UNIT 30 *Functions* Introduction to SIM

Learning objectives

This unit is focused on functions, looking at a more mathematical way of analysing their properties. It is an important topic for students continuing on to further study in mathematics and is also central to the CXC syllabus. After completing this unit of STUDENT INSTRUCTIONAL MATERIAL (SIM) you should

- understand the concepts of the domain and range of a function and 1 : 1 mappings
- understand and be able to use composite functions
- have confidence in finding the inverse of a function.

Key points and principles

- A function is defined for a specified range of values; this is called the *domain* of the function
- The *range* of a function is the set of values that the function maps onto.
- A function has an inverse only if it is a 1 : 1 mapping.

Facts to remember

- An alternative function notation is $f : x \to f(x)$

  For example, you could write $y = x^2$ as $f : x \to x^2$

- The composite function fg (or $fg(x)$) means $f(g(x))$

- The inverse function of *f* is denoted by f^{-1} and

  $$f f^{-1}(x) = f^{-1} f(x) = x$$

Glossary of terms

The *domain* of a function is the value of *x* for which the function is defined.

For example, $f(x) = x^2 + 1$ for $x \geq 0$ (domain is $x \geq 0$)

$$g(x) = \frac{1}{x-1} + \frac{1}{\text{x - 2}} \quad \text{for } 1 < x < 2 \quad (\text{domain is } 1 < x < 2)$$

The *range* of a function is the set of values thaat the function maps onto.

For example, if $f(x) = x^2,\ 0 \leq x \leq 5$,

the range of *f* is $0 \leq f(x) \leq 25$.

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1 : 1 mapping is a function for which every value of $f(x)$ is unique; that is, if $f(a) \neq f(b)$ unless $a = b$.

For example, $f(x) = x + 1$ is a 1 : 1 mapping, but

$f(x) = x^2$ is *not* a 1 : 1 mapping as, for example,

$$f(2) = 4 = f(-2)$$

The *composite function* fg or $fg(x)$ is defined as $f(g(x))$

For example, if $f(x) = x + 1$ and $g(x) = x^2$, then

$$fg(x) = f\left(x^2\right) = x^2 + 1$$

and

$$gf(x) = g(x + 1) = (x + 1)^2$$

The *inverse function* of f is denoted by f^{-1} and is such that

$$ff^{-1}(x) = x = f^{-1}f(x)$$

The inverse only exists if the function is a 1 : 1 mapping.

