

UNIT 35 *Trigonometric Problems*

Introduction to SIM

Learning objectives

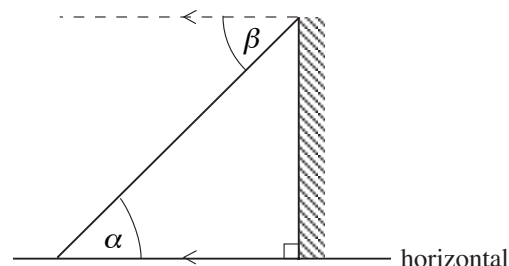
This unit continues the theme of finding angles and lengths in a triangle. After completing this unit of STUDENT INSTRUCTIONAL MATERIAL (SIM) you should

- be able to calculate sides and angles in trigonometrical problems
- understand how to use the SINE and COSINE rules in non right angled triangles and apply to bearings
- be able to find the area of any triangles, given two sides and the included angle
- be able to use Heron's formula to calculate the area of any triangle, given the lengths of its three sides.

Key points and principles

- *angle of elevation* (α) is the angle made with the horizontal, as shown in the diagram (angle β is called the *angle of depression*).

Angle α = angle β (alternate angles).



- for any triangle ABC , the sine rule states

$$\frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin C}{c}$$

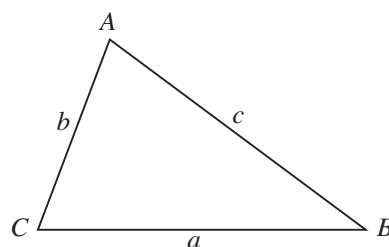
and the cosine rule states

$$c^2 = a^2 + b^2 - 2ab \cos C,$$

and similarly,

$$a^2 = b^2 + c^2 - 2bc \cos A$$

$$b^2 = c^2 + a^2 - 2ca \cos B$$



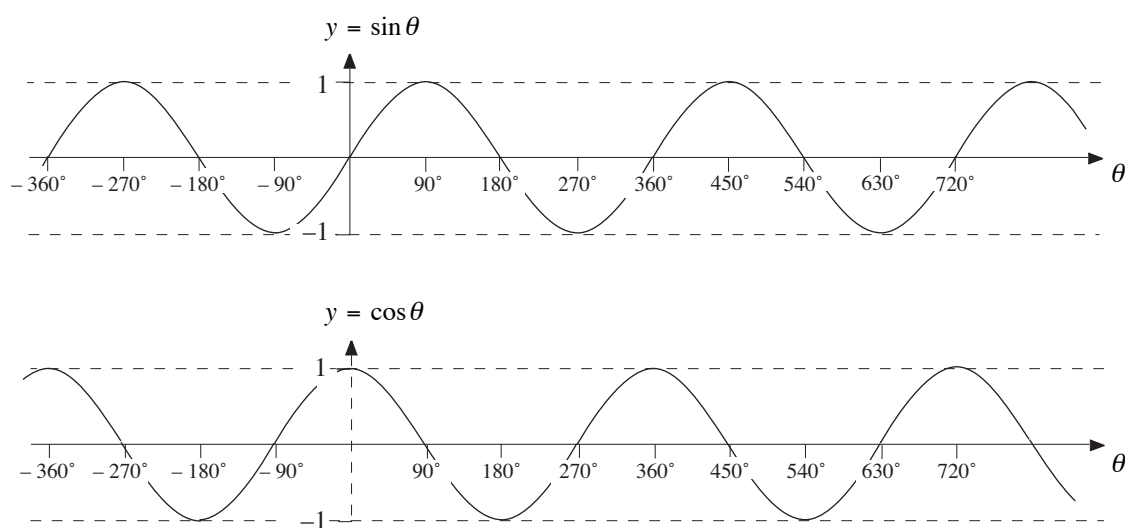
- Important values of sin, cos and tan are shown in the table below.

θ	$\sin \theta$	$\cos \theta$	$\tan \theta$
0°	0	1	0
30°	$\frac{1}{2}$	$\frac{\sqrt{3}}{2}$	$\frac{1}{\sqrt{3}}$
45°	$\frac{1}{\sqrt{2}}$	$\frac{1}{\sqrt{2}}$	1
60°	$\frac{\sqrt{3}}{2}$	$\frac{1}{2}$	$\sqrt{3}$
90°	1	0	infinite

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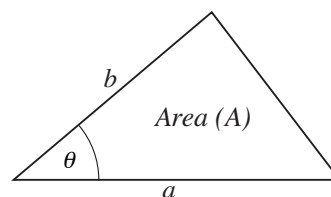
- The graphs of $y = \sin \theta$ and $y = \cos \theta$ are shown below.



Note when the functions are positive and when they are negative.

- The area of any triangle is given by

$$A = \frac{1}{2}ab \sin \theta$$



- The area of a triangle with side lengths a, b, c is given by the formula

$$A = \sqrt{s(s-a)(s-b)(s-c)}$$

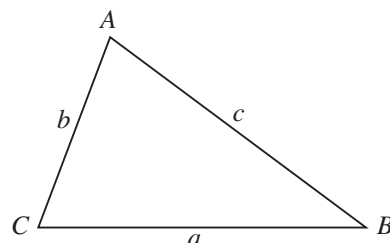
where

$$s = \frac{a + b + c}{2}$$

This is known as Heron's formula.

Facts to Remember

- sine rule $\frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin C}{c}$
- cosine rule $c^2 = a^2 + b^2 - 2ab \cos C$
 $a^2 = b^2 + c^2 - 2bc \cos A$
 $b^2 = c^2 + a^2 - 2ca \cos B$

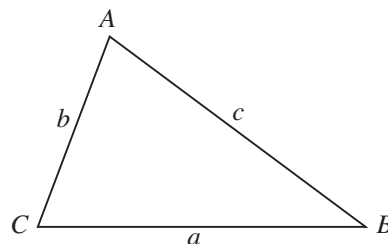


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- special angles

θ	$\sin \theta$	$\cos \theta$	$\tan \theta$
0°	0	1	0
30°	$\frac{1}{2}$	$\frac{\sqrt{3}}{2}$	$\frac{1}{\sqrt{3}}$
45°	$\frac{1}{\sqrt{2}}$	$\frac{1}{\sqrt{2}}$	1
60°	$\frac{\sqrt{3}}{2}$	$\frac{1}{2}$	$\sqrt{3}$
90°	1	0	infinite



- area of triangle $A = \frac{1}{2}ab\sin C$

or $A = \sqrt{s(s-a)(s-b)(s-c)}$ where $s = \frac{a+b+c}{2}$

Glossary of terms

- angle of elevation* (α) is the angle made with the horizontal, as shown in the diagram (angle β is called the *angle of depression*).

Angle α = angle β (alternate angles).

