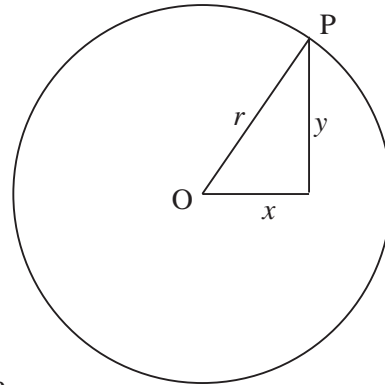


13 Graphs

13.11B Circles and Their Equations

In this section, we focus on loci associated with circles centred on the origin, their algebraic equations in particular.

The diagram shows a circle with the origin $(0, 0)$ as its centre and with radius r . $P(x, y)$ is a point on the circumference of the circle, so the length OP is equal to the radius r .



In Section 13.2D you learned how to use Pythagoras' Theorem to calculate the length of the line segment joining two points. Using Pythagoras' Theorem gives the standard equation

$$x^2 + y^2 = r^2$$

for the circle with centre the origin and radius r .

For example, the equation $x^2 + y^2 = 100$ represents a circle with radius 10 units.

The equation $x^2 + y^2 = 36$ represents a circle with radius 6 units. Both these circles have the origin as their centre.



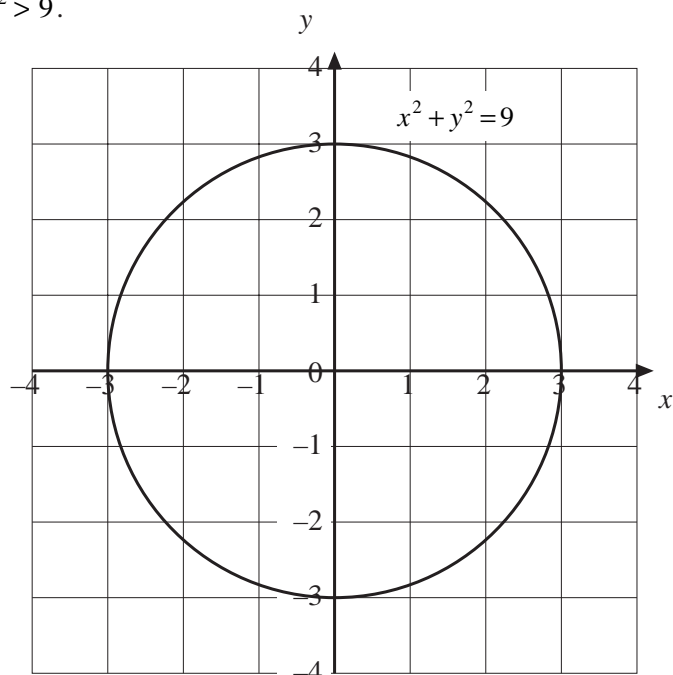
Worked Example 1

- Draw a graph showing the locus $x^2 + y^2 = 9$.
- Describe the region $x^2 + y^2 > 9$.



Solution

- The locus $x^2 + y^2 = 9$ represents a circle with centre the origin and radius 3 units. This has been drawn using a pair of compasses.
- The region $x^2 + y^2 > 9$ is the area outside the circle.





Worked Example 2

Calculate the area between the circles $x^2 + y^2 = 4$ and $x^2 + y^2 = 64$.

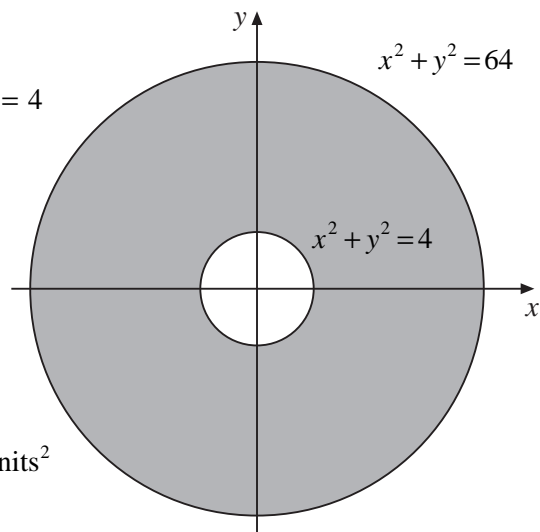


Solution

The first circle, $x^2 + y^2 = 4$, has radius 2 units and the second circle, $x^2 + y^2 = 64$, has radius 8 units.

The area of the small circle is $\pi \times 2^2 = 4\pi$
 $\approx 12.6 \text{ units}^2$

The area of the large circle is $\pi \times 8^2 = 64\pi$
 $\approx 201.1 \text{ units}^2$



Since the smaller circle lies inside the larger one, the area between the circles is

$$64\pi - 4\pi = 60\pi \text{ units}^2$$

$$\approx 188.5 \text{ units}^2 \text{ (to 1 d.p.)}$$



Note

The shaded area between the circles in Worked Example 1 can be represented by the inequality $4 < x^2 + y^2 < 64$.

If the two circles are included then the inequality is $4 \leq x^2 + y^2 \leq 64$.

The equation $x^2 + y^2 = r^2$ represents a circle with radius r and centre the origin.

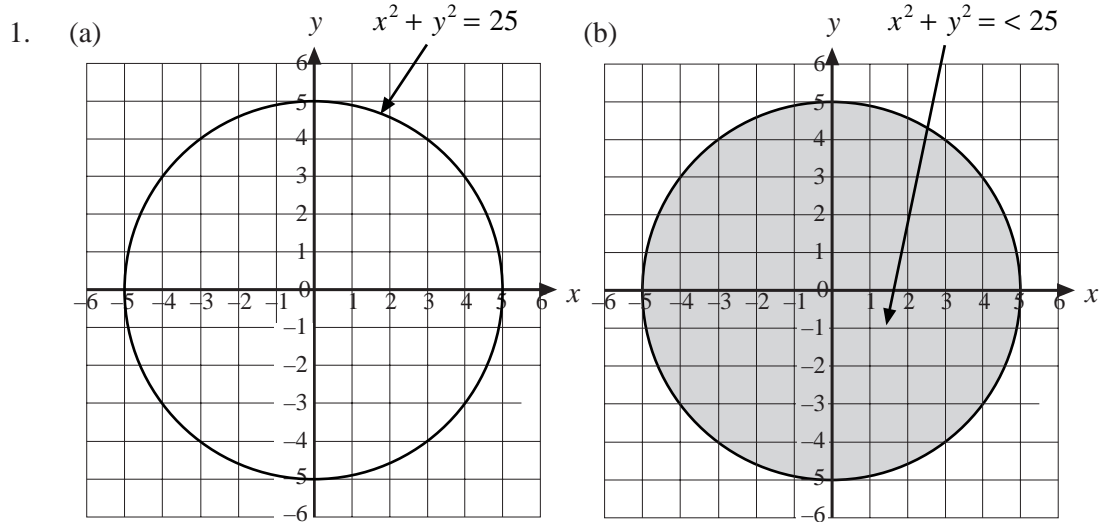


Exercises

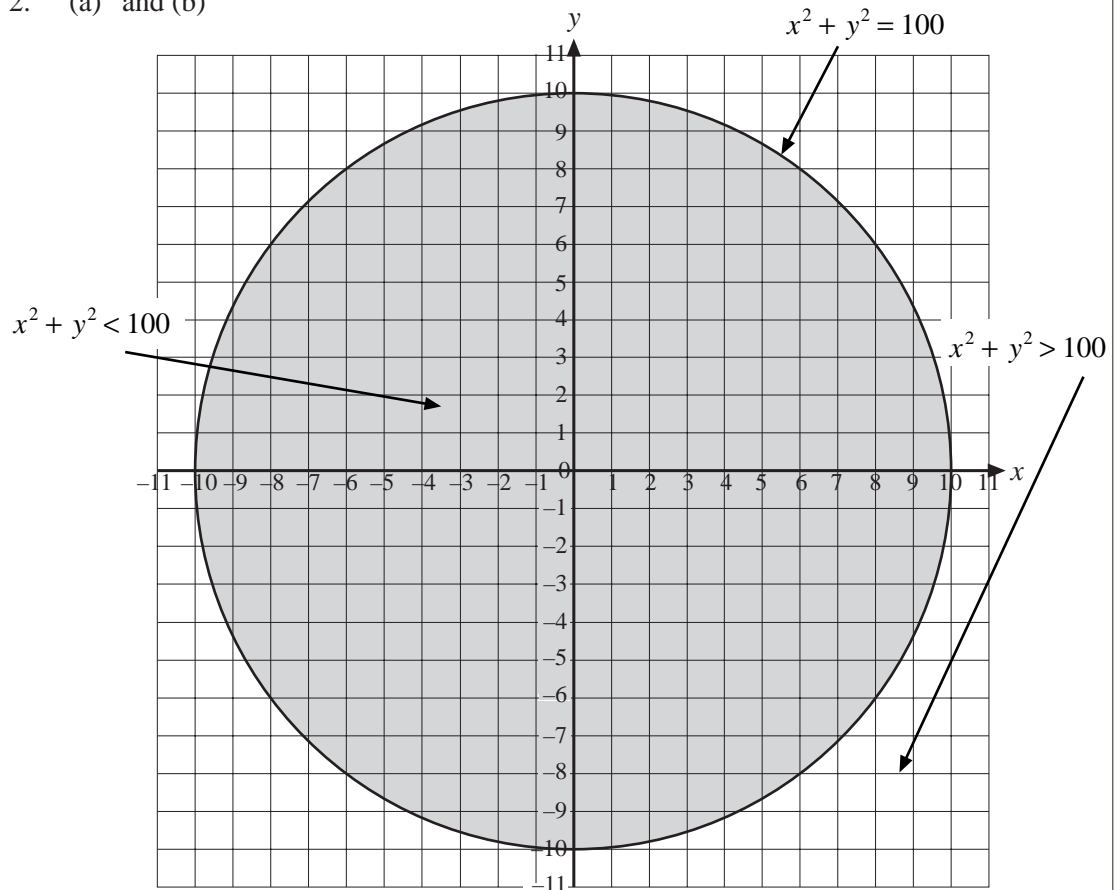
- Draw a diagram to show the locus $x^2 + y^2 = 25$.
 - On the diagram, shade the region $x^2 + y^2 < 25$.
- Draw a diagram to show the locus $x^2 + y^2 = 100$.
 - On the diagram, shade the region $x^2 + y^2 > 100$.
- Draw a diagram to show the loci $x^2 + y^2 = 16$ and $x^2 + y^2 = 36$.
 - On the diagram, shade the region $16 < x^2 + y^2 < 36$.
- Draw centrally-placed x - and y -axes and scale them using the ranges $-10 \leq x \leq 10$, $-10 \leq y \leq 10$.
 - Accurately construct the locus $36 \leq x^2 + y^2 \leq 81$ and calculate its area, leaving your answer as a multiple of π .

Answers

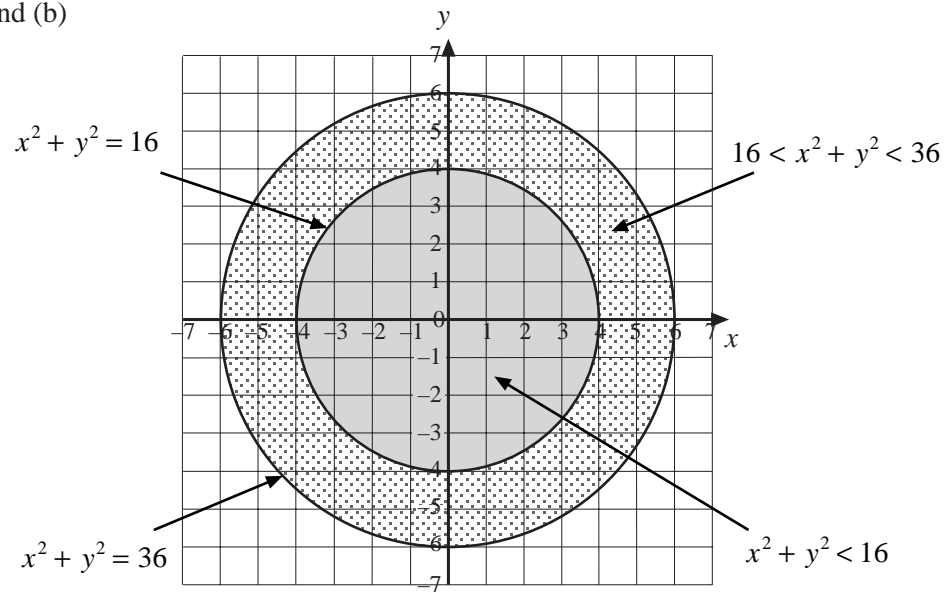
13.11B Circles and Their Equations



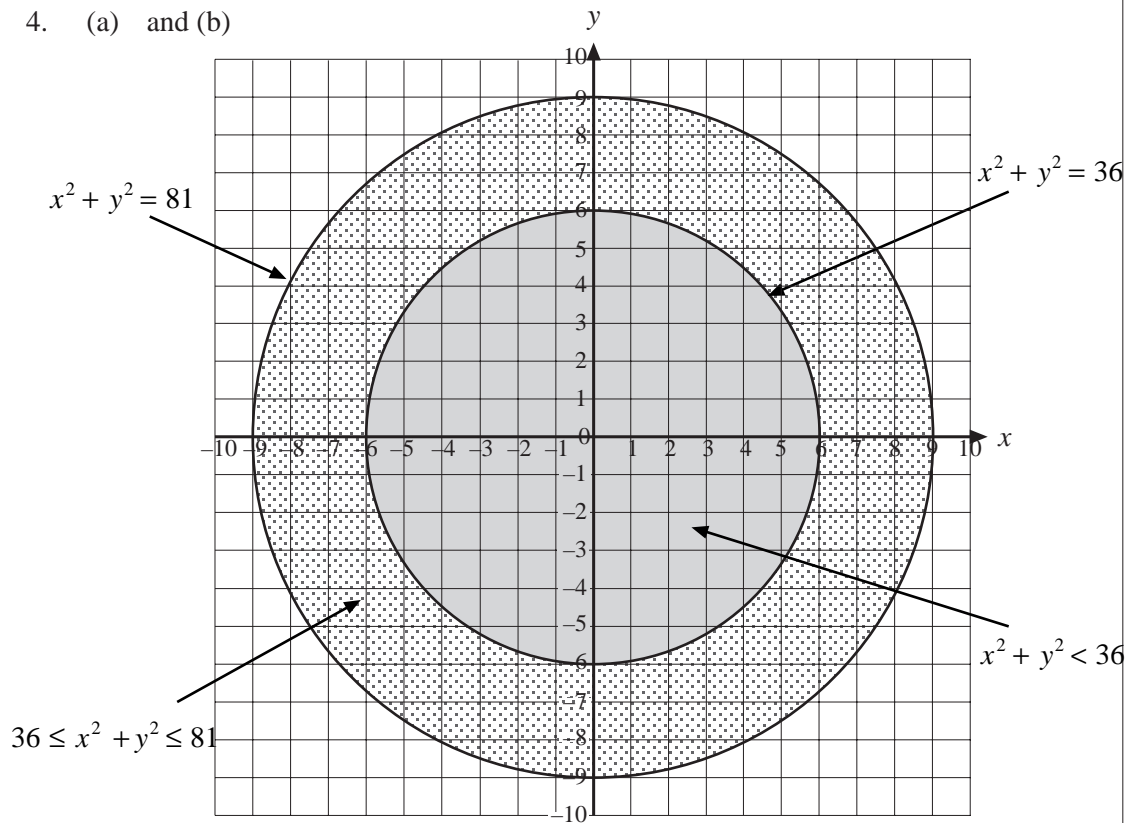
2. (a) and (b)



3. (a) and (b)



4. (a) and (b)



The area of the locus $36 \leq x^2 + y^2 \leq 81$ is 45π units².