

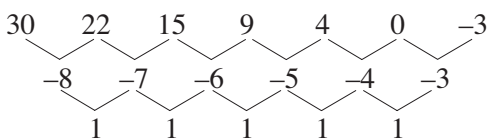
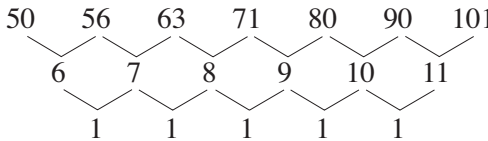
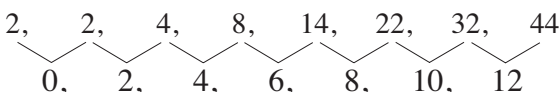
Student Text *UNIT 8 Number Sequences*

Answers

8.1 Simple Number Patterns

1. (a) 11, 13, 15, 17 (b) 24, 28, 32, 36 (c) 30, 35, 40, 45
 (d) 42, 49, 56, 63 (e) 54, 63, 72, 81 (f) 36, 42, 48, 54
 (g) 60, 70, 80, 90 (h) 66, 77, 88, 99 (i) 48, 56, 64, 72
 (j) 120, 140, 160, 180 (k) 90, 105, 120, 135 (l) 300, 350, 400, 450
2. (a) +3; 20, 23 (b) +8; 42, 50 (c) +5; 32, 37 (d) +11; 61, 72
 (e) +7; 43, 50 (f) $+\frac{1}{2}$; 5, $5\frac{1}{2}$ (g) +9; 49, 58 (h) -3; 11, 8
 (i) -4; 0, -4 (j) -4; -2, -6 (k) -3; -4, -7 (l) -3; -20, -23
3. (a) (i) 22 (ii) 242 (iii) 2442 (iv) 24442 (v) 244442
 (b) (i) 1089 (ii) 10989 (iii) 109989 (iv) 1099989 (v) 10999989
 (c) (i) 968 (ii) 9768 (iii) 97768 (iv) 977768 (v) 9777768
 (d) (i) 63 (ii) 693 (iii) 6993 (iv) 69993 (v) 6999993
4. (a) (i) $11 \times 11 \times 11 = 1331$, $11 \times 11 \times 11 \times 11 = 14641$
 (ii) Each one is a symmetric number (these are found in Pascal's triangle).
 (b) $11 \times 11 \times 11 \times 11 \times 11 = 11^5 = 161051$. This is *not* a symmetric number.
5. (a) (i) 5, 4, 3, 2, 1 (ii) 4, 5, 6, 7, 8
 (iii) In (i), the digits *increase* by 1 each time; in (ii), the numbers *decrease* by 1 each time.
 (b) The numbers sum to 9.

8.2 Recognising Number Patterns

- (d) 0, -3;
- 
- (e) 90, 101;
- 
- (f) 32, 42;
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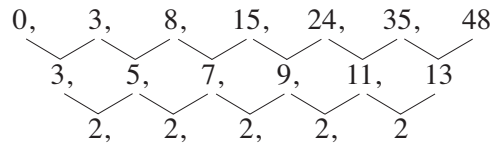
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Answers

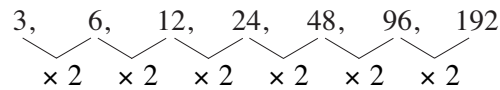
8.2

(g) 31, 36; +5

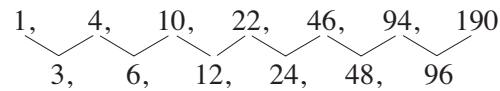
(h) 35, 48;



(i) 96, 192;

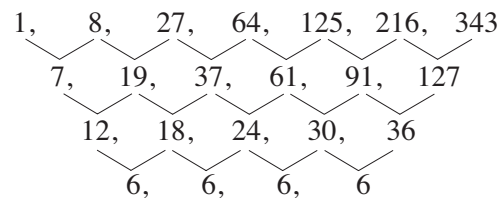
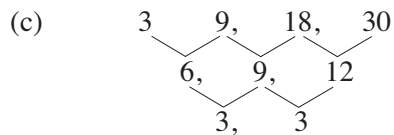


(j) 94, 190;

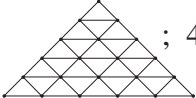
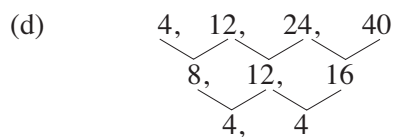


(k) -71, -101; -25, -30

2. 216, 343;

3. (a) (i) 3, 5, 7, 9 (ii) +2 (iii)  ; 11(b) 4, 7, 10, 13 (ii) +3 (iii)  ; 16

(ii) +15

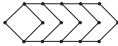
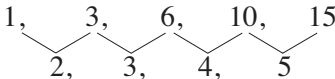
(iii)  ; 45

(ii) +20

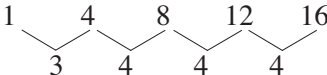
(iii)  ; 60

(e) 6, 10, 14, 18

(ii) +4

(iii)  ; 224. (a) 

+1 to difference to get next number; 21

(b) If 3 is ignored, the sequence is constant,
+4 for next number; 20

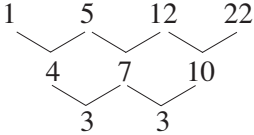
Student Text *UNIT 8 Number Sequences*

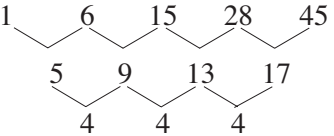
Answers

8.2

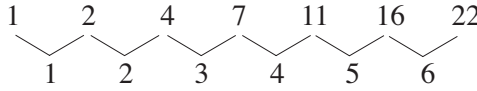
(c) 5, 8, 11, 14, 17 +3 for next number; 20

(d) 4, 6, 8, 10 +2 for next number; 12

(e)  +3 for next difference; 35

(f)  +4 for next difference; 66

5. (a) $1^2, 2^2, 3^2, 4^2, \dots$ (b) (i) $_n = n^2 + 2$ (ii) $_n = n^2 - 2$ (iii) $_n = 2n^2$ (iv) $_n = 4n^2$ 6. (a) (ii) $_n = (i)_n + 3$ (iii) $_n = (i)_n - 2$ (b) (i) $13 + 21 = 34$ (iii) $16 + 24 - 3 = 37$ (iii) $11 + 19 + 2 = 32$

7. Circle 8 

8. (a) 35, 42 (b) The next term is obtained by adding 7 to the last term.

9. (a) 41, 122 (b) 125, 216

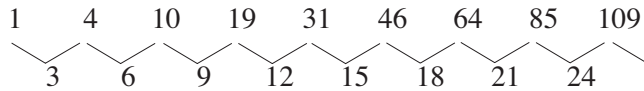
10. (a) $p = 15$ (b) $q = 52 + 15 = 67$

11. (a) It is increasing by 4. (b) (i) Sequence of square numbers (ii) 49
(iii) 47; minus 2

12. (a) (i) 5, 10, 15, 20, 25, 30, 35, 40 (ii) 5, 0, 5, 0, ..., alternate

(b) Terms are the positive multiples of 3.

(c) $9 \rightarrow 9 + 1 = 10, \quad 10 \times 2 = 20 \rightarrow 20$ $12 \rightarrow 12 + 1 = 13, \quad 13 \times 2 = 26 \rightarrow 26$ $15 \rightarrow 15 + 1 = 16, \quad 16 \times 2 = 32 \rightarrow 32$ $18 \rightarrow 18 + 1 = 19, \quad 19 \times 2 = 38 \rightarrow 38$

(d) 85, 109 

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Answers

8.3 Extending Number Patterns

- 40; 80
 - 50; 100
 - 101; 201
 - 25; 45
 - 41; 81
 - 11; 1
 - 4; -64
 - 49; 79
 - 1; -11
 - 32; 72
 - 52; 102
 - 42; -92
- $10^2 + 2 = 102$
 - $4 + 10 \times 1 = 14$ or $5 + 9 \times 1 = 14$
 - 116
 - 1428
 - 88
- 99
 - 41
 - 4059
- 90
 - 990
- 28 dots
 - 40 dots
 - 60 dots
- 17 sticks
 - Pattern 58
 - 401 sticks
 - n th pattern needs $(4n + 1)$ sticks

8.4 Formulae for Number Sequences

- 5, 9, 13, 17, 21, 25
 - 2, 3, 8, 13, 18, 23
 - 12, 22, 32, 42, 52, 62
 - 0, 3, 8, 15, 24, 35
 - 3, 9, 19, 33, 51, 73
 - 2, 4, 8, 16, 32, 64
- $u_n = n^3 - 1$
 - $u_n = 3n + 5$
 - $u_n = 8n - 2$
 - $u_n = n^2 + 1$
- 80
 - 10
 - 87
 - 0
 - 396
 - $\frac{1}{2}$
- Because the difference sequence is 7, 7, 7, ...,
 - $u_n = 7n + 1$
- $u_n = 5n - 1$
 - $u_n = 3n + 8$
 - $u_n = 6n - 8$
 - $u_n = -2n + 102$
 - $u_n = \frac{1}{2}n + \frac{1}{2}$
 - $u_n = -7n + 12$
 - $u_n = -\frac{1}{2}n + 10\frac{1}{2}$
- $u_n = 0.1n + 0.9$
 - $u_n = 4n - 1$
- 1, 4, 9, 16, 25, 36
 - $u_n = n^2 + 2$
 - $u_n = n^2 - 5$
 - $u_n = 2n^2$
 - $u_n = 2n^2 - 2$
 - $u_n = -n^2 + 100$
- $u_n = n^3 - 1$
 - $u_n = n^3 + 9$
 - $u_n = -n^3 + 200$
- $u_n = 4n + 10$; 14, 18, 22, 26, 30

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8.4

10. $u_n = 6n + 8$; 68

11. $u_n = 5n + 2$; 7

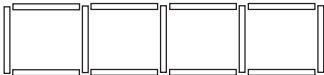
12. (a) $u_8 = 29$, add 4 (b) $n = 100$; 100th term is 397

13. $u_n = 4n + 3$

14. (b)

<i>Number of enclosures</i>	4	5	6	7	8
<i>Number of posts</i>	15	18	21	24	27

(c) 63 posts (d) $u_n = 3n + 3$

15. (a) (i)  (ii) 13 sticks (b) 3 more sticks

(c) number = $3s + 1$

16. (a) (i) add 5 (ii) $u_n = 5n + 2$

(b) area = $n(n + 1) = n^2 + n$

8.5 General Laws

1. (a) $u_n = \frac{n^2}{n + 9}$ (b) $u_n = \frac{2n + 3}{n + 1}$ (c) $u_n = 2 \times 3^{n-1}$ (d) $u_n = 0.9^{n-1}$

(e) $u_n = 1.2^{n-1}$ (f) $u_n = \frac{2}{3^{n-1}}$ (g) $u_n = 2^n$ (h) $u_n = 2^n + 1$

(i) $u_n = \frac{3n}{n + 3}$

(b), (d), (f), (i) converge; (a), (c), (e), (g), (h) diverge;

$$\begin{array}{cccc} \downarrow & \downarrow & \downarrow & \downarrow \\ 2 & 0 & 0 & 3 \end{array}$$

2. (a) $u_1 = 8$; $u_{n+1} = u_n - 3$ (b) $u_1 = 5$; $u_{n+1} = 4u_n$

(c) $u_1 = 2$; $u_{n+1} = 2u_n - 1$ (d) $u_1 = 4000$; $u_{n+1} = \frac{u_n}{2}$

(e) $u_1 = 3$, $u_2 = 3$; $u_{n+2} = u_n + u_{n+1}$

(f) $u_1 = 1$, $u_2 = 1$, $u_3 = 1$
 $u_{n+3} = u_n + u_{n+1} + u_{n+2}$

3. (a) 1, 3.5, 2.6071, 2.4543, 2.4495 (b) $\sqrt{6} \approx 2.449489743$

4. Yes; converges to 3. If u_1 is a different value it still converges to 3.

5. 0.1, 0.13, 0.1417, 0.14284777, 0.142857142; $\frac{1}{7}$

Student Text *UNIT 8 Number Sequences*

Answers

8.5

6. $\frac{1}{3}$; 0.5, 0.25, 0.3125, 0.33203125

7. (a) (i) $40 = 5 \times 8 = 5 \times (5 + 3)$ (ii) $n(n + 3)$

(b) (i) $40 = 6 \times 7 - 2$ (ii) $(n + 1)(n + 2) - 2$

(c) $(n + 1)(n + 2) - 2 = n^2 + 3n + 2 - 2 = n^2 + 3n = n(n + 3)$

8. (a) 64 (b) $u_n = 2^{n-1}$ (c) $u_{11} = 1024$, $u_{10} = \frac{1024}{2} = 512$

9. (a) Row 4: 13 15 17 19 Sum = $64 = 4^3$

(b) Row 10 $\rightarrow 10^3 = 1000$ (c) Row 20 $\rightarrow 20^3 = 8000$ (d) $x + 2$