STRAND A: Computation

Unit 5 *Ratio and Proportion*

Student Text

Contents

5 Ratio and Proportion

5.1 Simple Ratios

Ratios are used in many situations to describe how two quantities are related. For example, a cake recipe requires twice as much flour as margarine. The ratio of flour to margarine is 2 : 1. Sometimes ratios can be simplified; for example, the ratio 100 : 50 is the same as the ratio 2 : 1. Ratios are simplified in a very similar way to fractions.

Worked Example 1

Simplify each of the following ratios.

(a) 48 : 12 (b) 27 : 9 (c) 35 : 49

Solution

(a) Both numbers in the ratio can be divided by 12. This gives

$$48 : 12 = 4 : 1$$

Alternatively, the ratio can be simplified in a number of steps.

$$\begin{aligned} 48 : 12 &= 24 : 6 \\ &= 12 : 3 \\ &= 4 : 1 \end{aligned}$$

(b) Here both numbers in the ratio can be divided by 9. This gives

$$27 : 9 = 3 : 1$$

Alternatively, the ratio can be simplified in steps to give

$$\begin{aligned} 27 : 9 &= 9 : 3 \\ &= 3 : 1 \end{aligned}$$

(c) Here both numbers can be divided by 7 to give

$$35 : 49 = 5 : 7$$

Worked Example 2

A school class contains 12 girls and 20 boys. Find the ratio of:

(a) girls to boys,

(b) boys to girls.

Solution

(a) The ratio of girls to boys is:

12 to 20 or 12 : 20

This can be simplified by dividing both numbers by 4, to give

$$12 : 20 = 3 : 5$$

(b) To find the ratio of boys to girls, reverse the ratio of girls to boys, to give 5 : 3.

Worked Example 3

A glass contains 300 cm^3 of drink. The drink is made by mixing 50 cm^3 of concentrate with water. Find the ratio of concentrate to water.

Solution

Amount of water $= 300 - 50$

$= 250\ cm^3$

The ratio of concentrate to water is

$$50 : 250$$

which simplifies to

$$1 : 5$$

Worked Example 4

The ratio of blue sweets to other coloured sweets in one packet is 1 : 12. How many sweets would there be in the packet if it contained:

(a) 3 blue sweets?

(b) 5 blue sweets?

Solution

The ratio of 1 : 12 means that for every blue sweet there are 12 sweets of other colours.

(a) This packet contains 3 blue sweets and $3 \times 12 = 36$ other sweets.
In total the packet contains $3 + 36 = 39$ sweets.

(b) This packet contains 5 blue sweets and $5 \times 12 = 60$ other sweets.
This give a total of 65 sweets.

Challenge!

A cashier of a bank was given one million one cent coins to count. How long will he take if he can count five coins in one second?

Exercises

1. Simplify each of the following ratios.

(a)	4 : 2	(b)	8 : 2	(c)	3 : 6
(d)	9 : 12	(e)	5 : 30	(f)	8 : 42
(g)	3 : 18	(h)	25 : 75	(i)	8 : 100
(j)	30 : 240	(k)	64 : 80	(l)	21 : 15
(m)	81 : 48	(n)	32 : 100	(o)	50 : 49
(p)	4.8 : 1.2	(q)	10.5 : 3.5	(r)	8.6 : 30.1

2. A school contains 300 girls and 320 boys. Find:
   (a) the ratio of girls to boys,
   (b) the ratio of boys to girls.

3. The shape of a room is a rectangle with sides of length 5 m and 3.5 m. Find the ratio of:
   (a) the length to the width,
   (b) the width to the length.

4. Two different bus companies in Miami have different pricing policies.
   For Company A on one route the adult fare is \$1.20 and the child fare is 40 cents.
   For Company B on a different route, the adult fare is \$1.40 and the child fare is 70 cents.
   (a) Find the ratio of the child fare to the adult fare for each company.
   (b) Which company gives children the better deal?

5. A library contains 720 non-fiction books and 400 fiction books.
   (a) Find the ratio of fiction books to non-fiction books.
   (b) Find the new ratio if 40 new fiction books are bought for the library.

6. A drink contains lemonade and 60 cm^3 of orange juice. Find the ratio of juice to lemonade in:
   (a) a 100 cm^3 drink,
   (b) a 300 cm^3 drink.

7. A car park contains 400 parking spaces. Of these spaces, 60 are outdoor and the rest under shade. Find the ratio of outdoor spaces to shaded spaces.

8. In a season a football team played 60 matches. They won 18, lost 20 and the rest were draws. Find the following ratios:

   (a) number of matches won to number of matches drawn,

   (b) number of matches won to other matches,

   (c) number of matches lost to number of matches won.

9. Orange syrup is mixed with water in the ratio 1 : 8, that is, 1 part orange syrup to 8 parts water. How much water is mixed with:

   (a) 100 cm^3 of syrup,

   (b) 20 cm^3 of syrup,

   (c) 5 cm^3 of syrup?

10. In a school the ratio of teachers to students is 1 : 20. If there are 12 teachers, how many students are there in the school?

11. In a class the ratio of left-handed students to right-handed pupils is 1 : 12. There are 2 left-handed students in the class. How many students are there in the class?

12. In packets of sweets, chocolate covered peanuts are mixed with solid chocolate sweets in the ratio 1 : 3. How many sweets are there in a packet that contains 20 chocolate covered peanuts?

13. In a herd of cattle, the ratio of bulls to cows is 2 : 25. How many cattle would be in the herd of it contained 10 bulls?

5.2 Proportion and Ratio

When solving problems that contain ratios like 4 : 5 or 3 : 7, it is often useful to write the ratios in the form $n : 1$ or $1 : m$.

So $4 : 5$ is equivalent to $1 : \frac{5}{4}$ or $\frac{4}{5} : 1$

and $3 : 7$ is equivalent to $1 : \frac{7}{3}$ or $\frac{3}{7} : 1$.

Worked Example 1

In a fruit drink, orange juice and pineapple juice are mixed in the ratio 3 : 7. Find how much pineapple juice would be mixed with 500 cm^3 of orange juice.

Solution

The ratio of orange juice to pineapple juice is

$$3 : 7 \text{ or } 1 : \frac{7}{3}$$

So for every 1 cm^3 of orange juice, $\frac{7}{3}$ cm^3 of pineapple juice is needed.

For 500 cm^3 of orange juice, the amount of pineapple juice needed is

$$500 \times \frac{7}{3} = 1166\frac{2}{3}\ cm^3$$

Worked Example 2

Alvin buys 8 m of wire netting to make a chicken coop. This costs him \$504.

Mark is also making a chicken coop. He needs 6.8 m of wire netting. How much will this cost?

Solution

The cost per metre of the wire netting is

$$\frac{J\$504}{8} = J\$63$$

$$= J\$63 \text{ per m}$$

Mark needs 6.8 m. This will cost

$$6.8 \times J\$63 = J\$428.40$$

$$= J\$428 \quad \text{(to the nearest J\$)}$$

This is equivalent to using the ratio 6.8 : 8; so solving this problem as a ratio, you take

$$\frac{6.8}{8} \times J\$504 = J\$428, \text{ to the nearest J\$ (as before).}$$

Worked Example 3

Three men are digging trenches to install cables to connect new houses to the electricity supply. Working together they can dig 12 m of trench each day.

(a) How long will it take 2 men to dig 120 m of trench?

(b) How many men will be needed to dig 80 m of trench in 2 days?

Solution

As 3 men dig 12 m each day, 1 man digs 4 m each day.

(a) As each man digs 4 m per day, 2 men dig 8 m per day. The time taken to dig 120 m is given by

$$\frac{120}{8} = 15 \text{ days}$$

Alternatively, 2 men can dig

$$\frac{2}{3} \times 12 = 8 \text{ m}$$

of trench each day, so it will take

$$\frac{120}{8} = 15 \text{ days}$$

to dig the trench.

(b) As each man digs 4 m per day, each man digs 8 m in 2 days, The number of men needed is given by

$$\frac{80}{8} = 10 \text{ men}$$

Exercises

1. The ratio of the length to width of a photograph is 3 : 2. The photograph is enlarged so that the length is 18 cm.

   What is the width of the enlarged photograph?

2. The ratio of flour to sugar in a recipe is 5 : 4.

   (a) How much flour should be mixed with 100 grams of sugar?

   (b) How much sugar should be mixed with 200 grams of flour?

3. The manager of a music store estimates that the ratio of sales of DVDs to CDs is 4 : 7.

   (a) In one day, 92 DVDs were sold. How many CDs were sold that day?

   (b) On another day, 84 CDs were sold. How many DVDs were sold that day?

4. A 6 m length of rope costs J$270. Find the cost of the following lengths of rope.

   (a) 1 m (b) 15 m (c) 22 m

5. A greengrocer sells sweet potatoes by the kilogram. She sells 5 kg of sweet potatoes for J$650. Find the cost of:

   (a) 7 kg of sweet potatoes,

   (b) 20 kg of sweet potatoes,

   (c) 2 kg of sweet potatoes.

6. Ribbon is sold from a roll. Rachel buys 3 m for J$171. Find the cost of each length of ribbon below.

   (a) 5 m (b) 12 m (c) 70 cm

7. A recipe uses 200 grams of flour to make 8 buns.

   (a) How much flour would be needed for 20 buns?

   (b) How many buns could be made with 325 grams of flour?

8. Ben bought 12 litres of gasoline for \$7.08.
   (a) How much would 40 litres of gasoline cost?
   (b) How much gasoline could be bought for \$10?

9. Paul adds the following spices to a recipe for 4 people.

   2 *teaspoons curry powder*

   1 *teaspoons paprika*

   $\frac{1}{2}$ *teaspoon garlic powder*

   How much of each ingredient should he use for a recipe for:
   (a) 8 people, (b) 10 people, (c) 3 people?

10. Mr Plummer employs 5 men who together can build a wall 12 m long in 3 days.
    (a) How long would it take the men to build a wall 20 m long?
    (b) How long would it take 3 men to build the 12 m wall?
    (c) If the 12 m wall must be built in 2 days, how many more men must Mr Plummer employ?

11. A school with 700 pupils employs 25 teachers. Use this ratio of teachers and pupils to answer the following questions.
    (a) How many teachers are needed for a school of 560 pupils?
    (b) How many pupils are there in a school with 22 teachers?

12. Two people can unload a truck containing 200 boxes in 5 hours.
    (a) If four people are to unload the truck instead of 2, how much time will be saved?
    (b) A larger truck contains more boxes. If 6 people can unload it in $2\frac{1}{2}$ hours, how many boxes does it contain?

13. Three photocopiers can produce 5400 copies in one hour.
    (a) How long would it take for one photocopier to make 16 200 copies?
    (b) How long would it take for two photocopiers to make 1200 copies?
    (c) How many photocopiers would be needed to make 3600 copies in 30 minutes?

14. A decorator find that 2 people can paint 80 m^2 per day. A new warehouse has 560 m^2 of walls to be painted. The decorator wants to complete the work in a number of whole days and does not want to employ more than 10 people to work on the job. How can the work be completed in the shortest time?

15. Two litres of fruit syrup cost J$170.

What is the cost of 5 litres of fruit syrup at the same price per litre?

16. Here is a list of ingredients for making some Greek food.

These amount make enough for 6 people.

2 cloves of garlic.

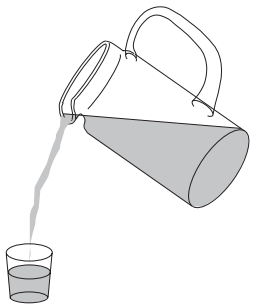
4 ounces of chick peas.

4 tablespoons of olive oil.

5 fluid ounces of Tahina paste.

Change the amount so that there will be enough for 9 people.

17. (a) An orange drink is made by mixing water with orange syrup.



$\frac{3}{4}$ of the orange drink is water.

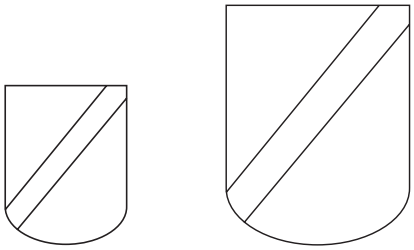
How many litres of water will be in 12 litres of orange drink?

(b) It takes 100 g of flour to make 15 shortbread biscuits.

(i) Calculate the weight of flour needed to make 24 shortbread biscuits.

(ii) How many shortbread biscuits can be made from 1 kg of flour?

18. A school badge is made in two sizes.



Not to scale

The width of the small size is 3 cm.

The large size is an enlargement of the small size in the ratio 2 : 3.

Calculate the width of the large size badge.

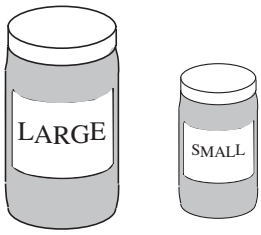
19. (a) A shopkeeper orders one hundred 60 g packets of peanuts.

What is the total weight of the order in kilograms?

(b) A 60 g packet of peanuts costs J$48.

Calculate the cost of an 80 g packet of peanuts at the same price per gram.

20. Guava jelly is sold in two sizes.



The large size costs J$288 and weighs 822 g.

The small size costs J$147 and weighs 454 g.

Which size is better value for money?

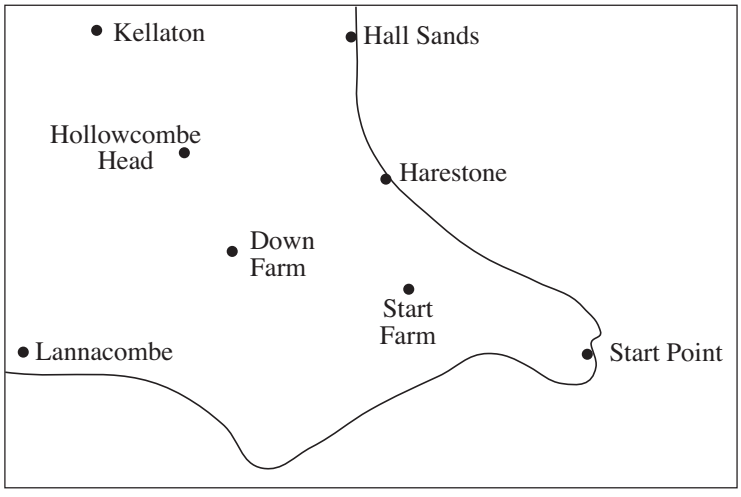
You **must** show all your working.

5.3 Map Scales and Ratios

Maps have a scale which relates the distance on the map to the corresponding distance on the ground. For example, on some ordnance survey maps the scale 1 : 50 000 is quoted on the cover. This means that a distance of 1 cm on the map corresponds to 50 000 cm, or 500 m, on the ground.

Worked Example 1

The map shown below has been drawn with a scale of 1 : 50 000*(There may be variations in distances on the diagram, due to printing.)*



Find the distances between:

(a) Down Farm and Start Farm,

(b) Kellaton and Harestone.

Give your answer in kilometres.

Solution

As the scale is 1 : 50 000, each centimetre on the map represents 50 000 cm in reality

(a) From the map the distance between Down Farm and Start Farm can be measured as 2.4 cm.

The actual distance between the two points is given by:

$$2.4 \times 50\,000 = 120\,000 \text{ cm}$$

$$= 1200 \text{ m}$$

$$= 1.2 \text{ km}$$

(b) The distance between Kellaton and Harestone can be measured as 4.3 cm.

The actual distance can then be calculated:

$$4.3 \times 50\,000 = 215\,000 \text{ cm}$$

$$= 2150 \text{ m}$$

$$= 2.15 \text{ km}$$

Worked Example 2

The distance between two places is 12 km. A map scale is 1 : 25 000. Find the distance between the two places on the map, in centimetres.

Solution

This map will use 1 cm for every 25 000 cm in reality

First convert 12 km to centimetres.

$$12 \text{ km} = 12\,000 \text{ m}$$

$$= 1\,200\,000 \text{ cm}$$

To find the distance on the map divide 1 200 000 by 25 000. This gives

$$\frac{1\,200\,000}{25\,000} = 48$$

So on the map the two places will be 48 cm apart.

Worked Example 3

Two places are 4.5 km apart. On a map they are 15 cm apart. What is the scale of the map?

Solution

First convert 4.5 km to cm, so that both distances are given in the same units.

$$4.5 \text{ km} = 4500 \text{ m}$$

$$= 450\,000 \text{ cm}$$

The scale can then be expressed as the ratio

$$15 : 450\ \ 000$$

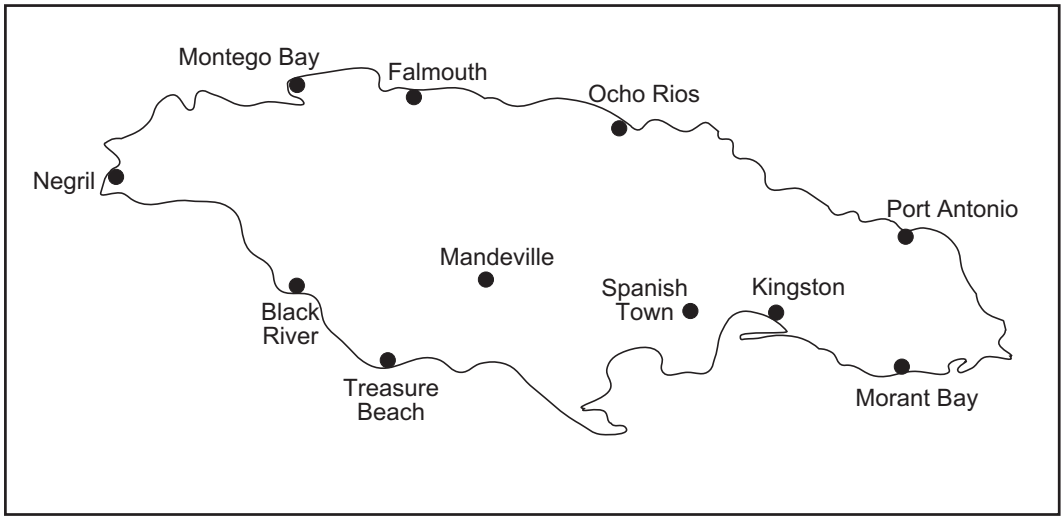
because 15 cm on the map represents 450 000 cm in reality This ratio can be simplified by dividing both numbers by 15 to give

$$1 : 30\ \ 000$$

Exercises

1. The sketch map below has a scale of 1 : 2 000 000*(There may be variations in distances on the map, due to printing.)*

   Use the map to find the distances between:



   (a) Montego Bay and Ocho Rios,

   (b) Kingston and Ocho Rios,

   (c) Negril and Montego Bay,

   (d) Falmouth and Morant Bay.

2. A map has a scale of 1 : 400 000. Copy and complete the table below which gives the distances between various towns.

Towns	*Distance on map*	*Actual distance*
Kingston and Montego Bay	34 cm	km
Negril and Spanish Town	37 cm	km
Mandeville and Falmouth	13 cm	km
Ocha Rios and Port Antonio	18 cm	km

3. On a map of Jamaica with a scale of 1 : 3 000 000 the distance between the towns of Kingston and Port Antonio, Portland is 1.4 cm. What is the actual distance between the two towns?

4. A ship is to sail from Kingston, Jamaica to Bridgetown, Barbados. On a map with a scale of 1 : 12 500 000, the distance between the two ports is 15 cm. Find the actual distance between the ports in kilometres.

5. The flying distance between Kingston and Havana is 810 km. What would be the distance between these two ports on maps with scales:

   (a) 1 : 3 000 000,

   (b) 1 : 1 000 000,

   (c) 1 : 500 000?

6. On a map with a scale of 1 : 3 000 000 the distance between the UK cities of Bristol and Bath is 0.6 cm. Find the actual distance between the two places and the distance between them on a map with a scale of 1 : 60 000.

7. On a map with a scale of 1 : 300 000 the distance between May Pen and Old Harbour is 5.5 cm.

   (a) Find the distance between the two towns in km.

   (b) How far apart would the two towns be on a map with a scale of 1 : 50 000?

8. Kingston and Spanish Town are 24 km apart by road. Find the scale of a map that represents this distance by:

   (a) 6 cm, (b) 60 cm,

   (c) 40 cm, (d) 5 cm.

9. A road atlas has maps on each page. The width of each page is 18 cm and this represents a distance of 63 km. Find the scale of the map.

   Some pages of the road atlas are photocopied and their size changed. Find the scale of the map that is produced if the widths of the pages are reduced to:

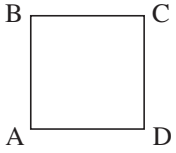
   (a) 9 cm,

   (b) 12 cm,

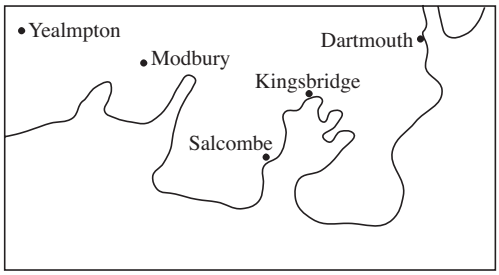
   (c) 16 cm.

10. A grid square on an ordnance survey map has sides of length 2 cm. The map has a scale of 1 : 5000.

    Four points are marked on the ground at the corners of a grid square. Find the actual distance between AC, correct to the nearest metre.



11. (a) Find the scale of the map shown below if the actual distance between Kingsbridge and Salcombe is 5.4 km.



(b) Find the actual distance between Modbury and Dartmouth.

12. On a map with a scale of 1 : 25 000 a plot of land is represented by a rectangle 1.5 cm by 1.2 cm. Find the area of the plot of land.

5.4 Proportional Division

Sometimes a quantity has to be divided in a certain ratio. For example, two waiters may divide their tips in the ratio 2 : 3 because one has worked longer than the other. If they had a total of \$5 in tips, one would get \$2 and the other \$3. If they had \$20 in tips, one would get \$8 and the other \$12.

Worked Example 1

Mary and Nikki earn \$285 by making curtains. Because Mary did more of the work they decide to divide the \$285 in the ratio 3 : 2. How much do they earn each?

Solution

This problem is solved by dividing the \$285 into 5 parts and giving 3 parts to Mary and 2 parts to Nikki. It is divided into 5 parts because the ratio is 3 : 2.

$$\frac{\$285}{5} = \$57$$

$$\text{Mary's share} = 3 \times \$57$$
$$= \$171$$

$$\text{Nikki's share} = 2 \times \$57$$
$$= \$114$$

Worked Example 2

Pineapple, orange and apple juices are mixed in the ratio 2 : 3 : 5 to make a new drink. Find the volume of each type of juice contained in 250 cm^3 of the new drink.

Solution

Adding the terms of the ratio gives

$$2 + 3 + 5 = 10$$

So the volume of the drink must be divided into 10 parts.

$$\frac{250}{10} = 25 \text{ cm}^3$$

Now the volume of each type of juice can be calculated.

Volume of pineapple juice $= 2 \times 25$

$= 50 \text{ cm}^3$

Volume of orange juice $= 3 \times 25$

$= 75 \text{ cm}^3$

Volume of apple juice $= 5 \times 25$

$= 125 \text{ cm}^3$

Exercises

1. The ratio of the volume of oxygen to nitrogen in the air is 1 : 4. Find the volume of oxygen and the volume of nitrogen in 10 litres of air.

2. The ratio of boys to girls in a school choir is 2 : 5. Find the numbers of boys and girls in the choir if there are 63 students in total.

3. Ben and Cheryll are given 140 stickers. They share them out in the ratio 4 : 3. How many stickers do they get each?

4. Andrea and Laura work as waitresses. Each week, Andrea works on 5 evenings and Laura on 4 evenings. They share any tips in the ratio 5 : 4 at the end of each week. How much do they get each if the total of tips for the week is:
   (a) J$1269,
   (b) J$3357,
   (c) J$2400? (Give the answer to the nearest dollar.)

5. In packs of *Fruit and Nut*, raisins and peanuts are mixed. The ratio of the weight of nuts to the weight of raisins is 5 : 3. Find the weight of nuts and the weight of raisins in:
   (a) 800 grams,
   (b) 200 grams,
   (c) 300 grams of the mix.

6. A football team arranges for J$100 000 to be divided among its players in the ratio of the goals scored. Goals are scored by the three players listed below.

Ricardo Smith	5
Ian Townsend	6
Fabian Allan	9

How much cash does each player get?

7. Hannah, Adam, Lucy and Jim are left $20 000 by a long lost relative.They divide the money in the ratio 4 : 3 : 2 : 1. How much does each of them get?

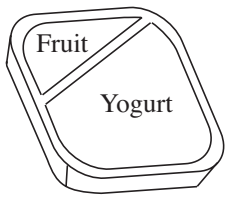
8. Ahmed and Afzal win a jar containing 500 sweets in a competition. They divide the sweets in the ratio 3 : 7. Ahmed shares his in the ratio 3 : 2 with his wife and Afzal shares his is the ratio 3 : 4 with his girlfriend.

How many sweets do Ahmed and Afzal have each?

9. Apples, bananas and oranges are mixed in the ratio 5 : 6 : 4 respectively by weight, to make a fruit salad. What weight of each type of fruit would be needed to make 6 kg of fruit salad?

10. A shoe shop sells a total of 240 pairs of shoes and sandals in one week. The ratio of sandals to shoes is 1 : 2. For the shoes, the ratio of high-heeled to flat is 7 : 1. For sandals, the ratio of flat to high-heeled is 1 : 3. Find the total number of flat shoes sold.

11. A fruit corner yogurt weighs 175 g altogether.



The ratio of the weight of fruit to the weight of yogurt is 2 : 5.

Calculate the weight of the fruit.