UNIT 4 *Trigonometry* Activities

Activities

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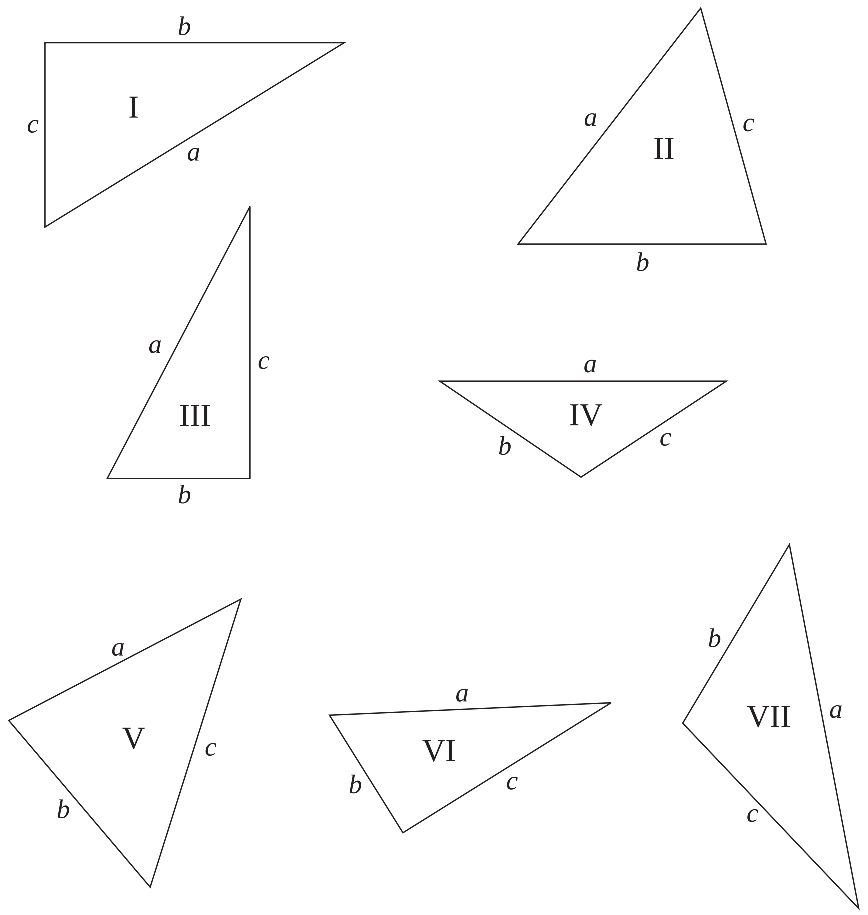
ACTIVITY 4.1 *Pythagoras' Theorem*

For each of the triangles below, measure the angles and the sides of each triangle. Measure the sides in cm, to one decimal place. Record your answers in the table.



Triangle	Right Angled?	a	b	c	a^2	b^2	c^2	$a^2 - b^2 - c^2$
I								
II								
III								
IV								
V								
VI								
VII								

What can you infer from the final column of results?

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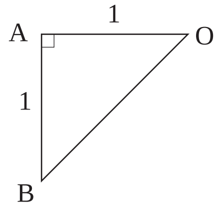
ACTIVITY 4.2 *Spirals*

Spirals of various kinds are found throughout nature in both plants and animals. Examples include *Snail's shell* *Sheep's horns* *Seashells* *Fossils.*

We will show here how to construct spirals and how they can be used.

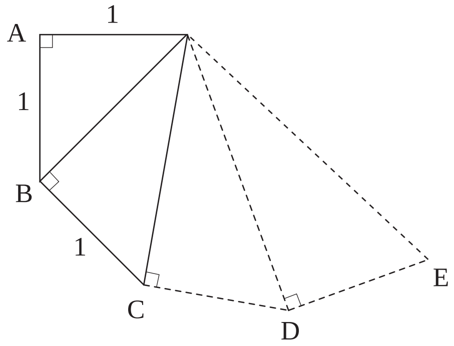
Geometric Construction

Step 1 Construct an isosceles triangle with two sides, OA and AB, of unit length.



Step 2 Add another line, BC, of unit length, perpendicular to the longest side, OB, of the triangle and complete the next triangle by drawing OC.

Step 3 Continue in this way, each time adding a unit length.

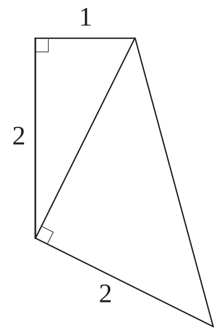


1. Draw the spiral in this way with 15 linked sides. What is the length of

   (a) OB (b) OC (c) OD? (Use Pythagoras' Theorem.)

2. From your construction, estimate the value of $\sqrt{14}$. Check your accuracy using a calculator.

3. Repeat the construction, starting with a right angled triangle as shown, adding on sides of length 2 units.
   How many sides do you need to construct in order to estimate $\sqrt{33}$?

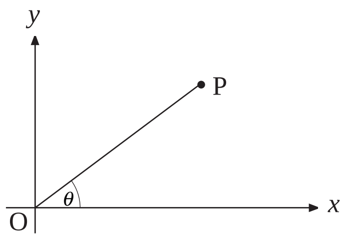


Extension

As it consists of straight lines, the construction used above produces only an *approximation* to a true spiral. We can, though, obtain a true spiral using the equation

$$r = \left(\frac{\theta}{360}\right)$$

Here *r* is the distance of the point P from the origin, where OP makes an angle θ with the *x* axis. Plot the points which correspond to $\theta = 0°,\ 45°,\ 90°, \ldots\ldots, 360°$, and join up the points with a smooth curve.
Continue the curve by plotting points corresponding to $\theta = 405°,\ 450°, \ldots\ldots,$ etc.



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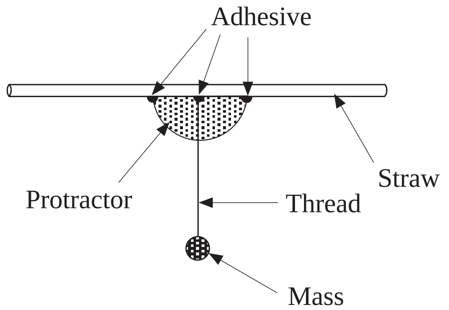
ACTIVITY 4.3 *Clinometers*

A *clinometer* is an instrument for measuring the angle of an incline (slope).

To make your clinometer, you need a drinking straw, a protractor, some adhesive, a small mass and a piece of thread.

Use adhesive to attach the straw to the protractor. The thread with a mass at its end will serve as a plumb line.

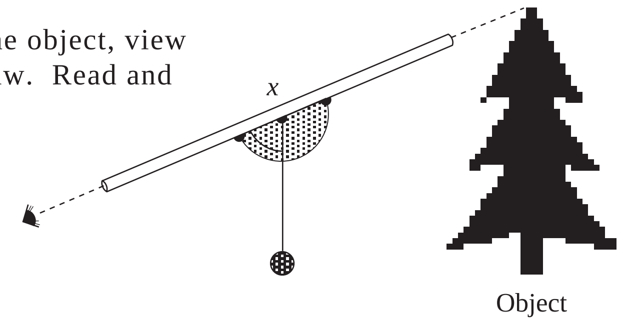


You can now use your clinometer to find the height of an object such as a tree or a building.

Follow this method.

1. Choose your object.

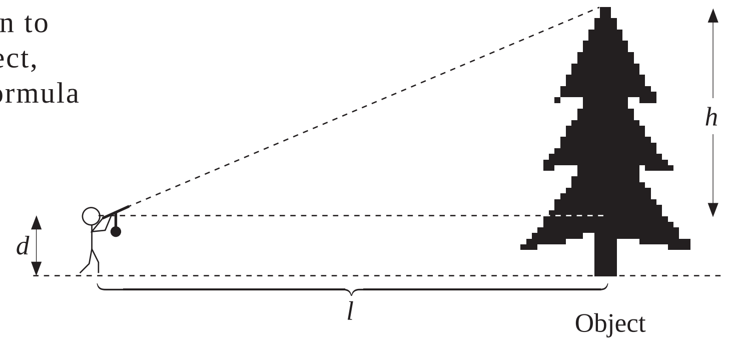
2. Standing some distance away from the object, view the top of it through the drinking straw. Read and record the value of angle x.



3. Measure and record your distance, l, from the object.

4. Use the readings you have taken to calculate the height of the object, above your eyeline, from the formula

$$\tan x = \frac{l}{h} \Rightarrow h = \frac{l}{\tan x}$$



5. The height, H, of the object is now given by

$$H = \frac{l}{\tan x} + d$$

when d is the height of your eyes from the ground.

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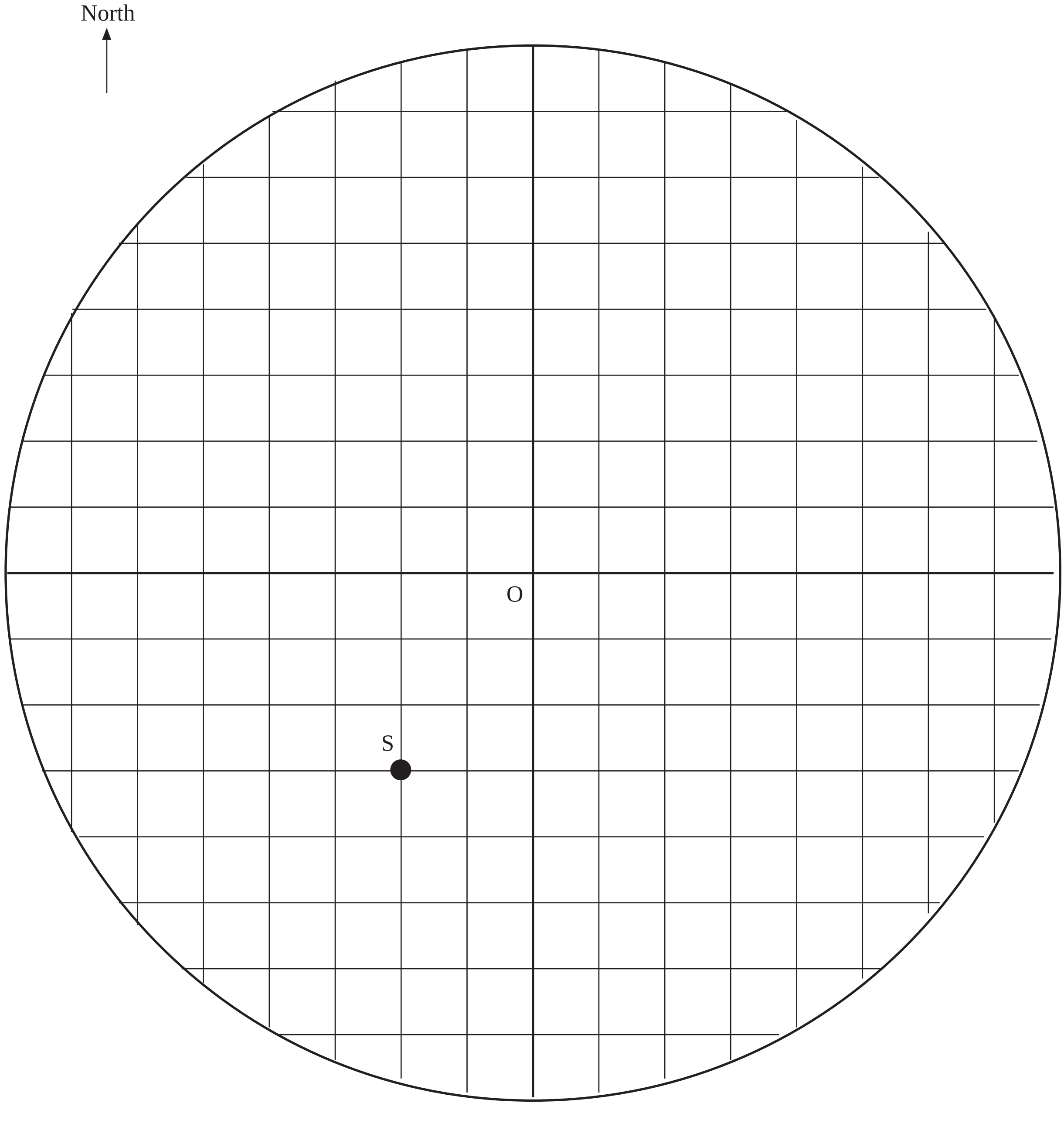
ACTIVITY 4.4 *Radar*

A ship is anchored at sea. The following diagram represents the circular screen of the ship's radar display. It is divided into squares. Each side of a square represents 10 km. The centre of the screen, O, represents the ship's own position. Another ship, S, is anchored at (–2, –3). Its position is indicated by a blip on the screen. (In the diagram, we use a dot to represent the blip.)



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ACTIVITY 4.4 (continued) *Radar*

1. What is the distance between the two ships, correct to the nearest km?

2. At 0600, another blip appears at (5, 6) with respect to O, travelling on a bearing of $200°$ at an estimated speed of 40 km/h.

   (a) Use a dot to indicate the position of this unidentified vessel on the screen. What is its distance from the ship at O, correct to the nearest km?

   (b) Draw a line clearly on the screen to show the course of navigation taken by this unidentified vessel.

   (c) If this unidentified vessel continues with the same course, what will be the shortest distance between it and the ship at O?
   Give your answer correct to the nearest km.

   (d) How long will it be from the time the blip first appears to the time the unidentified vessel moves out of the radar display? How far will it then be from O?

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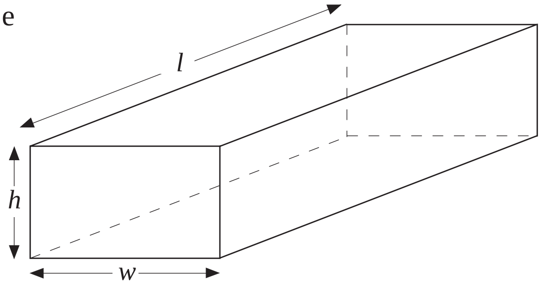
ACTIVITY 4.5 *Posting Parcels*

The *Royal Mail Data Post International* parcel service accepts parcels up to a maximum size as given in the rules below.

Rule 1 *Length* + *height* + *width* must not exceed 900 mm.

Rule 2 None of the *length, height, width* must exceed 600 mm.



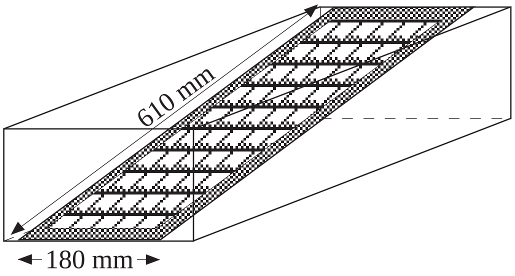
1. Which of the following parcels would be accepted for this service?

(a) $l = 620$ mm, $h = 120$ mm, $w = 150$ mm

(b) $l = 500$ mm, $h = 350$ mm, $w = 150$ mm

(c) $l = 550$ mm, $h = 100$ mm, $w = 150$ mm

2.

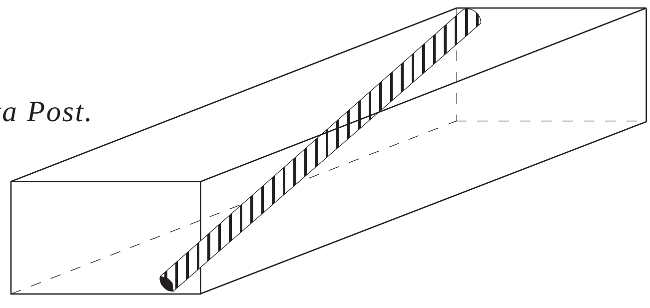


A picture with frame, 610 mm by 180 mm, is to be placed diagonally in a rectangular box as shown.

Find suitable dimensions for the box so that it would be accepted for the *Data Post* service.

3. A long, thin tube is to be sent by *Data Post*.

What is the largest possible length that can be sent?



Extension
What are the dimensions of the rectangular box of *maximum volume* that can be sent through the *Data Post* service?

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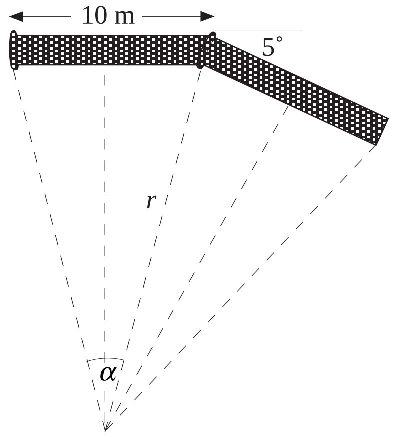
ACTIVITY 4.6 *Interlocking Pipes*

Pipes which are used to drain away surface rainwater from roads are designed with a *lip* in such a way that 10 metre pipes can have a change in angle of at most $5°$.

The Highways Department wants to lay these pipes along a road which has a curvature of radius 150 metres. Is this possible?

You can answer this problem by finding the minimum radius that can be obtained by using the maximum angle change of $5°$. The situation is illustrated below.



1. What is the value of the angle α as shown in the diagram?

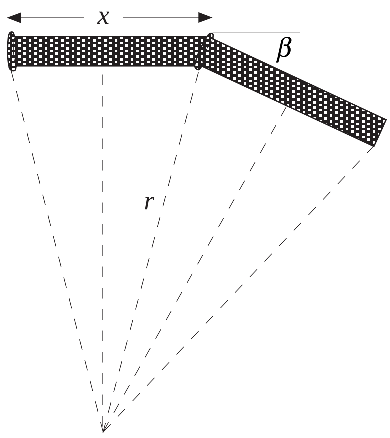
2. Show that $r = \dfrac{5}{\sin \frac{1}{2}\alpha}$.

   Hence find the minimum radius possible.

3. Can a curvature of 150 metres be achieved? If so, what should be the angle of change between pipes?

Although you have solved a particular problem, it would be very useful for the highway engineer to have a way of finding a *general* expression for the minimum radius possible, say r metres, for pipes of a given length, say x metres, which has a maximum angle change of $\beta°$, as shown below.

4. Find the relationship between r, x and β.



r \ x	2	5	10	20
50	...	...	11.48	...
75	...	...	...	...
100	...	...	...	...
125	...	...	...	...
150	...	...	3.82	...

5. Use this relationship to complete the ready reckoner table as shown above for the required angle β, given r and x.

6. If pipes are available only in sizes of 2, 5, 10 and 20 metres, find the maximum size to produce a radius of curvature of

   (a) 125 metres (b) 75 metres.

   (Assume that $5°$ is the maximum change in angle possible.)

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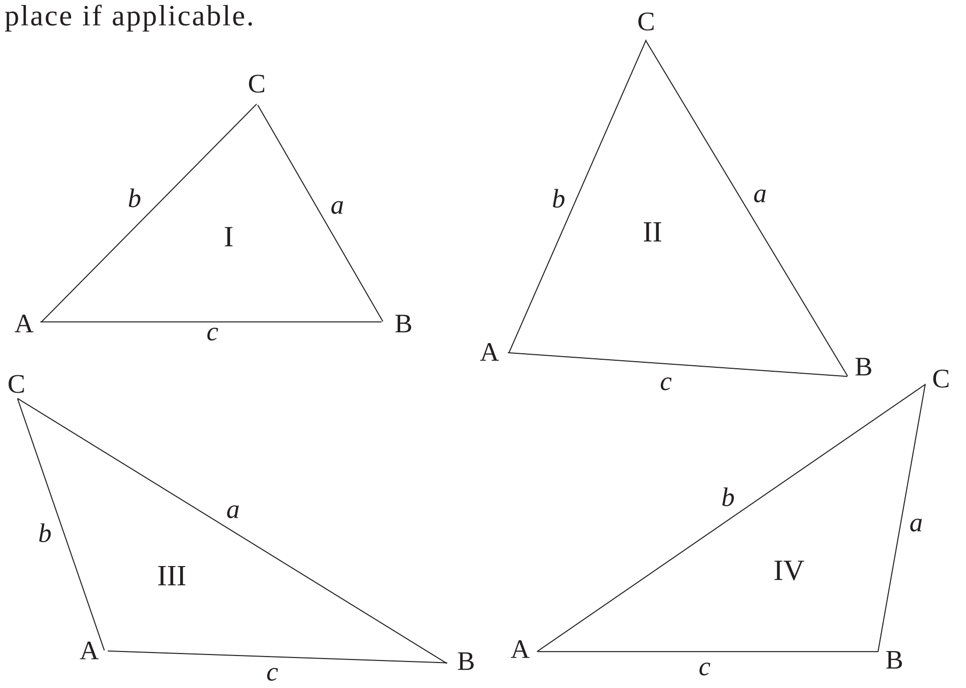
ACTIVITY 4.7 *Sine Rule*

Measure the sizes of all three angles and the lengths of all three sides of each of the following triangles. Complete the table that follows. Give your answers correct to one decimal place if applicable.



Triangle	∠A	∠B	∠C	a	b	c	$\frac{a}{\sin A}$	$\frac{b}{\sin B}$	$\frac{c}{\sin C}$
I									
II									
III									
IV									

What can you conjecture from these results?

Extension

Draw a circle of a certain diameter and draw any triangle ABC with its vertices on the circumference of the circle. Measure the sides and angles of the triangle, and compare the values of

$$\frac{a}{\sin A}, \quad \frac{b}{\sin B}, \quad \frac{c}{\sin C}$$

to the diameter. Repeat with a circle of different diameter.

What can you conjecture?

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