# UNIT 32 *Angles, Circles and Tangents* Introduction to SIM

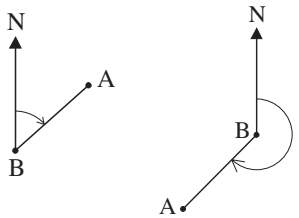
## *Learning objectives*

This unit continues the angle geometry theme, bringing in the important topic of *bearings* and also using circles and tangents. After completing this unit of STUDENT INSTRUCTIONAL MATERIAL (SIM) you should

- be able to identify and use bearings in practical problems
- understand and be able to use angle geometry results for circles, chords, diameters and tangents
- appreciate and use the subtended angle results in a circle and the concept of a cyclic quadrilateral
- understand and be confident in using the alternate segment theorem and the length formula for intersecting chords.

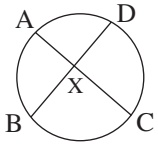
## *Key points and principles*

- In finding bearings, say A from B, you must find the angle, measured *clockwise*, made by the N direction with the line from B to A.



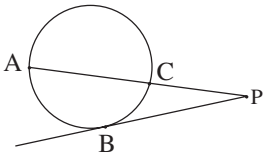

- The angle subtended at the circumference of a circle by a diameter is a right angle.
- The angles subtended at the circumference on the same side of a circle by a chord are equal.
- The angle subtended at the centre of a circle by an arc is twice the angle subtended at a point on the circumference (alternate segment theorem).
- Opposite angles of a cyclic quadrilateral sum to $180°$.
- The angle subtended between a tangent and a chord equals any angle on the circumference subtended by the chord.
- For the circle,

  $AX \times CX = BX \times DX$




- For the circle,

  $BP^2 = AP \times CP$



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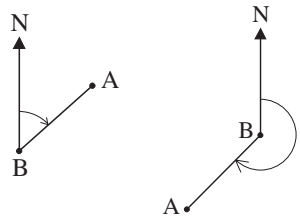
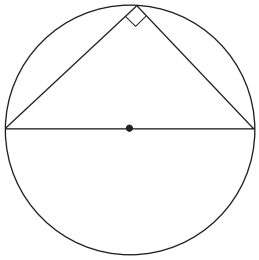
### *Facts to remember*

- *Bearings* are of the form of angles, expressed as three-digit numbers and are measured from the N, in a *clockwise* direction.

  In finding bearings, say A from B, you must find the angle, (measured clockwise) made by the N direction with the line from B to A.

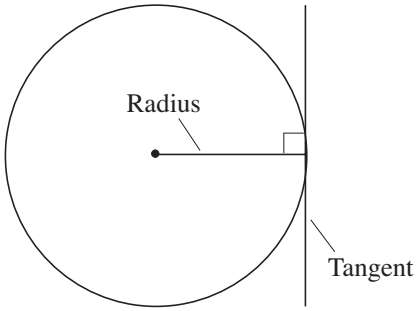



- When a triangle is drawn in a semi-circle as shown, the angle at the perimeter is always a right angle.

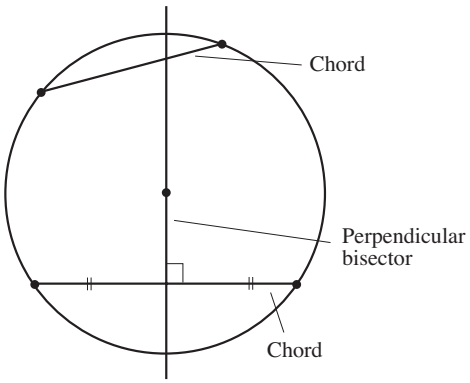


- A *tangent* is a line that touches only one point on the circumference of a circle. The point is known as the *point of tangency*.

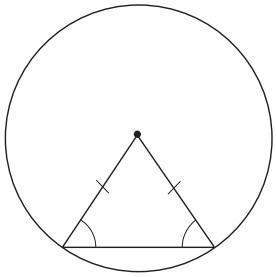
  A tangent is always perpendicular to the radius of the circle.




- A *chord* is a line joining any two points on the circle.
- The *perpendicular bisector* is a second line that divides the chord in half and is at right angles to it.
- The perpendicular bisector of a chord always passes through the centre of the circle.




- When the ends of a chord are joined to the centre of a circle, an isosceles triangle is formed, so the two angles marked are equal.



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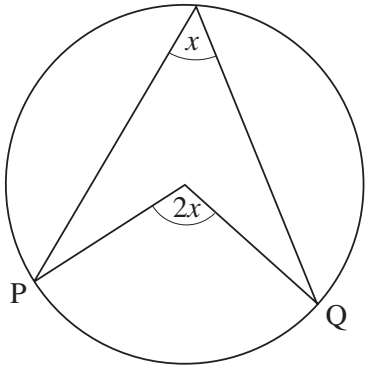
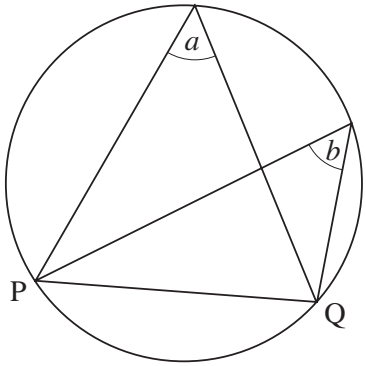
- The angle subtended by an arc, PQ, at the centre is twice the angle subtended at the circumference.



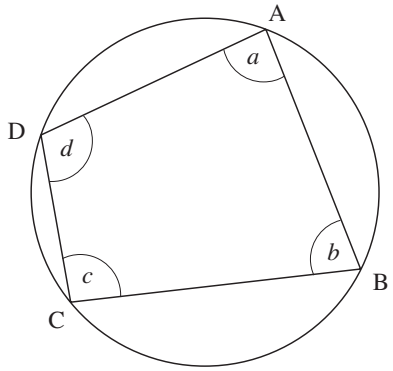

- Angles subtended at the circumference by a chord (on the same side of the chord) are equal; that is, in the diagram $a = b$.




- In *cyclic quadrilaterals* (quadrilaterals where all 4 vertices lie on the circumference of a circle), opposite angles sum to 180°; that is

$$a + c = 180°$$

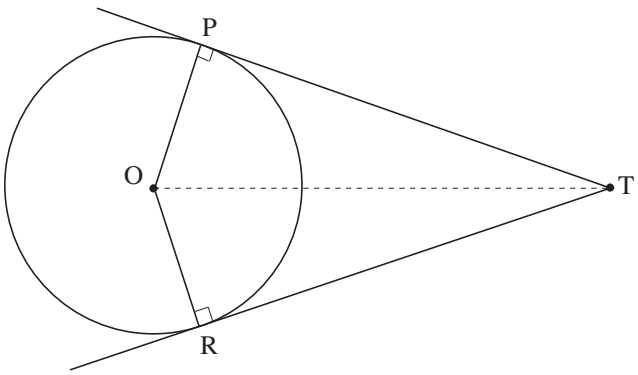
and $$b + d = 180°$$




- If two tangents are drawn from a point T to a circle with centre O, and P and R are the points of tangency, then, using symmetry

$$PT = RT$$

and triangles TPO and TRO are congruent.



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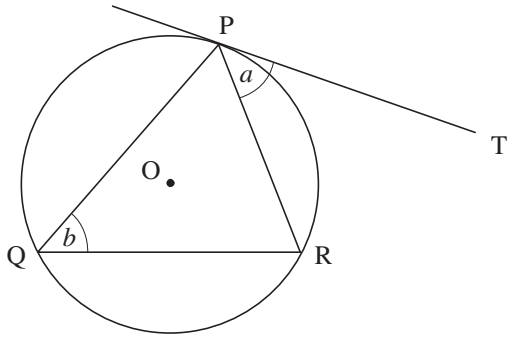
- The angle between a tangent and a chord equals an angle at the circumference subtended by the same chord;
  e.g. $a = b$ in the diagram.

  This is known as the

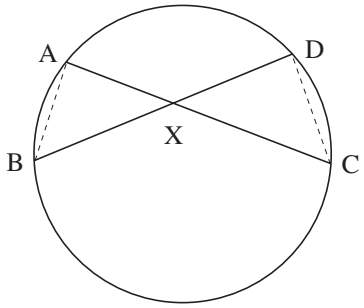
  *alternate segment theorem*.




- For any two intersecting chords, as shown,

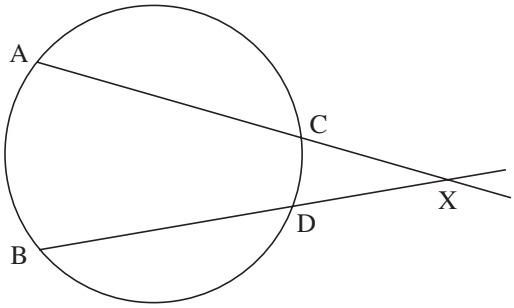
  $$AX \times CX = BX \times DX$$

  The proof is based on similar triangles.




This result will still be true even when the chords intersect *outside* the circle, as illustrated opposite.

*How can this be proved?*



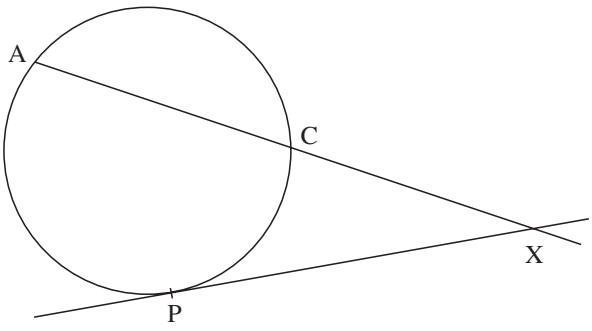

When the chord BD becomes a tangent,

and B and D coincide at the point P, then

$$AX \times CX = PX \times PX$$

or

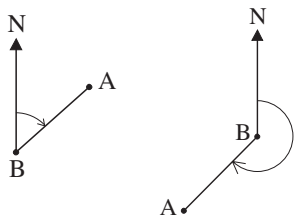
$$AX \times CX = PX^2$$



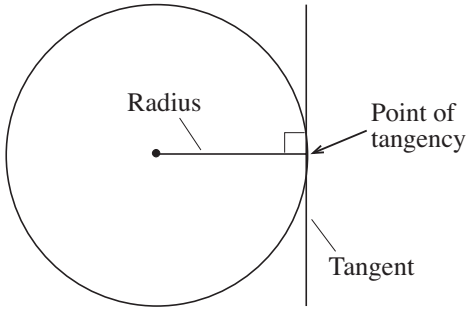
# UNIT 32 *Angles, Circles and Tangents* Introduction to SIM

## *Glossary of terms*

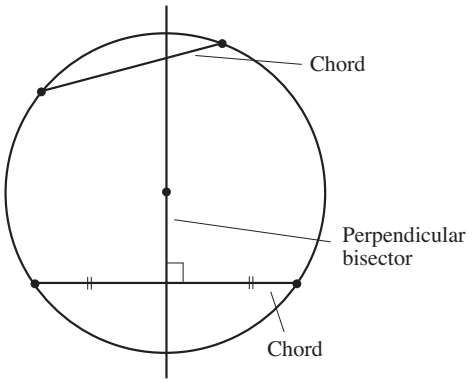
- A *bearing* gives the direction or position of something, or the direction of movement, relative to a fixed point. Bearings are of the form of angles, expressed in degrees as three-digit numbers; they are measured from north in a clockwise direction.
  For example, 060°, 210°.



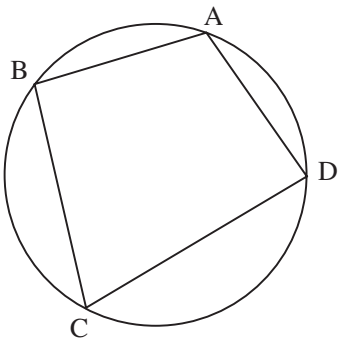

- A *tangent* is a line that touches only one point on the circumference of a circle.
  A tangent is always perpendicular to the radius of the circle.
- The *point of tangency* is the point where a tangent touches the circumference of a circle.




- A *chord* is a line joining any two points on the circle.
  The *perpendicular bisector* is a second line that divides the first line in half and is at right angles to it.
  The perpendicular bisector of a chord always passes through the centre of the circle.




- A *cyclic quadrilateral* is a quadrilateral with all four vertices on the circumference of a circle.




- An a*ngle subtended by a chord* in a circle is shown in the diagram opposite.