores of the students who took a maths test in his school. These scores are summarised in the table below.

Score	*Frequency*
$0 \le t < 35$	3
$35 \le t < 40$	7
$40 \le t < 55$	20
$55 \le t < 70$	42
$70 \le t \le 100$	16

Draw a histogram for this data.

4. The distribution of the ages of inhabitants of a village is shown in the table below.

Age	*Frequency*
0 - 4	10
5 - 9	12
10 - 19	19
20 - 39	36
40 - 59	30
60 - 64	9
65 - 79	11
80 - 99	3

(a) Explain why the width of the first class interval is 5.

(b) Find the width of all the other class intervals.

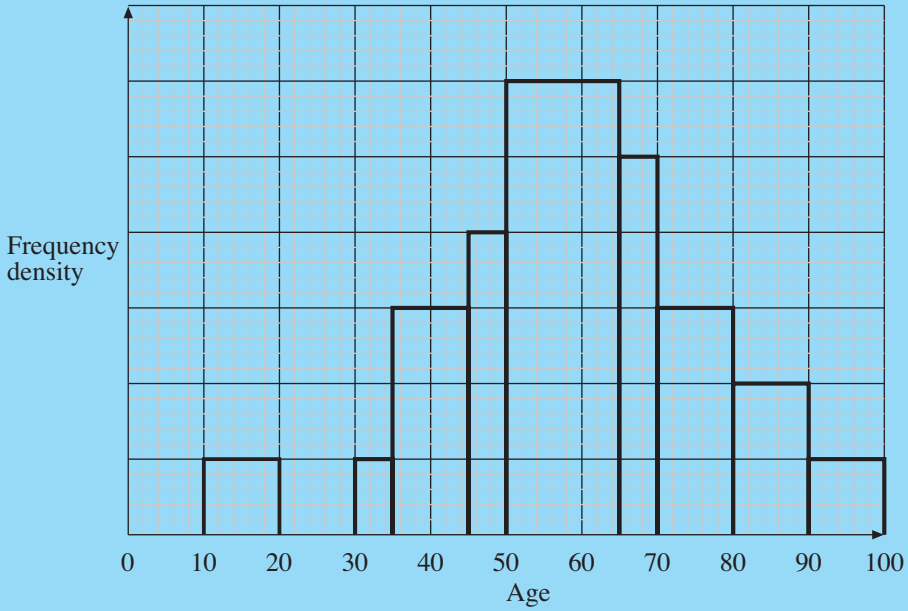
(c) Draw a histogram to show this data.

5. The finishing times to the nearest minute for the competitors in a half-marathon to complete the race are given below.

Group the data into suitable intervals and draw a histogram.

135	103	123	116	177	101
107	117	126	117	156	133
127	95	134	110	201	115
202	113	170	105	132	105
155	98	137	115	129	117
152	93	116	92	152	97
169	112	163	124	151	143
160	121	176	100	84	114
122	98	96	118	153	112
153	136	123	80	170	109

6. The age of each person at a party is illustrated in the histogram below.



There are 6 people in the 70-80 age range.

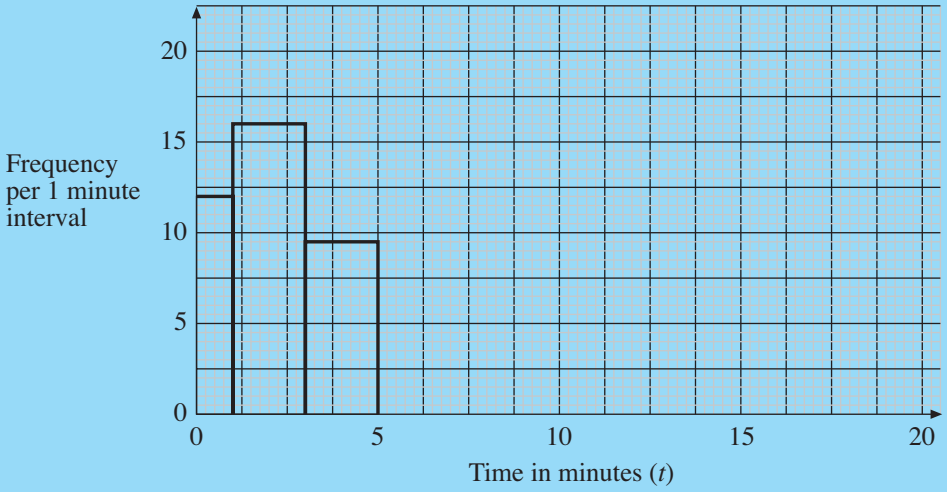
(a) How many people are there in the 45-50 age range?

(b) How many people are there in the 50-70 age range?

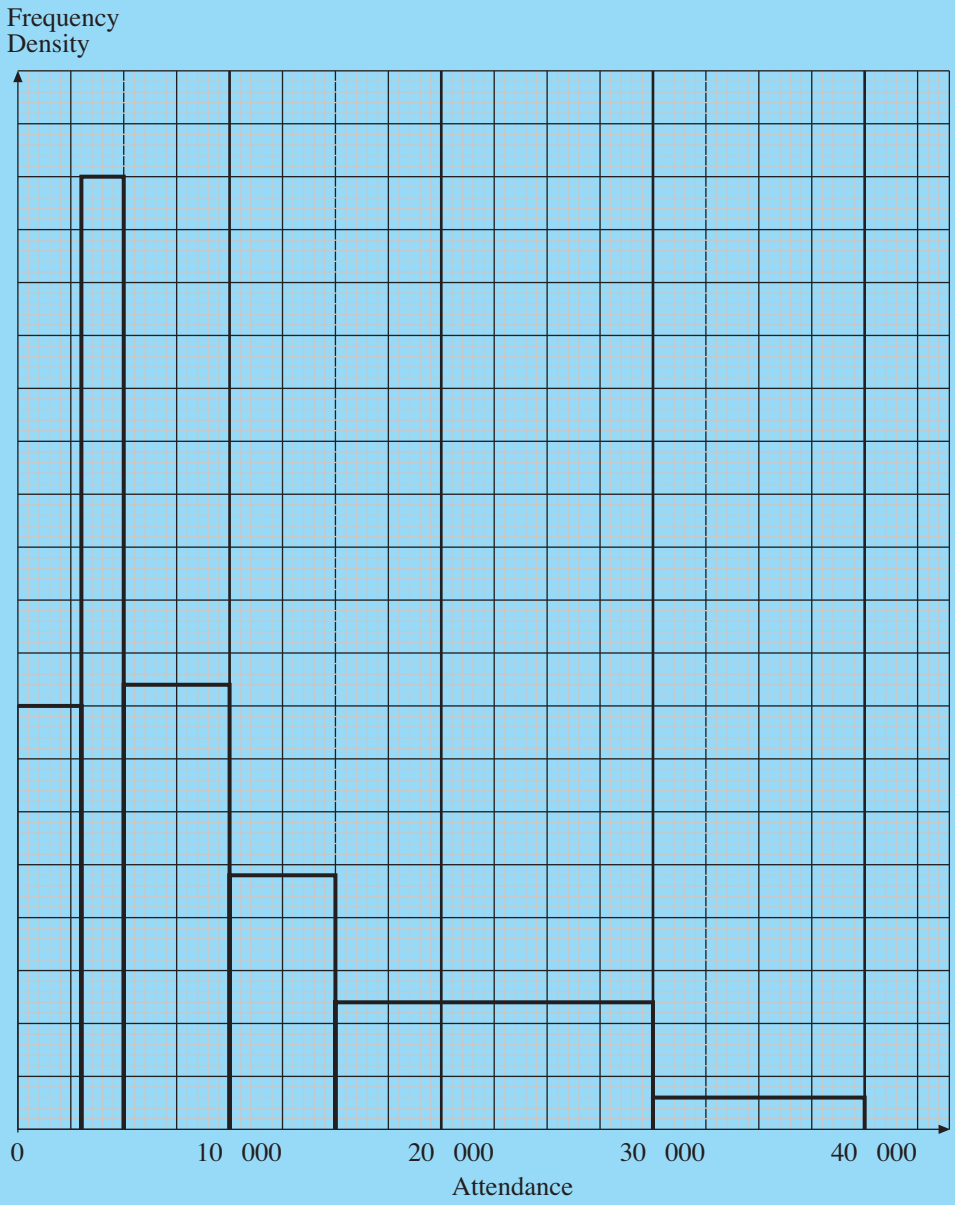
7. A sample was taken of the telephone calls to a school switchboard. The lengths of the telephone calls are recorded, in minutes, in this table.

Time in minutes (t)	$0 < t \leq 1$	$1 < t \leq 3$	$3 < t \leq 5$	$5 < t \leq 10$	$10 < t \leq 20$
Number of calls	12	32	19	20	15

Copy and complete the histogram to show this information.



8. The following histogram below represents the number of spectators at professional football matches in England one Saturday.



No match had more than 40 000 spectators.

At 4 matches the number of spectators was greater than or equal to 10 000 and less than 15 000.

(a) Use the information in the histogram to complete a copy of the following frequency table.

Number of spectators (n)	Frequency
$0 \leq n < 3000$	
$3000 \leq n < 5000$	
$5000 \leq n < 10\,000$	
$10\,000 \leq n < 15\,000$	4
$15\,000 \leq n < 30\,000$	
$30\,000 \leq n < 40\,000$	

(b) Calculate the total number of professional football matches played in England on that Saturday.