STRAND A: Computation

Unit 2 *Percentages*

Student Text

Contents

2 Percentages

2.1 Fractions, Decimals and Percentages

Percentage is a way of expressing a number as a fraction of 100: the term 'percentage' simply means 'per hundred'. Converting percentages to fractions is a simple process. Percentages can also be converted very easily to decimals, which can be useful when using a calculator. Fractions and decimals can also be converted back to percentages.

Worked Example 1

Convert each of the following percentages to fractions.

(a) 50% (b) 40% (c) 8%

Solution

(a) $50\% = \frac{50}{100} = \frac{1}{2}$

(b) $40\% = \frac{40}{100} = \frac{2}{5}$

(c) $8\% = \frac{8}{100} = \frac{2}{25}$

Worked Example 2

Convert each of the following percentages to decimals.

(a) 60% (b) 72% (c) 6%

Solution

(a) $60\% = \frac{60}{100} = 0.6$

(b) $72\% = \frac{72}{100} = 0.72$

(c) $6\% = \frac{6}{100} = 0.06$

Worked Example 3

Convert each of the following decimals to percentages.

(a) 0.04 (b) 0.65 (c) 0.9

Solution

(a) $0.04 = \frac{4}{100} = 4\%$

(b) $0.65 = \frac{65}{100} = 65\%$

(c) $0.9 = \frac{9}{10} = \frac{90}{100} = 90\%$

Worked Example 4

Convert each of the following fractions to percentages.

(a) $\frac{3}{10}$ (b) $\frac{1}{4}$ (c) $\frac{1}{3}$

Solution

To convert fractions to percentages, multiply the fraction by 100%. This gives its value as a percentage.

(a) $\frac{3}{10} = \frac{3}{10} \times 100\%$

$= \frac{3}{10} \times \frac{100}{1}\%$

$= \frac{300}{10}\%$

$= 30\%$

(b) $\frac{1}{4} = \frac{1}{4} \times 100\%$

$= \frac{1}{4} \times \frac{100}{1}\%$

$= \frac{100}{4}\%$

$= 25\%$

(c) $\frac{1}{3} = \frac{1}{3} \times 100\%$

$= \frac{1}{3} \times \frac{100}{1}\%$

$= \frac{100}{3}\%$

$= 33\frac{1}{3}\%$

Information

'Per cent' probably comes from the Latin , 'per centum', which means 'for each hundred'.

Exercises

1. Convert each of the following percentages to fractions, giving your answers in their simplest form.

(a)	10%	(b)	80%	(c)	90%	(d)	5%
(e)	25%	(f)	75%	(g)	35%	(h)	38%
(i)	4%	(j)	12%	(k)	82%	(l)	74%

2. Convert each of the following percentages to decimals.

(a)	32%	(b)	50%	(c)	34%	(d)	20%
(e)	15%	(f)	81%	(g)	4%	(h)	3%
(i)	7%	(j)	18%	(k)	75%	(l)	73%

3. Convert the following decimals to percentages.

(a)	0.5	(b)	0.74	(c)	0.35	(d)	0.08
(e)	0.1	(f)	0.52	(g)	0.8	(h)	0.07
(i)	0.04	(j)	0.18	(k)	0.4	(l)	0.3

4. Convert the following fractions to percentages.

(a) $\frac{1}{2}$ (b) $\frac{7}{10}$ (c) $\frac{1}{5}$ (d) $\frac{3}{4}$

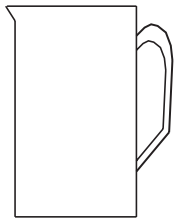
(e) $\frac{1}{10}$ (f) $\frac{9}{10}$ (g) $\frac{4}{5}$ (h) $\frac{4}{50}$

(i) $\frac{8}{25}$ (j) $\frac{7}{20}$ (k) $\frac{7}{25}$ (l) $\frac{2}{3}$

5. (a) Complete the equation $\frac{2}{3} = \frac{?}{15} = \frac{16}{?}$

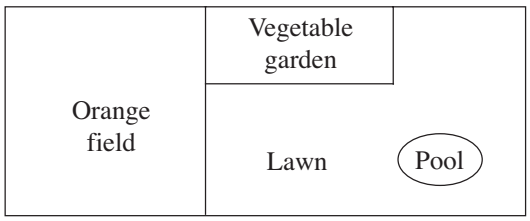
(b) Change $\frac{7}{40}$ to a percentage.

6. (a) Water is poured into this jug.
Copy the diagram and show accurately the water level when the jug is three-quarters full.



(b) What percentage of the jug is filled with water?

7. *Plan of a garden*



Not to scale

(a) In the garden the vegetable garden has an area of 46.2 m^2. The orange field has an area of 133.6 m^2.
What is the total area of the vegetable garden and the orange field? Give your answer to the nearest square metre.

(b) The garden has an area of 400 m^2.

(i) The lawn is 30% of the garden. Calculate the area of the lawn.

(ii) A pool in the garden has an area of 80 m^2. What percentage of the garden is taken up by the pool?

2.2 Fractions and Percentages of Quantities

Percentages are often used to describe changes in quantities or prices. For example,

'30% extra free' '10% discount' 'add $16\frac{1}{2}$% GCT'

This section deals with finding fractions or percentages of quantities.

Worked Example 1

Find 20% of \$84.

Solution

This can be done by converting 20% to either a fraction or a decimal.

Converting to a fraction

Note that $20\% = \frac{20}{100} = \frac{1}{5}$

Therefore $20\% \text{ of } \$84 = \frac{1}{5} \times \$84$

$= \$16.80$

Converting to a decimal

Note that $20\% = 0.2$

Therefore $20\% \text{ of } \$84 = 0.2 \times \$84$

$= \$16.80$

Worked Example 2

A shopkeeper decides to increase some prices by 10%. By how much would she increase the price of:

(a) a bar of soap costing 90 cents (b) a packet of rice costing \$2.00?

Solution

First note that $10\% = \frac{1}{10}$.

(a) $10\% \text{ of } 90 \text{ cents} = \frac{1}{10} \times 90 \text{ cents}$

$= 9 \text{ cents}$

So the cost of a bar of soap will be increased by 9 cents.

(b) $10\% \text{ of } \$2 = \frac{1}{10} \times \$2$

$= \$0.20 \text{ or } 20 \text{ cents}$

So the cost of a packet of rice is increased by 20 cents.

Worked Example 3

A farmer decides to sell 25% of his herd of 500 cattle. How many cows does he sell?

Solution

First note that $25\% = \frac{1}{4}$.

$$25\% \text{ of } 500 = \frac{1}{4} \times 500$$
$$= 125$$

So he sells 125 cows.

Worked Example 4

Natasha invests J$20 000 in a building society account. At the end of the year she receives 5% interest. How much interest does she receive?

Solution

First convert 5% to a fraction. $5\% = \frac{5}{100} = \frac{1}{20}$

$$5\% \text{ of J\$20 000} = \frac{1}{20} \times \text{J\$20 000}$$
$$= \text{J\$1000}$$

So she receives J$1000 interest.

Exercises

1. Find

(a)	10% of 200	(b)	50% of $5	(c)	20% of $8
(d)	25% of J$10 000	(e)	40% of $500	(f)	90% of 200
(g)	$33\frac{1}{3}\%$ of $12	(h)	75% of 800	(i)	75% of 1000
(j)	80% of 20 kg	(k)	70% of 5 kg	(l)	30% of 50 kg
(m)	5% of 100 m	(n)	20% of 50 m	(o)	25% of $30

2. Find

(a)	$\frac{2}{5}$ of 80	(b)	$\frac{3}{4}$ of 120	(c)	$\frac{1}{5}$ of 90
(d)	$\frac{1}{4}$ of 360	(e)	$\frac{4}{5}$ of 150	(f)	$\frac{3}{10}$ of 500

3. A firm decides to give 20% extra free in their packets of soap powder. How much extra soap powder would be given away free with packets which normally contain
   (a) 2 kg of powder (b) 1.2 kg of powder?

4. A picture costs J$30 000. A buyer is given a 10% discount. How much money does the buyer save?

5. John has invested $500 in a building society. He gets 5% interest each year. How much interest does he get in a year?

6. Lora bought an antique vase for J$12 000. Two years later its value had increased by 25%. What was the new value of the vase?

7. Khenan wants to replace a storm shutter in his house. The cost of the shutter is J$24 000. The hardware company has a special offer of a 25% discount. How much money does he save by using this offer?

8. When Maria walks to school she covers a distance of 1800 m. One day she discovers a short cut which reduces this distance by 20%. How much shorter is the new route?

9. Chen earns J$3000 per week from his part-time job. He is given a 5% pay rise. How much extra does he earn each week?

10. George weighed 90 kg. He went on a diet and tried to reduce his weight by 10%. How many kilograms did he try to lose?

11. Kina's mother decided to increase her pocket money by 40%. How much extra did Kina receive each week if previously she had been given J$400 per week?

12. A newborn baby girl weighed 4 kg. In the first three months her weight increased by 60%. How much weight had the baby gained?

13. Work out

(a) $\frac{7}{10}$ of $8 (b) 20% of $25 (c) $\frac{3}{8}$ of 6 metres.

14. (a) Calculate 15% of $600.

(b) List these fractions in order of size, starting with the smallest.

$$\frac{1}{3},\ \frac{2}{9},\ \frac{5}{6},\ \frac{1}{6}$$

15. In a certain school, 58% of the students are girls. If there are 406 girls in the school, calculate the total number of students in the school.

16. An athletics stadium has 35 000 seats. 4% of the seats are fitted with headphones to help people hear the announcements. How many headphones are there in the stadium?

17. Janeka wants to buy a computer costing $1800 in the USA. The deposit is $\frac{2}{5}$ of the price of the computer. Janeka's father gives her 30% of the price.

Will this be enough for her deposit?

You must explain your answer fully.

2.3 Quantities as Percentages

To answer questions such as,

Is it better to score 30 out of 40 or 40 out of 50?

it is helpful to express the scores as percentages.

Worked Example 1

Express '30 out of 40' and '40 out of 50' as percentages. Which is the better score?

Solution

'30 out of 40' can be written as $\frac{30}{40}$ and '40 out of 50' can be written as $\frac{40}{50}$.

Changing these fractions to percentages,

$$\frac{30}{40} = \frac{30}{40} \times 100\% \qquad \text{and} \qquad \frac{40}{50} = \frac{40}{50} \times 100\%$$

$$= 75\% \qquad\qquad = 80\%$$

So '40 out of 50' is the better score, since 80% is greater than 75%.

Worked Example 2

A student scores 6 out of 10 in a test. Express this as a percentage.

Solution

'6 out of 10' can be written as $\frac{6}{10}$. Changing this fraction to a percentage,

$$\frac{6}{10} = \frac{6}{10} \times 100\% = 60\%$$

Worked Example 3

Robyn and Rachel bought a CD for \$20. Robyn paid \$11 and Rachel paid \$9. What percentage of the total cost did each girl pay?

Solution

Robyn paid \$11 out of \$20, which is

$$\frac{11}{20} = \frac{11}{20} \times 100\% = 55\%$$

Rachel paid \$9 out of \$20, which is

$$\frac{9}{20} = \frac{9}{20} \times 100\% = 45\%$$

Worked Example 4

David earns \$200 per week and saves \$15 towards the cost of a cell phone.
What percentage of his earnings does he save?

Solution

He saves \$15 out of \$200, which is

$$\frac{15}{200} = \frac{15}{200} \times 100\% = 7.5\%$$

Exercises

1. Express each of the following as percentages.

(a)	8 out of 50	(b)	3 out of 25	(c)	8 out of 20
(d)	3 out of 10	(e)	6 out of 50	(f)	6 out of 40
(g)	12 out of 80	(h)	9 out of 30	(i)	27 out of 30
(j)	120 out of 300	(k)	84 out of 200	(l)	260 out of 400
(m)	28 out of 70	(n)	18 out of 60	(o)	51 out of 60

2. In a class of 25 students there are 10 girls. What percentage of the class are girls and what percentage are boys?

3. In the USA, the price of a bar of chocolate is 25 cents and includes 5 cents profit. Express the profit as a percentage of the price.

4. The value of a house is J\$4 000 000 and the value of the contents is J\$320 000. Express the contents value as a percentage of the house value.

5. In the crowd at a cricket match between Jamaica and Trinidad there were 14 000 *Jamaica* supporters and 11 000 *Trinidad* supporters. What percentage of the crowd supported each team?

6. A school won a prize of J\$200 000. The staff spent J\$160 000 on a new computer and the rest on software. What percentage of the money was spent on software?

7. A book contained 80 black and white pictures and 120 colour pictures.
What percentage of the pictures were in colour?

8. In a survey of 300 people it was found that 243 people watched *Days of Our Lives* regularly. Express this as a percentage.

9. Jamar needs another 40 stamps to complete his collection. There is a total of 500 stamps in the collection. What percentage of the collection does he have already?

10. A 600 ml bottle of shampoo contains 200 ml of free shampoo. What percentage is free?

11. Adrian finds that in a delivery of 500 bricks there are 20 broken bricks.
What percentage of the bricks are broken?

12. A glass of drink contains 50 ml of fruit juice and 200 ml of lemonade. What percentage of the drink is lemonade?

13. Research shows that there are 20 000 different types of fish in the world. People catch only 9000 different types. What percentage of the different types of fish do people catch?

14. Two recipes for making chocolate drinks are shown in the table below.

	Cups of Milk	**Cups of Chocolate**
Recipe A	3	2
Recipe B	2	1

(a) What percent of the mixture using Recipe A is chocolate?

(b) By showing suitable calculations, determine which of the two recipes, A or B, is richer in chocolate.

(c) If the mixtures from Recipe A and Recipe B are combined, what is the percent of chocolate in the new mixture?

(d) A vendor makes chocolate drink using Recipe A. 3 cups of milk and 2 cups of chocolate can make 6 bottles of chocolate drink. A cup of milk costs \$0.70 and a cup of chocolate costs \$1.15.

(i) What is the cost of making 150 bottles of chocolate drink?

(ii) What should be the selling price of each bottle of chocolate drink to make an overall profit of 20%?

2.4 More Complex Percentages

Not all percentages can be expressed as simple fractions and often figures such as 4.26% may need to be used. In these cases it is often better to work with decimals.

Worked Example 1

The cost of a hotel bill is J\$20 000. GCT at 16.5% has to be added to this bill. Find the GCT and the total bill.

Solution

Use 16.5% = 0.165.

Then

$$16.5\% \text{ of J\$20 000} = 0.165 \times \text{J\$20 000}$$

$$= \text{J\$3300}$$

So the total bill is

$$\text{J\$20 000} + \text{J\$3300} = \text{J\$23 300}$$

Worked Example 2

Jamie has $486.27 in his building society account which earns interest of 8.21% per year. How much interest does he get and how much money does he have in his account after the first year?

Solution

Writing 8.21% as a decimal gives 0.0821.

$$\begin{aligned} 8.21\% \text{ of } \$486.27 &= 0.0821 \times \$486.27 \\ &= \$39.92 \quad \text{(to the nearest cent)} \end{aligned}$$

So the account now contains

$$\$486.27 + \$39.92 = \$526.19$$

Worked Example 3

The cost of one load of concrete blocks is J$28 800 plus GCT at 16.5%. Find the total cost of the concrete blocks.

Solution

100% represents the original cost (J$28 800).

16.5% is the increase due to GCT.

$$100\% + 16.5\% = 116.5\%$$

So the total cost can be found in one stage by finding 116.5% of J$28 800.

Note that 116.5% is 1.165 as a decimal.

So

$$\begin{aligned} 116.5\% \text{ of J\$28 800} &= 1.165\% \times \text{J\$28800} \\ &= \text{J\$33 552} \end{aligned}$$

The total cost of the load of blocks is J$33 552.

Worked Example 4

Jessica's salary of J$48 000 is to be increased by 2.5%. Find her new salary.

Solution

Her new salary is 102.5% of her old salary.

$$\begin{aligned} 102.5\% \text{ of J\$48 000} &= 1.025 \times \text{J\$48000} \\ &= \text{J\$49200} \end{aligned}$$

Her new salary is J$49 200.

Worked Example 5

A holiday cruise from New York for a family costs $9995, but a special offer gives an 8.5% discount. Find the price of the cruise with the discount.

Solution

With an 8.5% discount, 91.5% of the original price must be paid. $(100\% - 8.5\% = 91.5\%)$

So

$$91.5\% \text{ of } \$9995 = 0.915 \times \$9995$$
$$= \$9145.43 \quad \text{(to the nearest cent)}$$

The discounted price is \$9145.43.

Worked Example 6

Janet's gross salary is \$2400 per month. Her tax-free allowances are shown below.

National Insurance	5% of gross salary
Personal Allowance	\$3000 per year

Calculate

(a) her gross yearly salary

(b) her total tax-free allowances for the year

(c) her taxable yearly income.

(d) A 10% tax is charged on the first \$20 000 of taxable income. A 20% tax is charged on the portion of taxable income above \$20 000.
Calculate the amount of income tax Janet pays.

Solution

(a) Janet's gross yearly salary $= \$2400 \times 12$
$= \$28\,800$

(b) National Insurance $= \frac{5}{100} \times \$28\,800$ per year
$= \$1440$ per year

Personal Allowance $= \$3000$ per year

Total tax-free allowances for the year $= \$1440 + \$3000$
$= \$4440$

(c) Taxable yearly income $= \$28\,800 - \$4440$
$= \$24\,360$

(d) Tax on first \$20 000 of taxable income $= \frac{10}{100} \times \$20\,000$

$= \$2000$

Tax on remaining \$4360 of taxable income $= \frac{20}{100} \times \$4360$

$= \$872$

Total tax paid $= \$2000 + \$872$

$= \$2872$ per year

Exercises

1. Find each of the following, giving your answers to the nearest cent.

   (a) 32% of \$50 (b) 15% of \$83 (c) 12.6% of \$40

   (d) 4.7% of \$30 (e) 6.9% of \$52 (f) 3.7% of \$18.62

2. (a) Add 16.5% GCT to \$415.

   (b) Add 3.2% interest to \$1148.

   (c) Increase a salary of \$15 000 by 1.6%.

   (d) Increase a price of \$199 by 3.2%.

   (e) Decrease \$420 by 7%.

3. A CD has a normal price of J\$1500.

   (a) In a sale its normal price is reduced by 12%. Find the sale price.

   (b) After the sale, normal prices are increased by 2.5%. Find the new price of the CD.

4. Bacteria killed 70% of the fish in a pond. If 150 fish survived, calculate how many fish were originally in the pond.

5. Peter earns \$9000 per year in his new job in Boston, USA. He does not pay tax on the first \$3500 he earns and pays 25% tax on the rest. How much tax does he have to pay?

6. A man living in the US with his wife and two children, earns \$32 000 a year.
   His annual tax-free allowances are shown in Table 1.

   Calculate

   (a) his TOTAL annual tax-free allowances

   (b) his annual taxable income.

Table 1

	Allowance
Adult	\$900 each
Child	\$400 each
Housing	\$2500 per family

Table 2 shows the taxes that are due annually.

Table 2

Taxable Income	Taxes Due
First \$20 000	\$1200
Remainder	30% of the remainder

(c) Calculate the taxes that he should pay annually.

7. A sound system costs J\$186 000 plus GCT at $16\frac{1}{2}\%$. Its price is increased by 4%. How much would you have to pay to buy the sound system at the new price?

8. A company pays a Christmas bonus of J\$12 000 to each of its employees. This is taxed at 25%. One year the bonus is increased by 5%. How much does an employee take home?

9. An electricity supplier offers a 20% discount on the normal price and a further 5% discount off the normal price if customers pay directly from their banks. For one household the electricity bill is normally \$200. Find out how much they have to pay after both discounts.

10. A cell phone costs J\$3500 plus GCT.

GCT is charged at $16\frac{1}{2}\%$.
How much is the GCT?

11.

CASH
A discount of 15% off the marked price if you pay cash

SUPER DVD PLAYER
\$276

TERMS
A deposit of $\frac{1}{4}$ of the marked price then 24 monthly payments of \$9.45 each

(a) Mr. Smith buys the DVD player for cash. How much discount is he allowed?

(b) Mr. Jones buys the DVD player on terms.

(i) How much must he pay as a deposit?

(ii) Multiply 945 by 24 without using a calculator.
S*how all your working*.

(iii) Work out the total price that Mr. Jones pays for his DVD player.

12. The usual price of a car radio is \$298 plus GCT at $16\frac{1}{2}\%$.

(a) (i) Work out the exact value of $16\frac{1}{2}\%$ of \$298.

(ii) What is the usual price of this car radio?

Gannet Store and *Berries' Store* are selling car radios with CD players at reduced prices. The usual price of these in both stores is \$466 (\$400 plus \$66 GCT).

(b) (i) Calculate the difference between the reduced prices in the two stores. Give your answers to the nearest cent.

Show your working clearly.

(ii) Which of the stores gives the bigger reduction?

2.5 Percentage Increase and Decrease

Percentage *increases* are calculated using

$$\text{Percentage increase} = \frac{\text{actual increase}}{\text{initial value}} \times 100\%$$

Similarly, percentage *decreases* are calculated using

$$\text{Percentage decrease} = \frac{\text{actual decrease}}{\text{initial value}} \times 100\%$$

Worked Example 1

The population of a village increased from 234 to 275 during one year. Find the percentage increase.

Solution

$$\text{Actual increase} = 275 - 234 = 41$$

$$\text{Percentage increase} = \frac{41}{234} \times 100\%$$

$$= 17.52\% \quad \text{(to 2 decimal places)}$$

Worked Example 2

When a beaker of sand is dried in a hot oven its mass reduces from 450 grams to 320 grams. Find the percentage reduction in its mass.

Solution

$$\begin{aligned} \text{Actual reduction} &= 450 \text{ grams} - 320 \text{ grams} \\ &= 130 \text{ grams} \\ \text{Percentage reduction} &= \frac{130}{450} \times 100\% \\ &= 28.9\% \end{aligned}$$

Worked Example 3

John buys pens for \$5 each and then sells them to other students for \$6.90. Find his percentage profit.

Solution

$$\begin{aligned} \text{Actual profit} &= \$6.90 - \$5 \\ &= \$1.90 \\ \text{Percentage profit} &= \frac{1.90}{5} \times 100\% \\ &= 38\% \end{aligned}$$

Exercises

1. A baby weighed 5.6 kg and six weeks later her weight had increased to 6.8 kg. Find the percentage increase.

2. A factory produces pens at a cost of 88 cents and sells them for \$1.10. Find the percentage profit.

3. A boat which cost J\$115 000 was sold one year later for J\$99 950. Find the percentage reduction in the value of the boat.

4. An investor bought some shares at a price of \$4.88 each. The price of the shares dropped to \$3.96 each. Find the percentage loss.

5. A supermarket offers a \$10 discount to all customers spending \$40 or more. Kate spends \$42.63 and John spends \$78.82. Find the percentage saving for Kate and John.

6. After a special offer the price of baked beans was increased from J\$150 per tin to J\$210 per tin. Find the percentage increase in the price.

7. The size of a school increased so that it had 750 students instead of 680 and 38 teachers instead of 37. Find the percentage increases in the number of teachers and students. Comment on your answers.

8. In a science experiment the length of a spring increased by 4 cm to 20 cm. Find the percentage increase in the length of the spring.

9. The average cost of a local telephone call for one customer dropped by 80 cents to J$2.40. Find the percentage reduction in the average cost of a local call.

10. In a year, the value of a house in the USA increased from $460 000 to $480 000. Find the percentage increase in the value of the house and use this to estimate the value after another year.

11. A battery was tested and found to power a CD player for 12 hours. An improved version of the battery powered the CD player for an extra 30 minutes. Find the percentage increase in the life of the batteries.

12. The value of an IT system depreciates as shown in the table.

IT System	*Value*
New	$12 000
After 1 year	$10 000
After 2 years	$ 8 800
After 3 years	$ 8 000

During which year is the percentage decrease in the value of the IT system the greatest?

13.

Quality Garden Supplies

SUMMER SALE!

Save 20% on goods totalling J$300 or more

(a) Deon bought a plant marked J$350. How much did he save?

(b) Kenton needs a large plant pot. He can buy pot A which is marked J$279.50 or pot B which is marked J$324.50.

(i) Calculate 20% of J$324.50.

(ii) How much cheaper would it be for Kenton to buy pot B than to buy pot A?

(c) Kentons wife suggests that he buys pot A, together with some seeds costing J$20.50 which she wants, so that he gets the 20% saving.

If he buys the seeds and pot A, express his saving as a percentage of the cost of pot A.

14.

Super Ace Games System

Normal Price $120

Sale Price $\frac{1}{3}$ off

(a) Work out the sale price of the *Super Ace Games System*.

Mega Ace Games System
Normal Price $320
Sale Price $272

(b) Find the percentage reduction on the *Mega Ace Games System* in the sale.

15. Liron paid J$720 for a CD gift set. He sold it for J$630. What was his loss as a percentage of the price he paid?

Investigation

The ancient Egyptians were the first to use fractions. However, they only used fractions with a numerator of one. Thus they wrote $\frac{3}{8}$ *as* $\frac{1}{4}+\frac{1}{8}$, *etc.*

What do you think the Egyptians would write for the fractions $\frac{3}{5}$, $\frac{9}{20}$, $\frac{2}{3}$ *and* $\frac{7}{12}$?