

UNIT 6 *Number System*

Activities

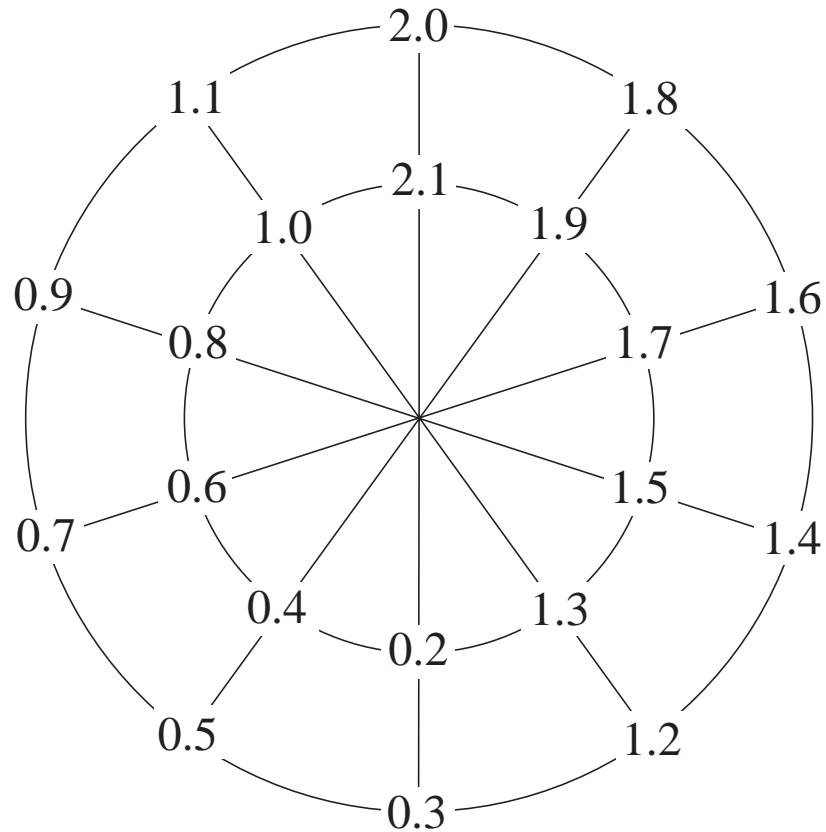
Activities

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ACTIVITY 6.1

Magic Circle

In this magic circle, there are two 'magic' totals.



- What are they?
- Explain how you found them.

Extension

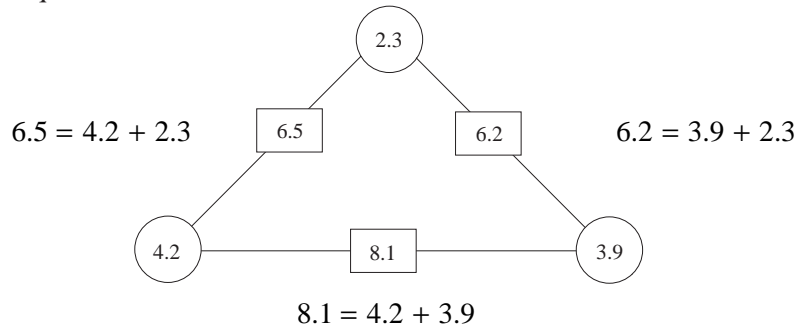
Design another magic circle of your own and ask a friend to solve it.

ACTIVITY 6.2

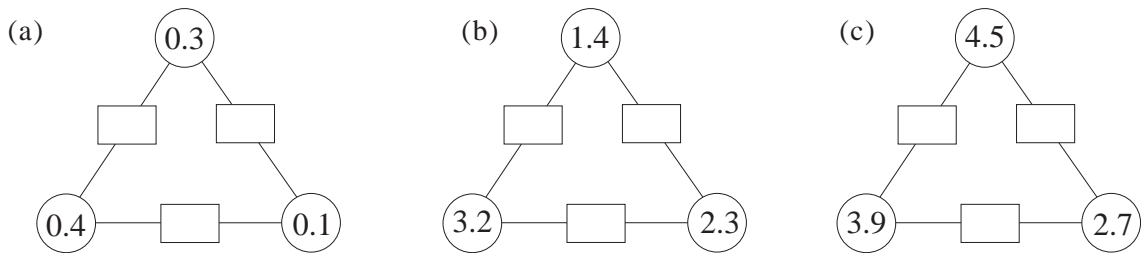
Decimal Arithmagons

In these arithmagons, the number in each **square** is the *sum* of the numbers in the **circles** on either side of the square.

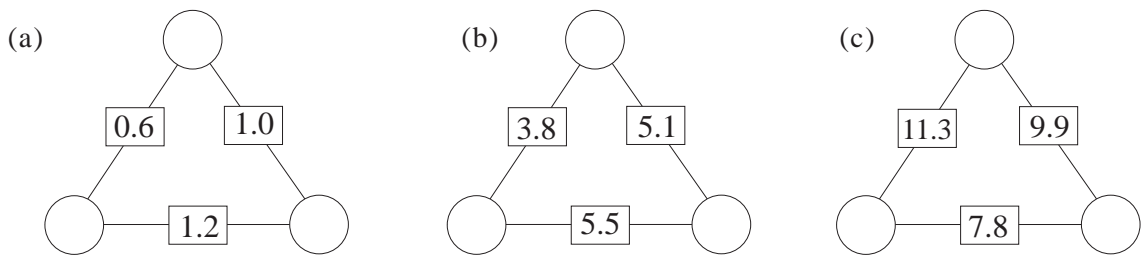
For example,



1. Find the numbers missing from the squares.

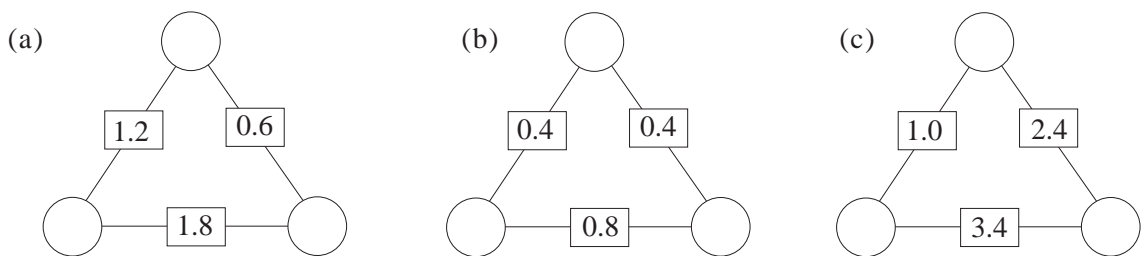


2. Find the numbers missing from the circles.



Extension

If the number in each square is the *difference* between the numbers each side of it, find the missing numbers



ACTIVITY 6.3

Dominoes

0.75	0.75
$\frac{1}{2}$	$\frac{1}{2}$
$\frac{1}{2}$	$\frac{1}{2}$

0.125	0.125
$\frac{1}{2}$	$\frac{1}{2}$
$\frac{1}{2}$	$\frac{1}{2}$

0.3	0.3
$\frac{1}{4}$	$\frac{1}{4}$
$\frac{1}{4}$	$\frac{1}{4}$

0.6	0.6
$\frac{1}{4}$	$\frac{1}{4}$
$\frac{1}{4}$	$\frac{1}{4}$

0.6	0.6
$\frac{3}{4}$	$\frac{3}{4}$
$\frac{3}{4}$	$\frac{3}{4}$

0.5	0.5
$\frac{3}{4}$	$\frac{3}{4}$
$\frac{3}{4}$	$\frac{3}{4}$

0.3	0.3
$\frac{1}{5}$	$\frac{1}{5}$
$\frac{1}{5}$	$\frac{1}{5}$

0.8	0.8
$\frac{1}{5}$	$\frac{1}{5}$
$\frac{1}{5}$	$\frac{1}{5}$

0.625	0.625
$\frac{2}{5}$	$\frac{2}{5}$
$\frac{2}{5}$	$\frac{2}{5}$

0.375	0.375
$\frac{2}{5}$	$\frac{2}{5}$
$\frac{2}{5}$	$\frac{2}{5}$

1.0	1.0
$\frac{3}{5}$	$\frac{3}{5}$
$\frac{3}{5}$	$\frac{3}{5}$

0.8	0.8
$\frac{3}{5}$	$\frac{3}{5}$
$\frac{3}{5}$	$\frac{3}{5}$

0.75	0.75
$\frac{4}{5}$	$\frac{4}{5}$
$\frac{4}{5}$	$\frac{4}{5}$

0.625	0.625
$\frac{4}{5}$	$\frac{4}{5}$
$\frac{4}{5}$	$\frac{4}{5}$

0.2	0.2
$\frac{1}{8}$	$\frac{1}{8}$
$\frac{1}{8}$	$\frac{1}{8}$

0.5	0.5
$\frac{1}{8}$	$\frac{1}{8}$
$\frac{1}{8}$	$\frac{1}{8}$

0.125	0.125
$\frac{3}{8}$	$\frac{3}{8}$
$\frac{3}{8}$	$\frac{3}{8}$

0.4	0.4
$\frac{3}{8}$	$\frac{3}{8}$
$\frac{3}{8}$	$\frac{3}{8}$

0.2	0.2
$\frac{5}{8}$	$\frac{5}{8}$
$\frac{5}{8}$	$\frac{5}{8}$

0.25	0.25
$\frac{5}{8}$	$\frac{5}{8}$
$\frac{5}{8}$	$\frac{5}{8}$

0.4	0.4
$\frac{1}{10}$	$\frac{1}{10}$
$\frac{1}{10}$	$\frac{1}{10}$

0.375	0.375
$\frac{1}{10}$	$\frac{1}{10}$
$\frac{1}{10}$	$\frac{1}{10}$

1.0	1.0
$\frac{3}{10}$	$\frac{3}{10}$
$\frac{3}{10}$	$\frac{3}{10}$

0.25	0.25
$\frac{3}{10}$	$\frac{3}{10}$
$\frac{3}{10}$	$\frac{3}{10}$

ACTIVITY 6.4

Estimation

Do not use a calculator, except to check your answers.

For each of these sums, draw a circle around the number which you think is nearest the correct answer.

For example, $\frac{7.1 \times 20.5}{3.5}$ is about { 4.2 (42) 84 420 }

1. $\frac{5.5 \times 13}{7}$ is about { 0.1 1 10 100 }
2. $\frac{42 \times 39}{16}$ is about { 1 10 100 1000 }
3. $\frac{210 \times 37}{17}$ is about { 4.5 45 450 4500 }
4. $\frac{6.5 \times 4.2}{2.2}$ is about { 1.2 12 120 1200 }
5. $\frac{12.7 \times 2.9}{3.7}$ is about { 0.1 1 10 100 }
6. A drink costs 55 p.
About how many drinks could you buy for £5? { 90 9 19 }
7. At a fairground, rides cost 40 p each.
About how many rides can you go on for £7? { 7 17 70 }
8. Oranges cost 15 p each.
About how many oranges can you buy for £50? { 33 330 3300 }
9. Stamps cost 24 p each.
About how many stamps can you buy for £10? { 4 40 400 }
10. Petrol costs £2.40 per gallon.
About how many gallons can you buy for £25? (10 25 100)

ACTIVITY 6.5

Russian Multiplication

Once upon a time, so legend has it, Russian peasants could add, multiply and divide only by 2, and so they developed a clever method of multiplying numbers together.

METHOD

- Put the 1st number in a left-hand column and the 2nd number in a right-hand column.
- Divide the number in the left-hand column by 2, ignoring any remainder, and multiply the number in the right-hand column by 2.
- Repeat Steps 2 until the number 1 is reached on the left-hand side.
- Delete any row which has an even entry on the left-hand side.
- Add the remaining right-hand side numbers. This final total is the answer.

Worked Example

To multiply 27 by 137:

27	137
13	274
<i>Even row deleted</i> → 6	548
3	1096
1	2192
$27 \times 137 = 3699$	

- Use the Russian method of multiplication to find:
 - 13×250
 - 16×135
 - 25×49

To see why the method works, it might become clearer if we write the multiplication sum in the *Worked Example* as

$$\begin{aligned}
 27 \times 137 &= (26 + 1) \times 137 \\
 &= 13 \times 274 + 1 \times 137 \\
 &= (12 + 1) \times 274 + 1 \times 137 \\
 &= 6 \times 548 + 1 \times 274 + 1 \times 137 \\
 &= 3 \times 1098 + 1 \times 274 + 1 \times 137 \\
 &= (2 + 1) \times 1098 + 1 \times 274 + 1 \times 137 \\
 &= 1 \times 2192 + 1 \times 1098 + 1 \times 274 + 1 \times 137 \\
 &= 3699
 \end{aligned}$$

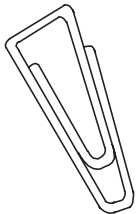
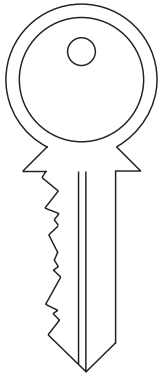
- Write out other multiplication sums in this way and use a calculator to check your answers.
- Explain in your own words why the method works.

ACTIVITY 6.6

Upper and Lower Bounds

Use *Ruler A* and *Ruler B* in turn to measure the lengths of the objects.

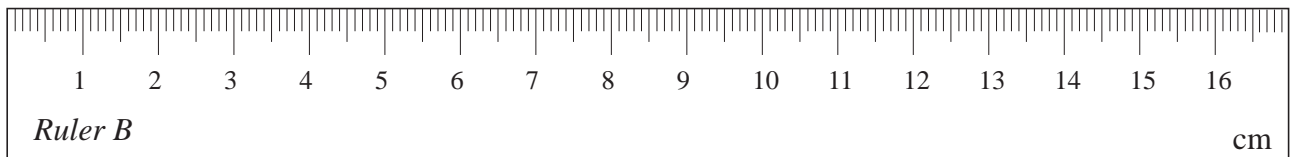
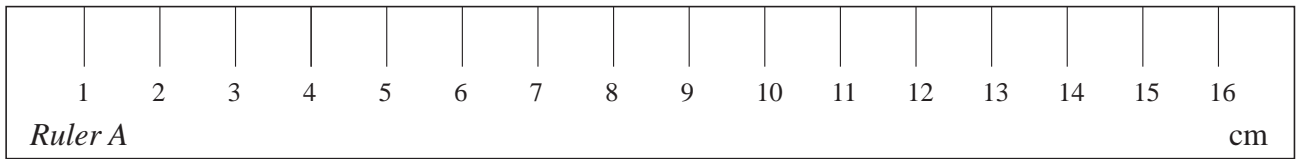
Record your measurements, giving the largest possible error and the upper and lower bounds.

Object	Measurement	Unit of Measure	Largest possible error	Lower Bound	Upper Bound
	<i>Ruler A</i>				
	<i>Ruler B</i>				
	<i>Ruler A</i>				
	<i>Ruler B</i>				
Length of this book	<i>Ruler A</i>				
	<i>Ruler B</i>				
Breadth of this book	<i>Ruler A</i>				
	<i>Ruler B</i>				

ACTIVITY 6.6

Rulers for Measurement

Photocopy and cut out for use in measuring objects in Activity 6.4.



ACTIVITY 6.7

Decimal Equivalents

Use a computer, calculator, or long division, to find the decimals equivalent for all the fractions $\frac{1}{1}, \frac{1}{2}, \dots, \frac{1}{20}$.

If the decimal equivalent is recurring, state the length of the cycle, i.e. the number of digits which repeat.

e.g. $\frac{3}{11} = 0.272\ 727\ 27\ \dots$ has a cycle of length 2, because 2 and 7 are repeated.

Be very careful with $\frac{1}{17}$ and $\frac{1}{19}$ – they may be longer than you expect!

Fraction	Decimal Equivalent	Recurring	Length of cycle
$\frac{1}{1}$	1.000 000 ...	✗	
$\frac{1}{2}$			
$\frac{1}{3}$	0.333 333 ...	✓	1
$\frac{1}{4}$			
$\frac{1}{5}$			
$\frac{1}{6}$			
$\frac{1}{7}$			
$\frac{1}{8}$			
$\frac{1}{9}$			
$\frac{1}{10}$			
$\frac{1}{11}$			
$\frac{1}{12}$			
$\frac{1}{13}$			
$\frac{1}{14}$			
$\frac{1}{15}$			
$\frac{1}{16}$			
$\frac{1}{17}$			
$\frac{1}{18}$			
$\frac{1}{19}$			
$\frac{1}{20}$			

ACTIVITY 6.8

Recurring Decimals

Sometimes a calculator answer is more complicated than it needs to be. For example, the display of

7.3333333333

is almost certainly $7\frac{1}{3}$. This is easy to recognise, but what about

0.09090909091 ?

It should not take you too long to recognise this as $\frac{1}{11}$.

1. Rewrite the decimal numbers below as fractions.

(a) 0.666 666 666 7

(b) 0.363 636 363 6

(c) 0.5555555556

(d) 0.142 857 142 9

The first three are all relatively straightforward, but (d) is a much more complicated recurring decimal. Its cycle consists of 6 recurrent numbers (142857).

2. Now consider

0.0958904109589

You have to be very familiar with decimal approximations to spot this one!

There is, though, a method for finding the fraction equivalent, as shown below.

$$\text{Let } x = 0.0958904109589\dots$$

$$\text{so that } 10^8 x = 9589041.09589\dots$$

Subtracting the first equation from the second gives

$$(10^8 - 1)x = 9589041 \Rightarrow x = \frac{9589041}{99999999}$$

Cancel out the common factors in the expression for x to find its fraction value in lowest terms.

3. Use the same procedure to find the fraction equivalent of

(a) 0.5714285714 ...

(b) 0.027027027 ...

(c) 0.07692307692 ...