STRAND I: Geometry and Trigonometry

Unit 36 *Constructions and Enlargements*

Student Text

Contents

36 Constructions and Enlargements

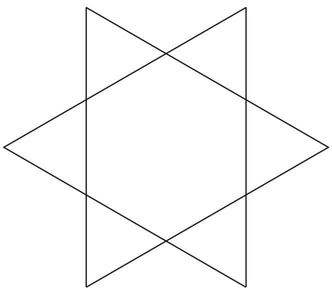
36.1 Drawing and Symmetry

This section revises the ideas of symmetry first introduced in Unit 31 and gives you practice in drawing simple shapes.

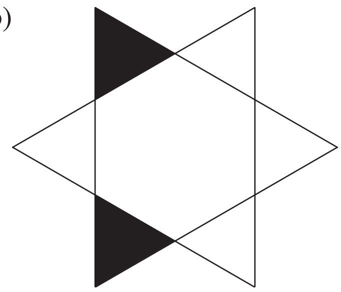
Worked Example 1

Describe the symmetries of each shape below.

(a)

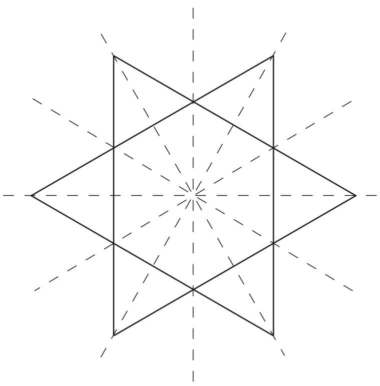


(b)



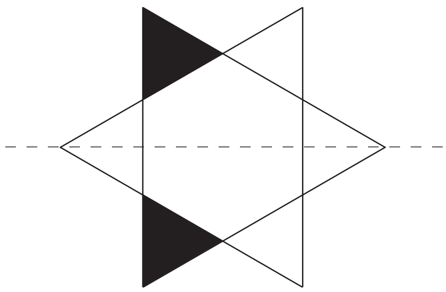
Solution

(a) This shape has 6 lines of symmetry, as shown in the diagram.



It has rotational symmetry of order 6 as it can be rotated about its centre to 6 different positions.

(b) This shape has one line of symmetry as shown below.



It has rotational symmetry of order 1, since it can rotate a full 360° back to its original position.

Worked Example 2

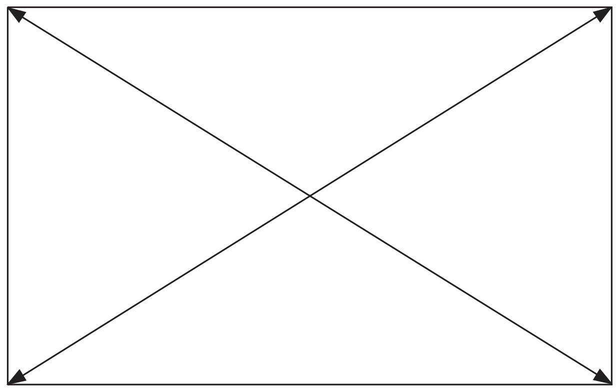
Draw accurately a rectangle with sides of length 8 cm and 5 cm.

Solution

First draw a line 8 cm long:.

Then draw lines 5 cm long at each end, making sure they are at right angles to the base line.

Finally, join these two lines to complete the rectangle.



Measure the diagonals and check that they are both the same length.

Exercises

1. Draw accurately rectangles with the following sizes.

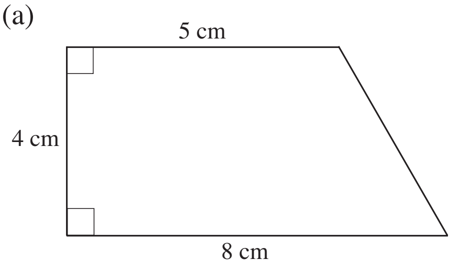
   (a) 3 cm by 8 cm (b) 10 cm by 3 cm

   (c) 6 cm by 7 cm (d) 6 cm by 4 cm

   For each rectangle check that both diagonals are the same length.

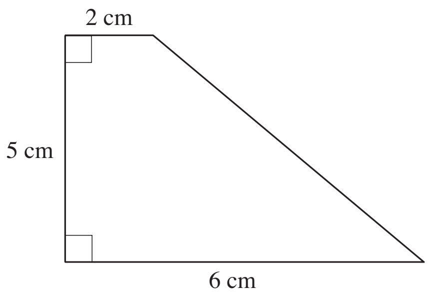
2. Make accurate drawings of each of the shapes shown below and answer the question below each shape.

   (a)



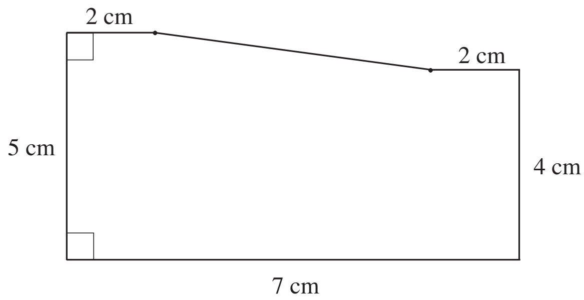
   What is the length of the sloping side?

   (b)



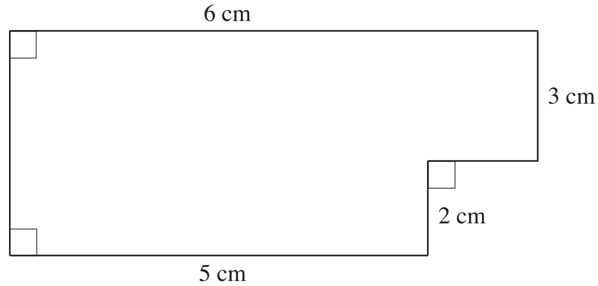
   What is the length of the longest diagonal?

(c)



What is the length of the sloping side?

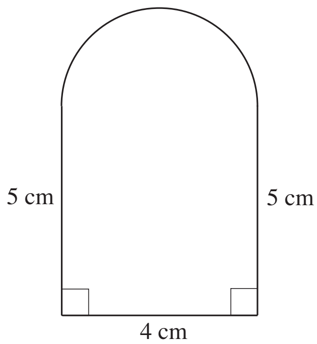
(d)



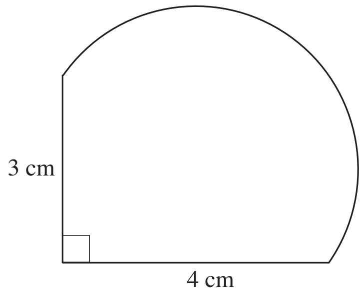
What is the length of the longest straight line which can be drawn inside the shape?

3. Each shape below includes a semi-circle. Make an accurate drawing of each shape and state the radius of the semi-circle.

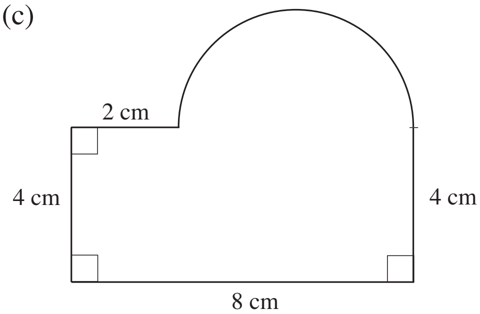
(a)



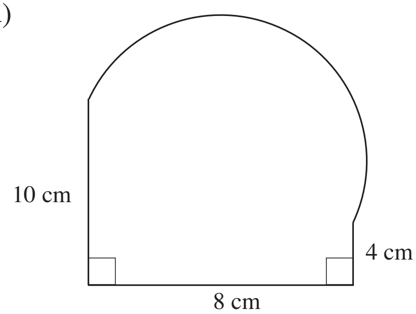
(b)



(c)



(d)

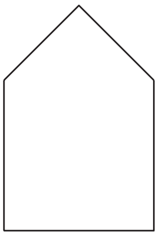


4. For each shape below:

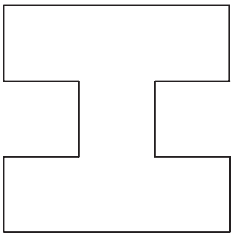
   (i) state the order of rotational symmetry,

   (ii) copy the shape and draw any line of symmetry.

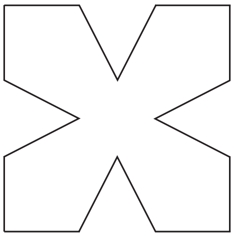
(a)



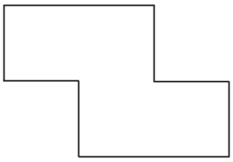
(b)



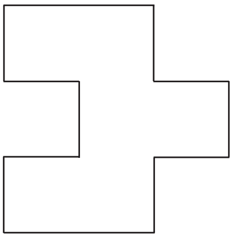
(c)



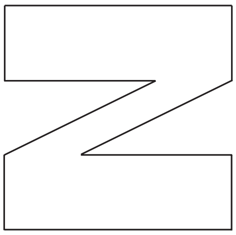
(d)



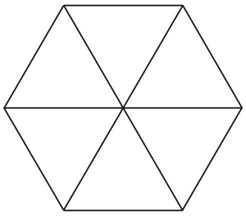
(e)



(f)



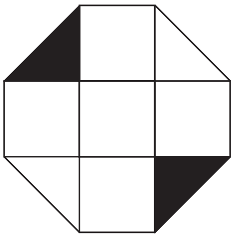
5. (a) Copy and shade part of the shape below so that it has 3 lines of symmetry.



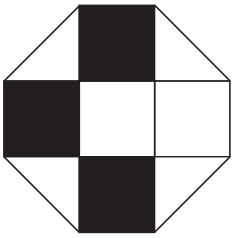
   (b) What is the order of rotational symmetry of the shape?

6. State the number of lines of symmetry and the order of rotational symmetry for each shape below.

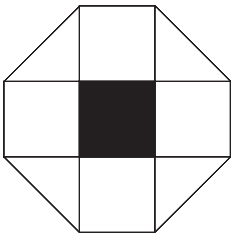
(a)



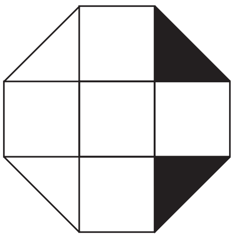
(b)



(c)

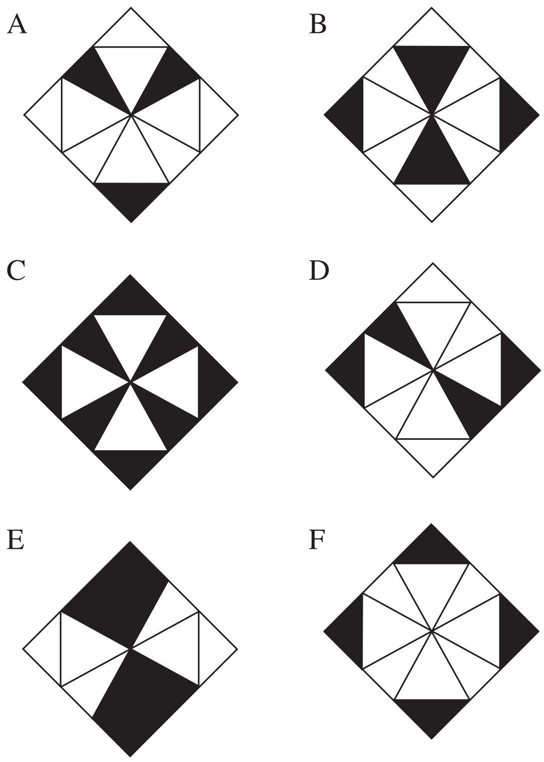


(d)

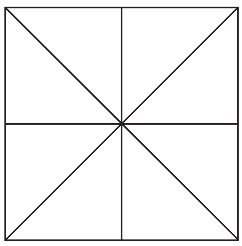


7. Which of the shapes below have:

(a) rotational symmetry of order 1
(b) no lines of symmetry
(c) more than two lines of symmetry
(d) rotational symmetry of order 2
(e) rotational symmetry of an order greater than 2?



8. Make 4 copies of the shape below.



Shade triangles in the shape to produce shapes with:

(a) 2 lines of symmetry
(b) one line of symmetry
(c) rotational symmetry of order 2,
(d) rotational symmetry of order 4.

Investigation

Look at an atlas and find out the scales used in maps of different countries. Are the same scales used for all the maps? If not, why not?

36.2 Constructing Triangles and Other Shapes

A protractor and a compass can be used to produce accurate drawings of triangles and other shapes.

We first recap some basic constructions that you will have met before.

Worked Example 1

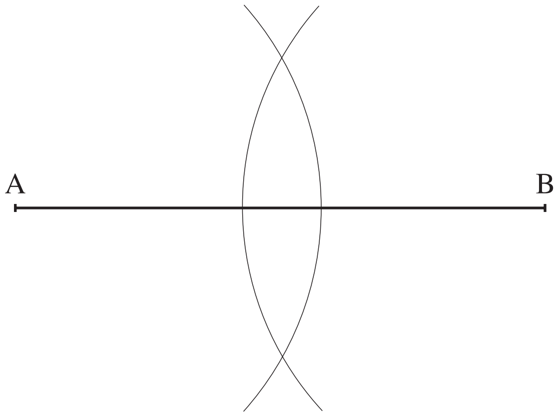
Construct the perpendicular bisector of the line AB.

Then label the midpoint of AB, M.

Solution

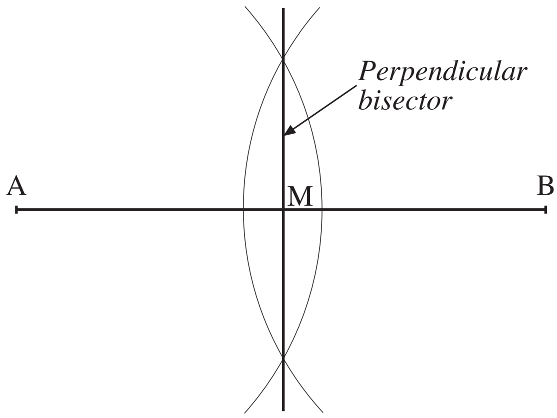
There are many lines that cut AB exactly in half. We have to construct the one that is perpendicular to AB.

We begin by drawing arcs of equal radius, centred on the points A and B, as shown in the diagram.
The radius of these arcs should be roughly $\frac{2}{3}$ to $\frac{3}{4}$ of the length AB.



Then draw a line through the intersection points of the two arcs.

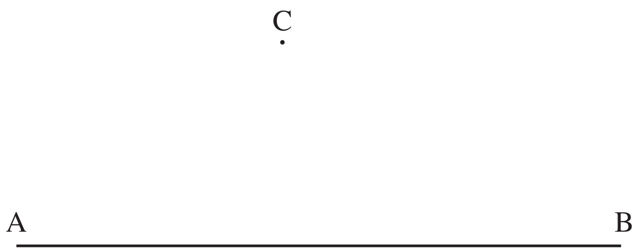
The point where the bisector intersects AB can then be labelled M.



Worked Example 2

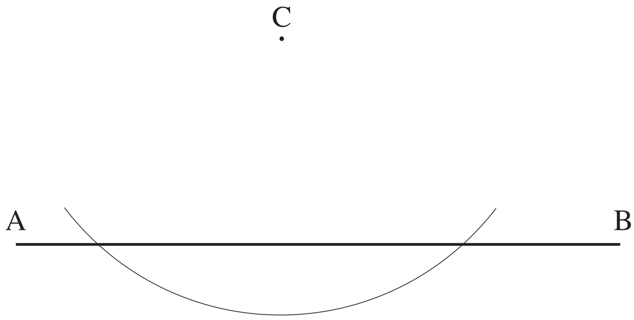
The diagram shows the line AB and the point C.

Draw a line through C that is perpendicular to AB.

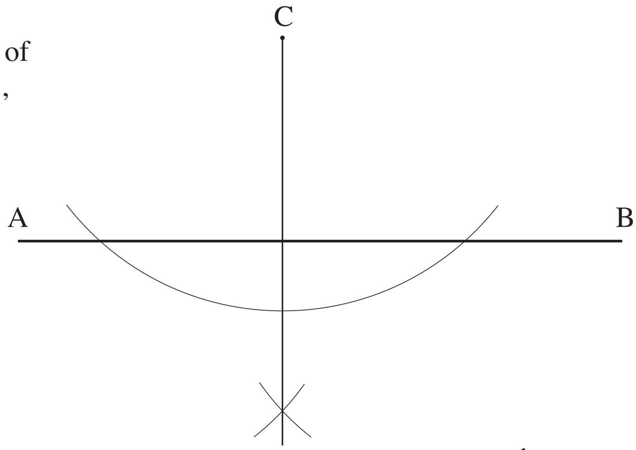


Solution

Using C as the centre, draw an arc as shown.

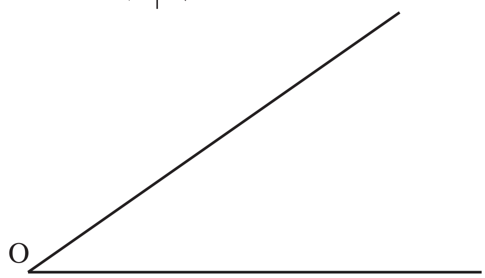


Then using the intersection points of this arc with the line AB as centres, draw two further arcs with radii of equal length. The perpendicular line can then be drawn from C through the point where these two new arcs cross.



Worked Example 3

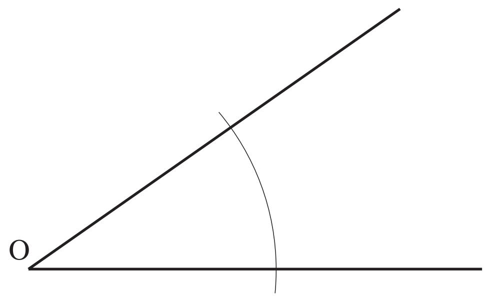
Bisect this angle.



Solution

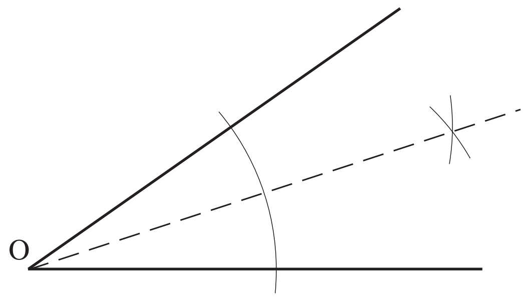
To bisect an angle you need to draw a line that cuts the angle in half.

First draw an arc using O as the centre.



Then draw two further arcs of equal radius, using the points where the arc intersects the lines as the centres.

The bisector can then be drawn from O through the point where these two new arcs cross.



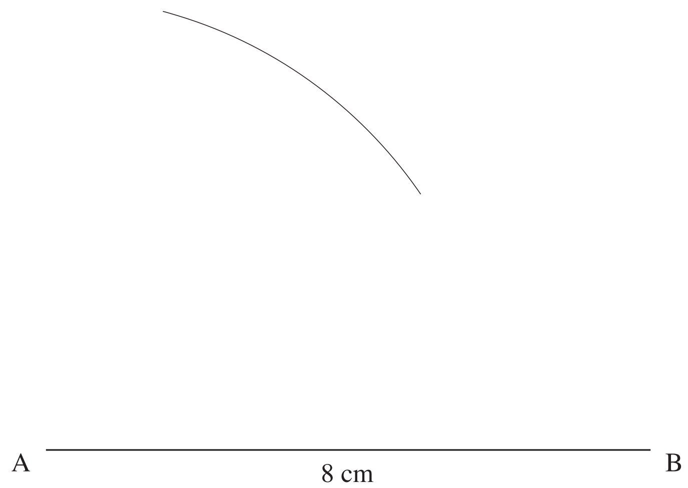
Worked Example 4

Construct a triangle with sides of length 8 cm, 6 cm and 6 cm.

Solution

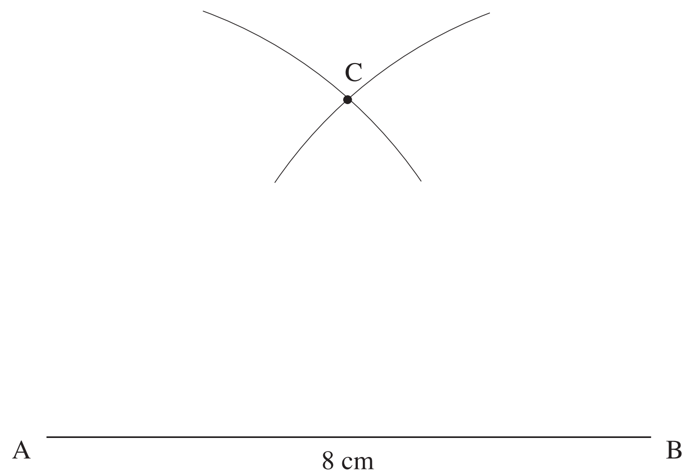
First draw a line of length 8 cm.

Then set the distance between the point and pencil of your compass to 6 cm and draw an arc with centre A as shown below.



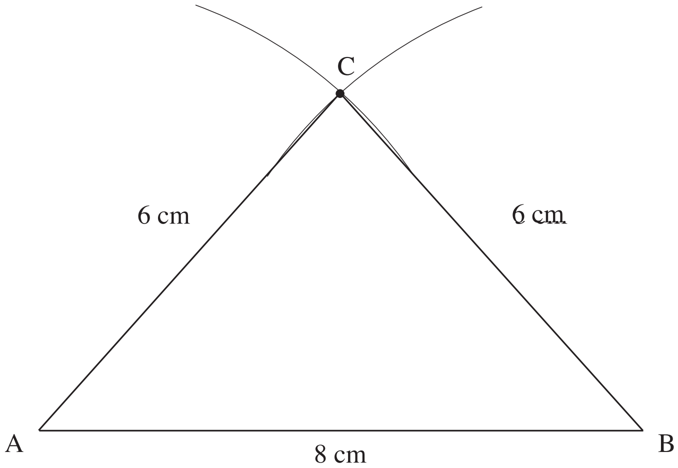
The arc is a distance of 6 cm from A.

With your compass set so that the distance between the point and the pencil is still 6 cm, draw an arc centred at B, as shown below.



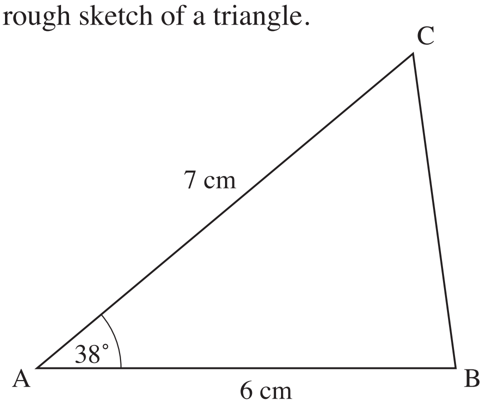
The point, C, where the two arcs intersect is the third vertex of the triangle.

The triangle can now be completed.



Worked Example 5

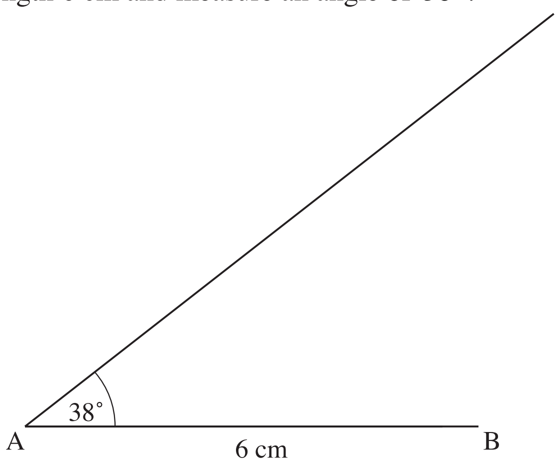
The diagram shows a rough sketch of a triangle.



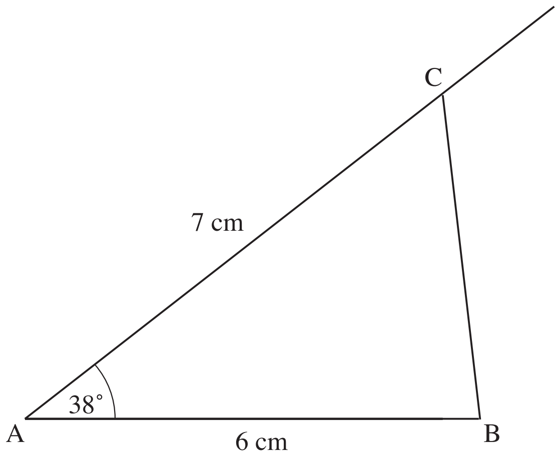
Make an accurate drawing of the triangle, using a ruler and protractor, and find the length of the third side.

Solution

First draw a line of length 6 cm and measure an angle of $38°$.



Then measure 7 cm along the line and the triangle can be completed.



The third side of the triangle can then be measured as approximately 4.3 cm.

Worked Example 6

(a) Draw a line segment, *PQ*, 7 cm long.

(b) **Using only a ruler, a pencil and a pair of compasses**, construct a line segment, *LM*, the perpendicular bisector of *PQ*, such that

LM cuts PQ at O, and $OL = OM = 4$ cm.

(c) Form parallelogram *PLQM* by joining the points *P*, *L*, *Q* and *M*.

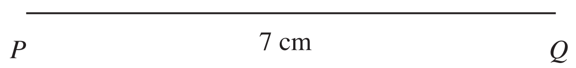
(d) Measure and state the size of the angle *MPL*.

(e) What type of parallelogram is *PLQM*? Give a reason for your answer.

(CXC)

Solution

(a)



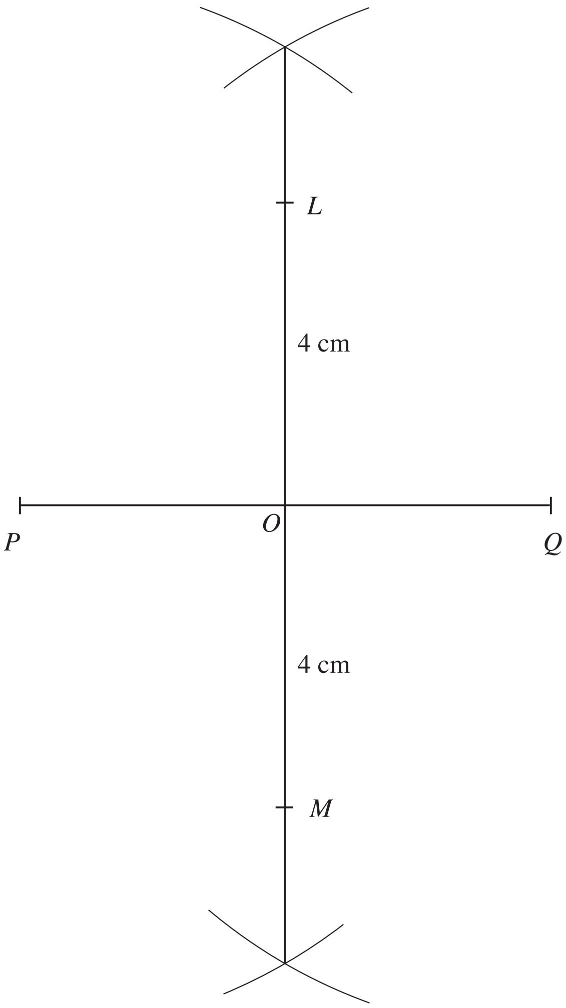
(b) With compass centre *P* and centre *Q* and radius, say 7 cm, draw circle segments. The points of intersection form the perpendicular to *PQ*, crossing at point *O*. Use a ruler to find points *L* and *M* on this line.

(c) See diagram on next page.

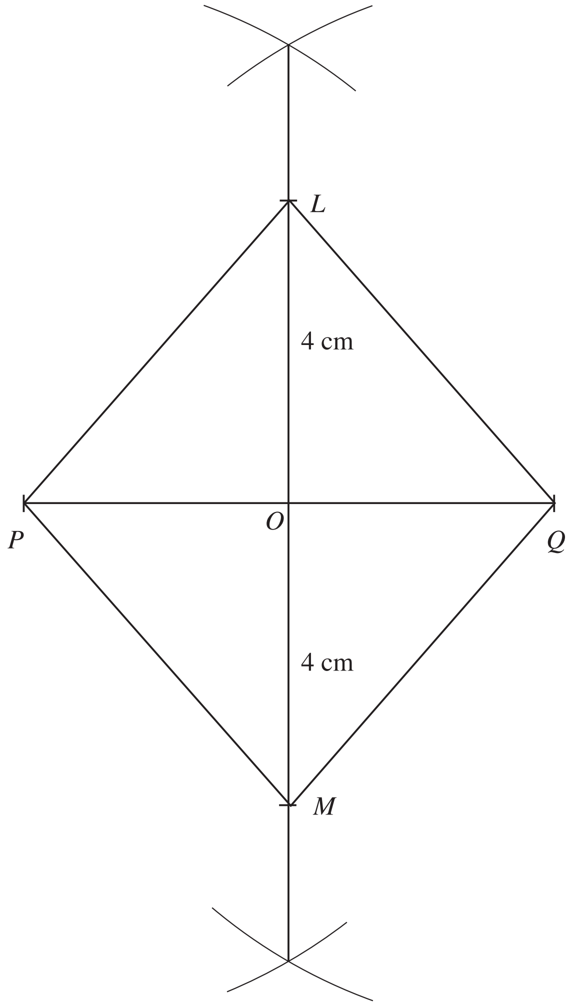
(d) Angle $MPL \approx 98°$

(e) *PLQM* is a rhombus as all sides are equal and angles are not equal to $90°$.

(b)

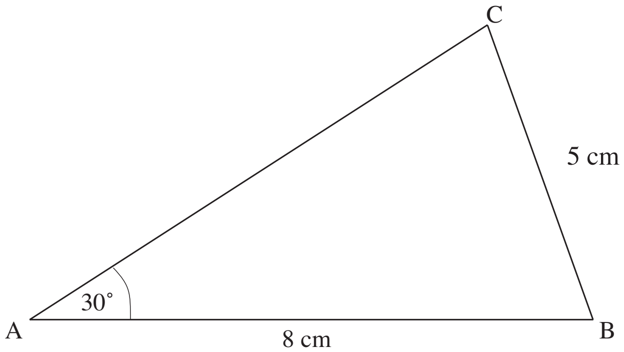


(c)



Worked Example 7

The diagram shows a rough sketch of a triangle.

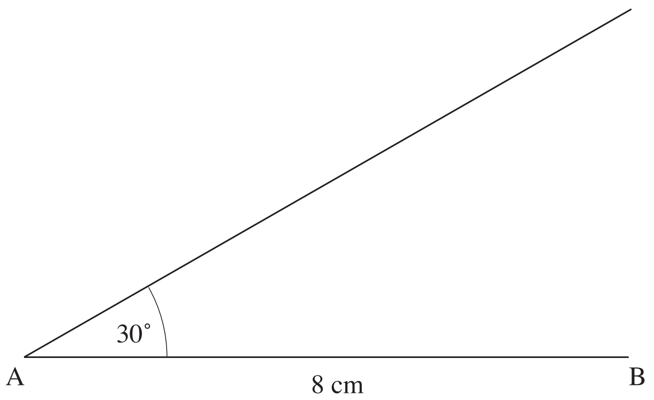


Make an accurate drawing of the triangle.

What is the length AC?

Solution

First draw the side of length 8 cm and measure the angle of $30°$, using a protractor, as shown below.



[Alternatively, you can construct an angle of $60°$ and then bisect it to obtain an angle of $30°$.

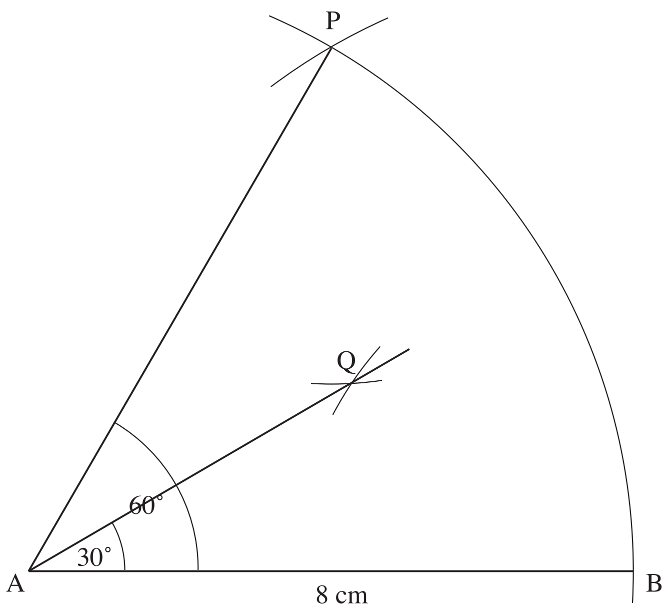
To construct an angle of $60°$:

Draw the line AB of length 8 cm.

Using compasses, draw a sector of the circle, centre A, radius 8 cm, as shown below.

With your compass point placed at point B, mark off the intersection of this circle with a second circle of radius 8 cm, centre B. Call this point P.

Join A to P. Angle PAB is $60°$.

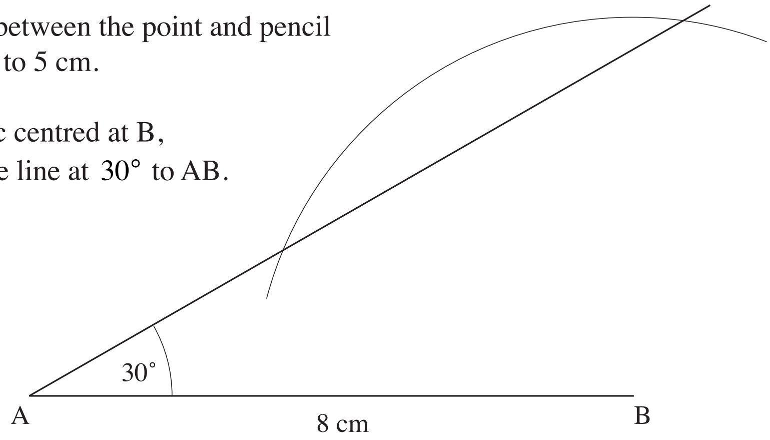


You can obtain an angle of $30°$ by bisecting angle PAB in the usual way (see diagram), by drawing sectors of circles, radii approximately 4.5 cm, centred on points P and B, marking the point of intersection as Q.

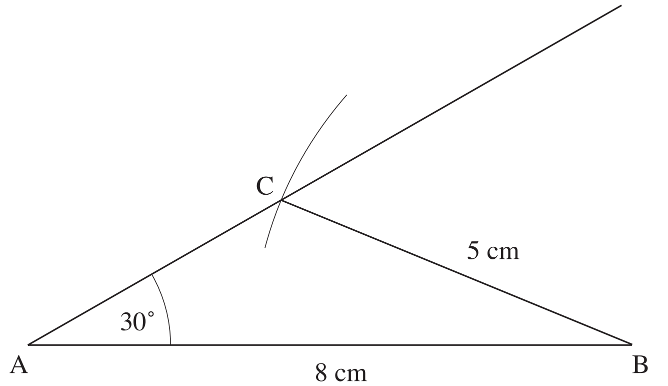
Join A to Q. Angle QAB is $30°$.]

Set the distance between the point and pencil of your compass to 5 cm.

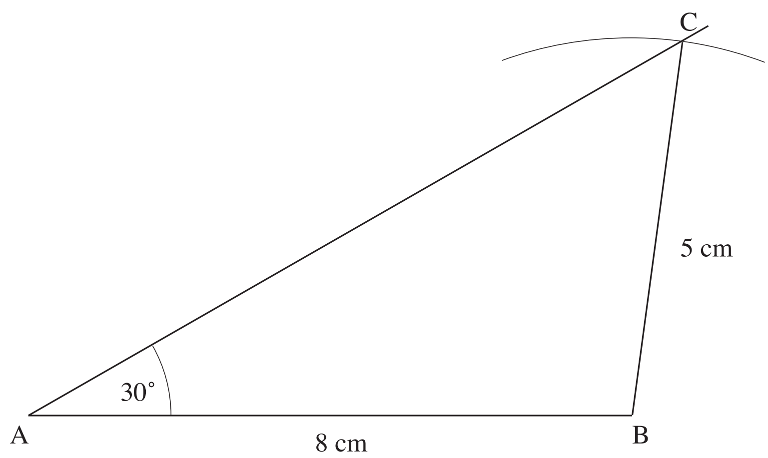
Then draw an arc centred at B, which crosses the line at $30°$ to AB.



As the arc crosses the line in two places, there are two possible triangles that can be constructed as shown below.



Both triangles have the lengths and angle specified in the rough sketch.



The possible lengths of AC are, approximately, 3.5 cm and 9 cm.

Note

An arc must be taken when constructing triangles to ensure that all possibilities are considered.

Exercises

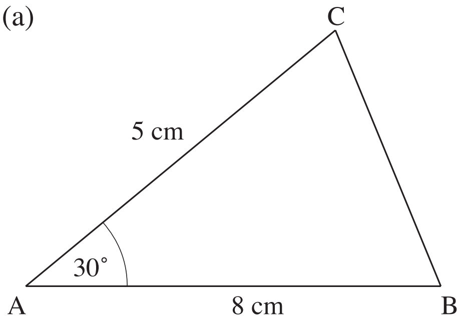
1. Draw triangles with sides of the following lengths.

   (a) 10 cm, 6 cm, 7 cm (b) 5 cm, 3 cm, 6 cm

   (c) 4 cm, 7 cm, 6 cm

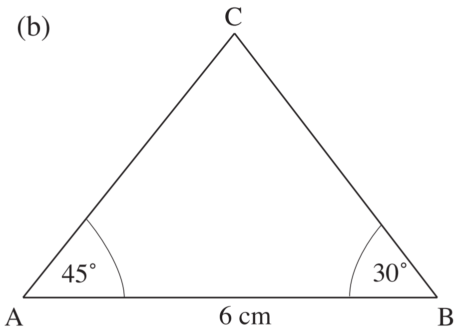
2. Draw accurately the triangles shown in the rough sketches below and answer the question given below each sketch.

(a)



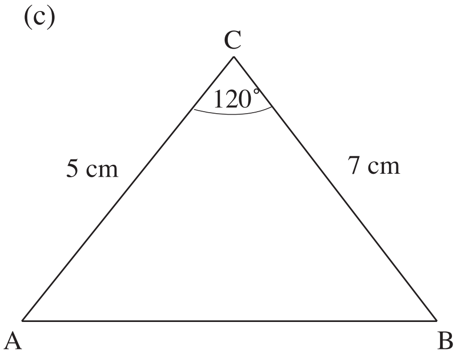
How long is the side BC?

(b)



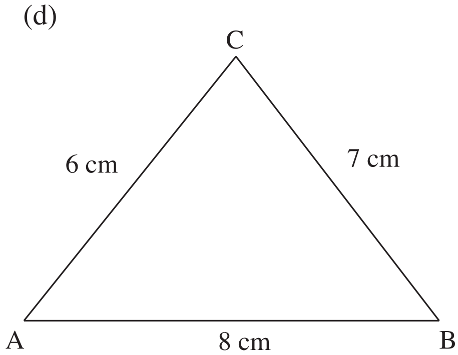
How long are the sides AC and BC?

(c)



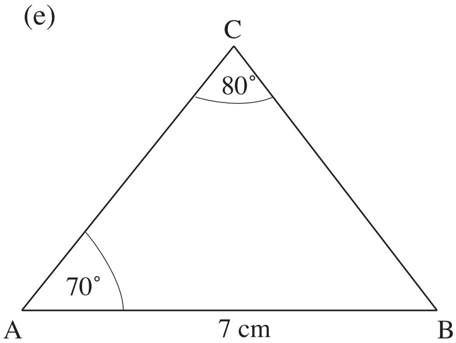
How long is the side AB?

(d)



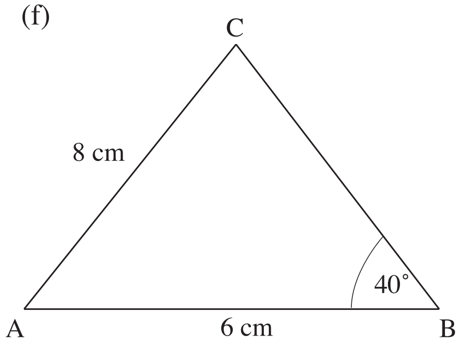
What is the size of the angle ABC?

(e)



How long is the side AC?

(f)



How long is the side BC?

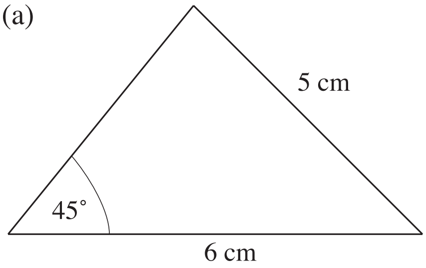
3. An isosceles triangle ABC has AB = 6 cm and angles ABC and CAB each equal to $50°$. Find the lengths of the other sides of the triangle.

4. An isosceles triangle has 2 sides of length 8 cm and one side of length 4 cm. Find the sizes of all the angles in the triangle.

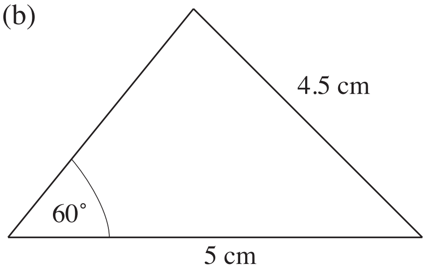
5. Draw an equilateral triangle with sides of length 5 cm.

6. For each rough sketch shown below, draw two possible triangles.

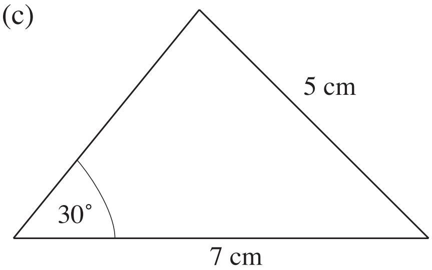
(a)



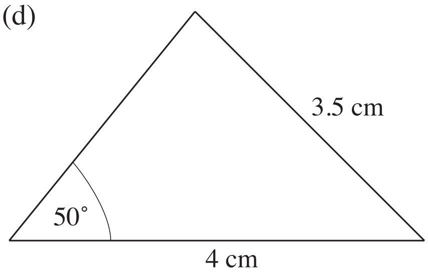
(b)



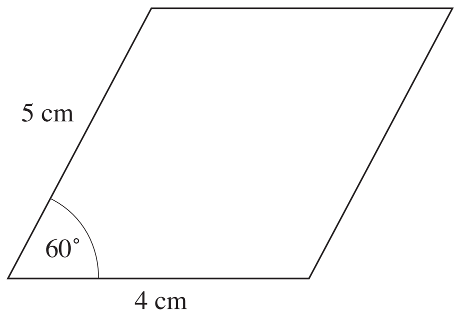
(c)



(d)



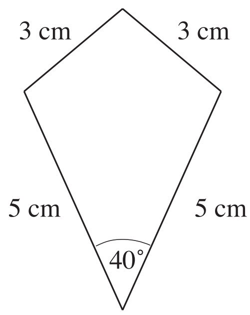
7. (a) Draw accurately the parallelogram shown below.



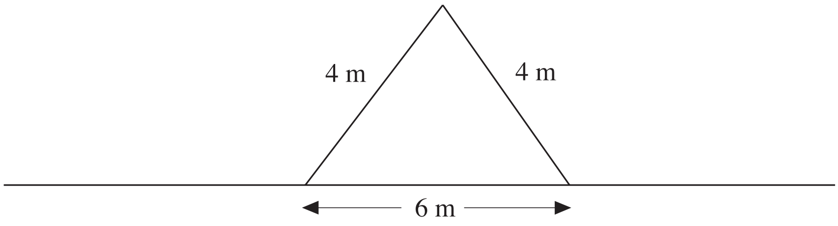
(b) Measure the two diagonals of the parallelogram.

8. (a) Draw the kite shown in the rough sketch opposite.

(b) Check that the diagonals of the kite are at right angles.

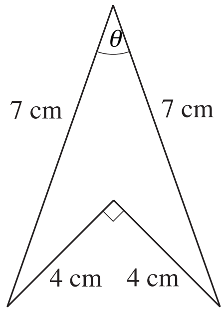


9. A pile of sand has the shape shown below. Using an accurate diagram, find its height.



10. Draw accurately the shape shown opposite.

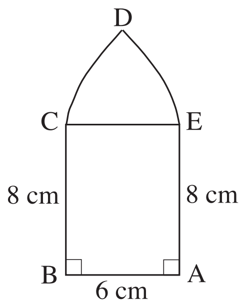
Find the size of the angle marked θ.



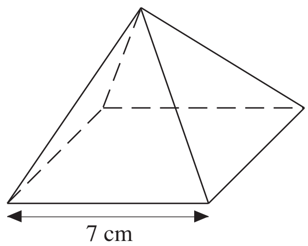
11. The sketch shows the design for a church window.

CB and EA are perpendicular to BA.
CD is part of a circle, centre E.
DE is part of a circle, centre C.

Using a ruler and compasses draw the design accurately.

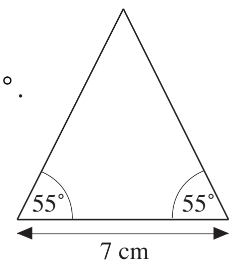


12. John is required to construct a pyramid with a square base, as shown below.



(a) Each sloping face is a triangle with base angles of $55°$.

Construct one of these triangles accurately and to full size.



(b) Construct the square base of the pyramid accurately and to full size.

13. A rectangle has sides of 8 cm and 5 cm.

    (a) Calculate the perimeter of the rectangle.

    (b) Construct the rectangle accurately.

14. Construct a rhombus ABCD with the line $\text{AB} = 4 \text{ cm}$ as base and with $\text{B}\hat{\text{A}}\text{D} = 50°$.

15. Using ruler and compasses ONLY:

    (a) Construct the triangle *FGH* with $FG = 7.5 \text{ cm}$, angle $FGH = 120°$ and angle $GFH = 30°$.

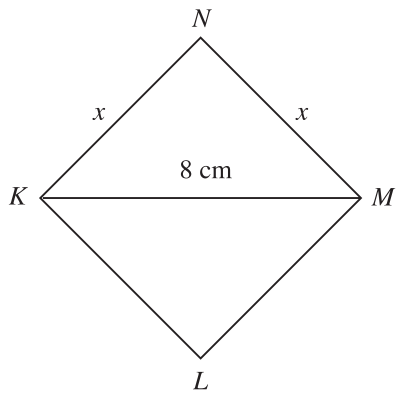
    (b) Locate on *FG,* the point *M,* the midpoint of *FG*.

    Show all construction lines.

    Measure and state the size of angle *GMH*.

(CXC)

16. (a)



The diagram above, **not drawn to scale**, shows a square *KLMN,* where $KM = 8 \text{ cm}$ and $KN = MN = x \text{ cm}$.

Show that $x^2 = 32$.

(b) (i) **Using ruler and compasses only**,

a) draw the diagonal $KM = 8 \text{ cm}$

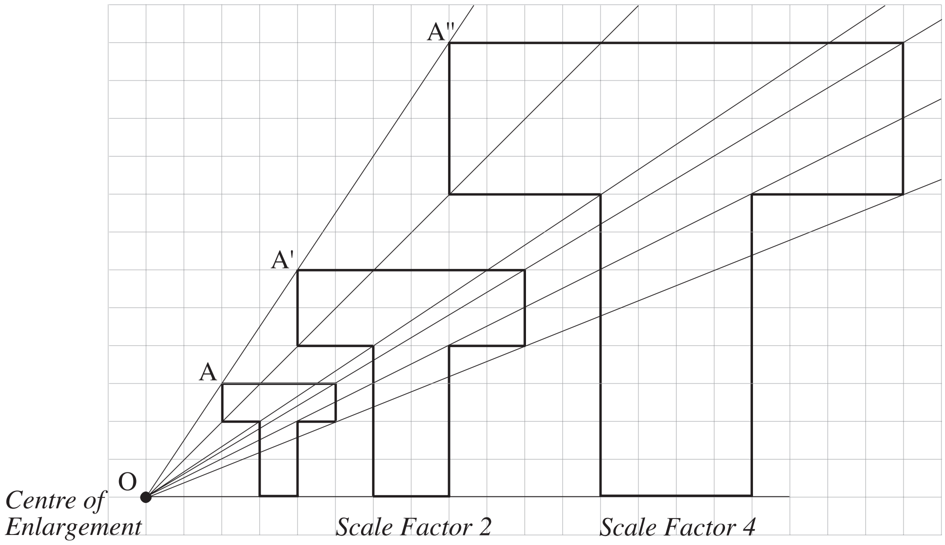
b) construct the perpendicular bisector of *KM*.

(ii) Hence, draw the square *KLMN*.

(CXC)

36.3 Enlargements

An *enlargement* is a transformation which enlarges (or reduces) the size of an image. Each enlargement is described in terms of a *centre of enlargement* and a *scale factor*.



The example shows how the original, A, was enlarged with scale factors 2 and 4. A line from the *centre of enlargement* passes through the corresponding vertex of each image.

Note

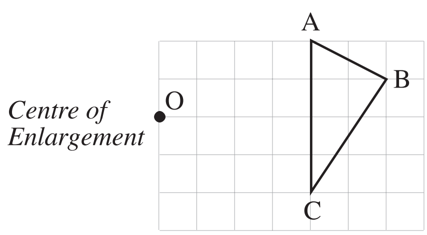
The distances, OA' and OA'', are related to OA:

$$OA' = 2 \times OA$$
$$OA'' = 4 \times OA$$

The same is true of all the other distances between O and corresponding points on the images.

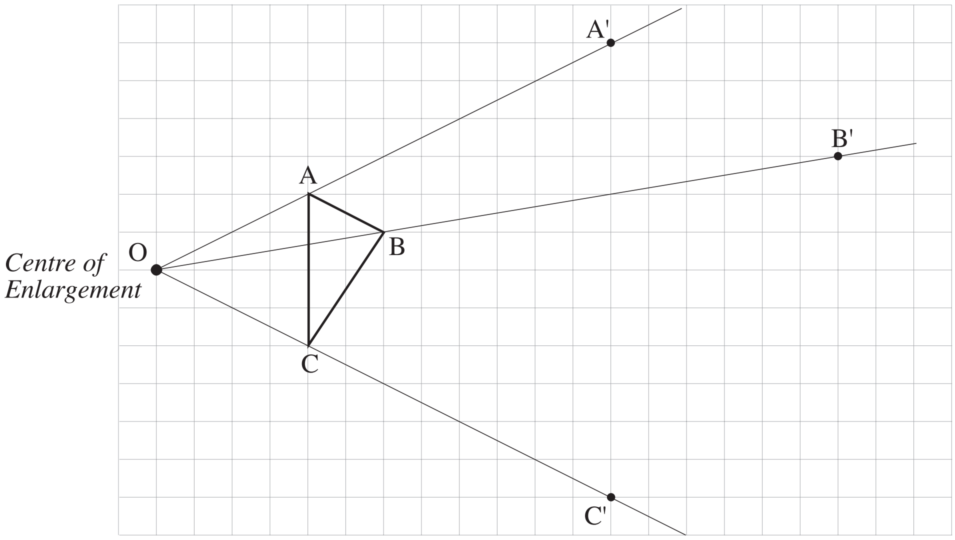
Worked Example 1

Enlarge the triangle shown using the centre of enlargement marked and scale factor 3.



Solution

The first step is to draw lines from the centre of enlargement through each vertex of the triangle as shown below.



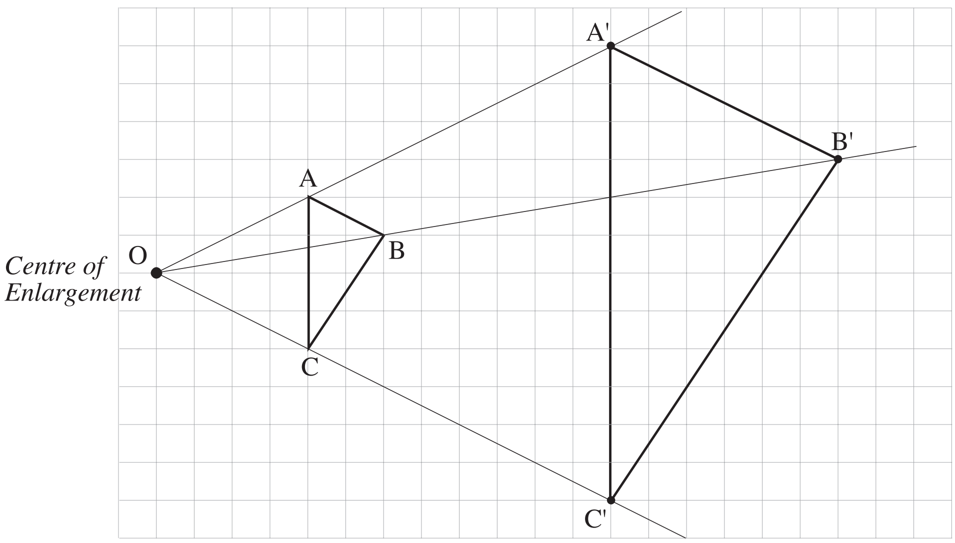
As the scale factor is 3, then

$$OA' = 3 \times OA$$

$$OB' = 3 \times OB$$

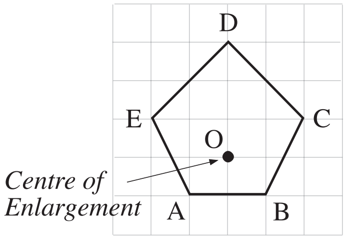
$$OC' = 3 \times OC$$

The points A', B' and C' have also been marked on the diagram. Once these points have been found they can be used to draw the enlarged triangle.



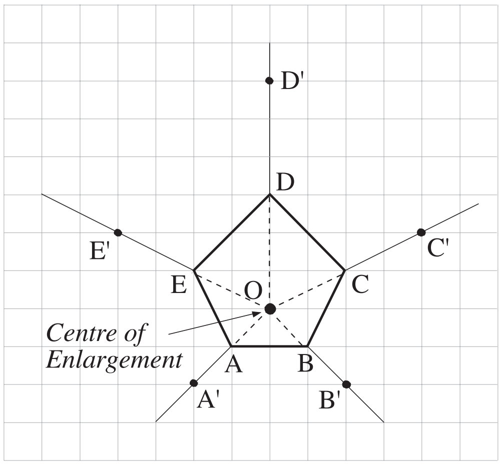
Worked Example 2

Enlarge the pentagon with scale factor 2 using the centre of enlargement marked on the diagram.



Solution

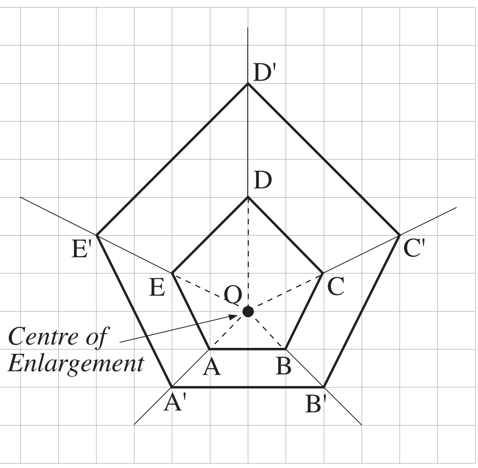
The first step is to draw lines from the centre of enlargement which pass through the five vertices of the pentagon.



As the scale factor is 2 the distances from the centre of enlargement to the vertices of the image will be

$$OA' = 2 \times OA$$
$$OB' = 2 \times OB$$
$$OC' = 2 \times OC$$
$$OD' = 2 \times OD$$
$$OE' = 2 \times OE\,.$$

These points can then be marked and joined to give the enlargement.

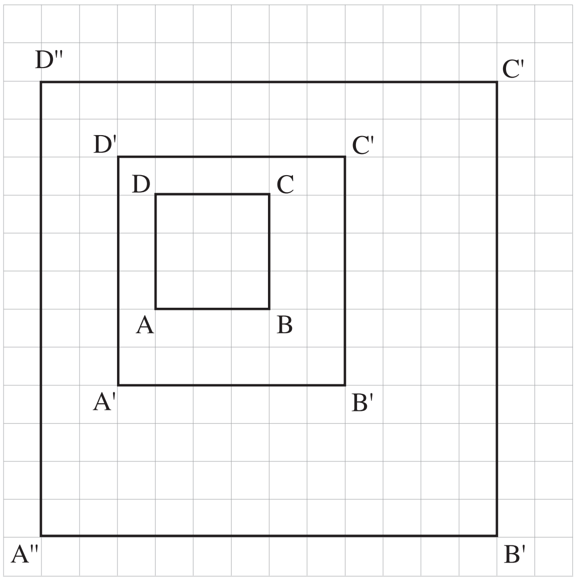


Worked Example 3

The diagram shows the square ABCD which has been enlarged to give the squares $A'B'C'D'$ and $A''B''C''D''$.

(a) Find the scale factor for each enlargement.

(b) Find the centre of enlargement.

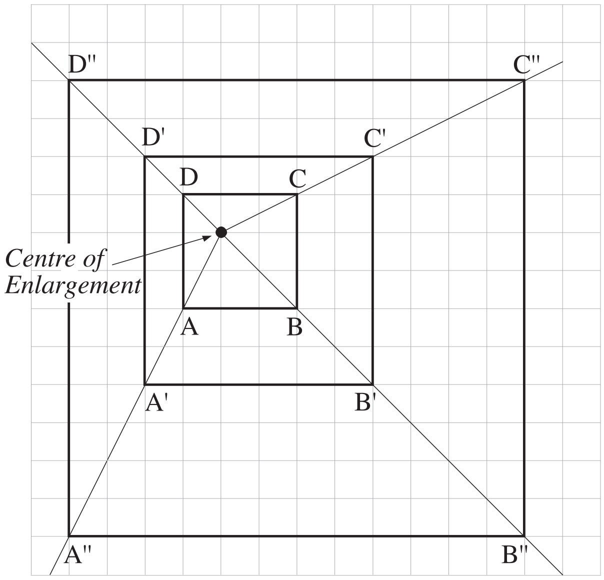


Solution

(a) The sides of the square ABCD are each 1.5 cm. The sides of the square $A'B'C'D'$ are 3 cm. As these are twice as long as the original, the scale factor for this enlargement is 2.

The sides of the square $A''B''C''D''$ are 6 cm, which is 4 times longer than the original square. So the scale factor for this enlargement is 4.

(b) To find the centre of enlargement draw lines through A, A' and A'', then repeat for B, B' and B'', C, C' and C'' and D, D' and D''.



These lines cross at the centre of enlargement as shown in the diagram.

Note

When the scale factor of an enlargement is a fraction, the size of the enlargement is reduced. The image of the original is then between the *centre of enlargement* and the original.

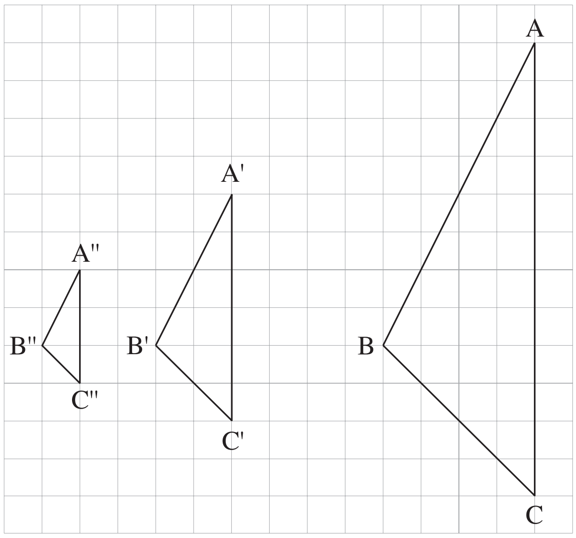
Worked Example 4

The diagram shows three triangles.

ABC was enlarged with different scale factors to give $A'B'C'$ and $A''B''C''$.

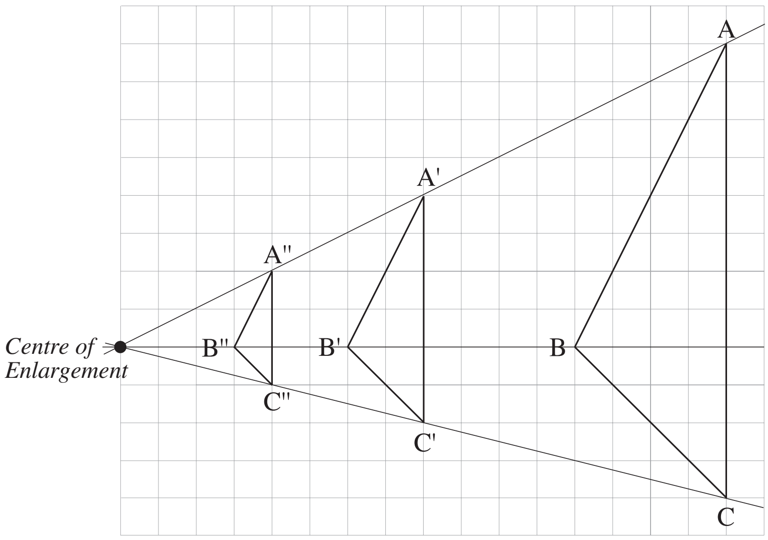
(a) Find the centre of enlargement.

(b) Find the scale factor for each enlargement.



Solution

(a) To find the centre of enlargement, lines should be drawn through the corresponding points on each figure.



(b) To find the scale factors, compare the lengths of sides in the different triangles. First consider triangles ABC and $A'B'C'$:

$$AC = 6 \text{ cm} \quad \text{and} \quad A'C' = 3 \text{ cm},$$

so $$A'C' = \frac{1}{2} \times AC$$

which means that the scale factor is $\frac{1}{2}$.

For triangles ABC and $A''B''C''$,

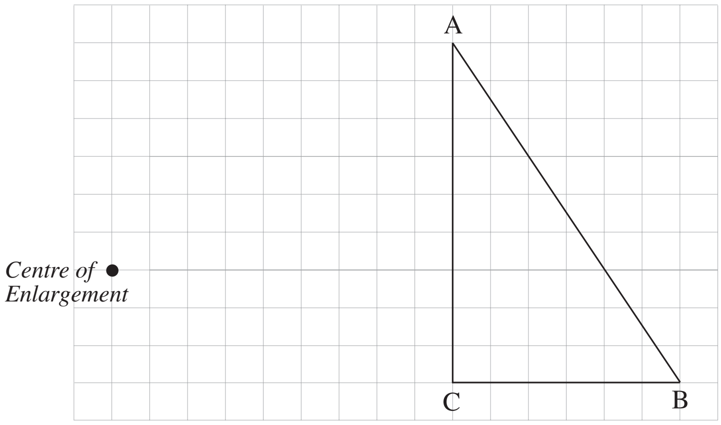
$$AC = 6 \text{ cm} \quad \text{and} \quad A''C'' = 1.5 \text{ cm},$$

so $$A''C'' = \frac{1}{4} \times AC$$

which means that the scale factor is $\frac{1}{4}$.

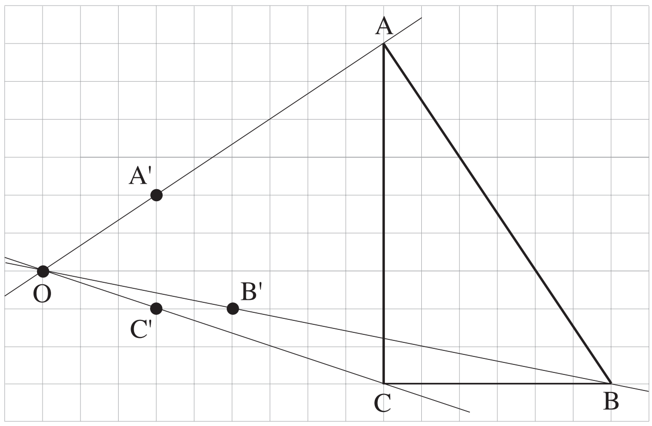
Worked Example 5

Enlarge the triangle shown with scale factor $\frac{1}{3}$ and centre of enlargement as shown.

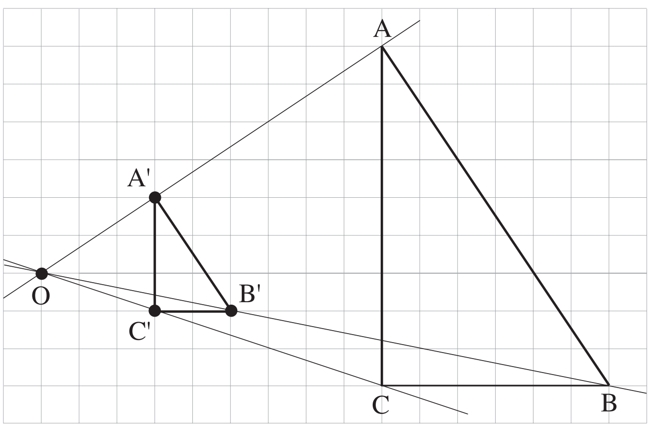


Solution

The first stage is to draw lines from each corner of the triangle through the centre of enlargement.



These points can then be joined to give the image.



Then the corners of the image should be fixed so that

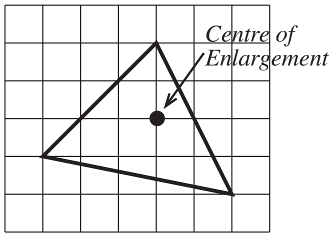
$$OA' = \frac{1}{3} \times OA$$

$$OB' = \frac{1}{3} \times OB$$

$$OC' = \frac{1}{3} \times OC$$

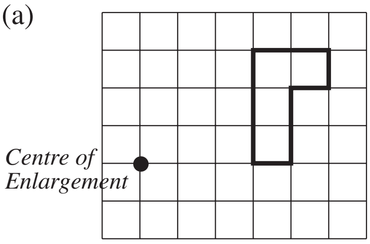
Exercises

1. Enlarge the triangle shown with scale factor 3 and the centre of enlargement shown.

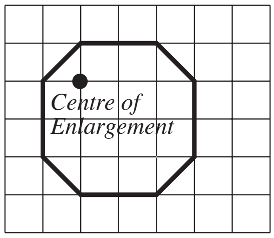


2. Copy the diagrams below on to squared paper. Enlarge each shape with scale factor 2 using the point marked as the centre of enlargement.

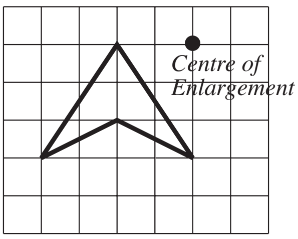
(a)



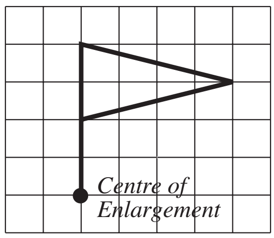
(b)



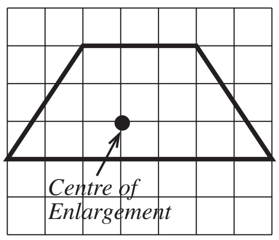
(c)



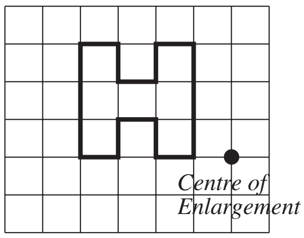
(d)



(e)

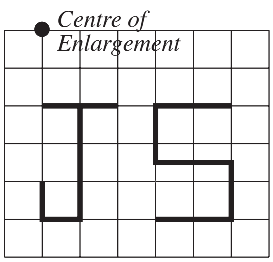


(f)



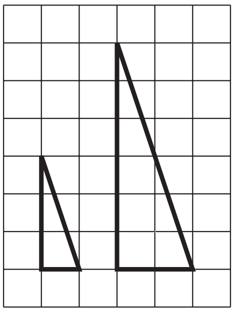
3. (a) A boy writes his initials as shown opposite. Use the marked centre of enlargement to enlarge his initials with scale factor 3.

   (b) Repeat part (a) using your own initials.

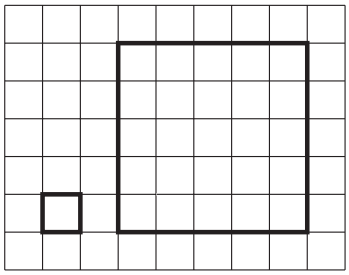


4. In each diagram below, the smaller shape has been enlarged to obtain the larger shape. For each example state the scale factor.

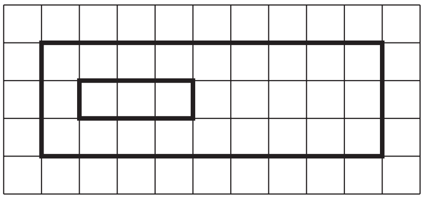
(a)



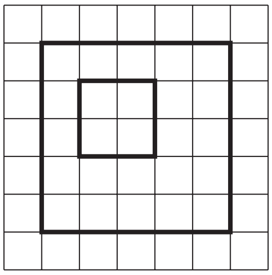
(b)



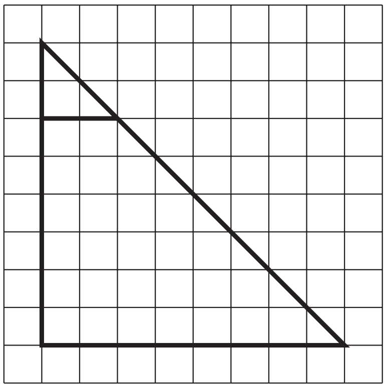
(c)



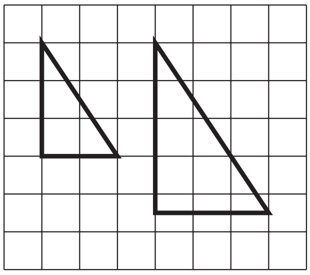
(d)



(e)

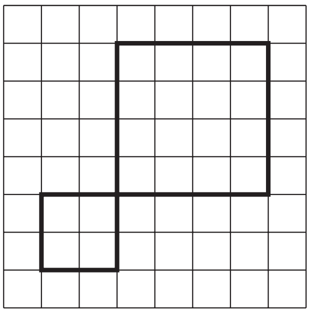


(f)

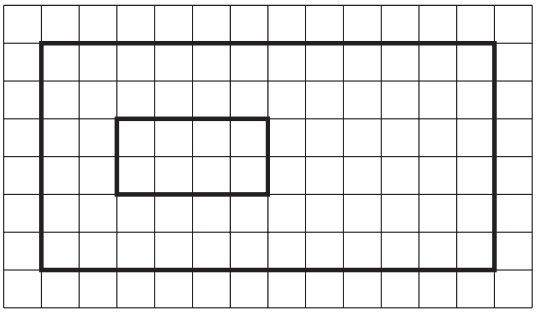


5. Copy each diagram on to squared paper. Then find the centre of enlargement and the scale factor when the smaller shape is enlarged to give the bigger shape.

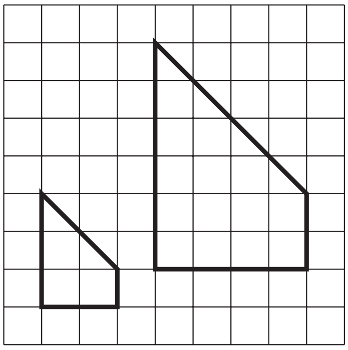
(a)



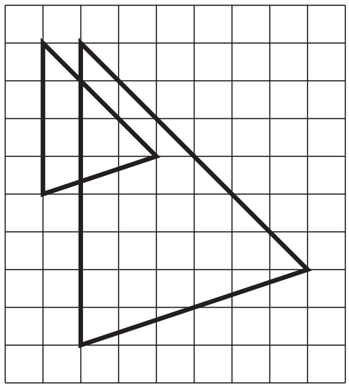
(b)



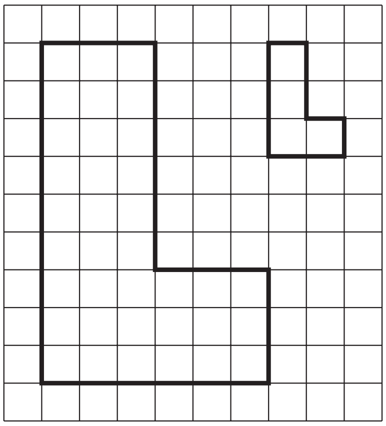
(c)



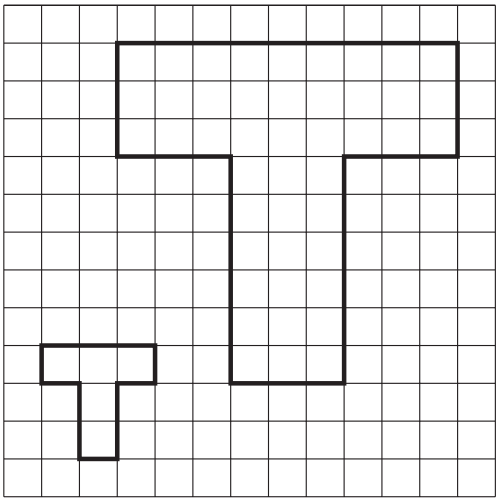
(d)



(e)

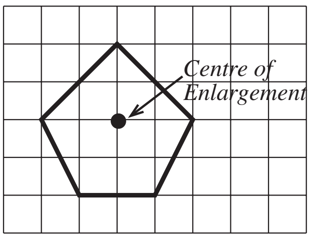


(f)

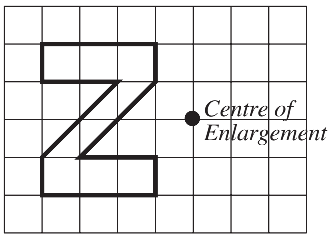


6. Copy each diagram below and enlarge it with scale factor 3.

(a)



(b)



7. (a) Draw a set of axes with x and y values from 0 to 15.

   (b) Plot the points $(2, 2)$, $(6, 2)$, $(6, 6)$ and $(2, 6)$, then join them to form a square.

   (c) Enlarge the square with scale factor 3, using the point with coordinates $(3, 3)$ as the centre of enlargement.

8. (a) Draw a circle with centre at $(4, 4)$ and radius 2.

   (b) Enlarge the circle with scale factor 2 and centre of enlargement $(4, 4)$.

   (c) Enlarge the circle with scale factor 3 and centre $(5, 5)$.

9. A triangle with vertices at the points with coordinates $(2, 1)$ $(7, 1)$ and $(7, 6)$ is enlarged to give the triangle with coordinates at the points $(6, 3)$, $(21, 3)$ and $(21, 18)$.

   (a) Draw both triangles.

   (b) What is the scale factor of the enlargement?

   (c) What are the coordinates of the centre of enlargement?

10. (a) On a set of axes draw a triangle with vertices at $(2, 0)$, $(4, 0)$ and $(3, 3)$.

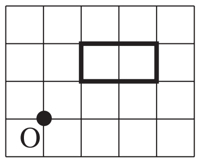
   (b) Enlarge the triangle with scale factor 2 using the point $(0, 0)$ as the centre of enlargement.

   (c) Write down the coordinates of both triangles. How do they compare?

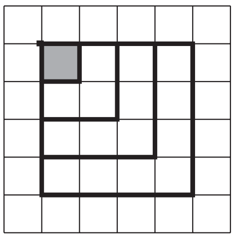
   (d) What would you expect to be the coordinates of your triangle if it were to be enlarged with scale factor 3 using $(0, 0)$ as the centre of enlargement?
   Check your answer by drawing the triangle.

   (e) Enlarge your original triangle with a different centre. Is there a simple relationship between the coordinates of the original and the enlargement, when the centre of enlargement is used?

11. Using the point O as the centre of enlargement, enlarge the rectangle with scale factor 3.



12.



The shaded square has sides of length 1 cm.
It is enlarged a number of times as shown.

   (a) Copy and complete the table below

.

Length of Side of Square	1 cm	2 cm	3 cm	4 cm
Perimeter of Square	4 cm	8 cm	12 cm	
Area of Square	1 cm^2	4 cm^2		16 cm^2

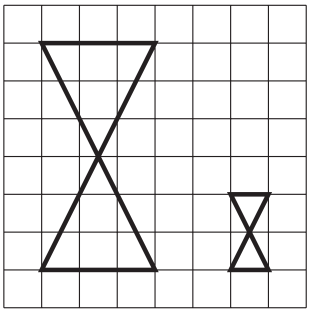
The shaded square continues to be enlarged.

   (b) Copy and complete the following table.

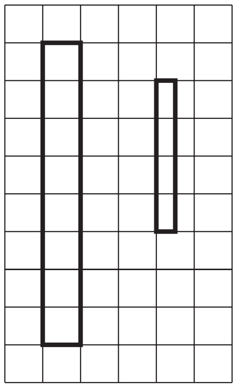
Length of Side of Square	
Perimeter of Square	
Area of Square	64 cm

13. For each pair of objects, state the scale factor of an enlargement which produces the smaller image from the larger one.

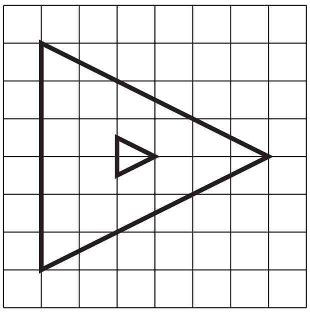
(a)



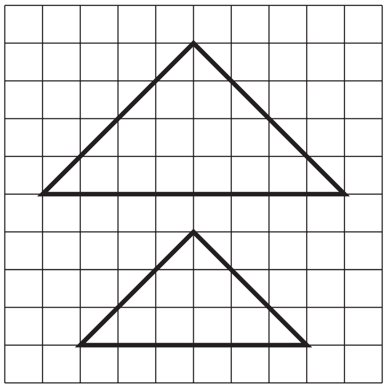
(b)



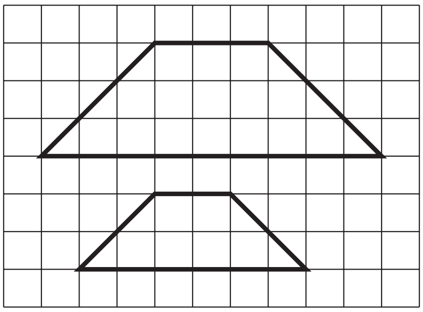
(c)



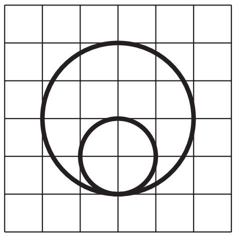
(d)



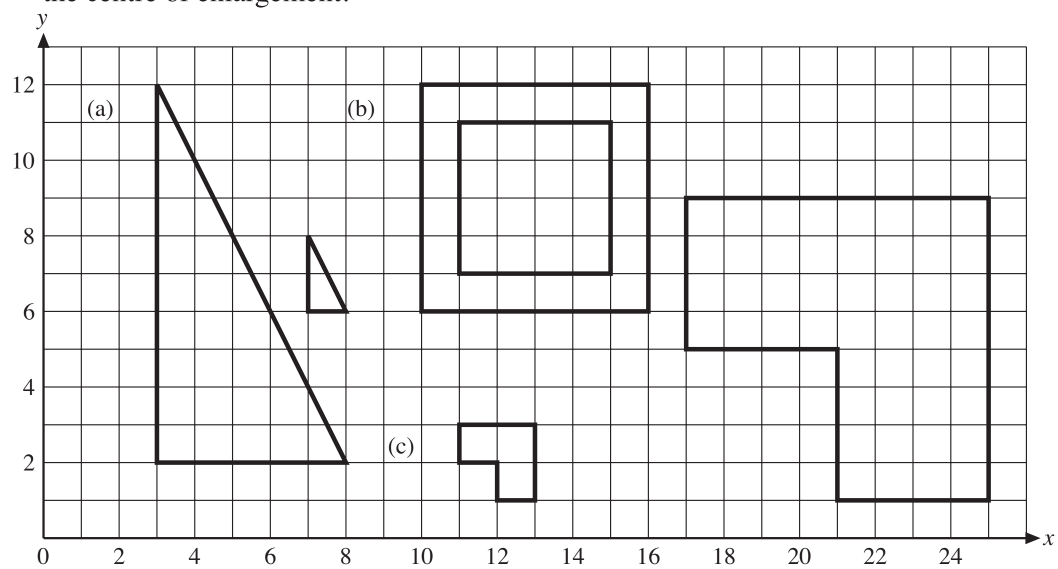
(e)



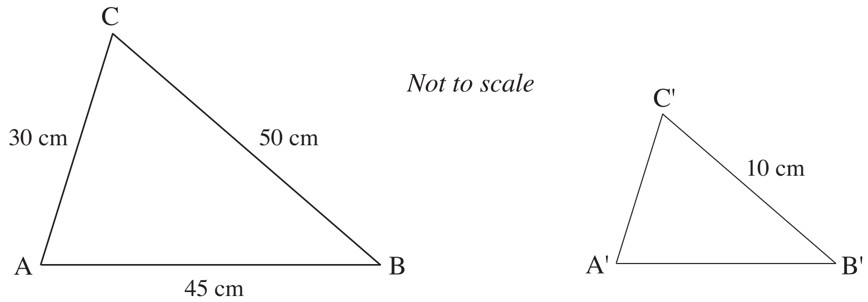
(f)



14. In each example below, the smaller shape has been obtained from the larger shape by an enlargement. For each example, state the scale factor and the coordinates of the centre of enlargement.



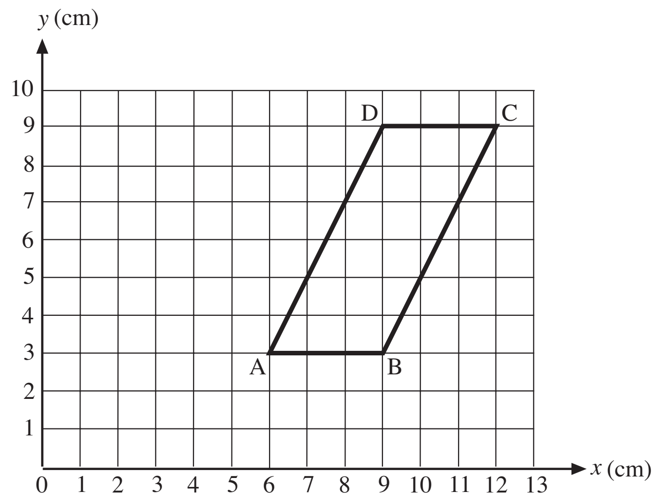
15. The larger triangle shown below is reduced in size by using a photocopier to give the smaller triangle.



(a) What is the scale factor of the enlargement which took place?

(b) What are the lengths of $A'C'$ and $B'C'$?

16. The parallelogram ABCD has vertices $(6, 3)$, $(9, 3)$, $(12, 9)$ and $(9, 9)$ respectively.



An enlargement scale factor $\frac{1}{3}$ and centre $(0, 0)$ transforms parallelogram ABCD onto parallelogram $A'B'C'D'$.

(a) (i) Draw the parallelogram $A'B'C'D'$.

(ii) Calculate the area of parallelogram $A'B'C'D'$.

(b) The side AB has length 3 cm. The original shape ABCD is now enlarged with a scale factor of $\frac{2}{5}$ to give $A''B''C''D''$.

Calculate the length of the side $A''B''$.

Investigation

Points X and Y lie on a straight line AB. Given that AX : XB = 1 : 2 and AY : YX = 2 : 3, write down the ratio AY : XB.