

UNIT 23 *Algebraic Manipulation*

Introduction to SIM

Learning objectives

This unit continues the algebraic concepts theme of the previous two units. After completing this unit of STUDENT INSTRUCTIONAL MATERIAL (SIM) you should be able to

- solve simultaneous equations
- fully expand brackets
- factorise expressions by taking out single terms and linear terms
- factorise quadratic expressions into two linear terms.

Key points and principles

- Simultaneous linear equations are of the form

$$ax + by = c$$

$$dx + ey = f$$

when a, b, c, d, e and f are constants.

- Simultaneous linear equations normally have unique solutions, but not when one equation is the multiple of another. For example, when

$$x + y = 1$$

$$2x + 2y = 2$$

there is no unique solution. Also when there are two inconsistent equations, for example,

$$x + y = 1$$

$$x + y = 3$$

there are NO solutions.

- Having obtained solutions for x and y for simultaneous linear equations, you should then always check this solution for x and y in the original equations.
- When expanding brackets, ensure that all terms are considered. For example,

$$\begin{aligned} 2x(x + 3) &= 2x \times x + 2x \times 3 \\ &= 2x^2 + 6x \end{aligned}$$

- When taking out simple factors from an equation, ensure that you take out all possible factors.

$$\begin{aligned} \text{For example, } 2x^2 + 6x &= 2(x^2 + 3x) && (\text{taking out the constant term } 2) \\ &= 2x(x + 3) && (\text{taking out the factor } x) \end{aligned}$$

- For quadratic factorisation of $x^2 + ax + b$, if factors of the form $(x + c)(x + d)$ exist, then

$$a = c + d, \quad b = cd$$

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Facts to remember

- $(a + b)(a - b) = a^2 - b^2$
- $(a + b)^2 = a^2 + 2ab + b^2$
- $(a - b)^2 = a^2 - 2ab + b^2$
- If $x^2 + ax + b = (x + p)(x + q)$, then

$$a = p + q \text{ and } b = pq$$

*Glossary of Terms**Simultaneous linear equations*

These are of the form

$$ax + by = c$$

$$dx + ey = f$$

when a, b, c, d, e and f are constants.

Expanding brackets

Expressions such as $a(x + b)$ are expanded out.

For example,

$$2(x + 3) = 2x + 6$$

$$x(4 - x) = 4x - x^2$$

$$2x(1 + 3x) = 2x + 6x^2$$

$$a(x + b) = ax + ab$$

Linear factorisation

Single factors are taken out of an expression.

For example,

$$3x + 9 = 3(x + 3)$$

$$4x^2 + x = x(4x + 1)$$

$$5x^2 + 10x = 5x(x + 2)$$

Quadratic factorisation

Quadratic expressions are factorised into linear factors.

For example,

$$x^2 + 5x + 6 = (x + 3)(x + 2)$$

$$x^2 - 4 = (x + 2)(x - 2)$$

$$(2x - 1)(x + 2) = 2x^2 + 3x - 2$$