

UNIT 6 Number System

NC: Number 2a,b and 3e,f

	St	Ac	Ex	Sp
TOPICS (Text and Practice Books)				
6.1 <i>Decimals</i>	✓	-	-	-
6.2 <i>Multiplying and Dividing with Decimals</i>	✓	-	-	-
6.3 <i>Fractions and Decimals</i>	✓	✓	-	-
6.4 <i>Long Multiplication and Division</i>	✓	✓	-	-
6.5 <i>Estimating Answers</i>	✓	✓	✓	✓
6.6 <i>Using Brackets and Memory on a Calculator</i>	✓	✓	✓	✓
6.7 <i>Upper and Lower Bounds</i>	×	✓	✓	✓
6.8 <i>Rational and Irrational Numbers</i>	×	×	✓	✓
6.9 <i>Working with Surds</i>	×	×	✓	✓
Activities (* particularly suitable for coursework tasks)				
6.1 <i>Magic Circle</i>	✓	✓	-	-
6.2 <i>Decimal Arithmagons</i>	✓	✓	-	-
6.3 <i>Dominoes</i>	✓	✓	-	-
6.4 <i>Estimation</i>	✓	✓	✓	✓
6.5* <i>Russian Multiplication</i>	✓	✓	✓	✓
6.6 <i>Upper and Lower Bounds</i>	×	✓	✓	✓
6.7 <i>Decimal Equivalents</i>	×	×	✓	✓
6.8* <i>Recurring Decimals</i>	×	×	✓	✓
OH Slides				
6.1 <i>Decimal Scales</i>	✓	✓	-	-
6.2 <i>Multiplying by Powers of 10</i>	✓	-	-	-
6.3 <i>Dividing by Powers of 10</i>	✓	-	-	-
6.4 <i>Fractions, Decimals and Percentages</i>	✓	✓	-	-
6.5 <i>Order of Calculation</i>	✓	✓	✓	
6.6 <i>Equivalent Expressions</i>	✓	✓	✓	✓
6.7 <i>Significant Figures</i>	✓	✓	✓	-
6.8 <i>Decimal Places</i>	✓	✓	✓	-
Mental Tests				
6.1	✓	✓	-	-
6.2	✓	✓	-	-
6.3	×	✓	✓	✓
6.4	×	×	✓	✓
Revision Tests				
6.1	✓	-	-	-
6.2	×	✓	-	-
6.3	×	×	✓	-

UNIT 6 *Number System*

Teaching Notes

Background and Preparatory Work

While visiting North Africa, *Leonardo of Pisa* (about 1170-1250), known as *Fibonacci*, learned from the Arabs about a number system which they had seen used by the Hindus in India. He returned to Pisa in 1202 and published a book called *Liber Abaci* which introduced the Arabic number system to Europe. This number system, now known as the decimal system, uses the ten symbols 0 to 9 and place value to represent numbers of any size. The system makes it possible for pupils in schools today to carry out calculations which were beyond the capabilities of learned mathematicians of Greek, Roman and medieval times.

Early number systems were based on, for example, 60 (Babylonian) or 12 (Roman) so that $2.3'5''$ would mean $2 + \frac{3}{12} + \frac{5}{144}$.

Decimal notation was introduced in the 16th and 17th centuries, with the Dutch engineer, *Simon Stevhus*, in 1585 using

$$25 \textcircled{0} 3 \textcircled{1} 7 \textcircled{2} 9 \textcircled{3} \text{ to mean } 25.379$$

and the Scottish mathematician, *John Napier*, using the notation

$$25,3'7''9''' \text{ for } 25.379.$$

Nowadays, most systems of measurement are based on a decimal system, although time is still measured using 60 as the base and in this country we continue to use miles and feet for distance!

Teaching Points

Introduction

This is a revision topic for *Standard* and *Academic* routes, covering decimals and estimation. The work on error bounds and rational and irrational numbers is probably new work and requires careful handling. This is also a useful time to make sure that all your students fully understand how to use calculators properly (including the 'memory' button). Other points to note are given below.

- A good starting point for looking at decimals and the number line is a ruler marked in centimetres and millimetres.
- Students may need to be informed that when reading decimal numbers the figures after the decimal point are read separately. e.g. 32.45 is read as thirty-two point four five, **not** thirty-two point forty-five, because the 4 is 4 tenths not 4 tens, and the 5 is 5 hundredths not 5 units.

OS 6.1

- It may be helpful to make use of the students' familiarity with decimal money when discussing notation and calculations with decimals.
- Students need to be aware that multiplying and dividing numbers by powers of ten has the effect of moving the numbers into different columns. It may be helpful to use column headings

$$\dots \text{Th H T U} \cdot \frac{1}{10} \frac{1}{100} \frac{1}{1000}$$

initially when explaining this.

- It should be pointed out that the methods used for multiplying and dividing by powers of 10 can also be used for multiplying and dividing by multiples of 10.

e.g. $32 \times 300 = 32 \times 3 \times 100 = 96 \times 100 = 9600$

- The implications of the notation $3.25 \leq x < 3.35$ need to be stressed.

OS 6.2 and 6.3

Language / Notation

- The correct notation for writing amounts of money needs to be emphasised. Common errors include writing

£3.4	instead of	£3.40
£2.57 p	instead of	£2.57
£0.53 p	instead of	53 p

- Students need to be aware that when writing numbers, spaces are now used instead of commas between thousands.

e.g. 2 300 not 2,300 and 6 000 000 not 6,000,000

- It should be pointed out that in a whole number, where there is no decimal point written, the point is at the end of the number.

Key Points

- The basic rules for the order of carrying out arithmetic operations need to be pointed out.

Brackets should be worked out first and then multiplication and division before addition and subtraction.

The mnemonic BODMAS is often used to remember

Brackets **O**f **D**ivide **M**ultiply **A**dd **S**ubtract.

- Students need to be taught how to use functions on scientific calculators. Since different calculators operate in different ways, it is important that each student understands how to operate his/her own calculator.

OS 6.5

Misconceptions

- Lack of awareness of rounding errors on a calculator.
e.g. 0.6666666666 keyed into the calculator may not give the same result to a calculation as the same number entered by calculating $2 \div 3$.

- Lack of understanding of the upper limit in rounded numbers.
e.g. Students may give the upper boundary of a length rounded to 9.7 cm to the nearest millimetre as 9.74 cm or 9.749 cm and not 9.749 or 9.75.
- That to multiply or divide a whole number by a power of 10, zeros are added or removed, and that to multiply or divide a decimal number by a power of 10 the decimal point is moved to the right or left.
- That decimals such as 3.4, 3.40 and 3.400 are essentially the same number.

Activities

6.1 Magic Circle

This is a useful introductory activity on decimals for *Standard* route.

6.2 Decimal Arithmagons

This is another revision activity for addition and subtraction of decimals. It can be used either with the whole class or as an exercise for homework.

6.3 Dominoes

Useful practice to reinforce the equivalence of fractions and decimals. Use with pairs of students; the sheet should be photocopied onto card.

6.4 Estimation

This is designed as a whole class activity to clarify the concept of approximation.

6.5 Russian Multiplication*

This is an interesting way of multiplying but the understanding of why the method works is not trivial and could be used as a starting point for coursework.

6.6 Upper and Lower Bounds

This is a whole class activity to introduce the concept of bounds and to show how errors in measurement can occur.

6.7 Decimal Equivalents

This is an activity either for the whole class or for individual students on the difference between recurring and non-recurring decimals.

6.8 Recurring Decimals*

This activity leads students through the method of finding fraction equivalents for recurring decimals. It could be a good starter for homework.

Applications

There are many obvious uses of decimal notation, e.g.

- money
- metric measurements
- foreign currency exchange
- wages.