

UNIT 7 *Number System and Bases*

Introduction to SIM

Learning objectives

The work in this unit is focused on the classification of numbers into different sets. After completing this unit of STUDENT INSTRUCTIONAL MATERIAL (SIM) you should be able to

- classify real numbers as rational or irrational
- appreciate the difference and overlap between the sets:

R real numbers

Q rational numbers

Q^+ positive rational numbers

N natural (counting) numbers

Z integers

- convert base 10 numbers to/from Binary (base 2)
- add and subtract numbers expressed in binary form
- multiply numbers expressed in binary form
- convert base 10 numbers to/from any other base.

Key points and principles

- Rational numbers are of the form $\frac{m}{n}$ where m and n are integers and $n \neq 0$.
- Irrational numbers are numbers that are not rational, that is, they cannot be expressed in the form $\frac{m}{n}$ where m and n are integers and $n \neq 0$.
- Numbers can be expressed uniquely in, for example, binary, using just the digits 0 and 1.

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Facts to remember

- Any rational number can be written as $\frac{m}{n}$ when m and n are integers and $n \neq 0$; if m and n are co-prime, the number is in its *simplest form*.

For example, $\frac{8}{6} \left(= \frac{4}{3} \right)$, $5 \left(= \frac{5}{1} \right)$, $-\frac{7}{2}$, etc.

- Any real number that is not a rational number is called an irrational number.

For example, $\sqrt{2}$, $\sqrt{3}$, $\sqrt{5}$, $2\sqrt{2}$, π , 3π , etc.

- In Base 2, the only digits used are 0 and 1.
- In Base 3, the only digits used are 0, 1 and 2; in Base 4, digits 0, 1, 2, 3, and so on.

Glossary of terms

Real number The real number system consists of whole numbers, fractions, decimals and irrational numbers (see below). These numbers may be positive, zero, or negative.
The set of real numbers is denoted by $\mathbb{Q}$.

Rational number Any number of the form $\frac{m}{n}$ when n and m are integers and $n \neq 0$.
The set of rational number is denoted by $\mathbb{Q}$.

Irrational number Any real number that is not rational.

Natural (or counting) numbers these are 1, 2, 3, ...; the set of natural numbers is denoted by $\mathbb{N}$.

Binary number system The number system in which any number is expressed in terms of powers of 2.
That is

$$\dots, 2^4, 2^3, 2^2, 2^1, 2^0 (= 1), 2^{-1}, 2^{-2}, \dots \text{ etc.}$$

The binary number system uses only the digits 0 and 1; for example,

$$101 = 1 \times 2^2 + 0 \times 2^1 + 1 \times 2^0 = 5$$

Integer The positive and negative whole numbers and zero; for example,

$$\dots, -3, -2, -1, 0, 1, 2, 3, \dots,$$

The set of integers is denoted by $\mathbb{Z}$.

Positive natural numbers the set of positive rational numbers is denoted by $\mathbb{Q}^+$.