Student Text *UNIT 23 Algebraic Manipulation* Answers

23.1 Simultaneous Linear Equations

1. (a) $x = 1, y = 2$ (b) $x = 5, y = 2$ (c) $x = 10, y = 3$

   (d) $x = 4, y = 2$ (e) $x = \frac{46}{43}, y = \frac{-13}{43}$ (f) $x = 5, y = -2$

   (g) $x = -2, y = 12$ (h) $x = 10, y = 12$ (i) $x = 5, y = 6$

   (j) $x = \frac{5}{7}, y = \frac{9}{28}$ (k) $x = \frac{7}{45}, y = \frac{13}{36}$ (l) $x = 7, y = \frac{1}{5}$

   (m) $x = \frac{1}{2}, y = 3$ (n) $x = -3, y = -1$ (o) $x = \frac{43}{33}, y = \frac{48}{11}$

   (p) $x = 0.2, y = 0.4$ (q) $x = 6, y = 12$ (r) $x = 4, y = 18$

2. (a) $(3, 5)$ (b) $(3, 7)$ (c) $\left(\frac{20}{9}, \frac{16}{9}\right)$

3. It is not possible to eliminate either x or y because of their coefficients. If the equations had been used to produce graphs it would be seen that the two straight lines produced are parallel. Therefore, there can be no point of intersection and it is impossible to solve the equations simultaneously.

4. (b) There is no way of eliminating x or y in order to find the value of the other variable.

   (c) $x + 2y = 12 \quad (1)$ gives $y = 6 - \frac{x}{2}$

   $3x + 6y = 36 \quad (2)$ gives $y = 6 - \frac{x}{2}$

   So equations (1) and (2) are different versions of the same equation.

5. $v + c = 3.7$; $v - c = 1.3$; $v = 2.5, c = 1.2$.

6. 72 return tickets and 28 single tickets were sold.

7. (a) 79 J$100 notes; 121 J$50 notes

   (b) 78 J$100 notes; 122 J$50 notes

8. 16

9. The length of the flight is 9 hours; the time difference is 5 hours.

10. $x = -11, y = 15$

11. $a = \frac{7}{2}, c = \frac{3}{2}$

12. (a) 2 metres

    (b) (i) $t = 1$ when $h = 37$, so $37 = (a \times 1) + (b \times 1) + 2$ and $35 = a + b$

    $t = 2$ when $h = 62$, so $62 = (a \times 4) + (b \times 2) + 2$ and $60 = 4a + 2b$

    (ii) $a = -5$; $b = 40$

    (c) 20.75 metres

Student Text *UNIT 23 Algebraic Manipulation* Answers

23.2 Expanding Brackets

1. (a) $x^2 + x$ (b) $2x - 3x^2$ (c) $6a^2 - 15a$ (d) $x^2 - x^3$
   (e) $4x^2 - 28x$ (f) $3x + 11$ (g) $x^2 - 2x$ (h) $x + x^3$
   (i) $-4x^2 - 3$ (j) $-5x$ (k) $5x^2 - 20x$ (l) $5x^2 - 8$
   (m) $5x^2 - x^3$ (n) $3x + 16$ (o) $7x - 3x^2$

2. (a) $x^2 + 7x + 6$ (b) $x^2 + 11x + 28$ (c) $x^2 + 6x - 16$
   (d) $x^2 + 3x - 4$ (e) $x^2 - 8x + 7$ (f) $6x^2 + x - 1$
   (g) $8x^2 + 34x + 21$ (h) $10x^2 + 11x - 6$ (i) $24x^2 + 14x - 3$
   (j) $12x^2 - 13x + 3$ (k) $15x^2 - 58x + 48$ (l) $15a^2 - 29a - 14$
   (m) $12a^2 - 35a + 25$ (n) $24n^2 - 8n - 2$ (o) $20x^2 - 17x - 24$

3. (a) $x^2 + 2x + 1$ (b) $x^2 - 2x + 1$ (c) $x^2 + 16x + 64$
   (d) $4x^2 + 4x + 1$ (e) $9x^2 + 24x + 16$ (f) $36x^2 - 12x + 1$
   (g) $4x^2 - 20x + 25$ (h) $36x^2 - 84x + 49$ (i) $25x^2 + 90x + 81$

4. (a) $ac + ad + bc + bd$ (b) $2a^2 + 3ac + c^2$ (c) $3a^2 - 14ad - 5d^2$
   (d) $12x^2 + xy - y^2$ (e) $4a^2 + 13ad + 3d^2$ (f) $2a^2 + ab + 8ac + 4bc$
   (g) $9xy - y^2 - 18x^2$ (h) $q^2 - 2p^2 - pq$ (i) $45x^2 - xy - 2y^2$
   (j) $4x^2 + 4xy - 2x - 2y$ (k) $4a^2 - 2ac + 2ab - bc$
   (l) $4x^2 - 29xy + 30y^2$ (m) $p^2 - 4q^2$ (n) $5a^2 - 13ab - 6b^2$
   (o) $10x^2 - 27xy + 18y^2$

5. (a) $x^2 - 1$; $x^2 - 16$; $x^2 - 36$ (b) $x^2 - 25$ (c) $a^2 - b^2$
   (d) $(5x)^2 - 2^2 = 25x^2 - 4$

6. (a) $x^2 + x - 2$ (b) $2x^2 - 6x + 4$ (c) $2x^2 + 4x + 2$

7. (a) $x^3 - 3x^2 - 10x$ (b) $x^3 + 4x^2 - 4x - 16$
   (c) $x^3 + 2x^2 - 5x - 6$ (d) $6x^3 + 37x^2 - 39x - 28$
   (e) $x^3 + 3x^2 + 3x + 1$ (f) $x^3 - 12x^2 + 48x - 64$

Student Text *UNIT 23 Algebraic Manipulation* Answers

23.3 Linear Factorisation

1. (a) $5x + 10 = 5(x + 2)$ (b) $6x - 8 = 2(3x - 4)$
   (c) $15x + 25 = 5(3x + 5)$ (d) $12x + 8 = 4(3x + 2)$
   (e) $18 - 6n = 6(3 - n)$ (f) $6x - 21 = 3(2x - 7)$
   (g) $16a + 24 = 8(2a + 3)$ (h) $33x - 9 = 3(11x - 3)$

2. (a) $6(x + 4)$ (b) $5(x - 4)$ (c) $8(2 - x)$ (d) $4(2n + 3)$
   (e) $2(6x - 7)$ (f) $3(a - 8)$ (g) $11(x - 6)$ (h) $5(2 + 5x)$
   (i) $20(5x - 2)$ (j) $10(5 - 4x)$ (k) $6(x - 5)$ (l) $5(y - 9)$
   (m) $12(1 + 3x)$ (n) $16(x + 2)$ (o) $3(9x - 11)$

3. (a) $x(x + 1)$ (b) $x(x + 2)$ (c) $a(2a - 5)$ (d) $x(4x + 1)$
   (e) $x(x + 4)$ (f) $x(a + b)$ (g) $3x(2x + 1)$ (h) $2x(2x - a)$

4. (a) $x(5x + 1)$ (b) $a(a + 3)$ (c) $n(5n + 2)$ (d) $3n(2n + 1)$
   (e) $5n(n - 2)$ (f) $3x(x + 2)$ (g) $15x(x + 2)$ (h) $7x(2x + 3)$
   (i) $8x(2x + 3)$ (j) $6x(5x - 3)$ (k) $5(1 + n^2)$ (l) $5(2n^2 - 3)$
   (m) $3n(n^2 + 3)$ (n) $9x(x - 3)$ (o) $5x^2(2x - 1)$

5. (a) $ax(1 + x)$ (b) $x(b + cx)$ (c) $2q(p - 2r)$ (d) $5y(3x - y)$
   (e) $8p(2q + 3p)$ (f) $6x(x + 3y)$ (g) $3p(p - 3x)$
   (h) $8x(3p + 7x)$ (i) $2xy(8x - 9y)$

6. (a) Yes; $2x(3x + 2)$ (b) Yes; $8x^2(2x + 1)$ (c) No
   (d) Yes; $3xy(x - 6y)$

23.4 Quadratic Factorisation

1. (a) $(x + 2)^2$ (b) $(x + 4)(x + 3)$ (c) $(x + 4)(x + 2)$
   (d) $(x + 6)(x + 1)$ (e) $(x + 8)(x + 2)$ (f) $(x + 3)(x + 1)$
   (g) $(x + 5)(x + 3)$ (h) $(x + 2)(x + 1)$ (i) $(x + 4)(x + 1)$
   (j) $(x + 3)(x + 8)$ (k) $(x + 11)(x + 1)$ (l) $(x + 8)(x + 7)$

Student Text *UNIT 23 Algebraic Manipulation* Answers

23.4

(m) $(x+3)(x+3)$ (n) $(x+5)(x+2)$ (o) $(x+7)(x+2)$

(p) $(x+6)(x+5)$ (q) $(x+8)(x+1)$ (r) $(x+8)(x+4)$

2. (a) $(x+2)(x-1)$ (b) $(x-4)(x+3)$ (c) $(x-5)(x+2)$

(d) $(x+5)(x-1)$ (e) $(x-7)(x+2)$ (f) $(x-4)(x+2)$

(g) $(x+5)(x-3)$ (h) $(x-2)(x-1)$ (i) $(x-5)(x-4)$

(j) $(x-7)(x-3)$ (k) $(x-7)(x-2)$ (l) $(x-5)(x-2)$

(m) $(x-8)(x+2)$ (n) $(x-9)(x-8)$ (o) $(x-8)(x+3)$

3. (a) $(x+1)(x-1)$ (b) $(x+4)(x-4)$ (c) $(x+9)(x-9)$

(d) $(3x+2)(3x-2)$ (e) $(4x+6)(4x-6)$ (f) $(2x+10)(2x-10)$

(g) $(x^2+10)(x^2-10) = (x^2+10)(x+\sqrt{10})(x-\sqrt{10})$

(h) $(x^2+2)(x^2-2) = (x^2+2)(x+\sqrt{2})(x-\sqrt{2})$

(i) $(2x^2+3)(2x^2-3) = (2x^2+3)(\sqrt{2x}+\sqrt{3})(\sqrt{2}x+\sqrt{3})$

4. (a) $(2x+1)(x+1)$ (b) $(3x+1)(x+2)$ (c) $(2x+3)(x+1)$

(d) $(3x+2)(x+4)$ (e) $(2x-1)(x+5)$ (f) $(4x+3)(x-2)$

(g) $(3x-5)(x+2)$ (h) $(3x-2)(x-7)$ (i) $(3x+1)(2x+5)$

(j) $(4x-1)(2x-1)$

5. (a) $(3x+1)(x-1)$ (b) $(3x+1)(x+1)$ (c) $(2x+1)(x+2)$

(d) $(3x+2)(x+2)$ (e) $(3x-1)(x+3)$ (f) $(4x+1)(x-3)$

(g) $(5x-2)(x+1)$ (h) $(3x-2)(x-2)$ (i) $(5x-2)(x+3)$

(j) $(3x+1)(2x+1)$ (k) $(2x-1)(3x-2)$ (l) $(5x+1)(2x-1)$

(m) $(4x-1)(2x+3)$ (n) $(2x+7)(3x-1)$ (o) $(3x-4)(2x-3)$

6. (a) $2x^2+9x+4$ (b) $2x(2x-3)$

7. (a) $3pq(4p-5q)$ (b) $2x^2+7x-15$ (c) $n = (C - 120) \div 40$

8. (a) $ab(5a+b)$ (b) $(3k-1)(3k+1)$ (c) $(2y-1)(y-2)$

9. (a) $(2g+f)(2g-f)$ (b) $(t+2p)(m-3)$

10. (a) $7mp(p+2m)$ (b) $(2y-5)(y-3)$