

STRAND H: Relations, Functions and Graphs

Unit 28 *Straight Lines*

Student Text

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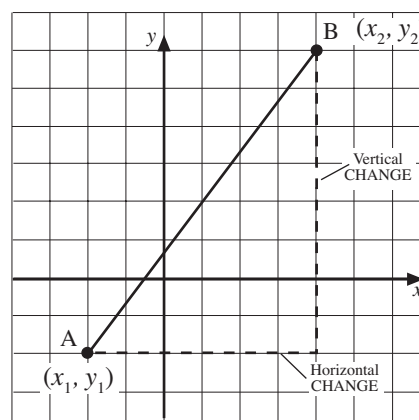
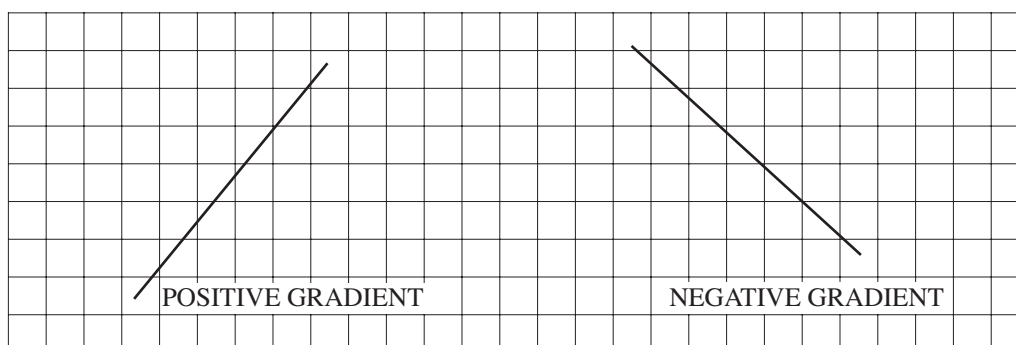
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28 Straight Lines

28.1 Gradient

The *gradient* of a line describes how steep it is.

The diagram below shows two lines, one with a *positive* gradient and the other with a *negative* gradient.



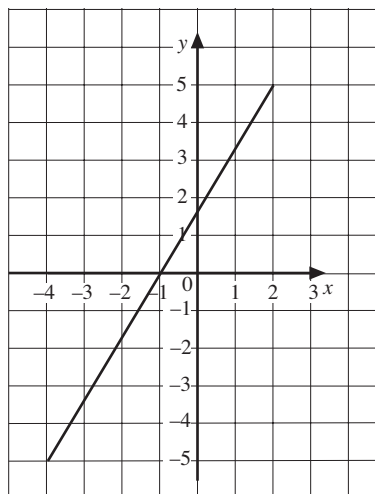
The gradient of a line between two points, A and B, is calculated using

$$\begin{aligned}
 \text{gradient of AB} &= \frac{\text{vertical change}}{\text{horizontal change}} \\
 &= \frac{(\text{y-coordinate of B}) - (\text{y-coordinate of A})}{(\text{x-coordinate of B}) - (\text{x-coordinate of A})} \\
 &= \frac{y_2 - y_1}{x_2 - x_1}
 \end{aligned}$$



Worked Example 1

Find the gradient of the line shown in the diagram.

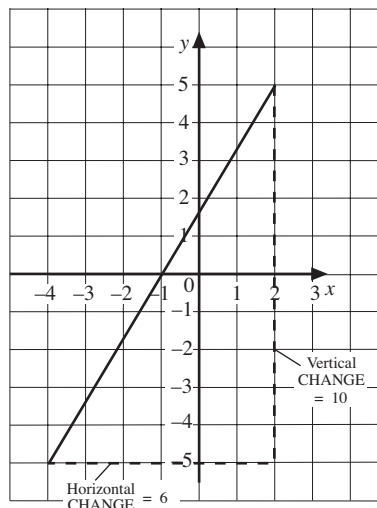


Solution

Draw a triangle under the line below to show the horizontal and vertical distances.

Here the vertical distance is 10 and the horizontal distance is 6.

$$\begin{aligned}\text{Gradient} &= \frac{\text{vertical change}}{\text{horizontal change}} \\ &= \frac{10}{6} \\ &= \frac{5}{3} \text{ or } 1\frac{2}{3}\end{aligned}$$



Worked Example 2

Find the gradient of the line joining the point A with coordinates (2, 4) and the point B with coordinates (4, 10).



Solution

$$\begin{aligned}\text{Gradient} &= \frac{(y\text{-coordinate of B}) - (y\text{-coordinate of A})}{(x\text{-coordinate of B}) - (x\text{-coordinate of A})} \\ &= \frac{10 - 4}{4 - 2} \\ &= \frac{6}{2} \\ &= 3\end{aligned}$$



Worked Example 3

Find the gradient of the line that joins the points with coordinates $(-2, 4)$ and $(4, 1)$.

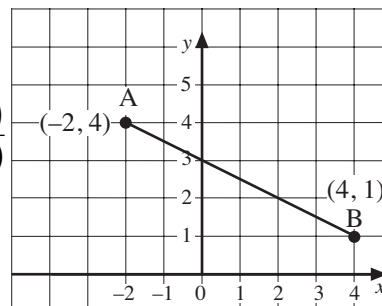


Solution

The diagram shows the line. It will have a negative gradient because of the way it slopes.

$$\begin{aligned}
 \text{Gradient} &= \frac{(y\text{-coordinate of B}) - (y\text{-coordinate of A})}{(x\text{-coordinate of B}) - (x\text{-coordinate of A})} \\
 &= \frac{1 - 4}{4 - (-2)} \\
 &= \frac{-3}{6} \\
 &= -\frac{1}{2}
 \end{aligned}$$

So the gradient is $-\frac{1}{2}$.



Parallel lines have the same gradient.



Worked Example 4

The coordinates of the points A, B, C and D are listed below.

A $(2, 4)$

B $(8, 7)$

C $(-1, -5)$

D $(5, -2)$

(a) Show that the line segments AB and CD are parallel.

(b) Are the line segments AC and BD parallel?



Solution

$$(a) \quad \text{Gradient of AB} = \frac{7 - 4}{8 - 2} = \frac{3}{6} = \frac{1}{2}$$

$$\text{Gradient of CD} = \frac{(-2) - (-5)}{5 - (-1)} = \frac{3}{6} = \frac{1}{2}$$

The line segments have the same gradients and so must be parallel.

$$(b) \quad \text{Gradient of AC} = \frac{(-5) - 4}{(-1) - 2} = \frac{-9}{-3} = 3$$

$$\text{Gradient of BD} = \frac{7 - (-2)}{8 - 5} = \frac{9}{3} = 3$$

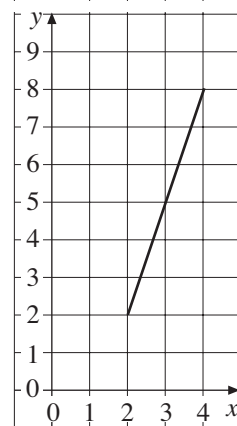
The line segments have the same gradient and so are parallel.

The results in Worked Example 4 mean that the quadrilateral ABDC is, in fact, a parallelogram.

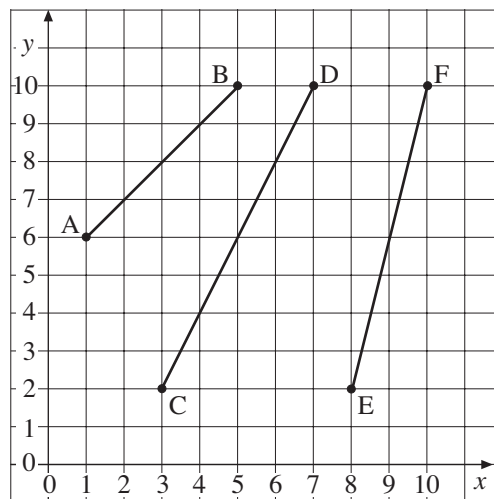


Exercises

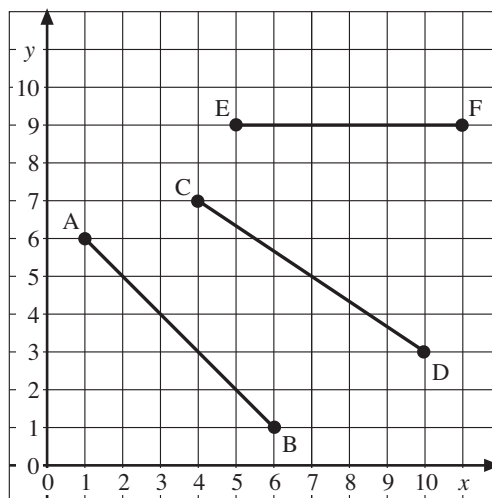
1. Find the gradient of the line shown on the graph opposite.



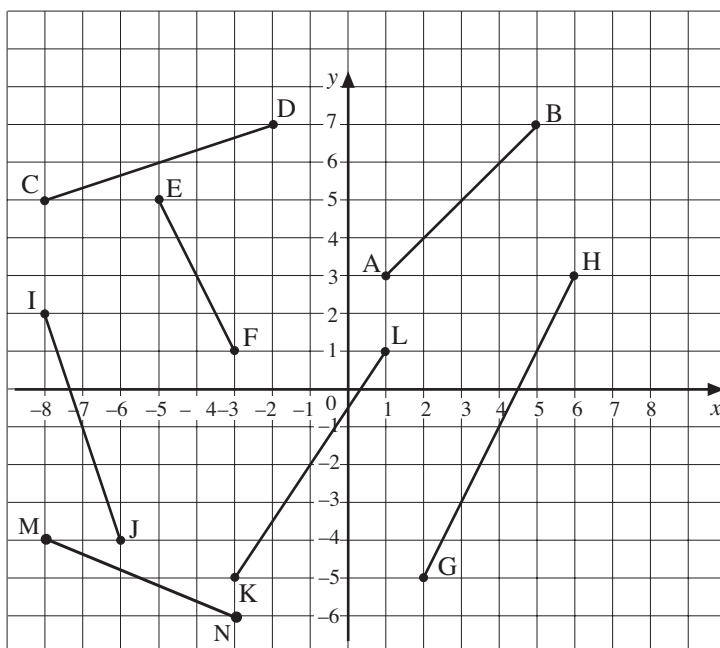
2. Find the gradient of each line in the diagram below.



3. Find the gradient of each line in the diagram below.

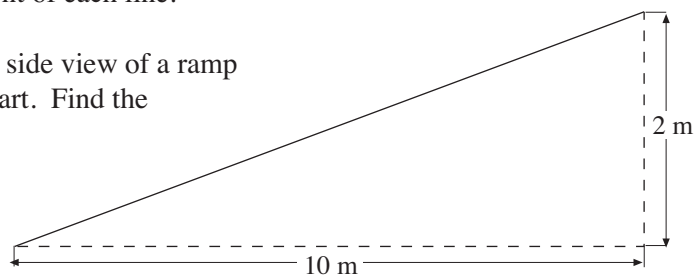


4. (a) Which of the lines in the diagram below have a positive gradient?

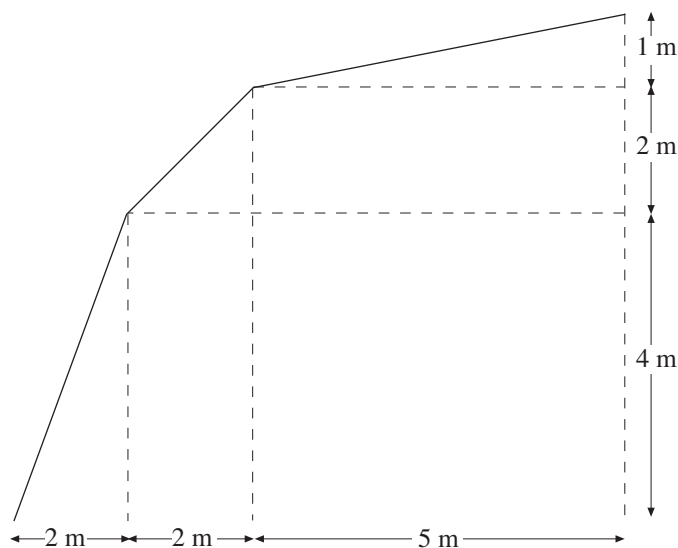


- (b) Which lines have a negative gradient?
 (c) Find the gradient of each line.

5. The diagram shows a side view of a ramp in a multistorey car part. Find the gradient of the ramp.



6. The diagram shows the cross-section of a roof. Find the gradient of each part of the roof.



7. Find the gradient of the line that joins the points with the coordinates:
- (a) (1, 1) and (9, 5),
 - (b) (2, 1) and (3, 6),
 - (c) (2, 2) and (3, 6),
 - (d) (0, 4) and (10, 12),
 - (e) (-1, 2) and (5, 8),
 - (f) (-2, -2) and (0, 12).
8. A quadrilateral is formed by joining the points A, B, C and D. The coordinates of each point are:
- A (2, 4) B (-1, 5) C (-4, 5) D (1, -1)
- Find the gradient of each side of the quadrilateral.
9. (a) Draw the line segment joining the points (4, 7) and (5, -1).
(b) Draw the line segment joining the points (-1, 3) and (0, -5).
(c) Are these lines parallel?
(d) Calculate the gradient of each line.
10. (a) Calculate the gradient of the line segment joining the points (1, 4) and (6, 3).
(b) Calculate the gradient of the line segment joining the points (3, 6) and (8, 5).
(c) Are the two line segments parallel?
11. In each case below the coordinates of the points A, B, C and D are listed. Determine whether or not AB is parallel to CD.
- | | | | | |
|-----|-----------|-----------|-----------|-----------|
| (a) | A (1, 2) | B (6, 7) | C (3, 6) | D (8, 11) |
| (b) | A (2, 4) | B (3, 1) | C (8, 2) | D (6, 3) |
| (c) | A (7, 2) | B (8, 10) | C (9, 1) | D (11, 3) |
| (d) | A (-1, 8) | B (3, -2) | C (7, 11) | D (11, 1) |
12. The vertices A, B, C and D of a quadrilateral have coordinates
- A (4, 7) B (8, 9) C (10, 1) D (6, -1)
- (a) Calculate the gradient of each side of the quadrilateral.
 - (b) Calculate the length of each side of the quadrilateral.
 - (c) What is the name of this type of quadrilateral?
13. A line segment joins the points A (2, 3) and B (5, -3).
- (a) Calculate the gradient of AB.
 - (b) The line segment CD is twice as long as AB and parallel to it.
If point C has coordinates (-2, 1), determine the coordinates of point D.

14. (a) Calculate the coordinates for three points on the line $y = 3x + 2$.
 (b) Plot these points and draw a straight line through them.
 (c) Find the gradient of the line that you have drawn.
 (d) Repeat (a) to (c) for the lines $y = 4x - 1$ and $y = 5x + 1$.
 (e) What is the connection between the equation of a line and its gradient?
 (f) What do you think the gradient of the line $y = 7x + 5$ will be?

28.2 Gradients of Perpendicular Lines

In this section we explore the relationship between the gradients of perpendicular lines and line segments.



Worked Example 1

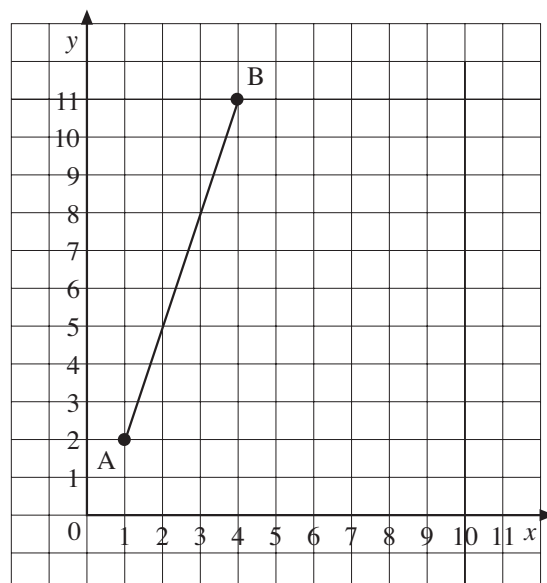
- (a) Plot the points A (1, 2) and B (4, 11), join them to form the line AB and then calculate the gradient of AB.
 (b) On the same set of axes, plot the points P (5, 4) and Q (8, 3), join them to form the line PQ and then calculate the gradient of PQ.
 (c) Measure the angle between the lines AB and PQ. What do you notice about the two gradients?



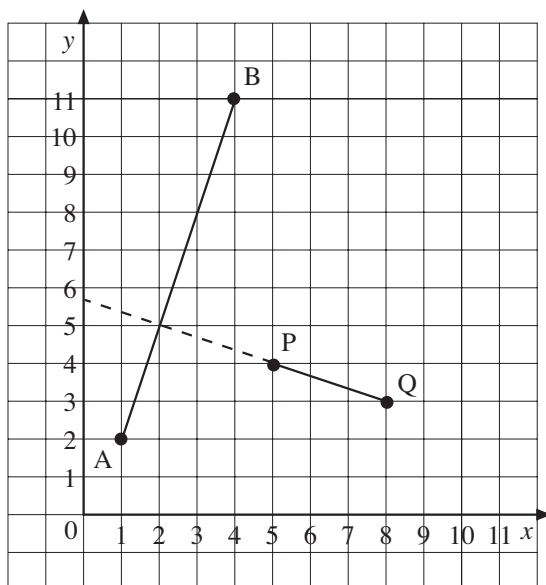
Solution

- (a) The points are shown in the diagram.

$$\text{Gradient of AB} = \frac{11 - 2}{4 - 1} = \frac{9}{3} = 3$$



- (b) The points P and Q can now be added to the diagram as shown below.



$$\text{Gradient of PQ} = \frac{3 - 4}{8 - 5} = -\frac{1}{3}$$

- (c) The line PQ has been extended on the diagram, so that the angle between the two lines can be measured.

The angle is 90° , a right angle.

In this case,

$$\text{the gradient of AB} = 3; \text{ the gradient of PQ} = -\frac{1}{3}$$

$$\text{and the gradients multiply to give } 3 \times \left(-\frac{1}{3}\right) = -1.$$



Note

The product of the gradients of two perpendicular lines will always be -1 , unless one of the lines is horizontal and the other is vertical.

In the example above,

$$\text{gradient of AB} = 3$$

$$\text{gradient of PQ} = -\frac{1}{3}$$

$$3 \times \left(-\frac{1}{3}\right) = -1$$



Note

$$\text{Gradient of PQ} = \frac{-1}{\text{gradient AB}}.$$

If the gradient of a line is m , and $m \neq 0$, then the gradient of a perpendicular line will be $-\frac{1}{m}$.



Worked Example 2

Show that the line segment joining the points A (3, 2) and B (5, 7) is perpendicular to the line segment joining the points P (2, 5) and Q (7, 3).



Solution

$$\text{Gradient of AB} = \frac{7 - 2}{5 - 3} = \frac{5}{2}$$

$$\text{Gradient of PQ} = \frac{3 - 5}{7 - 2} = \frac{-2}{5}$$

$$\text{Gradient of AB} \times \text{Gradient of PQ} = \frac{5}{2} \times \frac{-2}{5} = \frac{-10}{10} = -1.$$

So the line segments AB and PQ are perpendicular.



Exercises

- On a set of axes, draw the lines AB and PQ where the coordinates of these points are

A (1, 2)	B (10, 6)
P (1, 9)	Q (5, 0)
 - Are the lines perpendicular?
 - Calculate the gradient of AB.
 - Calculate the gradient of PQ.
 - Check that the product of these gradients is -1 .
- In each case, decide whether the lines AB and PQ are parallel, perpendicular or neither.

(a) A (4, 3)	B (8, 4)	P (7, 1)	Q (6, 5)
(b) A (-2, 0)	B (1, 9)	P (2, 5)	Q (6, 17)
(c) A (8, -5)	B (11, -3)	P (1, 1)	Q (-3, 7)
(d) A (3, 1)	B (7, 3)	P (-3, 2)	Q (1, 0)
- The points P (-3, 1), Q (1, 2), R (0, -1) and S (-4, -2) are the vertices of a quadrilateral.

 - Calculate the gradient of each side of the quadrilateral.
 - Is the quadrilateral a parallelogram?
 - Is the quadrilateral a rectangle?
- A triangle has vertices A (3, 1), B (7, 5) and C (1, 3). Show that the triangle is a right-angled triangle.

5. Show that the triangle with vertices A (4, 7), B (8, 2) and C (7, 3) is *not* a right-angled triangle.

6. The coordinates of the point A, B, C and D are listed below.

$$A (3, 0) \qquad B (0, 1)$$

$$C (1, 4) \qquad D (4, 3)$$

Show that ABCD is a square.

7. The points A (3, 2), B (6, 0), C (5, 4) and D (2, 6) are the vertices of a quadrilateral.

(a) Show that this is *not* a rectangle. (b) Show that this is a parallelogram.

8. The lines AB and PQ are perpendicular. The coordinates of the points are

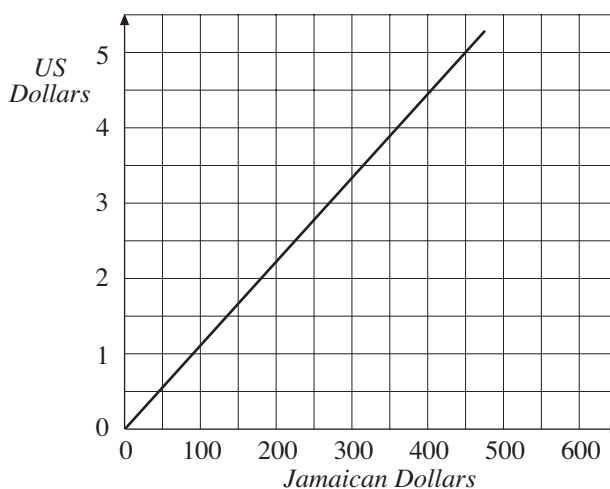
$$A (3, 2) \quad B (7, 4) \quad P (3, 7) \quad \text{and} \quad Q (6, q)$$

Determine the value of q .

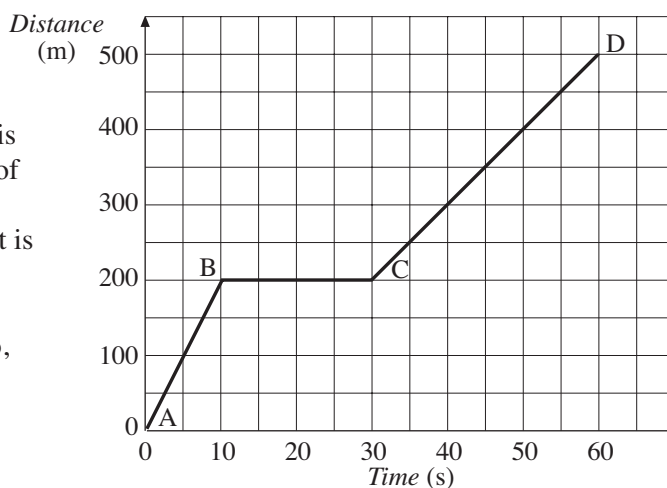
28.3 Applications of Graphs

In this section some applications of graphs are considered, particularly *conversion graphs* and graphs to describe *motion*.

The graph on the right can be used for converting US Dollars into and from Jamaican Dollars. (As currency exchange rates are continually changing, these rates might not be correct now.)

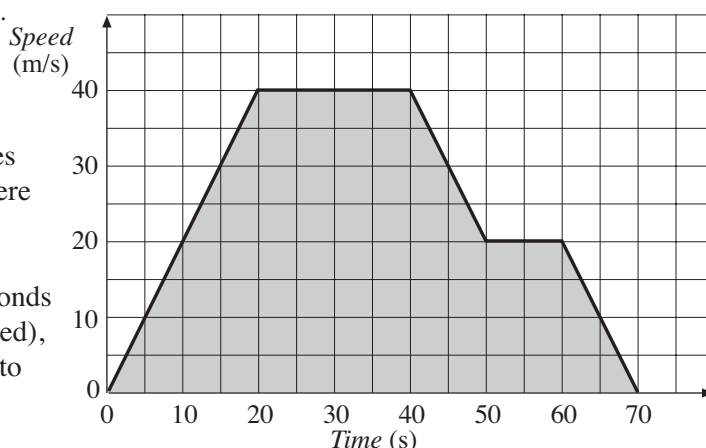


A distance-time graph of a car is shown opposite. The gradient of this graph gives the velocity (speed) of the car. The gradient is steepest from A to B, so this is when the car has the greatest speed. The gradient BC is zero, so the car is not moving.



The area under a velocity-time graph gives the distance travelled. Finding the shaded area on the graph shown opposite would give the distance travelled.

The gradient of this graph gives the *acceleration* of the car. There is constant acceleration from 0 to 20 seconds, then zero acceleration from 20 to 40 seconds (when the car has constant speed), constant deceleration from 40 to 50 seconds, etc.



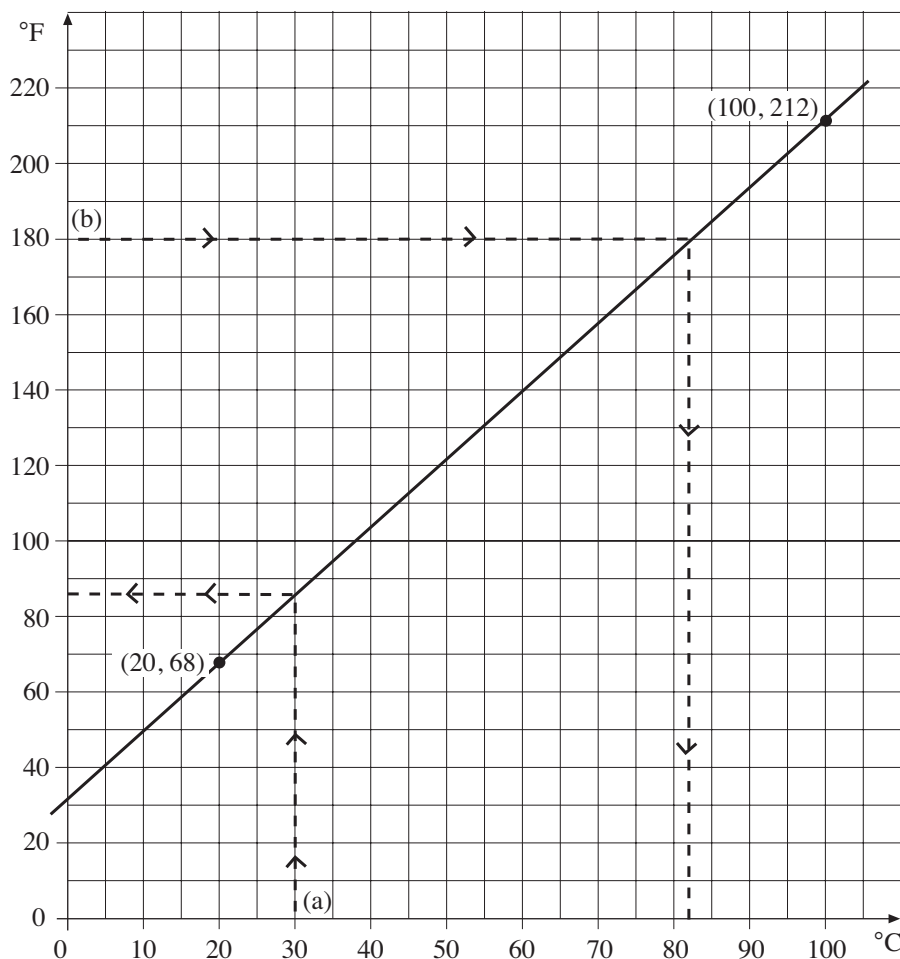
Worked Example 1

A temperature of 20°C is equivalent to 68°F and a temperature of 100°C is equivalent to a temperature of 212°F . Use this information to draw a conversion graph. Use the graph to convert:

- (a) 30°C to $^{\circ}\text{Fahrenheit}$, (b) 180°F to $^{\circ}\text{Celsius}$.

Solution

Taking the horizontal axis as temperature in $^{\circ}\text{C}$ and the vertical axis as temperature in $^{\circ}\text{F}$ gives two pairs of coordinates, $(20, 68)$ and $(100, 212)$. These are plotted on a graph and a straight line drawn through the points.

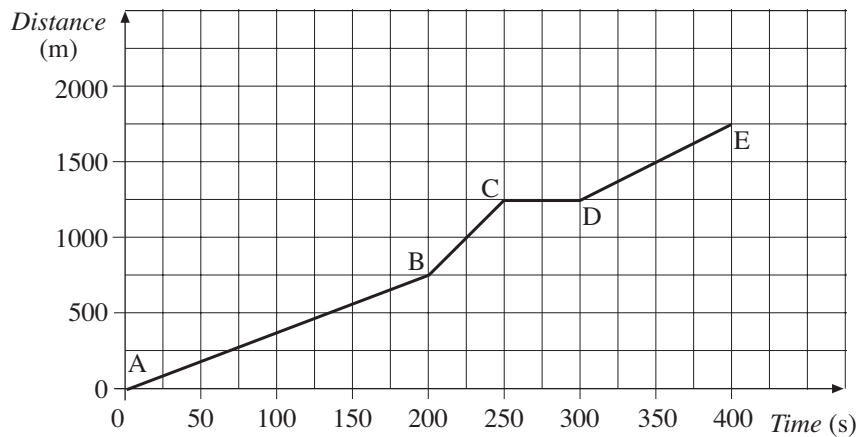


- (a) Start at 30°C , then move up to the line and across to the vertical axis, to give a temperature of about 86°F .
- (b) Start at 180°F , then move across to the line and down to the horizontal axis, to give a temperature of about 82°C .



Worked Example 2

The graph shows the distance travelled by a girl on a bike.



Find the speed she is travelling on each stage of the journey.



Solution

$$\begin{aligned}\text{For AB the gradient} &= \frac{750}{200} \\ &= 3.75\end{aligned}$$

So the speed is 3.75 m/s.



Note

The units are m/s (metres per second), as m are the units for distance and s the units for time.

$$\begin{aligned}\text{For BC the gradient} &= \frac{500}{50} \\ &= 10\end{aligned}$$

So the speed is 10 m/s.

For CD the gradient is zero and so the speed is zero

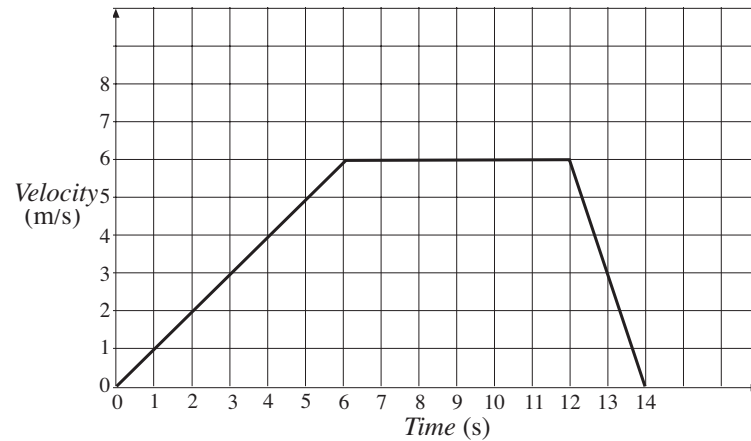
$$\begin{aligned}\text{For DE the gradient is} &= \frac{500}{100} \\ &= 5\end{aligned}$$

So the speed is 5 m/s.



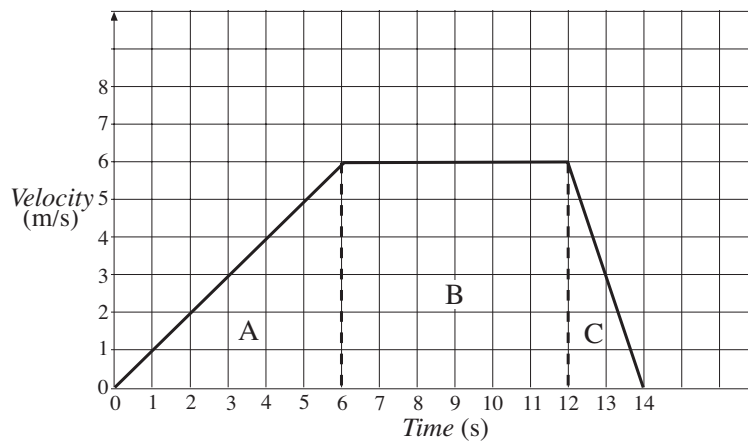
Worked Example 3

The graph shows how the velocity of a bird varies as it flies between two trees. How far apart are the two trees?



Solution

The distance is given by the area under the graph. In order to find this area it has been split into three sections, A, B and C.



$$\begin{aligned}\text{Area of A} &= \frac{1}{2} \times 6 \times 6 \\ &= 18\end{aligned}$$

$$\begin{aligned}\text{Area of B} &= 6 \times 6 \\ &= 36\end{aligned}$$

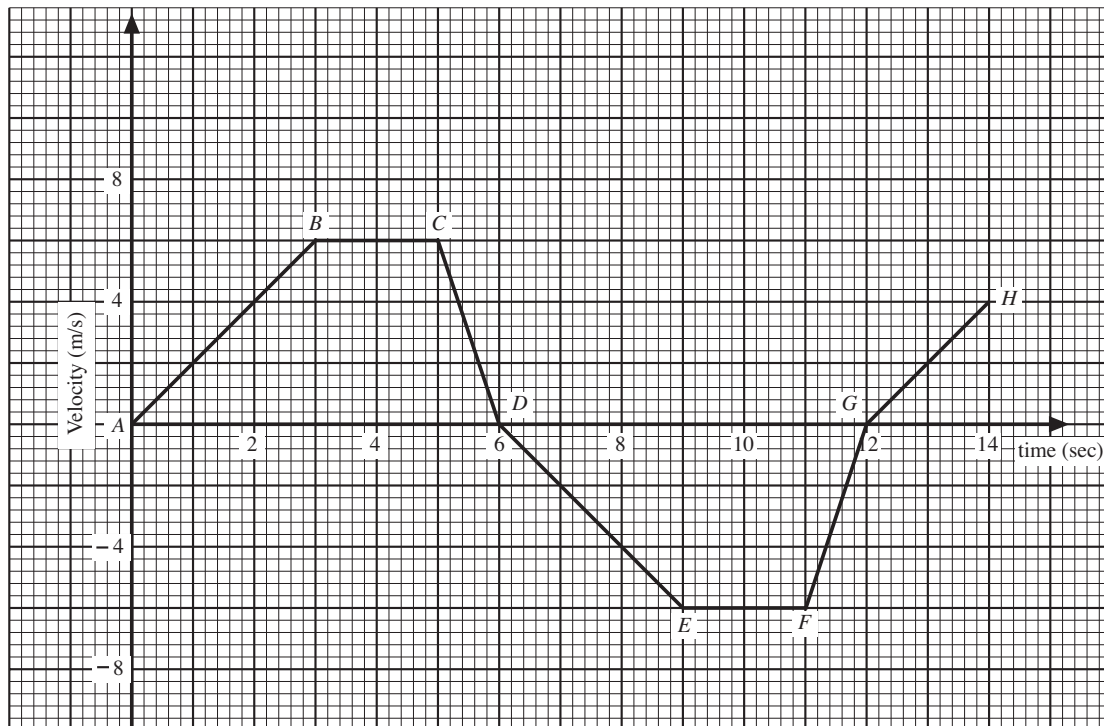
$$\begin{aligned}\text{Area of C} &= \frac{1}{2} \times 2 \times 6 \\ &= 6\end{aligned}$$

$$\begin{aligned}\text{Total Area} &= 18 + 36 + 6 \\ &= 60\end{aligned}$$

So the trees are 60 m apart. Note that the units are m (metres) because the units of velocity are m/s and the units of time are s (seconds).



Worked Example 4



The graph shows the velocity of a toy train for a time period of 14 seconds.

- (a) State
 - (i) the maximum velocity attained by the train
 - (ii) the total number of seconds, during which the train travelled at the maximum speed.
 - (iii) the time period during which the velocity is negative.
- (b) Calculate the acceleration of the train.
 - (i) during the first 3 seconds
 - (ii) for the time period of x seconds where $6 \leq x \leq 9$.
- (c) Calculate the distance travelled
 - (i) between 2 and 6 seconds
 - (ii) over the entire journey of 14 seconds.

(CXC)



Solution

- (a) (i) 6 m/s (from B to C)
 (ii) 2 seconds (as B is reached at 3 seconds and C at 5 seconds)
 (iii) Velocity is negative from C (time 5 seconds) to E (time 9 seconds)
 i.e. from $9 - 5 = 4$ seconds

- (b) (i) Acceleration is the gradient of $AB = \frac{6}{3} = 2 \text{ m/s}^2$
 (ii) For $6 \leq x \leq 9$, there is acceleration $= \frac{(-6)}{3} = -2 \text{ m/s}^2$
 (or deceleration of 2 m/s^2)

- (c) (i) Distance travelled from 2 to 6 seconds

$$= 1 \times \frac{(4 + 6)}{2} + 2 \times 6 + \frac{1}{2} \times 1 \times 6$$

$$= 5 + 12 + 3$$

$$= 20 \text{ m}$$

- (ii) Distance travelled from A to $D = \frac{1}{2} \times 2 \times 4 + 20$

$$= 24 \text{ m}$$

By symmetry, distance travelled from D to $G = 24 \text{ m}$

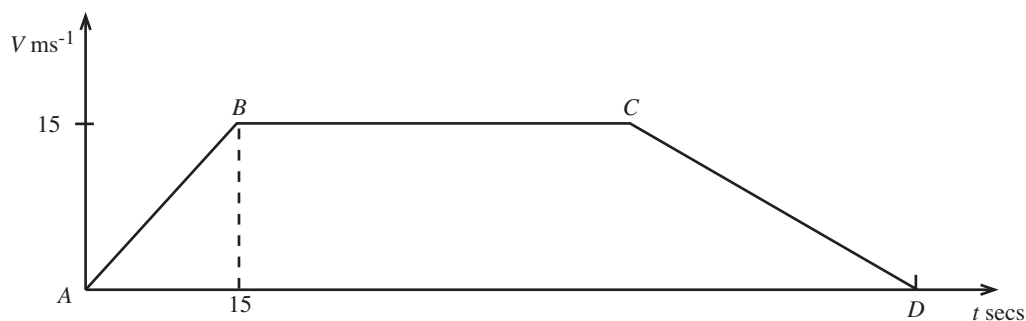
Distance travelled for G to $H = \frac{1}{2} \times 2 \times 4 = 4 \text{ m}$

So total distance travelled $= 24 + 24 + 4$
 $= 52 \text{ m}$



Worked Example 5

(a)



The velocity-time graph above, **not drawn to scale**, shows that a train stops at two stations, A and D . The train accelerates uniformly from A to B , maintains a constant speed from B to C and decelerates uniformly from C to D .

Using the information on the graph,

- (i) calculate, in ms^{-2} , the train's acceleration
 - (ii) show that the train took 30 secs from C to D if it decelerated at $\frac{1}{2}\text{ms}^{-2}$.
 - (iii) If the time taken from A to D is 156 seconds, calculate the distance in metres between the two stations.
- (b) An express train stops at station P and station Q . On leaving P , the train accelerates uniformly at the rate of 2ms^{-2} for 60 seconds. Then it decelerates uniformly for 90 seconds and comes to a stop at Q .
- (i) Using 1 unit to represent 20 units on BOTH axes, sketch the velocity-time graph to represent the information above.
 - (ii) Using your graph, determine
 - (a) the maximum velocity
 - (b) the rate at which the train decelerates.



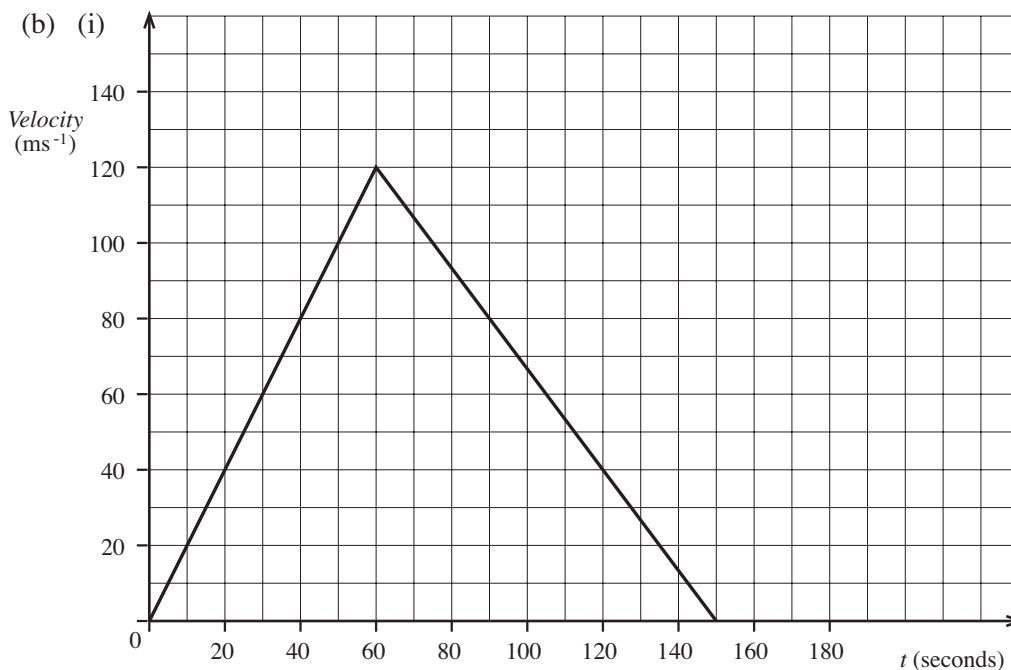
Solution

- (a) (i) Acceleration = $\frac{15}{15} = 1\text{ms}^{-2}$
- (ii) If it takes t seconds from C to D , then
- $$\frac{1}{2} = \frac{15}{t} \Rightarrow t = 30 \text{ seconds}$$
- (iii) Time from B to $C = 156 - (15 + 30) = 111$ seconds

Total distance = area under graph

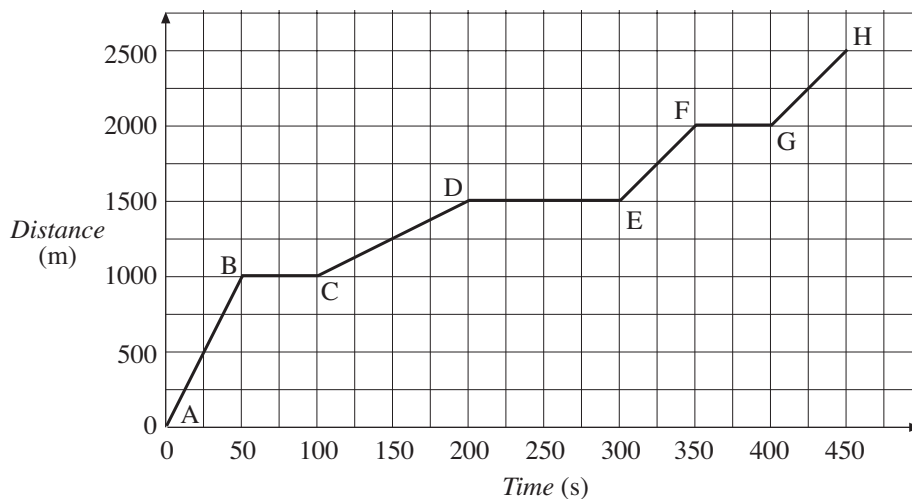
$$\begin{aligned}
 &= \left(\frac{1}{2} \times 15 \times 15 + 15 \times 111 + \frac{1}{2} \times 15 \times 30 \right) \text{ m} \\
 &= \left(\frac{225}{2} + 1665 + 225 \right) \text{ m} \\
 &= \frac{4005}{2} \text{ m} = 2002\frac{1}{2} \text{ m}
 \end{aligned}$$

- (b) (i) See diagram on following page.
- (ii) (a) Maximum velocity = 120ms^{-1}
- (b) Deceleration = $\frac{120}{90} = \frac{4}{3}\text{ms}^{-2}$

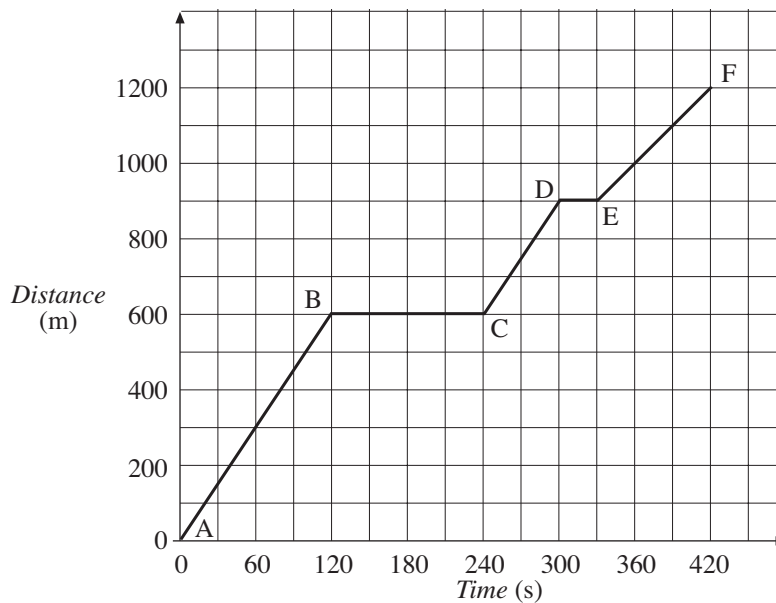


Exercises

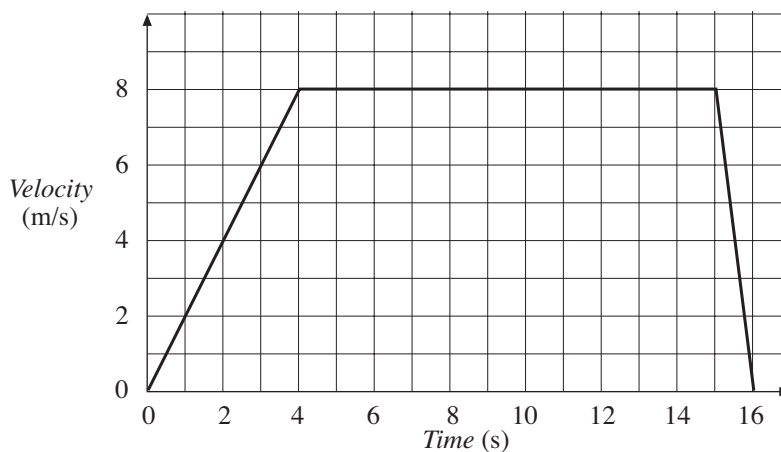
- Use the approximation that 10 kg is about the same as 22 lbs to draw a graph for converting between pounds and kilograms. Use the graph to convert the following:
 - 6 lbs to kilograms,
 - 8 lbs to kilograms,
 - 5 kg to pounds,
 - 3 kg to pounds.
- Use the approximation that 10 gallons is about the same as 45 litres to draw a conversion graph. Use the graph to convert:
 - 5 gallons to litres,
 - 30 litres to gallons.
- The graph shows how the distance travelled by a route taxi increased.



- (a) How many times did the taxi stop?
- (b) Find the velocity of the taxi on each section of the journey.
- (c) On which part of the journey did the taxi travel fastest?
4. The distance-time graph shows the distance travelled by a car on a journey to the shops.

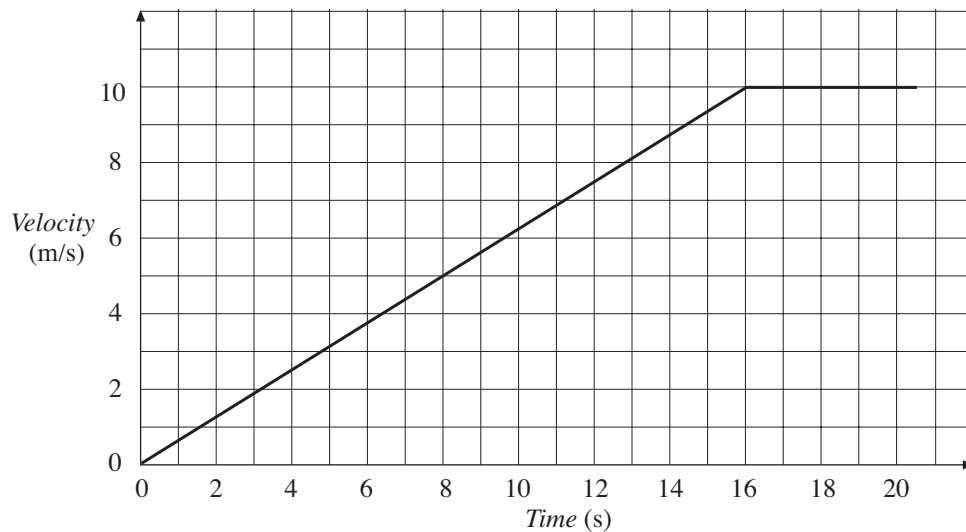


- (a) The car stopped at two sets of traffic lights. How long did the car spend waiting at the traffic lights?
- (b) On which part of the journey did the car travel fastest? Find its velocity on this part.
- (c) On which part of the journey did the car travel at its lowest velocity? What was this velocity?
5. The graph below shows how the speed of an athlete varies during a race.



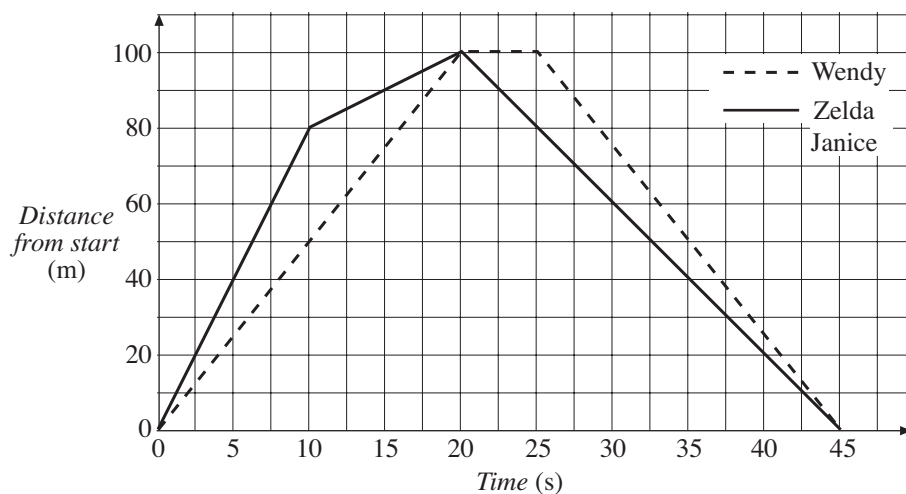
What was the distance of the race?

6. The graph below shows how the velocity of a truck varies as it sets off from a set of traffic lights.



Find the distance travelled by the truck after

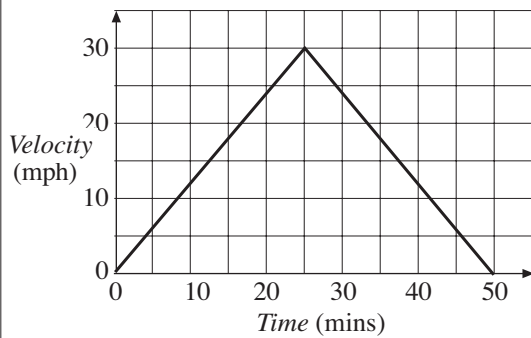
- (a) 8 seconds, (b) 16 seconds, (c) 20 seconds.
7. Khenan runs at a constant speed for 10 seconds. He has then travelled 70 m. He then walks at a constant speed for 8 seconds until he is 86 m from his starting point.
- (a) Find the speed at which he runs and the speed at which he walks.
- (b) If he had covered the complete distance in the same time, with a constant speed, what would that speed have been?
8. The graph shows how the distance travelled by Zelda and Janice changes during a race from one end of the school field to the other end, and back.



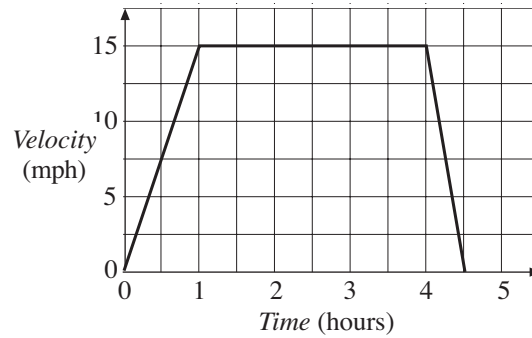
Describe what happens during the race.

9. Find the area under each graph below and state the distance that it represents.

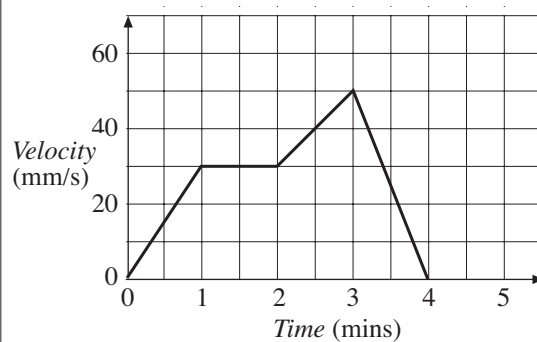
(a)



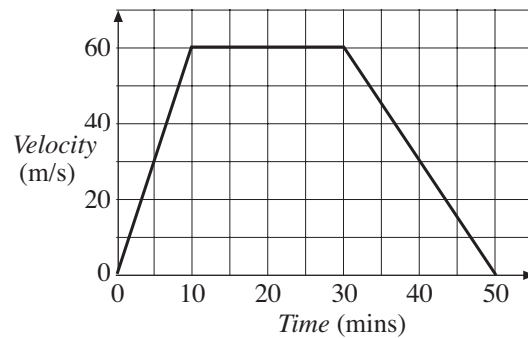
(b)



(c)

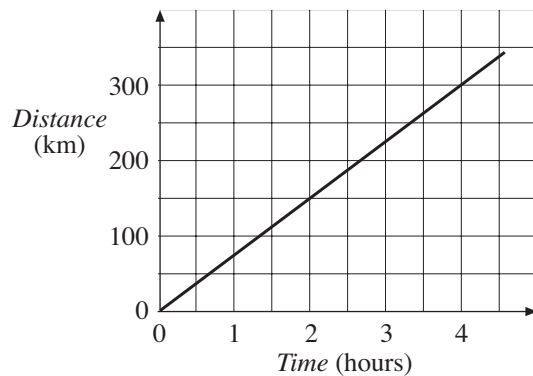


(d)

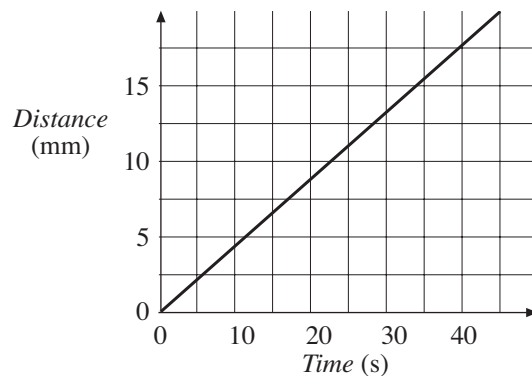


10. For each distance-time graph, find the velocity in the units used on the graph and in m/s.

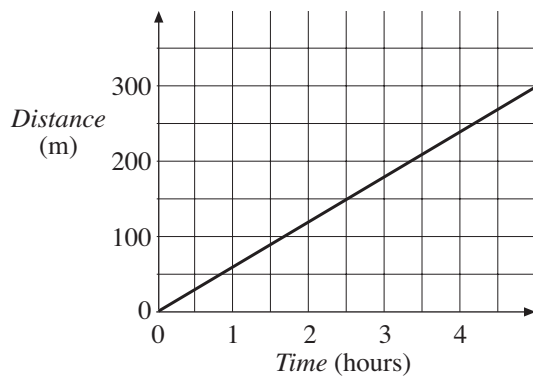
(a)



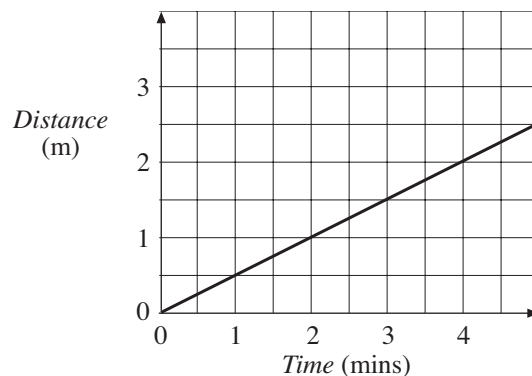
(b)



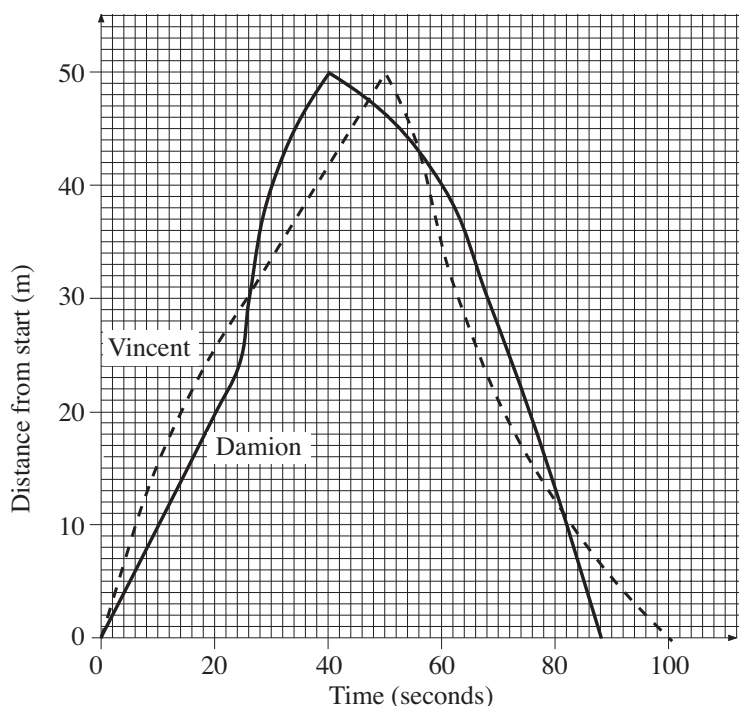
(c)



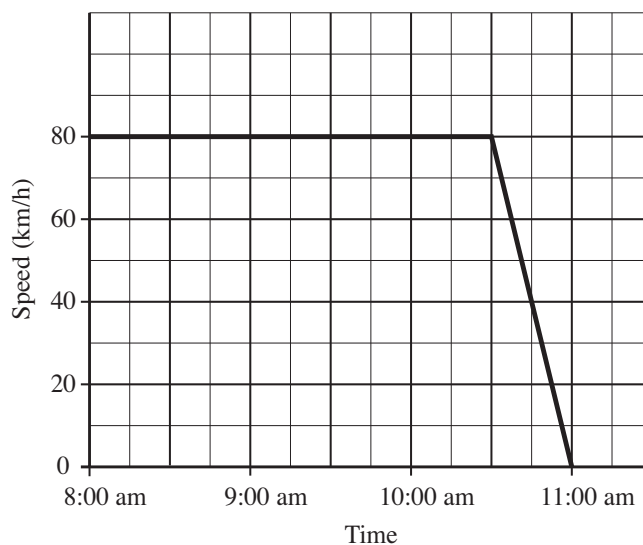
(d)



11. The graph represents a swimming race between Vincent and Damion.



- (a) At what time did Damion overtake Vincent for the second time?
- (b) What was the **maximum** distance between the swimmers during the race?
- (c) Who was swimming faster at 56 seconds? How can you tell?
12. The speed-time graph below shows the journey of a car from 8:00 am to 11:00 am.

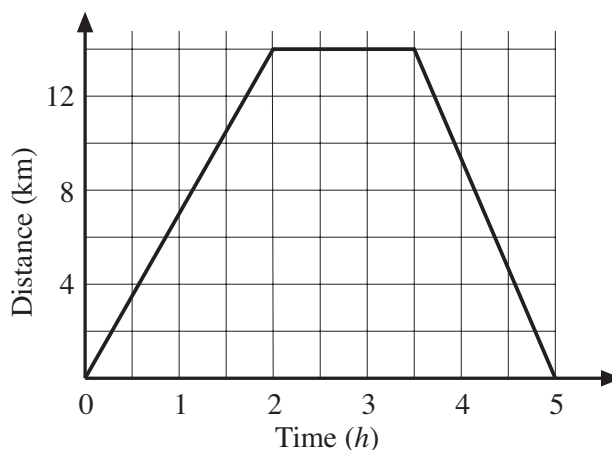


Using the graph, determine

- (a) the time at which the speed of the car was 40 km/h
- (b) the distance the car travelled for the entire journey
- (c) the average speed of the car for the entire journey.

(CXC)

13. The graph below represents the 5-hour journey of an athlete.



- What was the average speed during the first 2 hours?
- What did the athlete do between 2 and 3 hours after the start of the journey?
- What was the average speed on the return journey?

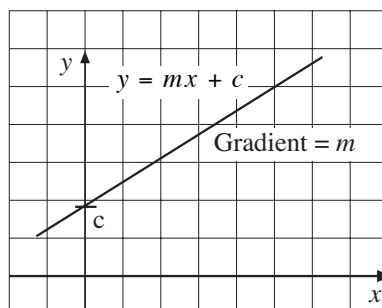
(CXC)

28.4 The Equation of a Straight Line

The equation of a straight line is usually written in the form

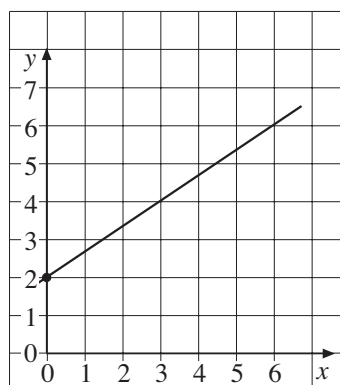
$$y = mx + c$$

where m is the gradient and c is the y intercept.



Worked Example 1

Find the equation of the line shown in the diagram.



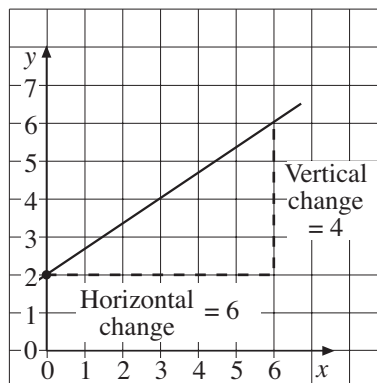


Solution

The first step is to find the gradient of the line.

Drawing the triangle shown under the line gives

$$\begin{aligned}\text{gradient} &= \frac{\text{vertical change}}{\text{horizontal change}} \\ &= \frac{4}{6} \\ &= \frac{2}{3}\end{aligned}$$



So the value of m is $\frac{2}{3}$. The line intersects the y-axis

at 2, so the value of c is 2. The equation of a straight line is

$$y = mx + c$$

In this case

$$y = \frac{2}{3}x + 2$$



Worked Example 2

Find the equation of the line that passes through the points (1, 5) and (3, 1).

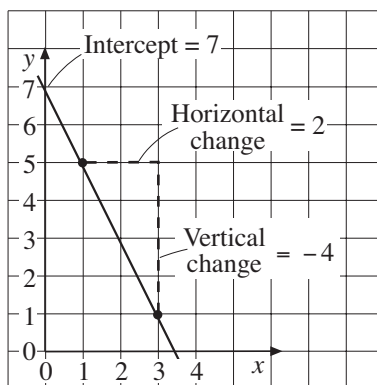


Solution

Plotting these points gives the straight line shown.

Using the triangle drawn underneath the line allows the gradient to be found.

$$\begin{aligned}\text{gradient} &= \frac{\text{vertical change}}{\text{horizontal change}} \\ &= \frac{-4}{2} \\ &= -2\end{aligned}$$



The y intercept is 7.

So $m = -2$ and $c = 7$ and the equation of the line is

$$y = -2x + 7 \text{ or } y = 7 - 2x$$



Worked Example 3

- Draw the line $x = 4$.
- Draw the line $y = 2$.
- Write down the coordinates of the point of intersection of these lines.



Solution

- (a) For the line $x = 4$ the x -coordinate of every point will always be 4. So the points

$$(4, 0) \quad (4, 3) \quad (4, 5)$$

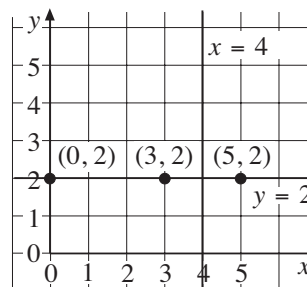
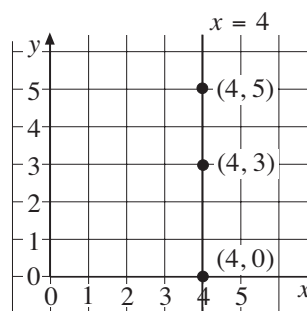
all lie on the line $x = 4$.

- (b) For the line $y = 2$ the y -coordinate of every point will always be 2. So the points

$$(0, 2) \quad (3, 2) \quad \text{and} \quad (5, 2)$$

all lie on the line $y = 2$.

- (c) The graph in (b) shows that the lines intersect at the point with coordinates $(4, 2)$.



Worked Example 4

The point with coordinates $(4, 9)$ lies on the line with equation $y = 2x + 1$. Determine the equation of the perpendicular line that also passes through this point.



Solution

The line $y = 2x + 1$ has gradient 2.

The perpendicular line will have gradient $-\frac{1}{2}$ and so its equation will be of the form

$$y = -\frac{1}{2}x + c$$

As the line passes through $(4, 9)$, we can use $x = 4$ and $y = 9$ to determine the value of c .

$$9 = -\frac{1}{2} \times 4 + c$$

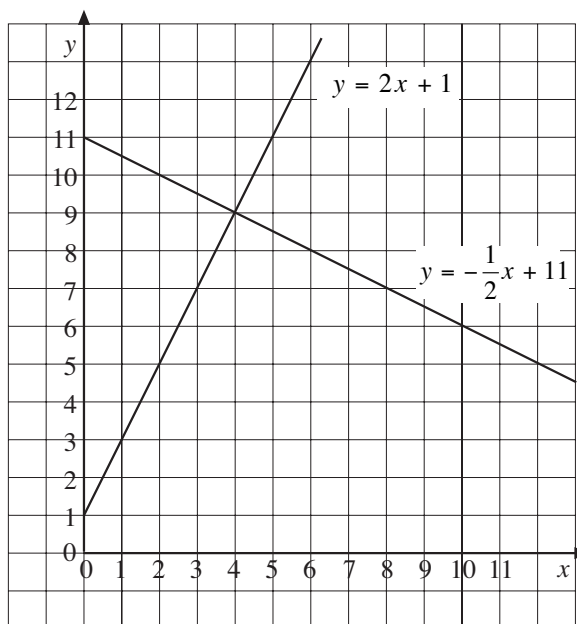
$$9 = -2 + c$$

$$c = 11$$

The equation of the perpendicular line is therefore

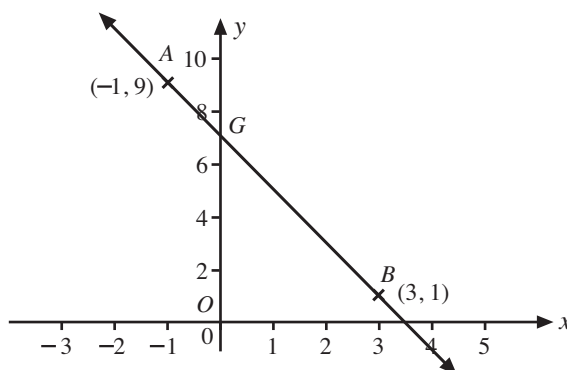
$$y = -\frac{1}{2}x + 11$$

The graph shows both lines.





Worked Example 5



In the diagram above, **not drawn to scale**, AB is the straight line joining $A (-1, 9)$ and $B (3, 1)$.

- Calculate the gradient of the line, AB .
- Determine the equation of the line, AB .
- Write the coordinates of G , the point of intersection of AB and the y -axis.
- Write the equation of the line through O , the origin, that is perpendicular to AB .
- Write the equation of the line through O that is parallel to AB .

(CXC)



Solution

$$(a) \quad \text{Gradient of } AB = \frac{9 - 1}{-1 - 3} = \frac{8}{(-4)} = -2$$

$$(b) \quad \text{Equation of line } AB \quad y = mx + c \Rightarrow y = -2x + c$$

As it passes through $(3, 1)$,

$$1 = -2 \times 3 + c \Rightarrow c = 7$$

$$\text{i.e. } y = -2x + 7$$

$$(c) \quad x = 0, \text{ so } y = -2 \times 0 + 7 = 7. \quad G \text{ is point } (0, 7).$$

$$(d) \quad \text{Gradient} = \frac{-1}{m} = \frac{-1}{-2} = \frac{1}{2}$$

$$\text{So } y = \frac{1}{2}x \quad (\text{as it passes through the origin})$$

$$(e) \quad \text{The line parallel to } AB \text{ has gradient } -2, \text{ so the line through } O \text{ has equation}$$

$$y = -2x$$



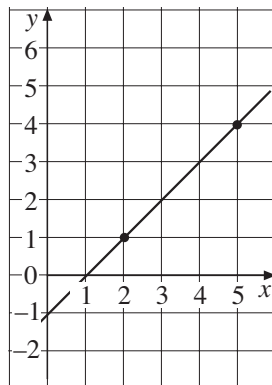
Exercises

1. Find the equation of the straight line with:
 - (a) gradient = 2 and y-intercept = 4,
 - (b) gradient = 3 and y-intercept = -5,
 - (c) gradient = $\frac{1}{2}$ and y-intercept = 2,
 - (d) gradient = -2 and y-intercept = 1,
 - (e) gradient = $\frac{3}{4}$ and y-intercept = -3.

2. Write down the gradient and y-intercept of each line.

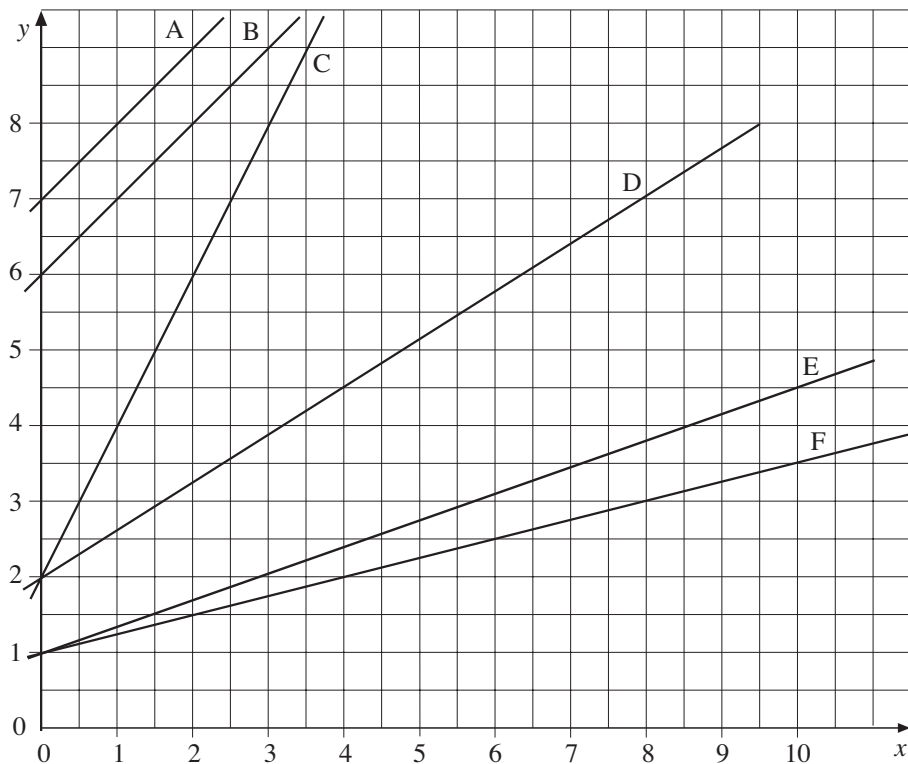
(a) $y = 2x + 3$	(b) $y = 4x - 2$
(c) $y = \frac{1}{2}x + 1$	(d) $y = \frac{2}{3}x - 4$
(e) $y = 4(x + 2)$	(f) $y = 3(x - 7)$
(g) $y = \frac{x + 5}{2}$	(h) $y = \frac{x - 10}{4}$

3. The diagram shows the straight line that passes through the points (2, 1) and (5, 4).



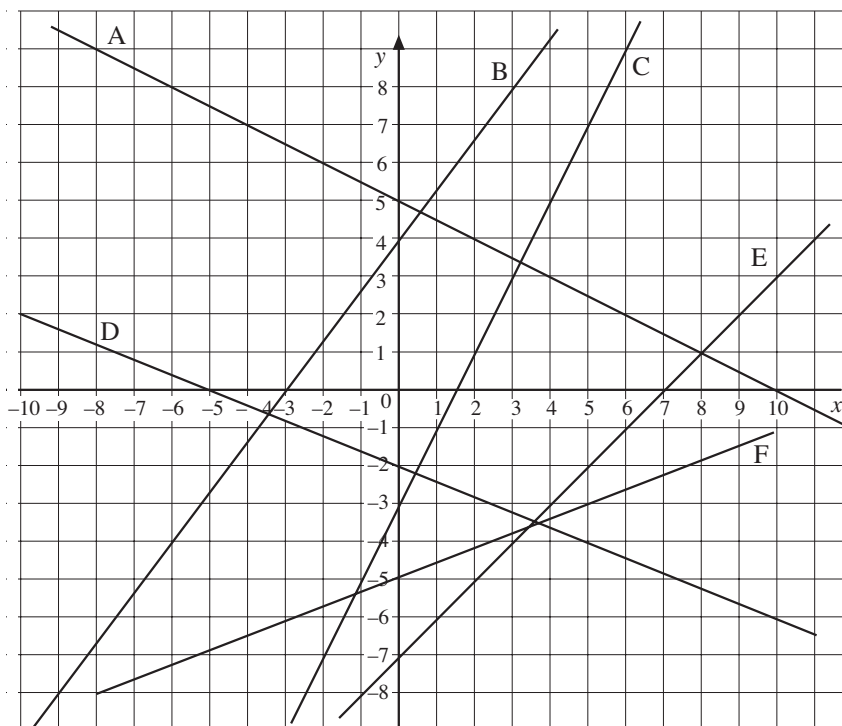
- (a) Find the gradient of the line.
- (b) Write down the y-intercept.
- (c) Write down the equation of the straight line.

4. Find the equation of each line shown in the diagram below.



5. Write down the gradient of each line and the coordinates of the y-intercept.
- | | |
|------------------|----------------------------|
| (a) $y = 2x - 8$ | (b) $y = -3x + 2$ |
| (c) $y = 4x - 3$ | (d) $y = \frac{1}{2}x + 2$ |
| (e) $y = 8 - 2x$ | (f) $y = 4 - 3x$ |
| (g) $x + y = 8$ | (h) $y = 3(5 - x)$ |
6. Find the equation of the line that passes through the points with the coordinates below.
- | | |
|------------------------|-----------------------|
| (a) (0, 2) and (4, 10) | (b) (4, 2) and (8, 4) |
| (c) (0, 6) and (6, 4) | (d) (1, 4) and (3, 0) |
7. The graph opposite can be used for converting gallons to litres.
- | | |
|---|--|
| (a) Find the equation of the line. | |
| (b) Draw a similar graph for converting litres to pints, given that 11 litres is approximately 20 pints. Use the horizontal axis for pints. | |
| (c) Find the equation of the line drawn in (b). | |

8. Find the equation of each line in the diagram below.

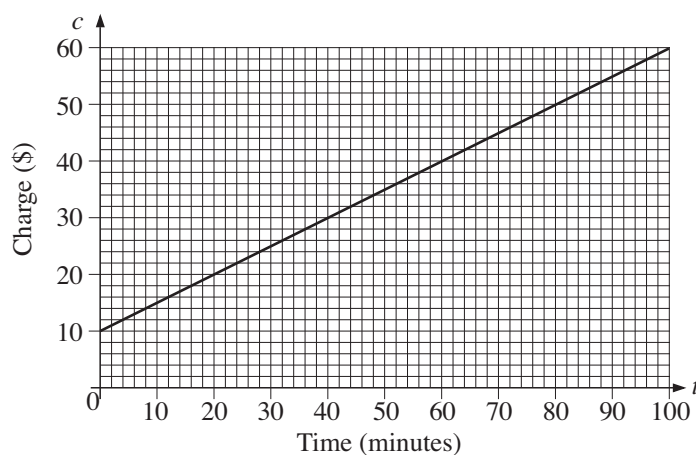


9. The velocity of a ball thrown straight up into the air was recorded at half second intervals.

Time (s)	0.5	1.0	1.5	2.0	2.5	3.0
Velocity (ms^{-1})	15	10	5	0	-5	-10

- (a) Plot a graph with time on the horizontal axis.
 - (b) Draw a line through the points and find its equation.
 - (c) What was the velocity of the ball when it was thrown upwards?
10. (a) The line $y = x + c$ passes through the point (4, 7). Find the value of c .
- (b) The point (5, -2) lies on the line $y = 2x + c$. Find the value of c .
- (c) The line $y = mx + 2$ passes through the point (3, 17). Find the value of m .

11. Television repair charges depend on the length of time taken for the repair, as shown on the graph.

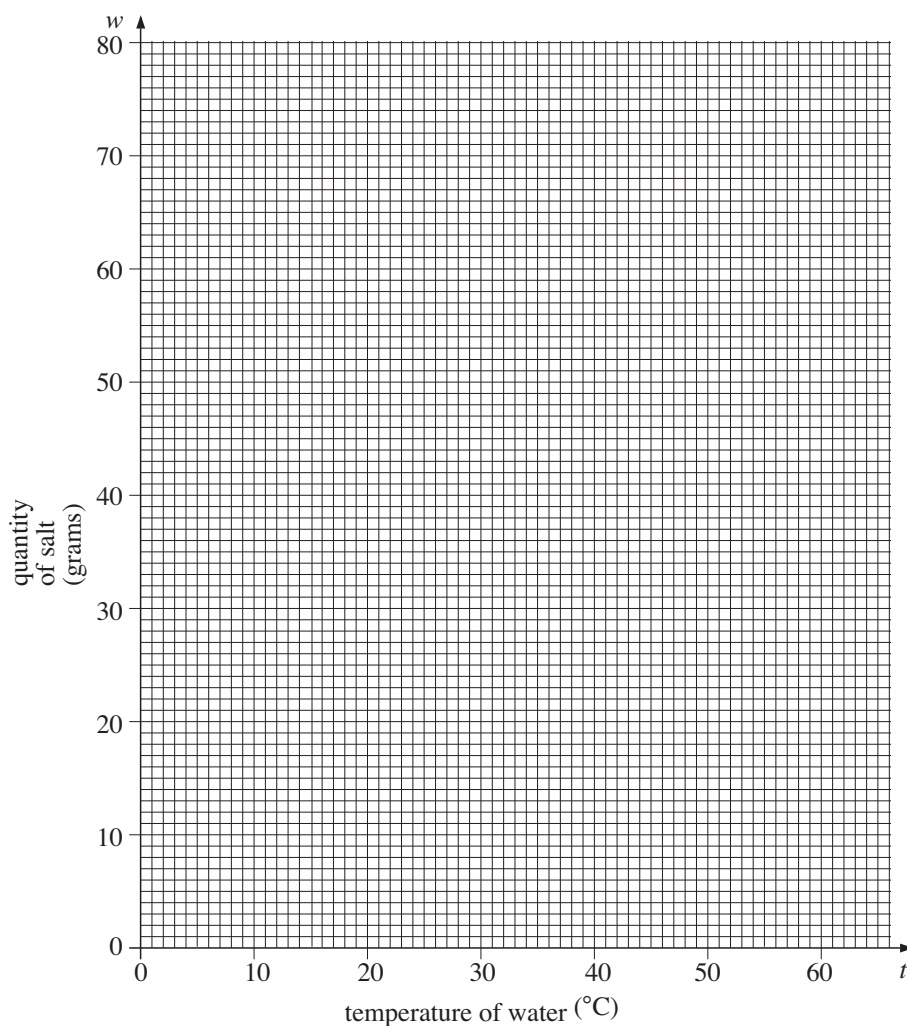


The charge is made up of a fixed amount plus an extra amount which depends on time.

- (a) What is the charge for a repair which takes 45 minutes?
 - (b)
 - (i) Calculate the gradient of the line.
 - (ii) What does the gradient represent?
 - (c) Write down the equation of the line.
 - (d) Mr Banks' repair will cost \$84 or less. Calculate the maximum amount of time which can be spent on the repair.
12. The table shows the largest quantity of salt, w grams, which can be dissolved in a beaker of water at temperature t °C.

t °C	10	20	25	30	40	50	60
w grams	54	58	60	62	66	70	74

- (a) On a copy of the following grid, plot the points and draw a graph to illustrate this information.



- (b) Use your graph to find
- the lowest temperature at which 63 g of salt will dissolve in the water.
 - the largest amount of salt that will dissolve in the water at 44 °C.
- (c) (i) The equation of the graph is of the form

$$w = at + b.$$

Use your graph to estimate the values of the constants a and b .

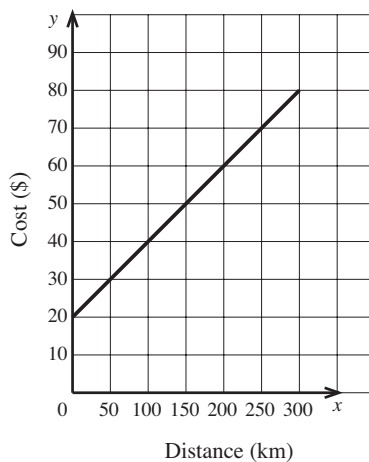
- (ii) Use the equation to calculate the largest amount of salt which will dissolve in the water at 95 °C.

13. A straight line passes through the point $P(-3, 5)$ and has a gradient of $\frac{2}{3}$.

- Write down the equation of this line in the form $y = mx + c$.
- Show that this line is parallel to the line $2x - 3y = 0$.

(CXC)

14. The cost of hiring a taxi consists of a basic charge plus a charge per km travelled. The graph below shows the total cost in dollars (y) for the number of km travelled (x).



- (a) What is the cost of hiring a taxi to travel a distance of
- (i) 250 km
 - (ii) 155 km?
- (b) What distance in km was travelled when the cost was \$40?
- (c) What is the amount of the basic charge?
- (d) Calculate the gradient of the line.
- (e) Write down the equation of the line in the form $y = mx + c$.
- (f) Calculate the cost of hiring a taxi to travel a distance of 330 km. (CXC)
15. A line has equation $y = 4x - 1$. Another line is parallel to this and passes through the point with coordinates (5, 2). Determine the equation of this line.