

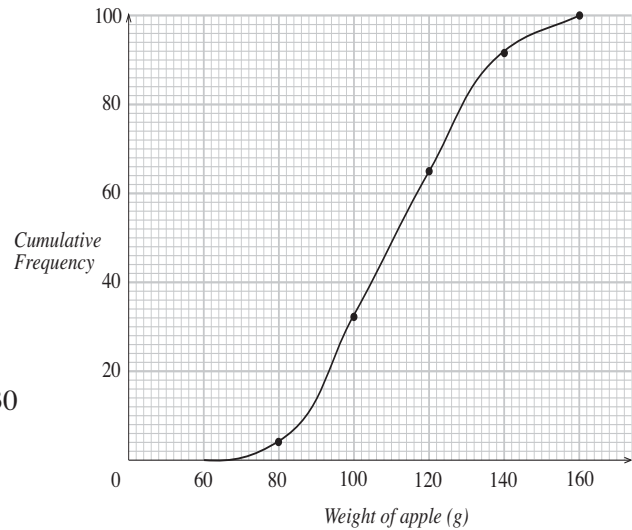
UNIT 18 Measures of Variation

Answers

18.1 Cumulative Frequency

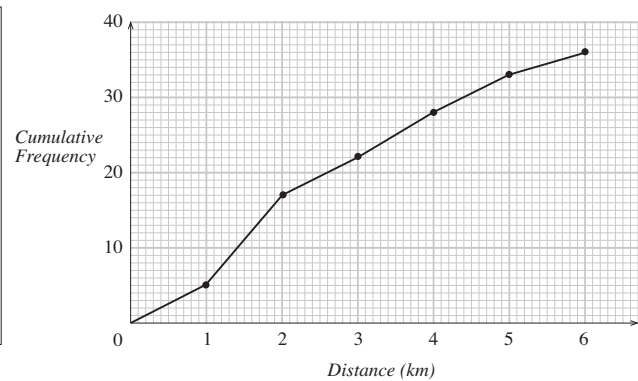
1. (a)

Weight in grams (w)	Cumulative Frequency
$60 < w \leq 80$	4
$80 < w \leq 100$	32
$100 < w \leq 120$	65
$120 < w \leq 140$	92
$140 < w \leq 160$	100

Median ≈ 111 , Inter-quartile range ≈ 30

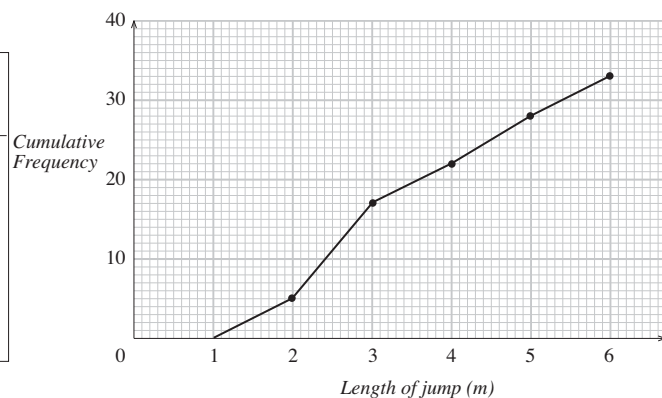
(b)

Distance in km (d)	Cumulative Frequency
$0 < d \leq 1$	5
$1 < d \leq 2$	17
$2 < d \leq 3$	22
$3 < d \leq 4$	28
$4 < d \leq 5$	33
$5 < d \leq 6$	36

Median ≈ 2.2 , Inter-quartile range – about 2.5.

(c)

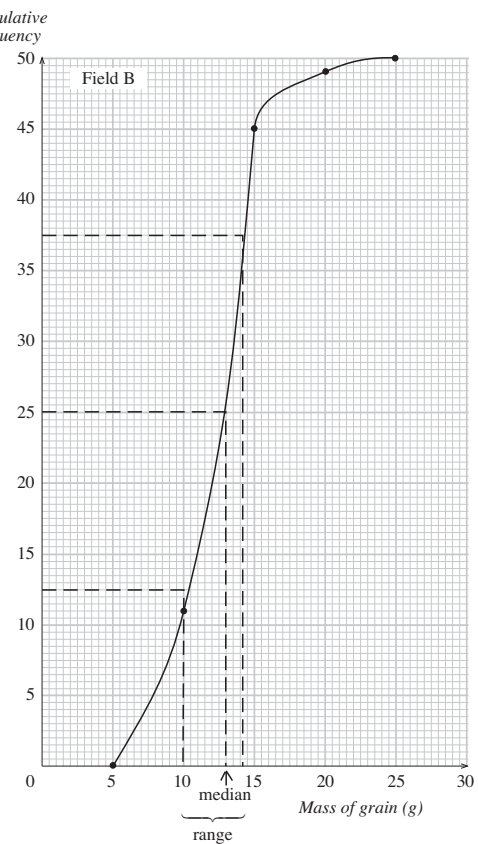
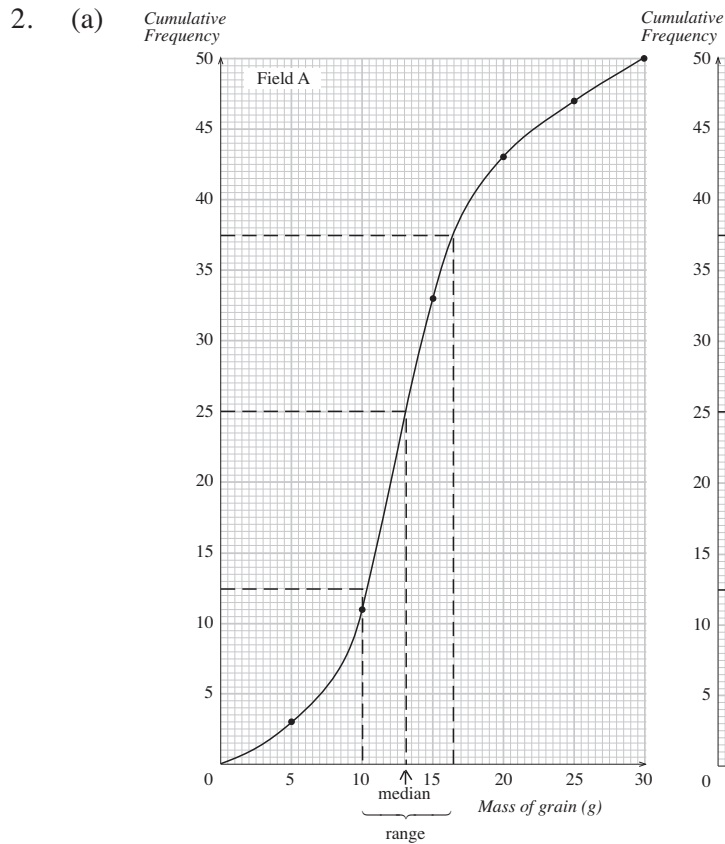
Length in metres (d)	Cumulative Frequency
$1 < d \leq 2$	5
$2 < d \leq 3$	17
$3 < d \leq 4$	22
$4 < d \leq 5$	28
$5 < d \leq 6$	33

Median ≈ 3 , Inter-quartile range – about 2.2.

UNIT 18 *Measures of Variation*

Answers

18.1



(b) The median of field A is 13g, the median of field B is 12g.

The inter-quartile range for field A is around 7, for B it is around 4.

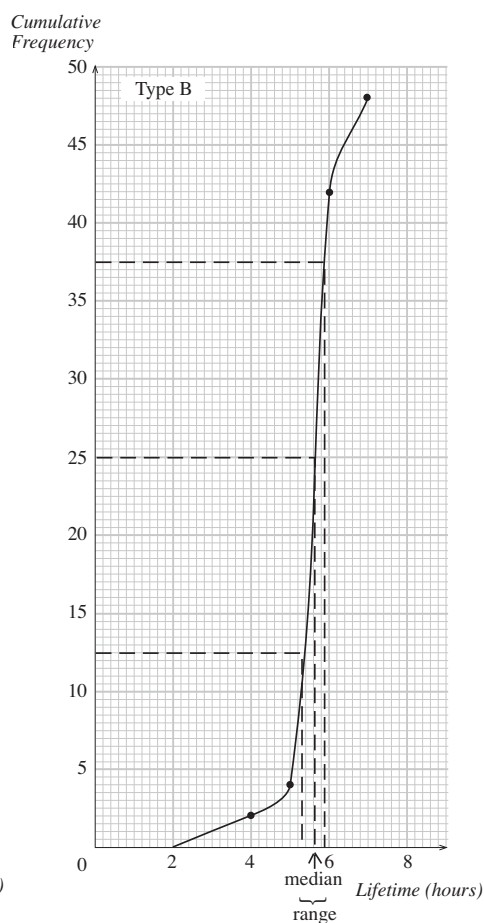
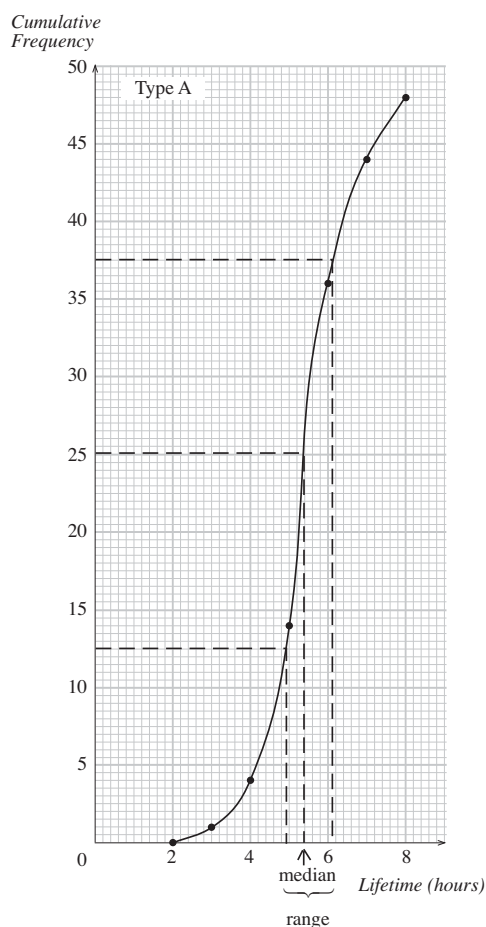
(c) Field B is more reliable than field A (its inter-quartile range is narrower), although it is less productive in 50% of the cases (its median is lower than the median of field B).

UNIT 18 Measures of Variation

Answers

18.1

3. (a)



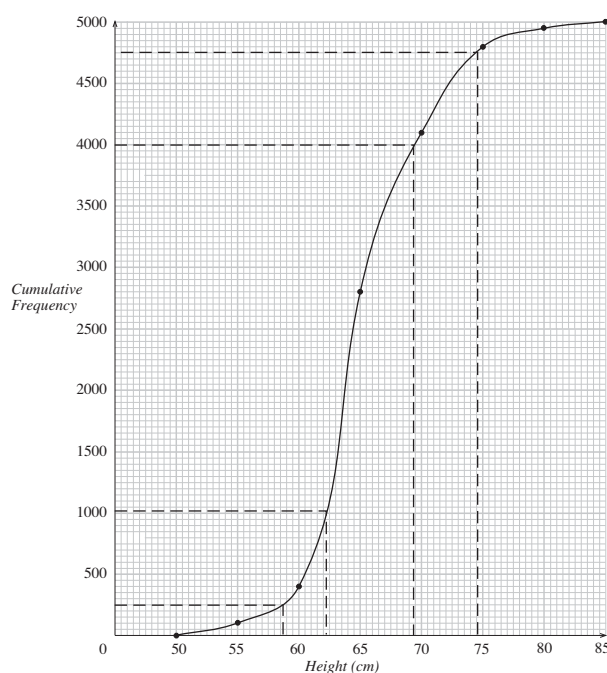
Median for type A ≈ 5.4 , Inter-quartile range for type A ≈ 1 .

Median for type B ≈ 5.6 , Inter-quartile range for type B ≈ 0.5 .

- (b) Type B - although both types have quite similar medians, type B is more predictable (its inter-quartile range is narrower) and most of the time its lifetime is above 5.

4. The heights of children in each category (to the nearest cm) are:

Very tall	-	$75 < h \leq 85$
Tall	-	$70 < h \leq 75$
Normal	-	$62 < h \leq 70$
Short	-	$58 < h \leq 62$
Very short	-	$50 < h \leq 58$

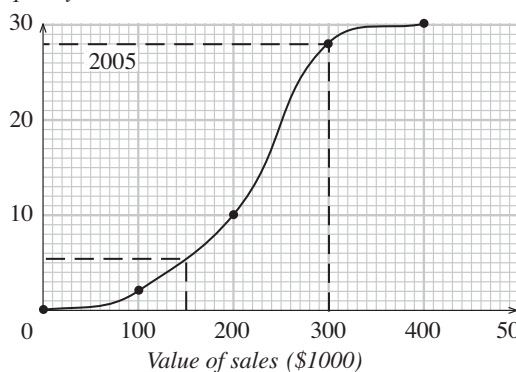
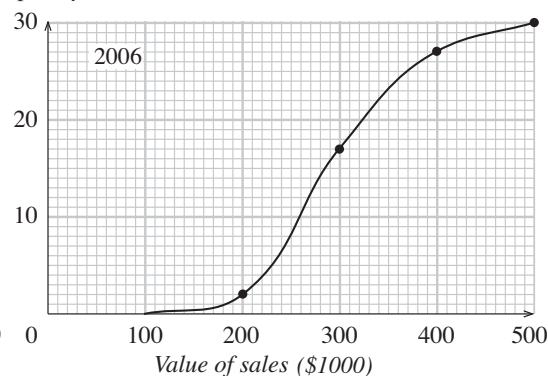


UNIT 18 Measures of Variation

Answers

18.1

5.

Cumulative
FrequencyCumulative
Frequency

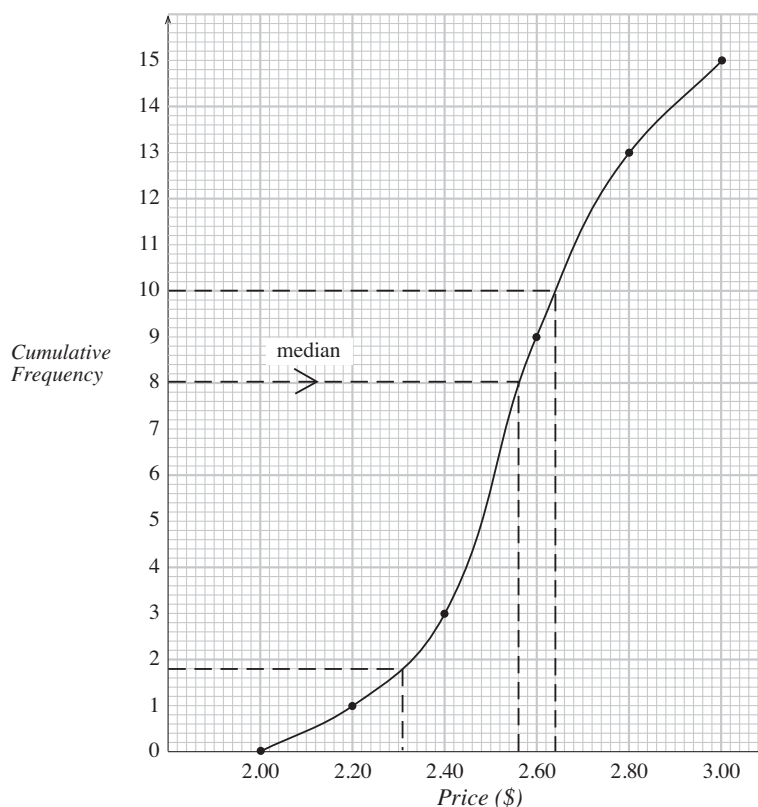
For 2005:

Bonus	Value of sales
\$50	$0 < V \leq 155$
\$250	$155 < V \leq 300$
\$500	$300 < V \leq 400$

For 2006:

Bonus	Value of sales
\$ 50	$0 < V \leq 235$
\$250	$235 < V \leq 420$
\$500	$420 < V \leq 500$

6. (a)



- (i) 5 or 6 shops (ii) About \$2.50 (iii) 2 shops
 (iv) 8 shops (v) 5 or 6 shops

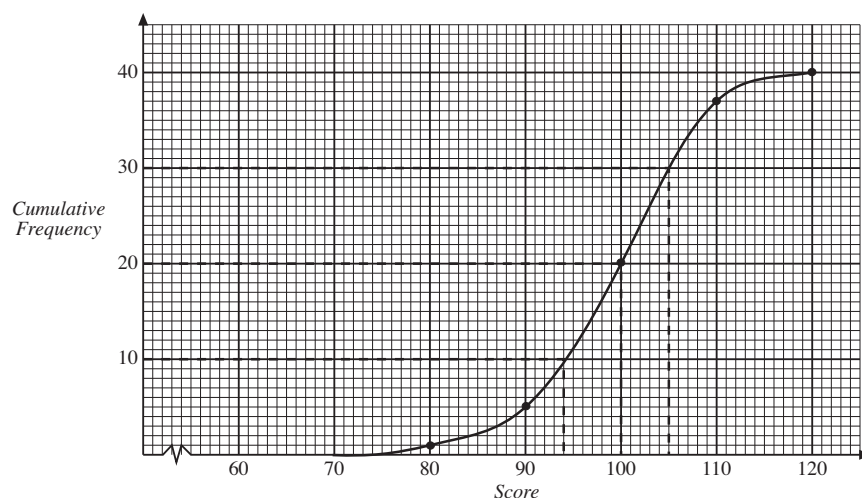
(b) The only exact answer is (iv). The other answers are estimates since they relate to prices for which we do not have exact information.

UNIT 18 Measures of Variation

Answers

18.1

7. (a)



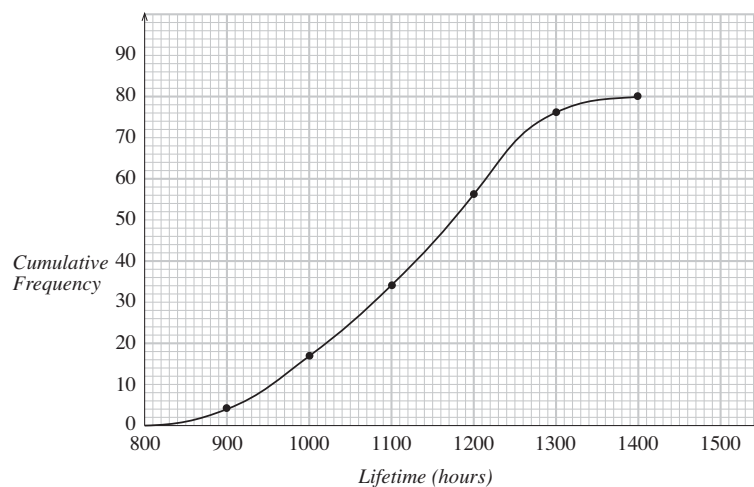
(b) (i) Derrice's median = 100 (ii) Inter-quartile range - about 11.

(c) (i) Jenevia was the more consistent player because her inter-quartile range is lower.

(ii) Derrice won most of the games. She scored less than 100 in 20 matches and more than 103 in only 16 matches, whereas Jenevia scored more than 103 in 20 matches and her inter-quartile scores were quite consistent.

8. (a) Cumulative frequencies - 34, 56, 76, 80, 80.

(b)



(c) 58 bulbs (d) Inter-quartile range – about 200.

(e) The bulbs from the second sample are more reliable than those from the first sample.

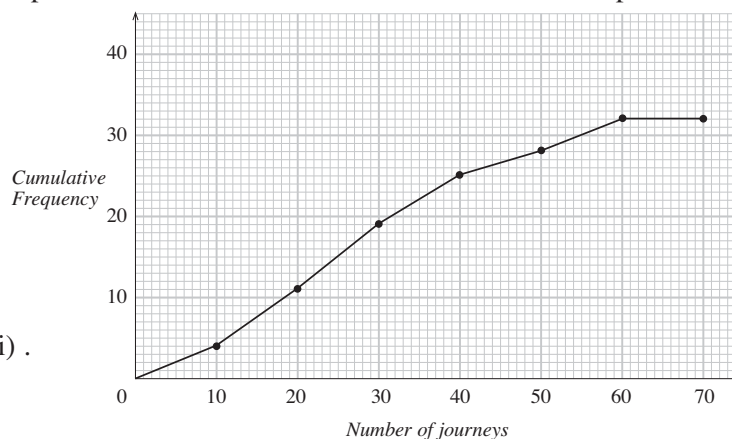
9. (a) Cumulative frequencies
4, 11, 19, 25, 28, 32, 32.

(b) (i) as graph.

(ii) Median – about 22

(iii) 6 people

(c) The second group travelled more (by route taxi) .



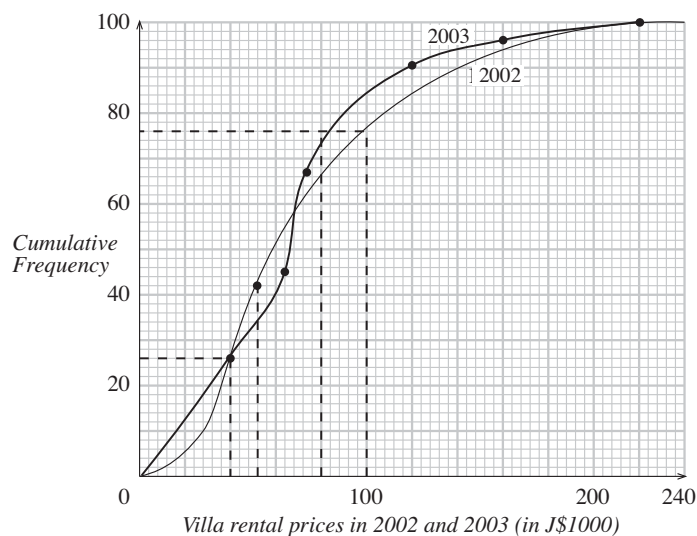
UNIT 18 Measures of Variation

Answers

18.1

10. (a) Cumulative frequencies – 26%, 45%, 67%, 82%, 91%, 96%, 100%.

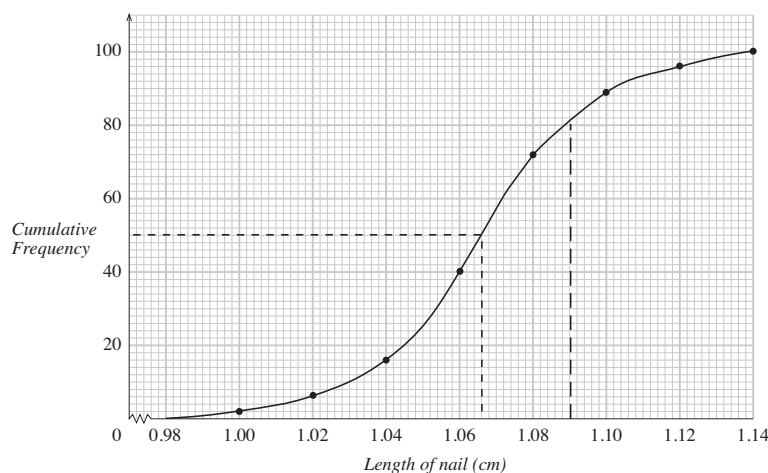
(b)



- (c) In 2002, 76% of villa rentals cost up to J\$100 000, whilst in 2003 76% of rentals were up to J\$80 000. Hence, in 2003 the rental for the holiday villa should be around J\$80 000.

11. (a) Cumulative frequencies: 2, 6, 16, 40, 72, 89, 96, 100.

(b)



- (i) Median: between 1.06 cm and 1.07 cm

- (ii) Inter-quartile range: about 0.04

12. (a) Mean distance
= 28.17 km (to 2 d.p.)

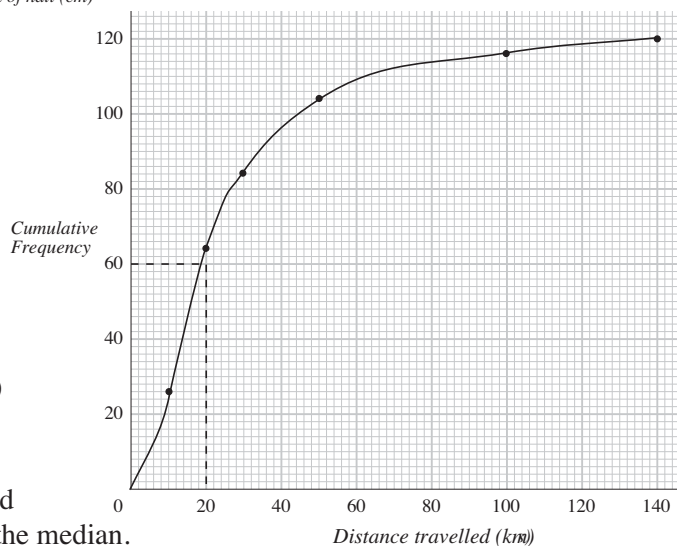
- (b) (i) Number of guests:
26, 64, 84, 104, 116, 120

(ii) as graph

- (c) Median: between 18 and 20 km

- (ii) The range is very large (140 km), and 36 people (who make up almost a third) travel more than 30 km (above the median).

These upper values influenced the mean which is far above the median.



UNIT 18 Measures of Variation

Answers

18.1

13. (a) (i) 18.5 (ii) $35 - 13 = 22$

(b)

0 -	10 -	20 -	30 -	40 -	50 -	60 -	70 - 80
(3	8	20)	34	50	57	59	60

- (c) cumulative frequency graph

- (i) 37 - 38 (ii) $47 - 26 = 21$

- (d) The median time is much increased, but there is a similar interquartile range.
The first sample might have been taken from one particularly good operator.

18.2 Box and Whisker Plots

1. (a) (i) 24.7 to 26.4 i.e. 1.7 (ii) $25.3 - 24.6 = 0.7$

- (iii) It is far from the other data values.

- (c) Firm C – as it is the only one to meet the requirements.

2. (a) 66 ; 52, 78 (c) Box plot quickly shows you the main statistical features.

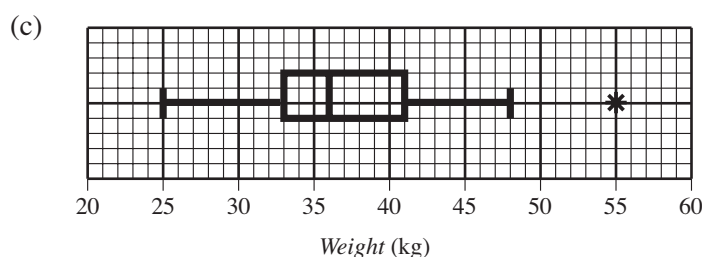
3. (a) 25 30 30 31 33 34 36 36 37 37 38 38 38 40 41 43 43 48 55

$\uparrow$ $\uparrow$ $\uparrow$
 LQ Median UQ

Hence $IQR = 41 - 33 = 8$

- (b) Any outliers will be in regions: $x < 33 - 1.5 \times 8 = 21$
 $x > 41 + 1.5 \times 8 = 53$

So there is only one outlier, 55.



- (d) Normal distribution, as it is symmetric.

- (e) They are not yet fully grown pigs.

4. (a) 0, 3, 5, 6, 7, 9, 10, 12, 13, 14, 16, 22, 24, 25, 26, 33, 35, 60, 64

$\uparrow$ $\uparrow$ $\uparrow$
 lower median upper
 quartile quartile

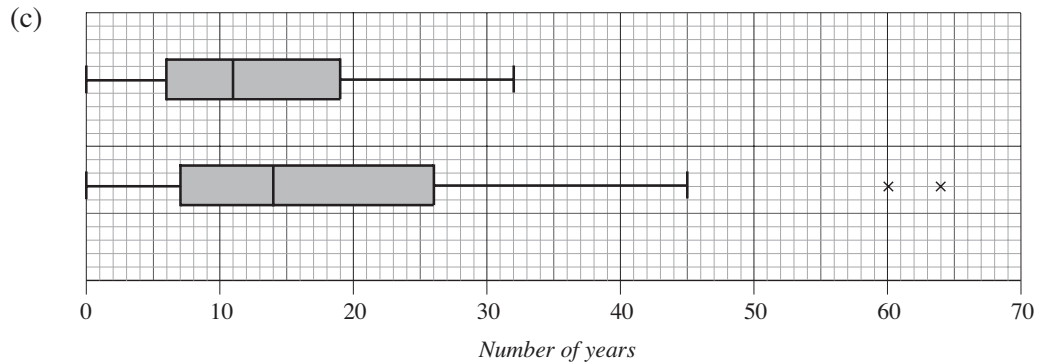
- (b) $IQR = 26 - 7 = 19$, so any outlier will be in regions: $x < 7 - 1.5 \times 19 = -21.5$
 $x > 26 + 1.5 \times 19 = 54.5$

There are two outliers, 60 and 64.

UNIT 18 Measures of Variation

Answers

18.2



(d) Generally, monarchs appear to reign longer than Popes, with a similar lower quartile, slightly greater median but significantly greater upper quartile, i.e. stretching to the right.

5. (a) (i) 9.5 (ii) $20 - 7 = 13$ (iii) About 8%
 (b) (i) 8, 52, 67, 76, 83, 120 (ii) 13, 35.5
6. (a) 19 (b) 3 (c) (i) 46 (ii) 30, 58
 (d) Graph

18.3 Standard Deviation

1. (a)

	Mean	Standard Deviation
A	53.67	4.31
B	73.67	4.31
C	107.33	8.62

(b) Adding same number to values: the mean increases but the s.d does not; values by a scale factor: the mean increases and so does the s.d.

2. (a)

	Mean	Standard Deviation
A	2.24 kg	0.04
B	2.64 kg	0.07

(b) On average, the boxes filled by A weigh less than the boxes filled by B, but A is more accurate ($0.04 < 0.07$).

3.

	Mean	Standard Deviation
A	10.02	0.49
B	9.6	0.14

The experiments done by B are more accurate and more reliable ($0.14 < 0.49$).

4. 45.58; 1.49
 5. 6.7; 1.85

UNIT 18 Measures of Variation

Answers

18.3

6. The estimated mean is 1.22, the estimated standard deviation is 0.96.
 7. The estimated mean is 61.30, the estimated standard deviation is 51.62.

8.

	<i>Estimated Mean</i>	<i>Estimated Standard Deviation</i>
A	1.57	1.15
B	3.2	1.99

On average, the sizes of the families in A are smaller and less spread out than in B.

9. (a) mean = 5, standard deviation = $\sqrt{2}$ (≈ 1.41)
 (b) Any five consecutive integers have the following pattern:

$n - 2, n - 1, n, n + 1, n + 2$, where n is the middle integer.

There are two integers which are 2 units away from n , ($n - 2$ and $n + 2$), and two integers which are 1 unit away from n , ($n - 1$ and $n + 1$). Since n is the mean, the standard deviation will be:

$$\sqrt{\frac{2^2 + 1^2 + 0^2 + 1^2 + 2^2}{5}} = \sqrt{\frac{10}{5}} = \sqrt{2}$$

10. (a) mean = 30, standard deviation = 16.11
 (b) The means in both Mathematics and English are identical, but the marks in English are less spread out ($3.6 < 16.11$) than in Mathematics.
11. (a) (i) 32 (ii) 11.92
 (b) On average, the girls performed less than boys ($30 < 32$), but their scores are less spread out ($6.5 < 11.32$) around the mean.
12. (a) 8.98
 (b) Class A has, on average, higher and less spread out I.Q. scores than Class B.
 (c) 0
13. (a) (i) 6 (ii) 2.16
 (b) The second test has 6 as mean and 1.48 as standard deviation; e.g. the means are identical on both tests but the second group performed more homogeneously than the first group. On average, the scores of the second group were closer to 6 than the scores of the first group.
14. (a) mean = 4.5, s.d. = 1.86
 (b) Yes it does. The values within one s.d. of the mean lie between 2.64 and 6.36. In our data these values are 3, 4, 5, 6. The total frequency of these values in our data is $7 + 8 + 9 + 10 = 34$. The percentage of values within one s.d. of the mean is therefore:

$$\frac{34}{50} \times 100 = 68\%$$