UNIT 26 *Solving Inequalities* Introduction to SIM

Learning objectives

This unit introduces the topic of *solving inequalities,* which is an important aspect of mathematics and a natural extension to solving equations. After completing this unit of STUDENT INSTRUCTIONAL MATERIAL (SIM) you should

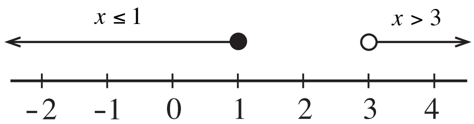
- be able to illustrate inequalities on a number line
- be able to solve linear inequalities
- be able to solve quadratic inequalities
- understand how to illustrate linear inequalities in two variables
- be able to illustrate and solve linear programming problems in two variables.

Key points and principles

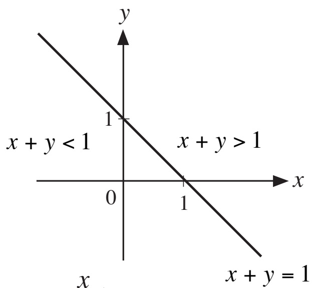
- You must be careful to differentiate between $<$ and $\leq$.
- Representing $<$ and $\leq$ on a number line must be made clear,

  i.e. use small unshaded circle, ○, for the end point of $<$

  use small shaded circle, ●, for the end point of $\leq$.

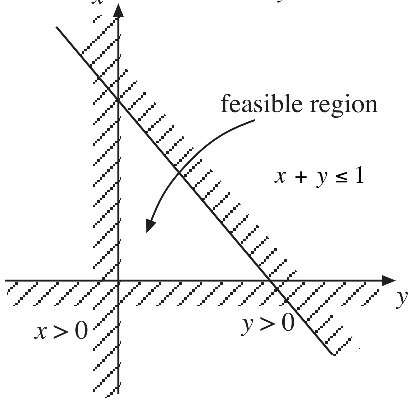


- A set of inequalities is solved by finding the values of the variables which satisfy all the inequalities.
- The equation $ax + by = c$ defines a line; one side of the line satisfies $ax + by > c$ and the other side satisfies $ax + by < c$. For example, $x + y = 1$.



- In linear programming, the feasible region is the region in which all the inequalities are satisfied and the solution to the optimisation problem is found at <u>one</u> of the vertices.

  For example, for the set of inequalities $x + y \geq 1$, $x \geq 0$, $y \geq 0$, the feasible region, is shown opposite.



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Facts to remember

- $x \geq a$ means that either $x > a$ or $x = a$
- Multiplying an inequality by a negative number reverses the inequality sign;
  for example, $x \geq 4 \Rightarrow -x \leq -4$.

Glossary of terms

$>$ means 'greater than'; for example, $5 > 3$.

$\geq$ means 'greater than or equal to'; for example, $5 \geq 3$, $5 \geq 5$.

$<$ means 'less than'; for example, $3 < 5$.

$\leq$ means 'less than or equal to'; for example, $3 \leq 5$, $5 \leq 5$.

$\neq$ means 'not equal to'; for example, $3 \neq 5$.

Linear inequalities are of the form

$$ax + b \leq c \quad \text{or} \quad ax + b < c\text{, etc.}$$

For example, $2x + 3 \leq 7$

Solving linear inequalities means manipulating the inequality to give $x \geq \ldots$ or $x \leq \ldots$

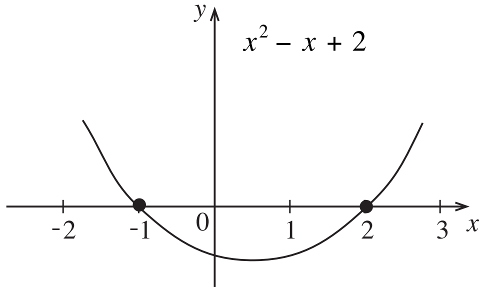
For example, $2x + 3 \leq 7 \Rightarrow 2x \leq 4 \Rightarrow x \leq 2$

Quadratic inequalities are of the form

$$ax^2 + bx + c \leq 0\text{, etc.}$$

For example, $x^2 - x + 2 \geq 0 \Rightarrow (x - 2)(x + 1) \geq 0$

and illustrating on a graph,



shows that the inequality is true for $x \geq 2$ or $x \leq -1$.

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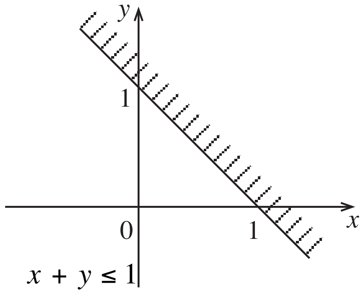
General linear inequality in two variables is of the form $ax + by \geq c$ and can be illustrated graphically.

For example, $x + y \leq 1$



Linear programming For example, maximum $P = 2x + y$ subject to

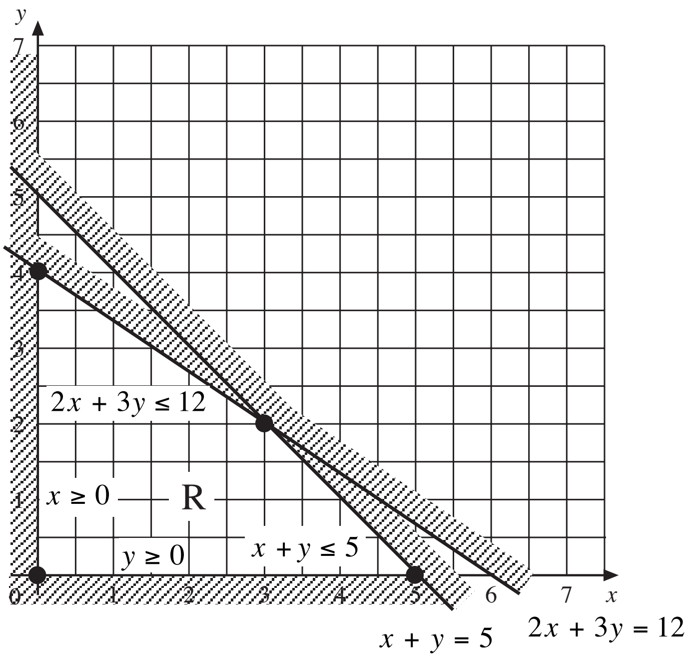
$$x + y \leq 5$$

$$2x + 3y \leq 12$$

$$x \geq 0, \quad y \geq 0$$

Feasible region This is the region in which all the inequalities are satisfied.

For the example above, the feasible region is labelled R; it is the region <u>not</u> shaded.



(**Note**: the maximum of $P = 2x + y$ will occur at one of the vertices marked by ● on the diagram; in fact, it will occur at $(5, 0)$ and has value $P = 10$.)