Answers

1 Indices

1.1 Multiplication and Division

1. (a) 20 (b) 21 (c) 36 (d) 42 (e) 45 (f) 18
   (g) 28 (h) 49 (i) 40 (j) 8 (k) 9 (l) 4
   (m) 7 (n) 7 (o) 9 (p) 0 (q) 0 (r) 0

2. (a) 3 (b) 7 (c) 4 (d) 8 (e) 3 (f) 4 (g) 9
   (h) 7 (i) 3 (j) 7 (k) 4 (l) 5 (m) 2 (n) 4
   (o) 7 (p) 0 (q) 0 (r) 0

3. 24

4. 27

5. (a) 16 (b) 28 (c) 32

6. (a) 6 (b) 3 (c) 4

7. 8

8. (a) 35 (b) 14 (c) 42

9. (a) Daniel 70p, Joel 56p (b) Daniel has 14p more than Joel

10. (a) 80 (b) 64 (c) 40

11. £6

12. (a) 9 (b) 7, with 1 left over

13. Team A: 7, Team B: 21, Team C: 14, Team D: 14 14. (a) 7 (b) 5

1.2 Squares, Cubes, Square Roots and Cube Roots

1. (a) 25 (b) 36 (c) 1 (d) 49 (e) 6 (f) 1 (g) 7
   (h) 5

2. (a) 27 (b) 64 (c) 216 (d) 1000 (e) 3 (f) 10
   (g) 6 (h) 4

3. (a) 100 (b) 4 (c) 16 (d) 49 (e) 64 (f) 81
   (g) 1 (h) 343 (i) 512 (j) 0 (k) 0 (l) 8

4. (a) 10 (b) 2 (c) 9 (d) 8 (e) 4 (f) 3

5. (a) 144 (b) 121 (c) 3375 (d) 2197 (e) 169 (f) 225
   (g) 400 (h) 1331 (i) 11 (j) 20 (k) 13 (l) 15
   (m) 15 (n) 13 (o) 12 (p) 11

6. (a) 52 (b) 5 (c) 116 (d) 25 (e) 16 (f) 72
   (g) 1001 (h) 100

1.3 Index Notation

1. (a) 4^5 (b) 3^3 (c) 6^7 (d) 7^4 (e) 18^3 (f) 19^2
   (g) 4^6 (h) 7^5 (i) 10^6 (j) 100^5

2. (a) 81 (b) 625 (c) 2401 (d) 10 000 (e) 1 (f) 729
   (g) 128 (h) 2 (i) 4096 (j) 4 (k) 1 (l) 25

3. (a) 2^{11} (b) 3^9 (c) 3^{13} (d) 4^5 (e) 5^4 (f) 5^5
   (g) 4^2 (h) 5^3 (i) 3^2 (j) 7^4 (k) 17^2 (l) 9^4
   (m) 4^5 (n) 4^{16} (o) 3^6 (p) $3^0 = 1$ (q) $3^1 = 3$
   (r) 3^5 (s) 3^7 (t) 4^7 (u) 5^0

4. (a) 2^2 (b) 2^3 (c) 2^4 (d) 2^6 (e) 3^3 (f) 5^2
   (g) 4^3 (h) 3^4 (i) 5^3

5. (a) 3^{13} (b) 2^8 (c) 4^{11} (d) 3^{10} (e) 2^9 (f) 2^{10}
   (g) 3^5 (h) 3^7 (i) 3^5 (j) 8^{10} (k) 7^3 (l) 9^2
   (m) 2^4 or 4^2 (n) 2^3 (o) 2^3

6. (a) 2^3 (b) 10^3 (c) 2^4 (d) 3^3 (e) 3^4 (f) 10^4
   (g) 5^4 (h) 4^3 (i) 6^4 (j) 2^0 (k) 6^2 (l) 5^0

7. (a) 2^6 (b) 3^4 (c) 6^6 (d) 5^6 (e) 2^8 (f) 4^6
   (g) 3^8 (h) 5^8 (i) 3^6

8. (a) 2^8 (b) 2^4 (c) 3^{10} (d) 5^3 (e) $\left(10^5\right)^3$ (f) $\left(7^5\right)^4$

9. (a) 3^6 (b) 2^{14} (c) 5^{12} (d) 7^3 (e) 7^4 (f) 2^7
   (g) $3^0 = 1$ (h) $4^1 = 4$ (i) $2^1 = 2$

10. (a) a^5 (b) a^{10} (c) x^9 (d) x^2 (e) y^3 (f) p^3
    (g) q^3 (h) x^8 (i) b^3 (j) b^6 (k) c^3 (l) x^5
    (m) y^2 (n) $x^0 = 1$ (o) x^8 (p) p^4 (q) x^3 (r) y^4
    (s) $x^0 = 1$ (t) $x^1 = x$ (u) x^{12} (v) x^8 (w) x^{15}
    (x) x^{54}

11. (a) $p = 3$ (b) $q = 0$

12. $2x^4$

1.4 Factors

1. (a) 1, 2, 7, 14 (b) 1, 3, 9, 27 (c) 1, 2, 3, 6 (d) 1, 3, 5, 15
   (e) 1, 2, 3, 6, 9, 18 (f) 1, 5, 25 (g) 1, 2, 4, 5, 8, 10, 20, 40
   (h) 1, 2, 4, 5, 10, 20, 25, 50, 100 (i) 1, 3, 5, 9, 15, 45
   (j) 1, 2, 5, 10, 25, 50 (k) 1, 2, 3, 4, 6, 9, 12, 18, 36 (l) 1, 2, 4, 7, 14, 28

2. (a) $1\times10,\ 2\times5,\ 5\times2,\ 10\times1$ (b) $1\times8,\ 2\times4,\ 4\times2,\ 8\times1$
   (c) $1\times7,\ 7\times1$ (d) $1\times9,\ 3\times3,\ 9\times1$
   (e) $1\times16,\ 2\times8,\ 4\times4,\ 8\times2,\ 16\times1$ (f) $1\times22,\ 2\times11,\ 11\times2,\ 22\times1$
   (g) $1\times11,\ 11\times1$
   (h) $1\times24,\ 2\times12,\ 3\times8,\ 4\times6,\ 6\times4,\ 8\times3,\ 12\times2,\ 24\times1$

3. (a) 4 (b) 3 (c) 3 (d) 4 (e) 5 (f) 4 (g) 11
   (h) 1

4. (a) 6, 10, 20, 8, 2, 24, 4 (b) 10, 20, 15, 55

5. (a) (i) 20, 22, 24, 26 (ii) 21, 24, 27 (iii) 20, 25
   (b) prime numbers

6. (a) (i) 16 (ii) 18 (b) (i) 25 (ii) 27

1.5 Prime Factors

1. 2, 3, 5, 7, 13, 19, 23

2. 53, 59

3. (a) 2×5 (b) $2\times3\times7$ (c) $2^2\times17$ (d) $2^3\times3\times7$
   (e) 2×5^3 (f) $2\times3^3\times5$ (g) $3\times11\times13$ (h) $3\times5^2\times11$
   (i) $7\times11\times13$

4. (a) $32=2^5$ and $56=2^3\times7$ (b) $2^3\ (=8)$

5. (a) $2\times3=6$ (b) $2\times3=6$ (c) $3\times5=15$ (d) 2
   (e) $2\times5=10$ (f) $5\times7=35$ (g) $2^3\times3=24$
   (h) $2\times3\times13=78$ (i) $3\times7^2=147$

6. (a) $45=3^2\times5,\quad 99=3^2\times11,\quad 135=3^3\times5$
   (b) (i) $3^2=9$ (ii) $3^2=9$ (iii) $3^2\times5=45$ (c) $3^2=9$

7. (a) 5 (b) $3^2\times5=45$ (c) $2^3=8$ (d) $2\times5=10$
   (e) $2^3\times3=24$ (f) $2\times3\times5=30$ (g) $2^2\times3^3=108$
   (h) $2^2\times11=44$ (i) $2^2\times3^2\times7=252$

1.6 Further Index Notation

1. (a) $\frac{1}{16}$ (b) $\frac{1}{8}$ (c) $\frac{1}{6}$ (d) $\frac{1}{7}$ (e) 3 (f) 8 (g) 2

   (h) 3 (i) 1 (j) $\frac{1}{25}$ (k) 8 (l) 32 (m) 2187

   (n) 125 (o) $\frac{1}{2}$

2. (a) -4 (b) -1 (c) -3 (d) $\frac{1}{2}$ (e) $-\frac{1}{2}$ (f) 2

   (g) 3 (h) -1 (i) $\frac{1}{3}$ (j) -1 (k) -2 (l) -2

   (m) -3 (n) $\frac{1}{2}$ (o) -2 (p) $\frac{1}{3}$ (q) $\frac{2}{3}$ (r) $\frac{2}{5}$

3. (a) 0.125 (b) 0.05 (c) 2 (d) 4 (e) $\frac{1}{225} = 0.004444...$

   (f) 0.000125 (g) 729 (h) 27 (i) $\frac{1}{2}$ (j) 1728

   (k) 62748517 (l) 1331

4. (a) $\frac{1}{a}$ (b) a^{10} (c) a^4 (d) $\frac{1}{a^6}$ (e) $\frac{1}{a^2}$ (f) $\frac{1}{a^6}$

   (g) a^8 (h) $a^{\frac{5}{2}}$ (i) $\frac{1}{a^{\frac{3}{2}}}$ (j) a^2 (k) $\frac{1}{a^3}$ (l) a^3

   (m) $\frac{a^2}{b^2}$ (n) $\frac{a^6}{b^{12}}$ (o) $a^{12}b^2$ (p) $\frac{b^4}{a^4}$ (q) $\frac{a^8}{b^{12}}$

   (r) $\frac{m^2}{n^6}$ (s) $\frac{a^3}{b^5}$ (t) $\frac{m^2}{a}$ (u) $\frac{c^3}{a^4b}$ (v) $\frac{x}{m^2}$

   (w) $\frac{z^{12}}{x^8y^4}$ (x) $\frac{b^{\frac{16}{3}}}{a^2}$

5. (a) $\frac{1}{9}$ (b) a^4 (c) $y = \frac{5}{2}$

1.7 Standard Form

1. (a) 4.7×10^4 (b) 5.21×10^4 (c) 3.2×10^7 (d) 3.241×10^5
   (e) 4.2×10^2 (f) 8.1×10^4 (g) 5×10^3 (h) 4.7×10^{10}
   (i) 3.2×10^9 (j) 6.2×10^{-4} (k) 5.71×10^{-2} (l) 2.0×10^{-7}
   (m) 1.24×10^{-1} (n) 3.71×10^{-2} (o) 2.1×10^{-4} (p) 7×10^{-5}
   (q) 4.71×10^{-1} (r) 3×10^{-4}
2. (a) 1×10^6 (b) 1.5×10^4 (c) 6.4×10^6 (d) 3.04×10^7
   (e) 4×10^6 (f) 4×10^5

3. (a) 600 000 (b) 431 (c) 58 600 000 (d) 0.00083 (e) 4172
   (f) 0.0000642 (g) 47 (h) 0.32 (i) 0.000847
   (j) 334 000 000 (k) 0.0003471 (l) 842.1 (m) 16.75
   (n) 0.0000084 (o) 0.000712

4. (a) Yes (b) No: 4.32×10^3 (c) No: 1.56×10^{-7}
   (d) No: 4×10^2 (e) Yes (f) No: 7×10^{-5} (g) Yes
   (h) No: 5.471×10^4 (i) Yes

5. (a) 6×10^4 (b) 1.6×10^7 (c) 1.25×10^5 (d) 4×10^{-3}
   (e) 2×10^{-3} (f) 1.6×10^{-3} (g) 2.8×10^{-3} (h) 8.8×10^3
   (i) 9×10^{-4} (j) 9×10^2 (k) 4×10^{-4} (l) 1×10^4

6. (a) 5×10^4 (b) 6.2×10^5 (c) 1.456×10^6

7. (a) $365 \times 24 = 8.76 \times 10^3$ (b) $7 \times 24 \times 60 = 1.008 \times 10^4$
   (c) $24 \times 60^2 = 8.64 \times 10^4$

8. (a) 6370 km (b) $6\,370\,000 = 6.37 \times 10^6$
   (c) $2 \times \pi \times 6.37 \times 10^6 = 4.0024... \times 10^7$

9. 5 900 000 000 000 000 000 000 000 10. 3×10^{-2} mm

11. 4.82×10^8 kg

12. (a) 153 000 000 km and 147 000 000 km
    (b) 1.53×10^{11} m and 1.47×10^{11} m

13 (a) $4.444\,355\,556 \times 10^9$ (b) 3.68785269×10^8 (c) 8×10^9
   (d) 1.536×10^{11} (e) 1.6544×10^{10} (f) 2.24×10^9

1.8 Calculations with Standard Form

1. (a) 6×10^{12} (b) 8×10^{8} (c) 9×10^{16} (d) 2×10^{12}
   (e) 2.4×10^{15} (f) 1.2×10^{7} (g) 2.88×10^{11} (h) 2.2×10^{2}
   (i) 5.832×10^{7} (j) 6.76×10^{-4} (k) 1.302×10^{-8}
   (l) 4.5×10^{-13}
2. (a) 4×10^{4} (b) 3×10^{3} (c) 2×10^{2} (d) 8×10^{2}
   (e) 1.2×10^{6} (f) 1.2×10^{9} (g) 2.7×10^{-1} (h) 5×10^{7}
   (i) 4.1×10^{19} (j) 1.1×10^{8} (k) 3×10^{9} (l) 1.1×10^{-10}

3. (a) 1.764×10^{13} (b) 1.369×10^{-3} (c) 1.728×10^{-15}
   (d) $2.846... \times 10^{4}$ (e) 7.44×10^{22} (f) 1.558×10^{-7}
   (g) $9.5833... \times 10^{-2}$ (h) 1.46875×10^{3} (i) 5.0625×10^{-10}
   (j) $2.449... \times 10^{2}$ (k) 5.12×10^{11} (l) 6.753×10^{12}

4. (a) 8.64×10^{5} (b) 6.048×10^{5} (c) 3.1536×10^{7}

5. (a) 2.73×10^{-12} kg (b) 3.64×10^{2} kg (c) 6.37×10^{-22} kg

6. 1.26×10^{12} mm^2

7. (a) (i) 3.32×10^{3} m (ii) 9.96×10^{5} m (iii) 1.328×10^{1} m
   (b) (i) $3.012... \times 10^{-2}$ s (ii) 6.024... s (iii) $6.024... \times 10^{-5}$ s

8. (a) 3×10^{10} m (b) 5×10^{2} s (8 min 20 s)

9. (a) 3.84×10^{8} m (b) 9.6×10^{5} s (266 hours 40 min)

10. (a) 1×10^{6} cm^3 (b) 1.3 kg (c) $2.30769... \times 10^{3}$ cm^3
    (d) 9×10^{-2} kg, $3.333... \times 10^{4}$ cm^3

11. $4.5 \times 10^{9} \times (1.03)^{10} = 6.0476... \times 10^{9}$

12. (a) 52 000 000 (b) 1.2×10^{-1} cm 13. 8.54×10^{8}

14. 1.845×10^{11} tonnes (2×10^{11} to 1 s.f.)

15. (a) 4.29981696 (b) 3×10^{-2}

16. (a) 1.496×10^{8} (b) $2.5752... \times 10^{6}$

17. (a) 5×10^{101} (b) 5×10^{-8}

2 Formulae

2.1 Using Formulae

1. (a) $A = 8,\ P = 12$ (b) $A = 30,\ P = 26$ (c) $A = 22,\ P = 26$
   (d) $A = 20,\ P = 18$

2. (a) 16 (b) 12 (c) 15 (d) 20

3. (a) 30 (b) 400

4. (a) 30 (b) 12 (c) 17

5. (a) 60 (b) 105 (c) 144

6. (a) 26 (b) 14 (c) 19 (d) 46 (e) 18 (f) 12
   (g) 4 (h) 2 (i) 26 (j) 50 (k) 30 (l) 40 (m) 6
   (n) 10

7. £130 8. 17.4 cm

2.2 Construct and Use Simple Formulae

1. (a) $P = 2a + b,\ P = 16$ (b) $P = 4a,\ P = 20$ (c) $P = 5a + b,\ P = 40$
   (d) $P = a + 2b + c,\ P = 27$ (e) $P = 6a,\ P = 60$
   (f) $P = 2a + 2b + 2c,\ P = 36$ (g) $P = 2a + 2b + c,\ P = 520$
   (h) $P = 3a + b,\ P = 21$

2. (a) $A = ab,\ A = 60\ \text{cm}^2$ (b) $A = a^2,\ A = 9\ \text{cm}^2$
   (c) $A = a^2 + ab,\ A = 20\ \text{cm}^2$ (d) $A = ab + bc,\ A = 48\ \text{cm}^2$
   (e) $A = \frac{1}{2}ab,\ A = 10\ \text{cm}^2$ (f) $A = \frac{1}{2}ab + b^2,\ A = 45\,000\ \text{cm}^2$

3. (a) $(x + 1)$ and $(x + 2)$ (b) $T = 3x + 3$

4. (a) $M = \frac{x + y}{2}$ (b) $M = \frac{p + q + r + s + t}{5}$

5. (a) $T = 3p + 2q$ (b) £190

6. (a) $P = 2x + 2(x + 3) = 4x + 6$ (b) $A = x(x + 3)$

7. (a) $x + 1$ (b) $x - 3$ (c) $S = 3x - 2$

8. (a) $C = 3 + 2n$ (b) £19

9. (a) $C = 1 + 2m$ (b) £7

10. (a) $2n$ (b) $2n + 6$

11. (a) $100 - 8x$ (b) $(20 - 2x)(30 - 2x)$

12. $C = 27n$

13. (a) $C = 45l$ (b) $C = xl$

14. (a) $S = P + Q$ (b) (i) $S = X + 3250$ (ii) $S = X + 650n$

2.3 Revision of Negative Numbers

1. (a) -2 (b) 4 (c) -3 (d) -8 (e) 24 (f) 54
(g) 8 (h) -8 (i) 27 (j) 8 (k) -13 (l) -35
(m) -24 (n) -11 (o) -5 (p) 3 (q) 11 (r) 3
(s) -8 (t) -6 (u) -1

2. (a) 1 (b) 16 (c) 25 3. -3 °C 4. 24 °C

2.4 Substitution into Formulae

1. (a) 50 (b) 68 (c) 14 (d) 23 (e) -4 (f) 59

2. (a) 10 (b) 40 (c) 11.25 (d) 4 (e) –10 (f) 7.04

3. (a) 19.6 (b) 18.4 (c) 18.08 (d) 18.8

4. (a) -280 (b) -40 (c) 80 (d) 800 ; 4

5. (a) 80 (b) 51 (c) ± 4 (d) ± 3 (e) -3 (f) ± 5
(g) 0 (h) $\frac{3}{4}$ (i) 1 (j) 10 (k) -2 (l) -10
(m) ± 10 (n) 0.18 (o) 0.38 (p) ± 5 (q) ± 8 (r) ± 15

6. (a) 3.8 (b) 0.225 (c) 2.6 (d) 7.5 (e) 9.7 (f) 2.4
(g) 0.5 (h) 7.12 (i) 3.7

7. -21.67 (2 d.p.)

8. -13

9. (a) $-\frac{13}{8}$ (b) $-\frac{5}{8}$

2.5 More Complex Formulae

1. (a) $\frac{12}{7}$ (b) -30 (c) $-\frac{21}{4}$ (d) $-\frac{20}{3}$

2. (a) ± 1.3 (b) ± 8 (c) ± 3.4

3. (a) $-\frac{3}{2}$ (b) $\frac{12}{25}$ (c) -10 (d) $\frac{10}{4}\left(=\frac{5}{2}\right)$ (e) -4

   (f) $-\frac{1}{3}$ (g) $-\frac{17}{7}$ (h) $\frac{7}{9}$ (i) 1

4. (a) ± 5 (b) ± 12 (c) ± 7 (d) ± 11 (e) ± 2 (f) ± 13

5. (a) $\frac{60}{11}$ (b) 588.24 (2 d.p.) (c) 572.67 (2 d.p.)

6. 33.5

7. (a) -32.3 (3 s.f.) (b) -30

8. (a) i) 0.2kg ii) kg per cm^3, $kgcm^{-3}$ or $\frac{kg}{cm^3}$ (b) 2.29kg (2dp)

2.6 Changing the Subject

1. (a) $x = \frac{y}{4}$ (b) $x = \frac{y - 3}{2}$ (c) $x = \frac{y + 8}{4}$ (d) $x = 4y - 2$

   (e) $x = 5y + 2$ (f) $x = y - a$ (g) $x = ya + b$ (h) $x = \frac{y - c}{a}$

   (i) $x = \frac{yc - b}{a}$ (j) $x = \frac{yb + c}{a}$ (k) $x = y - a - b$ (l) $x = yc + a - b$

   (m) $x = \frac{y}{ab}$ (n) $x = \frac{y - c}{ab}$ (o) $x = \frac{3cy + b}{4a}$ (p) $x = \frac{pd + bc}{a}$

   (q) $x = \frac{y}{b} - a$ (r) $x = \frac{4y}{a} - 3$ (s) $x = \frac{2q}{3} + 4$ (t) $x = \frac{4v}{5} - y$

   (u) $x = 4(z - a) + 3$

2. $I = \frac{V}{R}$; $R = \frac{V}{I}$

3. $m = \frac{F}{a}$; $a = \frac{F}{m}$

4. $r = \frac{C}{2\pi}$

5. (a) $t = \frac{v - u}{a}$ (b) $a = \frac{v - u}{t}$

6. $z = 3m - x - y$

7. (a) $a = \dfrac{v^2 - u^2}{2s}$ (b) $a = \dfrac{s}{(t + \frac{1}{2}t^2)}$

8. $z = \dfrac{v}{xy}$

9. (a) $r = +\sqrt{\dfrac{V}{\pi h}}$ (only a positive value because r is radius) (b) 2.82

10. (a) $h = \dfrac{V}{x^2}$; $h = \dfrac{A - 2x^2}{4x}$ (b) 2 (c) 2.5

11. (a) $a = \dfrac{2A}{h} - b$ (b) $A = \dfrac{1}{2} \times 3a \times h = \dfrac{3ah}{2}$; $a = \dfrac{2A}{3h}$

2.7 Further Change of Subject

1. (a) $x = \dfrac{5 - y}{3}$ (b) $x = \dfrac{8 - y}{6}$ (c) $x = \dfrac{a - y}{2}$ (d) $x = \dfrac{6 - 5y}{2}$

   (e) $x = \dfrac{8 - 2y}{7}$ (f) $x = \dfrac{3y + 5}{7}$ (g) $x = a - b - 2p$ (h) $x = 10 - aq$

   (i) $x = \dfrac{q - rb}{5}$

2. (a) $a = 4q^2$ (b) $a = bz^2$ (c) $a = \dfrac{c}{z^2}$ (d) $a = \dfrac{3y^2}{8}$

   (e) $a = 32bv^2$ (f) $a = \dfrac{\pi r^2}{25}$ (g) $a = 4p^2 - b$

   (h) $a = b - 12r^2$ (i) $a = \dfrac{18}{c^2} - b$

3. (a) $\dfrac{2}{2a - 1}$ (b) $\dfrac{1}{b + 2}$ (c) $\dfrac{1}{2 - x}$ (d) $\dfrac{3x}{3 - x}$ (e) $\dfrac{5p}{5 + p}$

   (f) $\dfrac{6x}{3 - x}$ (g) $\dfrac{4rv}{v - 2r}$ (h) $\dfrac{7q}{q - 7}$ (i) $\dfrac{ap}{p - a}$

4. (a) $g = \dfrac{4\pi^2 l}{T^2}$ (b) 10.07 (2 d.p.)

5. (a) $v = \dfrac{uf}{u - f}$ (b) -24

6. (a) $h = \dfrac{v^2}{2g}$ (b) 1.8 (c) $g = \dfrac{v^2}{2h}$ (d) 0.8

7. (a) $R = \dfrac{u^2}{g}$ (b) 14.4

8. (a) $X = \dfrac{RYZ}{YZ - RZ - RY}$ (b) 24

9. (a) $r = \sqrt[3]{\dfrac{3V}{4\pi}}$ (b) 2.62 (2 d.p.)

2.8 Expansion of Brackets

1. (a) $3x + 3$ (b) $4a + 8$ (c) $3x - 18$ (d) $15 - 5b$
   (e) $16 - 2x$ (f) $3x + 12$ (g) $10x - 24$ (h) $12x - 30$
   (i) $6x + 21$

2. (a) $-2x - 12$ (b) $-3x - 6$ (c) $-6x + 18$ (d) $-7x + 14$
   (e) $-8x - 4$ (f) $-15 + 10x$ (g) $-6x + 16$ (h) $12 + 3x$
   (i) $-16 + 32x$

3. (a) $x^2 + x$ (b) $x - x^2$ (c) $x^2 - 6x$ (d) $-3x^2 + 2x$
   (e) $-4x^2 + 6x$ (f) $4a^2 + 5a$ (g) $6a^2 - 15a$ (h) $12y^2 - 63y$
   (i) $30y - 12y^2$

4. (a) $2x - 13$ (b) $x^2 - 2x$ (c) $5x + 23$ (d) $6x + 6$
   (e) $11x - 28$ (f) $n^2 + 10n - 8$ (g) $2a + 28$
   (h) $3x^2 - 10x + 24$ (i) $3x^2 - 5x$

5. (a) $x^3 + x^2$ (b) $2x^3 - 10x^2$ (c) $2x + 6$ (d) $8x - 4$
   (e) $6x^3 - 12x$ (f) $4x^3 + 2x^2 + 4x$ (g) $2ap + aq + bp$
   (h) $3ny + 4xy - 5nx$ (i) $2xp$

6. (a) $x^2 + 4x$ (b) $a^2 - a$ (c) $x^2 - 2x$

7. (a) $2(x + 1)$ (b) $2x + 2$ (c) Double then add 2
   (d) $x(x + 1) = x^2 + x$ Think of a number, multiply it by itself then add the original number.

8. (a) b (b) $c - a$ (c) $b(c - a)$ (d) $bc - ba$

2.9 Factorisation

1. (a) 5 (b) 2 (c) 5 (d) $(3x+2)$ (e) $(3-n)$
   (f) $(2x-7)$ (g) $(2a+3)$ (h) $(11x-3)$

2. (a) $6(x+4)$ (b) $5(x-4)$ (c) $8(2-x)$ (d) $4(2n+3)$
   (e) $2(6x-7)$ (f) $3(a-8)$ (g) $11(x-6)$ (h) $5(2+5x)$
   (i) $20(5x-2)$ (j) $10(5-4x)$ (k) $6(x-5)$ (l) $5(y-9)$
   (m) $12(1+3x)$ (n) $16(x+2)$ (o) $3(9x-11)$

3. (a) x (b) x (c) a (d) $(4x+1)$ (e) $(x+4)$
   (f) $(2x+1)$ (g) $(a+b)$ (h) $(2x-a)$

4. (a) $x(5x+1)$ (b) $a(a+3)$ (c) $n(5n+2)$ (d) $3n(2n+1)$
   (e) $5n(n-2)$ (f) $3x(x+2)$ (g) $15x(x-2)$ (h) $7x(2x+3)$
   (i) $8x(2x+3)$ (j) $6x(5x-3)$ (k) $5(1+n^2)$ (l) $5(2n^2-3)$
   (m) $3n(n^2+3)$ (n) $9x(x-3)$ (o) $5x^2(2x-1)$

5. (a) $ax(1+x)$ (b) $x(b+cx)$ (c) $2q(p-2r)$ (d) $5y(3x-y)$
   (e) $8p(2q+3p)$ (f) $6x(x+3y)$ (g) $3p(p-3x)$
   (h) $8x(3p+7x)$ (i) $2xy(8x-9y)$

6. (a) $2x(3x+2)$ (b) $8x^2(2x+1)$ (c) No (d) $3xy(x-6y)$

7. (a) $6x(6+x)$ (b) $x=\dfrac{y-3}{5}$

2.10 Algebraic Manipulation

1. (a) $-a-b$ (b) $\dfrac{b-d}{a-c}$ (c) $\dfrac{-1}{a-b}$ (d) $4a+6$ (e) $\dfrac{c-b}{3}$
   (f) $\dfrac{a-c}{b-d}$ (g) $\dfrac{a-2}{3}$ (h) a (i) $\dfrac{p+q}{q-p}$ (j) $3a+2b$
   (k) $\dfrac{-5-a}{3}$ (l) $\dfrac{ab}{4-a}$

2. (a) $\dfrac{P}{1-P}$ (b) $\dfrac{b}{P-a}$ (c) $\dfrac{Qa+b}{Q-1}$ (d) $\dfrac{q^2y+y}{q^2-1}$
   (e) $\dfrac{-2-3a}{a-1}$ (f) $\dfrac{4c-b}{3}$ (g) $\dfrac{p^2}{1-p^2}$ (h) $\dfrac{-2}{w^2-1}$ or $\dfrac{2}{1-w^2}$
   (i) $\dfrac{w^2+2}{1-w^2}$ (j) $\pm\sqrt{\dfrac{2}{p-1}}$ (k) $\pm\sqrt{\dfrac{3p-2}{p-1}}$ (l) $\pm\sqrt{\dfrac{gy+y}{1-g}}$

2.11 Algebraic Fractions

1. (a) $\frac{9x}{20}$ (b) $\frac{11x}{28}$ (c) $\frac{8x}{15}$ (d) $\frac{41y}{21}$ (e) $\frac{23y}{20}$ (f) $\frac{13y}{7}$
   (g) $\frac{19x}{70}$ (h) $\frac{x}{6}$ (i) $\frac{9x}{8}$ (j) $\frac{27x}{24}$ (k) $\frac{5a+4b}{20}$
   (l) $\frac{8x+3y}{24}$ (m) $\frac{5a-3b}{15}$ (n) $\frac{10a+12b}{15}$ (o) $\frac{32a-27b}{36}$

2. (a) $\frac{4y+2x}{xy}$ (b) $\frac{6y-x}{xy}$ (c) $\frac{y+3x}{xy}$ (d) $\frac{5}{a}$ (e) $\frac{8b+3a}{2ab}$
   (f) $\frac{10b-3a}{6ab}$ (g) $\frac{25b+12a}{15ab}$ (h) $\frac{23}{15a}$ (i) $\frac{17}{28a}$
   (j) $\frac{21b-16a}{24ab}$ (k) $\frac{1}{12a}$ (l) $\frac{11}{8a}$

3. (a) $\frac{2x+1}{x(x+1)}$ (b) $\frac{3x+4}{x(x+2)}$ (c) $\frac{7x+3}{x(x+1)}$ (d) $\frac{4x+10}{x(x+2)}$
   (e) $\frac{4x+2}{x(x+2)}$ (f) $\frac{14x-12}{3x(x+3)}$ (g) $\frac{-6}{x(x+1)}$ (h) $\frac{6x-10}{x(x-5)}$
   (i) $\frac{27x+42}{5x(x+6)}$ (j) $\frac{17x-49}{2x(x-7)}$ (k) $\frac{23x-50}{3x(x-10)}$ (l) $\frac{7x-8}{3x(x-8)}$

4. (a) $\frac{2x+3}{(x+1)(x+2)}$ (b) $\frac{2x}{(x+1)(x+1)}$ (c) $\frac{7}{(x+2)}$
   (d) $\frac{6x-28}{(x-2)(x-6)}$ (e) $\frac{-x-2}{(x+3)(x+4)}$ (f) $\frac{x+35}{(x+7)(x-7)}$
   (g) $\frac{8x+28}{(x-4)(8+x)}$ (h) $\frac{-2x+30}{(x-4)(x+7)}$ (i) $\frac{8x+27}{(x+6)(x-1)}$
   (j) $\frac{5x-2}{(2x+6)(3x-8)}$ (k) $\frac{-6x}{(2x+5)(5-4x)}$ (l) $\frac{17x-7}{(2x-1)(3x-1)}$
   (m) $\frac{37x+13}{(2x+3)(5x-1)}$ (n) $\frac{18x+4}{(3x-7)(2x+3)}$ (o) $\frac{29x+9}{(5x-4)(2x+3)}$

5. (a) $\frac{3x^2}{(x+1)(x-2)}$ (b) $\frac{15x}{(x-7)(2x+1)}$ (c) $\frac{5x^2-3x}{(x-1)(3x-1)}$
   (d) $\frac{x^2+3x+12}{(x+3)(x-1)}$ (e) $\frac{8x^2+11x}{(x-3)(x+4)}$ (f) $\frac{5x}{(2-x)(4x-3)}$
   (g) $\frac{5x^2-13x}{(5-x)(x+1)}$ (h) $\frac{-4x^2-14x}{(4+x)(x+6)}$ (i) $\frac{x^2-4x-18}{(x+6)(x-1)}$

3 Angle Geometry

3.1 Measuring Angles

Note that measured angles are approximate answers

1. (a) 78˚ (b) 120˚ (c) 60˚ (d) 130˚ (e) 125˚ (f) 60˚

3. (a) 315˚ (b) 195˚ (c) 240˚ (d) 325˚ (e) 264˚
   (f) 350˚

5. (a) $a = 62^\circ$, $b = 118^\circ$ (b) $a = 58^\circ$, $b = 76^\circ$, $c = 46^\circ$
   (c) $a = 104^\circ$, $b = 76^\circ$ (d) $a = 42^\circ$, $b = 74^\circ$, $c = 64^\circ$
   The angles add up to 180˚

6. (a) 50˚, 60˚, 70˚ (b) 31˚, 59˚, 90˚ (c) 15˚, 19˚, 147˚
   (d) 33˚, 40˚, 107˚ The three angles add up to 180˚

7. (a) $a = 150^\circ$, $b = 90^\circ$, $c = 120^\circ$ (b) $a = 152^\circ$, $b = 116^\circ$, $c = 63^\circ$, $d = 29^\circ$
   (c) $a = 48^\circ$, $b = 154^\circ$, $c = 35^\circ$, $d = 123^\circ$ (d) $a = 45^\circ$, $b = 45^\circ$, $c = 270^\circ$
   In each case the angles add up to 360˚

8. (c) 7.7 cm and 6.4 cm, 90˚

9. (b) 11.5 cm, 34˚, 66˚

10. (a) 34˚, 34˚, 51˚, 241˚ (b) 25˚, 29˚, 98˚, 208˚
    In both cases the angles add up to 360˚

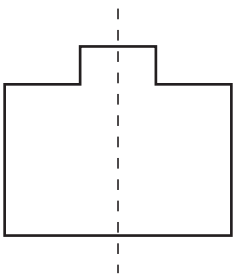
11. The interior angles will always add up to 540˚

3.2 Line and Rotational Symmetry

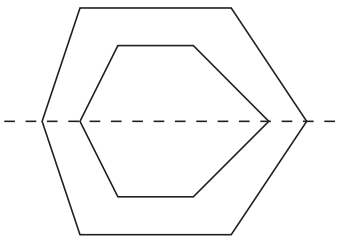
1. (a) B - 2 lines, D - 2 lines, E - 1 line, F - 1 line, G - 4 lines, I - 1 line
   (b) A - order 4, B - order 2, D - order 2, G - order 4, H - order 3

2. A - has symmetry, no lines, order 3
   B - has symmetry, 1 line, no order
   C - has symmetry, 1 line, no order
   D - has symmetry, 1 line, no order
   E - has symmetry, 1 line, no order
   F - has symmetry, 4 lines, order 8
   G - has symmetry, 1 line, no order
   H - has symmetry, no lines, order 4
   I - no symmetry, no lines, no order

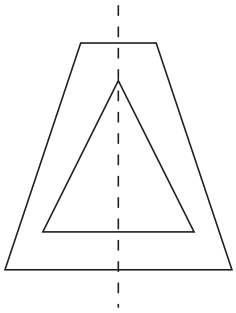
3. (a)



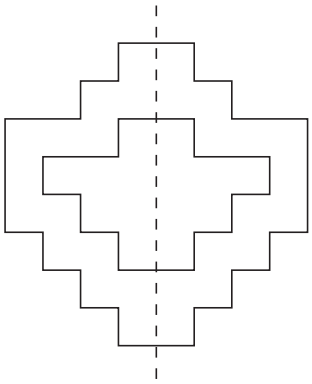
(b)



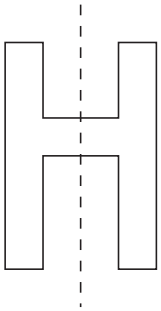
(c)



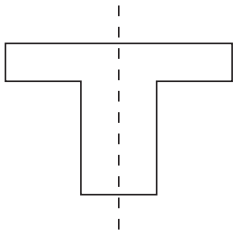
(d)



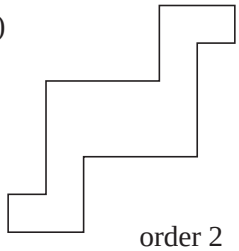
(e)



(f)

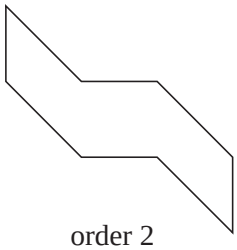


4. (a)



order 2

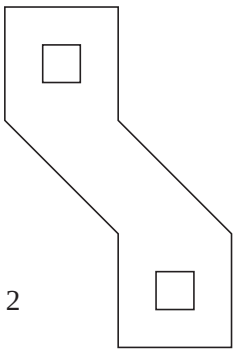
(b)



order 2

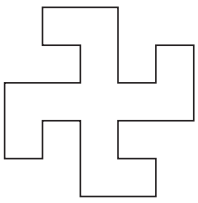
(c) Not possible

(d)



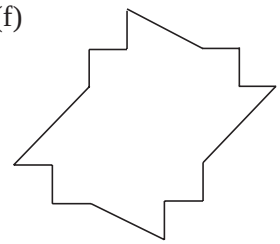
order 2

(e)



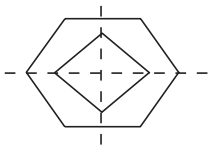
order 4

(f)



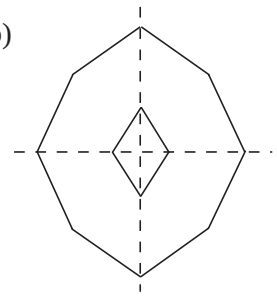
order 2

5. (a)



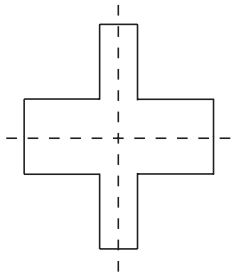
Order 2

(b)



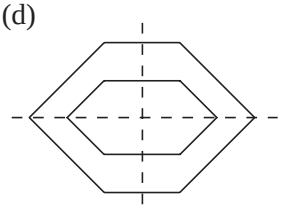
Order 2

(c)



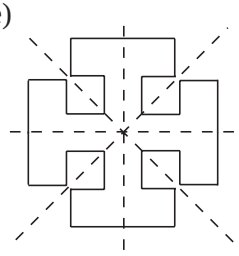
Order 2

(d)



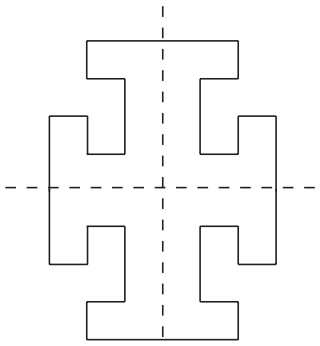
Order 2

(e)



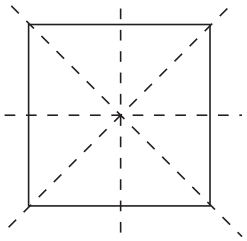
Order 4

(f)

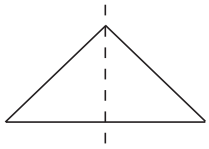


Order 2

6.

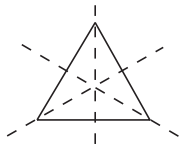


7 (a) (i)



(ie. any isosceles triangle)

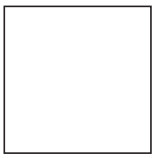
(ii)



(ie. any equilateral triangle)

(b) No.

8.

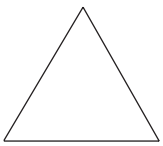


(eg. a square)

9. (a)

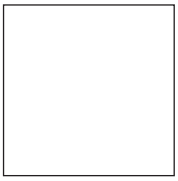
Rotational symmetry order 2

(b)



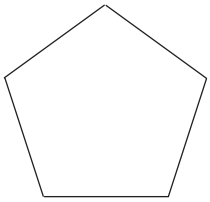
Rotational symmetry order 3

(c)

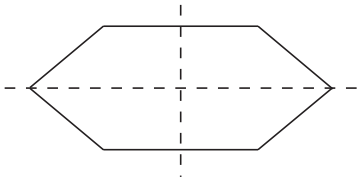


Rotational symmetry order 4

(d)

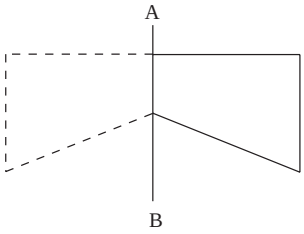


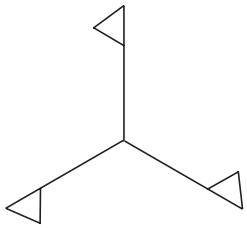
Rotational symmetry order 5

10. (a) No (b)  (c) No

11. Letter I has rotational symmetry.

12. Designs (a), (b) and (d) have line symmetry.

13. (a) 

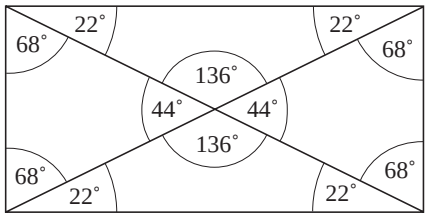
(b) 

(c) Rotational symmetry of order 2.

3.3 Angle Geometry

1. (a) $a = 50°$ (b) $x = 130°$ (c) $b = 92°$ (d) $a = 80°$
   (e) $a = 111°$ (f) $x = 82°$ (g) $x = 110°$ (h) $a = 45°$
   (i) $x = 55°$ (j) $a = b = 70°$ (k) $a = b = c = 60°$
   (l) $a = 50°, b = 80°$ (m) $a = 109°$ (n) $x = 114°$ (o) $x = 87°$

2. (a) (i) $a = 70°, b = 110°$ (ii) $a = 53°, b = 127°$ (iii) $a = 48°, b = 132°$
   (b) b is equal to the sum of the two opposite angles in the triangle.
   (c) (i) $b = 105°$ (ii) $b = 106°$ (iii) $b = 135°$

3. 

4. (a) $a = 75°, b = 75°, c = 30°, d = 75°$
   (b) $a = 60°, b = 60°, c = 30°, d = 60°, e = 60°, f = 60°, g = 30°$
   (c) $a = 80°, b = 45°, c = 45°, d = 55°, e = 80°$
   (d) $a = 30°, b = 20°, c = 10°, d = 80°, e = 80°, f = 60°$

5. (a) $a = 65°, b = 80°$ (b) $a = 40°,$
   (c) $a = 60°, b = 60°, c = 60°, d = 120°, e = 30°$
   (d) $a = 65°, b = 65°, c = 58°, d = 90°, e = 35°$
   (e) $a = 90°, b = 97°, c = 41.5°, d = 41.5°, e = 69°, f = 69°, g = 104°, h = 38°$
   (f) $a = 60°, b = 60°, c = 60°, d = 80°, e = 100°, f = 40°, g = 40°, h = 120°, i = 38°$

6. $a = 44°,\ b = 68°,\ c = 68°,\ d = 112°,\ e = 112°,\ f = 68°$

7. $a = 50°,\ b = 40°,\ c = 70°,\ d = 20°,\ e = 65°,\ f = 50°$

8. $a = 25°,\ b = 110°,\ c = 45°,\ d = 65°,\ e = 70°,\ f = 25°,\ g = 25°$

9. (a) $9x = 180°,\ x = 20°$ (b) $3x - 30 = 180°,\ x = 70°$
(c) $3x + 30 = 180°,\ x = 50°$ (d) $5x = 360°,\ x = 72°$
(e) $4x + 20 = 180°,\ x = 40°$ (f) $4x = 360°,\ x = 90°$
(g) $17x + 20 = 360°,\ x = 20°$ (h) $2x = 30°,\ x = 15°$
(i) $5x + 90 = 360°,\ x = 54°$ (j) $10x + 80 = 180°,\ x = 10°$
(k) $6x = 150°,\ x = 25°$ (l) $13x + 22 = 360°,\ x = 26°$

10. (a) order = 6 (b) (i) AOB = 60° (ii) Equilateral triangle

11. BCD = 134° ABC = 77°

3.4 Angles with Parallel and Intersecting Lines

1. (a) $a = 38°$, Opposite angles
(b) $a = 57°$, Opposite angles, $b = 123°$, Straight line
(c) $a = 60°$, Straight line, $b = 120°$, Opposite angles, $c = 60°$, Opposite angles
(d) $a = 100°$, Straight line, $b = 100°$, Opposite angles
(e) $a = 145°$, Straight line, $b = 35°$, Opposite angles, $c = 145°$, Opposite angles
(f) $a = 50°$, Corresponding angles
(g) $a = 40°$, Corresponding angles, $b = 140°$, Straight line
(h) $a = 60°$, Straight line, $b = 60°$, Corresponding angles, $c = 120°$, Straight line
(i) $a = 42°$, Opposite angles, $b = 138°$, Supplementary angles,
$c = 42°$, Corresponding angles
(j) $a = 100°$, Straight line, $b = 80°$, Opposite angles, $c = 100°$, Opposite angles
$d = 80°$, Corresponding angles
(k) $a = 25°$, Opposite angles, $b = 155°$, Straight line,
$c = 25°$, Corresponding angles
(l) $a = 124°$, Alternate angles, $b = 56°$, Straight line
(m) $a = 37°$, Corresponding angles, $b = 143°$, Straight line,
$c = 37°$, Opposite angles
(n) $a = 56°$, Corresponding then Opposite angles, $b = 124°$, Straight line,
$c = 124°$, Corresponding then Opposite angles,
(o) $a = 160°$, Straight line, $b = 160°$, Corresponding angles,
$c = 20°$, Alternate angles

2. (a) $a = 70°,\ b = 140°$
(b) $a = 60°,\ b = 110°,\ c = 70°,\ d = 120°$
(c) $a = 52°,\ b = 128°,\ c = 52°,\ d = 128°$
(d) $a = 75°,\ b = 105°,\ c = 75°,\ d = 105°$

(e) $a = 60°,\ b = 80°,\ c = 80°$
(f) $a = 70°,\ b = 50°,\ c = 60°,\ d = 70°,\ e = 70°$
(g) $a = 74°,\ b = 100°,\ c = 41°,\ d = 115°$
(h) $a = 48°,\ b = 48°,\ c = 132°,\ d = 138°,\ e = 42°,\ f = 48°$
(i) $a = 64°,\ b = 52°,\ c = 64°$
(j) $a = 38°,\ b = 52°,\ c = 52°$

3. (a) $4x = 180°,\ x = 45°$ (b) $10x = 360°,\ x = 36°$
(c) $8x = 180°,\ x = 22.5°$ (d) $9x = 180°, x = 20°$
(e) $6x = 180°,\ x = 30°$ (f) $8x = 180°,\ x = 22.5°$

4. AB is parallel to EF , GH is parallel to IJ

5. $a = 80°,\ b = 50°,\ c = 80°,\ d = 50°$

6. (a) AC and BD are parallel (b) BAC = 50° because AEC is isosceles

7. (a) Square, Rectangle, Rhombus and Parallelogram
(b) Rectangle, Parallelogram, Kite, Rhombus and Square

8. (a) 36°; alternate angles (b) 54°; angle POQ is 90°

9. (a) $p = 48°$ (b) $q = 84°$ (c) Alternate angles

3.5 Angle Symmetry in Polygons

1. (a) 108° (b) 120° (c) 135° (d) 144°

2. (a) 1260° (b) 1620°

3. (a) Square (b) Hexagon (c) Pentagon (d) Nonagon
(e) Triangle (f) Decagon

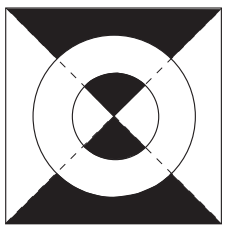
4. Many possible solutions

6. (a) (i) No (ii) No (iii) No (b) (i) Yes (ii) No (iii) No

7. (a) 1260° (b) 180°, 360°, 540°, 720°, 900°, 1080°
(c) $180n - 360$ (d) 2160° (e) 9

8. (a) $\frac{360}{n}$ (b) $180 - \frac{360}{n}$ (c) 162°

9. (a) 2 (c) (i) 5 (ii) 72°

10. 

11. (a) 45° (b) 135°

12. (b) (ii) 120° (c) (i) 2 (ii) 6 (d) Cube

3.6 Symmetry Properties of 3D Shapes

2. (a) 4 (b) 2 (c) 4 (d) 4

3. (d) Not possible

5. 8 planes of symmetry along solid , 1 plane of symmetry through cross section
   5 axes of symmetry

3.7 Compass Bearings

1. (a) Katie (b) James (c) Hester (d) Robin (e) Tom
   (f) Simon (g) NE

2. (a) 100˚ (b) 155˚ (c) 177˚ (d) 355˚ (e) 207˚
   (f) 180˚

3. (a) (i) 150˚ (ii) 090˚ (iii) 055˚ (b) 230 m

4. (a) 128˚ (b) 267˚ (c) 257˚ (d) 317˚ (e) 100˚

5. (a) 535 m, 450 m (b) 38 m (c) 348˚, 67 km (d) 293˚
   (e) 073˚, 465 m

6. (a) 315˚, 169 m (b) 203˚, 400 km (c) 015˚, 545 km
   (d) 043˚, 10 km (e) 348˚, 8.5 km (f) 015˚, 15.5 km
   (g) 175˚, 1430 m

7. 188˚, 7000 km

8. (a) Kendal (b) Taunton

9. (5 km , 50˚)

10. (a) (i) 20˚ (ii) alternate angles (iii) 48˚
    (b) (i) 052˚ (ii) 212˚ (iii) 312˚

12. (a) 103˚

3.8 Angles and Circles 1

1. (a) $a = 90^\circ$, $b = 65^\circ$
   (b) $a = 72^\circ$, $b = 90^\circ$
   (c) $a = 90^\circ$, $b = 76^\circ$, $c = 90^\circ$, $d = 74^\circ$
   (d) $a = 90^\circ$, $b = 58^\circ$, $c = 90^\circ$, $d = 32^\circ$
   (e) $a = 90^\circ$, $b = 49^\circ$, $c = 60^\circ$, $d = 60^\circ$, $e = 30^\circ$, $f = 30^\circ$, $g = 120^\circ$
   (f) $a = 40^\circ$, $b = 100^\circ$, $c = 50^\circ$, $d = 50^\circ$, $e = 80^\circ$
   (g) $a = 44^\circ$, $b = 44^\circ$, $c = 46^\circ$, $d = 46^\circ$, $e = 88^\circ$, $f = 44^\circ$
   (h) $a = 70^\circ$, $b = 40^\circ$, $c = 140^\circ$, $d = 20^\circ$, $e = 20^\circ$

3. (a) $a = 90^\circ$, $b = 65^\circ$ (b) $a = 74^\circ$ (c) $a = 90^\circ$, $b = 90^\circ$, $c = 50^\circ$, $d = 75^\circ$
(d) $a = 10^\circ$, $b = 170^\circ$

4. (a) $a = 20^\circ$, $b = 140^\circ$ (b) $a = 25^\circ$, $b = 25^\circ$ (c) $a = 30^\circ$, $b = 120^\circ$
(d) $a = 100^\circ$, $b = 40^\circ$, $c = 40^\circ$
(e) $a = 48^\circ$, $b = 84^\circ$, $c = 42^\circ$, $d = 42^\circ$, $e = 96^\circ$
(f) $a = 75^\circ$, $b = 75^\circ$, $c = 15^\circ$, $d = 15^\circ$, $e = 150^\circ$
(g) $a = 69^\circ$, $b = 69^\circ$, $c = 42^\circ$, $d = 69^\circ$, $e = 69^\circ$
(h) $a = 28^\circ$, $b = 124^\circ$, $c = 70^\circ$, $d = 40^\circ$, $e = 16^\circ$

5. (a) $a = 30^\circ$, $b = 70^\circ$, $c = 70^\circ$, $d = 80^\circ$ (b) $a = 110^\circ$, $b = 140^\circ$
(c) $a = 110^\circ$, $b = 140^\circ$, $c = 40^\circ$, $d = e = f = g = 70^\circ$
(d) $a = b = 65^\circ$, $c = 60^\circ$, $d = 115^\circ$

6. (a) 5 (b) 7.8 (c) 12 (d) 10

3.9 Angles and Circles 2

1. (a) 30° (b) 120° (c) $c = d = 35^\circ$ (d) 146°
(e) $f = 90^\circ$, $g = 55^\circ$ (f) $x = y = 43^\circ$
(g) $a = 65^\circ$, $b = 25^\circ$, $c = 25^\circ$, $d = 65^\circ$ (h) $a = 27^\circ$, $b = 126^\circ$, $c = 63^\circ$

2. (a) OAB = OBA, BAC = BFC, OGA = OAG, OFC = OCF,
OCB = OBC, FCB = ACD = GAC
(b) angles GAC , ACD , BCF

3. (a) $a = 120^\circ$, $b = 75^\circ$ (b) $c = 149^\circ$, $d = 123^\circ$
(c) $a = 55^\circ$, $b = 125^\circ$ (d) $c = 140^\circ$ (e) $a = 48^\circ$, $b = 75^\circ$
(f) $a = 75^\circ$, $b = 100^\circ$ (g) $a = 85^\circ$, $b = 30^\circ$ (h) $x = 160^\circ$

4. ABCD and PQR

5. (a) $a = 37^\circ$, $b = 108^\circ$, $c = 37^\circ$ (b) $a = 30^\circ$, $b = c = 75^\circ$, $d = 60^\circ$, $e = 30^\circ$
(c) $a = 32.5^\circ$, $b = 147.5^\circ$ (d) 34°

6. (a) 50° (b) 22.5° (c) 40°

9. $x = 94^\circ$, $y = 28^\circ$, $z = 19^\circ$

10. (a) x (b) $90 - x$ (c) x (d) $2x$

3.10 Circles and Tangents

1. (a) 40° (b) $b = 55^\circ$, $c = 35^\circ$ (c) $a = b = c = 70^\circ$
(d) $a = 11^\circ$, $b = 79^\circ$, $c = 79^\circ$, $d = 22^\circ$ (e) $a = 52^\circ$, $b = 104^\circ$
(f) $a = b = c = 24^\circ$, $d = 62^\circ$

2. (a) 4.8 (b) $\frac{8}{3}$ (c) $x = 3.9$, $y = 4$ (d) $x = 7$, $y = 3.5$
(e) $x = 4$, $y = 6$ (f) $x = 3$, $y = 2$

3. (c) 6

4. (b) 30°

5. $a = 74^\circ$, $b = 36^\circ$, $c = 32^\circ$

6. (a) $x = 65$, (b) $y = 130$, (c) $z = 50$

4 Trigonometry

4.1 Squares and Triangles

1. (a) Isosceles (b) Scalene (c) Equilateral (d) Isosceles
   (e) Scalene (f) Equilateral (g) Isosceles (h) Isosceles

2. (a) Isosceles (b) Scalene

3. (a) 25 cm^2 (b) 49 cm^2 (c) 100 m^2 (d) 1 cm^2

4. (a) 4 m^2 (b) 10 000 m^2 (c) 225 cm^2 (d) 289 cm^2

5. (a) 3 cm (b) 5 m (c) 10 m (d) 8 cm (e) 1 cm (f) 20m

6. 32 cm^2

7. 72 cm^2

8. 9 cm^2

4.2 Pythagoras' Theorem

1. (a) 5 m (b) 13 m (c) 9 cm (d) 24 m (e) 10 m (f) 8 cm
   (g) 15 m (h) 39 cm

2. (a) 13.04 cm (b) 20.52 cm (c) 8.94 cm (d) 8.60 m (e) 7.14 cm
   (f) 8.94 cm (g) 7.81 m (h) 11.83 m (i) 14.97 cm (j) 6.40 m
   (k) 10.47 m (l) 7.07 m (m) 7.22 cm (n) 4.89 m

3. (a) 320 m (b) 233.2 m (c) 86.8 m

4. 2.5 m

5. 2.44 m

6. 9.54 m

7. 10.77 m

8. 4.24 m

9. 2.06 m

10. (a) 10.44 km (b) 14.32 km

11. 6.71 m

12. (a) 295 m (b) X

4.3 Further Work with Pythagoras' Theorem

1. (a) 7.14 (b) 7.07 (c) 5.39 (d) 7.75 (e) 10.95
   (f) 14.28

2. (a) 14.14 (b) 1.94 (c) 1.29 (d) 3.12

3. (a) Yes (b) No (c) Yes (d) No

4. (a) 3.46 m (b) 1.73 m (c) 5.69 m

5. 122.47 cm

6. 332.75 cm

7. (a) 26.93 km (b) 26.93 km

8. 28.21 cm

9. 11.18 m , 11.18 m , 19.02 m

10. 41.22 m , 48.21 m

11. 7.75 cm ; 15.49 cm^2

12. (a) 27.71 cm^2 (b) 173.21 cm^2 (c) 1.73 cm^2

4.4 Sine, Cosine and Tangent

1. (a) hyp : BC ; adj : AC ; opp : AB
   (b) hyp : DF ; adj : DE ; opp : EF
   (c) hyp : GI ; adj : GH ; opp : HI
   (d) hyp : LK ; adj : LJ ; opp : JK
   (e) hyp : MO ; adj : MN ; opp : NO
   (f) hyp : PQ ; adj : RQ ; opp : PR

2. (a) $\sin\theta = \frac{3}{5}$ $\cos\theta = \frac{4}{5}$ $\tan\theta = \frac{3}{4}$

   (b) $\sin\theta = \frac{5}{13}$ $\cos\theta = \frac{12}{13}$ $\tan\theta = \frac{5}{12}$

   (c) $\sin\theta = \frac{15}{17}$ $\cos\theta = \frac{8}{17}$ $\tan\theta = \frac{15}{8}$

   (d) $\sin\theta = \frac{2}{2.5} = \frac{4}{5}$ $\cos\theta = \frac{1.5}{2.5} = \frac{3}{5}$ $\tan\theta = \frac{2}{1.5} = \frac{4}{3}$

   (e) $\sin\theta = \frac{48}{50}$ $\cos\theta = \frac{14}{50}$ $\tan\theta = \frac{48}{14}$

   (f) $\sin\theta = \frac{3.5}{12.5} = \frac{7}{25}$ $\cos\theta = \frac{12}{12.5} = \frac{24}{25}$ $\tan\theta = \frac{3.5}{12} = \frac{7}{24}$

3. (a) 0.500 (b) 3.732 (c) 1.308 (d) 0.407 (e) 0.649
(f) 1.000 (g) 0.754 (h) 1.000 (i) 0.707 (j) 0.669
(k) 0.686 (l) 0.707

4. (a) 60° (b) 90° (c) 24.2° (d) 55.2° (e) 48.6° (f) 23.1°
(g) 45° (h) 30° (i) 63.4° (j) 82.0° (h) 15.1° (i) 79.2°

6. (a) $\cos\theta = \frac{z}{x}$ (b) $\sin\alpha = \frac{z}{x}$ (c) $\tan\theta = \frac{y}{z}$ (d) $\cos\alpha = \frac{y}{x}$
(e) $\sin\theta = \frac{y}{x}$ (f) $\tan\alpha = \frac{z}{y}$

4.5 Finding Lengths in Right Angled Triangles

1. (a) 5.14 cm (b) 11.82 cm (c) 5.13 cm (d) 6.06 cm (e) 9 cm
(f) 8.21 cm (g) 10.63 cm (h) 18.38 cm (i) 15.59 cm (j) 6.68m
(k) 10.28 m (l) 20 m (m) 11.30 m (n) 4.16 m (o) 7.43 m

2. (a) 3.71 m (b) 1.50 m

3. (a) 1.73 m (b) 1.21 m (c) 1 m

4. 0.60 m

5. 143.4 m

6. (a) 386.4 km (b) 103.5 km

7. (a) 103.9 km (b) 60 km

8. 20.5 m to 35.3 m

9. (a) 12.11 cm (b) 13.46 cm (c) 6.55 cm (d) 7 cm
(e) 26.86 m (f) 38.83 m (g) 13.68 cm (h) 30.66 cm
(i) 1.51m

10. (a) 1.88 m (b) 2.92 m

11. 124.5 cm

12. (a) 3.83 cm , 22.98 cm^2 (b) $a\tan\theta$; $\frac{1}{2}a^2\tan\theta$

13. 10.34 m

14. (a) 4.44 cm (b) 7.56 cm

4.6 Finding Angles in Right Angled Triangles

1. (a) 53.1° (b) 71.6° (c) 75.5° (d) 47.0° (e) 33.1°
   (f) 18.6° (g) 29.1° (h) 14.5° (i) 45.6° (j) 14.5°
   (k) 45.5° (l) 23.8°

2. 60°

3. 11.5°

4. (a) 21.8° (b) 68.2°

5. (a) 48.2° (b) 11.18 m

6. 040°

7. 306°

8. (a) $\alpha = 33.7°,\ \beta = 19.4°$ (b) 7.21 m , 10 m

9. 5.74°

10. (a) 12.37 cm (b) 72.08°

11. (a) 7.62 m (b) 30.96°

4.7 Mixed Problems with Trigonometry

1. 8.82 m

2. 12.50 m

3. 7.13°

4. 1.03°

5. (a) 381.6 m (b) 1.91°

6. 7.85 m

7. 7.20 m

8. (a) 8.76 m (b) 6.02 m (c) 6.56 m (d) 5.09 m

9. (a) 57.15 m (b) 12.02 m 10. 23.58° ; 938.6 m

11. (a) 8.96 m (b) 38.5° (c) 6.72 m

12. (a) (i) 373.4 m (ii) 20.4° (b) 200.2 m

4.8 Sine and Cosine Rules

1. (a) 51.6° (b) 52.3° (c) 48.8° (d) 69.4° (e) 34.2° (f) 56.0°
2. (a) 5.43 (b) 9.05 (c) 6.01 (d) 30.13 (e) 9.84 (f) 4.77

3. (a) A = 52.8° B = 42.2° $a = 3.68$
   (b) A = 19.9° B = 50.1° $b = 12.16$
   (c) B = 73° $b = 4.45$ $c = 3.56$
   (d) A = 44.2° B = 56.8° $a = 4.33$

4. (a) Yes (b) No, only one (c) No, impossible even for one (d) Yes

5. B = 65.6°, C = 47.4°, $b = 123.6$

6. A = 34.1°, C = 64.9°, $a = 6.25$

7. (a) B = 52.4°, C = 67.6°, $a = 3.28$ (b) B = 25.1°, C = 4.9°, $a = 17.66$
   (c) A = 45.5°, B = 106.6°, C = 27.9°
   (d) A = 120.5°, B = 36.9°, C = 22.6°
   (e) $b = 8.41$, A = 64.92°, C = 60.08°
   (f) $c = 9.81$, A = 28.45°, B = 21.55°

8. (a) 263.7 m (b) 192.9 m

9. 2.65 miles

10. (a) 117.3° (b) 10.2 m

11. 60.02°

12. (a) 23.35 cm (b) 62.66°

13. 303.1°

14. 45.98 (if no allowance made for inaccurate measurements) or 46.87

15. 26.8 m

16. (a) 80° (b) 6.99 cm

4.9 Angles Larger than 90°

1. (a) $\frac{\sqrt{3}}{2}$ (b) $-\frac{1}{2}$ (c) $-\frac{1}{\sqrt{2}}$ (d) $-\frac{1}{2}$ (e) $-\frac{1}{\sqrt{2}}$ (f) -1
   (g) $\frac{1}{2}$ (h) $\frac{\sqrt{3}}{2}$ (i) $\frac{1}{\sqrt{2}}$ (j) $-\frac{\sqrt{3}}{2}$ (k) -1 (l) $-\frac{1}{2}$

3. (a) 0.766 (b) – 0.819 (c) – 0.766 (d) 0.643 (e) 0.940
   (f) – 0.766 (g) – 0.985 (h) – 0.985 (i) – 0.259 (j) 0.985
   (k) 0.707 (l) – 0.766

4. 6 ; -135°, -45°, 225°, 315°, 585°, 675°

5. 6 ; -300°, -60°, 60°, 300°, 420°, 660°

6. (a) -315.6°, -224.4°, 44.4°, 135.6°
   (b) -156.4°, -23.6°, 203.6°, 336.4° (c) -90°, 270°
   (d) -306.9°, -53.1°, 53.1°, 306.9°
   (e) -246.4°, -113.6°, 113.6°, 246.4° (f) -180°, 180°

7. (a) 14.0°, 194.0°, 374.0°, 554.0° (b) 45°, 225°, 405°, 585°
   (c) 153.4°, 333.4°, 513.4°, 693.4°

8. (a) 306.9° (b) 143.1° (c) 220° (d) 270°

9. (b) 180° (c) 120°, 720°

10. (a) $y = 5\cos 4x$ (b) $y = 4\sin\left(\frac{3x}{2}\right)$ (c) $y = 0.2\cos 10x$

    (d) $y = 0.7\sin\left(\frac{x}{3}\right)$

13. $y = 10.5\sin 30(x-4) + 10.5$; 19.6 °C, 1.4 °C

14. (b) $T = 98.6 + 0.3\cos 15(t-17)$

15. (a) $60^\circ, 300^\circ$ (b) 330° (d) $r = q + 180^\circ$

16. (b) 60° (c) 300°, 420°

5 Probability

5.1 Probabilities

1. (a) 0 (b) about 250 (c) about 250

2. (a) 50 (b) 50 (c) 0

3. (a) Impossible (b) Unlikely (c) Likely or Unlikely
(d) Likely or Unlikely (e) Unlikely (f) Likely (g) Likely

5. (a) 10 (b) 20 (c) 1000 (d) 600

6. About 900

7. (a) about 1500 (b) about 250

8. (a) 50 (b) 50 (c) 25 (d) 25

5.2 Simple Probability

1. (a) 0.1 (b) $\frac{1}{4}$ (c) $\frac{1}{2}$ (d) $\frac{4}{5}$

2. (a) 0.2 (b) 0.4

3. 0.98

4. $\frac{4}{5}$

5. (a) $\frac{4}{7}$ (b) not to snow

6. 0.99

7. (a) $\frac{2}{5}$ (b) 12

8. (a) 0.6 (b) 0.9 (c) 0.1

9. (a) $\frac{9}{20}$ (b) $\frac{10}{11}$ (c) $\frac{9}{13}$

10. No

11. 0.15

12. (a) C (b) B

13. (a) near to 0 (b) near to 1

5.3 Outcome of Two Events

2. VC, VS, MC, MS, RC, RS
3. GG, RG, GR, RR
4. (a) Nigel wins sit ups and Ben wins press ups
   (b) BJ, BN, BB, JB, JN, JJ, NB, NJ, NN (c) BJ, BB, JB, JJ
   (d) BJ, BN, BT, BB, JB, JN, JT, JJ, NB, NJ, NT, NN, TB, TN, TJ, TT
5. BS, BT, BD, BB, ST, SD, SS, TD, TT, DD
6. CD, CB, CC, DB, DD, BB
7.

	F	M
H	HF	HM
T	TF	TM
C	CF	CM

8. HHH ; HHT, HTH, THH ; HTT, THT, TTH ; TTT
9. (b) $\frac{4}{9}$

5.4 Finding Probabilities Using Relative Frequency

4. $\frac{1}{4}$
5. (a) $\frac{4}{5}$
6. (a) $\frac{2}{7}, \frac{7}{10}, \frac{1}{4}, \frac{1}{5}, \frac{1}{3}, \frac{2}{3}$ (b) Andrew (c) Rachel (d) Charles
7. (a) $\frac{3}{8}$ (b) $\frac{5}{12}$ (c) $\frac{5}{24}$
8. (a) $\frac{4}{5}$ (b) 96

5.5 Determining Probabilities

1. (a) $\frac{1}{4}$ (b) $\frac{1}{4}$ (c) $\frac{1}{13}$ (d) $\frac{1}{13}$ (e) $\frac{4}{13}$
2. (a) $\frac{13}{54}$ (b) $\frac{13}{54}$ (c) $\frac{2}{27}$ (d) $\frac{2}{27}$ (e) $\frac{8}{27}$
3. (a) $\frac{1}{6}$ (b) $\frac{1}{6}$ (c) $\frac{1}{2}$ (d) $\frac{1}{2}$
4. (a) $\frac{1}{8}$ (b) $\frac{1}{8}$ (c) $\frac{1}{2}$ (d) $\frac{5}{8}$ (e) $\frac{1}{4}$ 5. $\frac{3}{8}$
6. (a) $\frac{2}{5}$ (b) $\frac{4}{5}$ (c) $\frac{1}{5}$ (d) $\frac{4}{5}$ (e) $\frac{9}{49}$ (f) $\frac{39}{49}$ (g) $\frac{10}{49}$
7. (a) $\frac{9}{25}$ (b) $\frac{6}{25}$ (c) $\frac{3}{5}$ (d) $\frac{16}{25}$
8. (a) $\frac{2}{5}$ (b) $\frac{1}{5}$ (c) 1 (d) $\frac{4}{5}$

9. (a) $\frac{2}{5}$ (b) $\frac{1}{5}$ (c) $\frac{1}{5}$

10. (a) $\frac{3}{10}$ (b) $\frac{2}{9}$ (c) $\frac{1}{8}$ (d) $\frac{3}{8}$

11. (a) $\frac{7}{30}$ (b) $\frac{23}{30}$

12. (a) $\frac{1}{2}$ (b) $\frac{1}{6}$

13. (a) $\frac{1}{200}$ (b) 20

14. (a) $\frac{3}{10}$ (b) $\frac{4}{5}$

15. (a) Mint (b) $p(\text{mint}) = \frac{2}{3}$, $p(\text{toffee}) = \frac{1}{4}$, $p(\text{pen}) = \frac{1}{12}$ (c) $\frac{1}{12}$ (d) 0

5.6 Probability of Two Events

1. (a) $\frac{1}{4}$ (b) $\frac{1}{2}$ (c) $\frac{1}{2}$

2. (a) $\frac{1}{12}$ (b) $\frac{1}{4}$ (c) $\frac{1}{4}$ (d) $\frac{1}{3}$ (e) $\frac{1}{2}$

3. (a) (i) $\frac{1}{6}$ (ii) $\frac{1}{9}$ (iii) $\frac{1}{2}$ (iv) $\frac{5}{18}$ (v) $\frac{5}{18}$ (b) 7

4. (b) (i) $\frac{1}{9}$ (ii) $\frac{1}{3}$ (iii) $\frac{8}{9}$ (c) $\frac{1}{9}$

5. (a) $\frac{1}{4}$ (b) $\frac{1}{2}$

6. (a) $\frac{1}{8}$ (b) $\frac{5}{8}$ (c) 5

7. (a) 6 (b) $\frac{2}{3}$ (c) $\frac{1}{3}$

8. (a) 400 (b) $\frac{1}{400}$ (c) $\frac{1}{20}$ (d) $\frac{1}{25}$

9. (a) $\frac{1}{4}$ (b) $\frac{1}{8}$ (c) $\frac{7}{8}$

10. (a) (i) 1, 2 ; 1, 3 ; 2, 2 ; 2, 3 ; 3, 2 ; 3, 3 (ii) $\frac{1}{3}$ (iii) $\frac{2}{3}$ (b) (i) 9

11. (a) $\frac{1}{5}$ (b) $\frac{2}{5}$ (c) AX , AY , AZ , BW , BX , BY , BZ , CW , CX , CY , CZ , DW , DX , DY , DZ , EW , EX , EY , EZ

12. (b) $\frac{1}{8}$

13. (b) $\frac{4}{9}$

5.7 Use of Tree Diagrams

1. (b) $0.6 \times 0.6 = 0.36$; $0.6 \times 0.4 = 0.24$; $0.4 \times 0.6 = 0.24$; $0.4 \times 0.4 = 0.16$
   (c) 0.16 (d) 0.36 (e) 0.48

2. (a) $\frac{1}{6}$ (c) (i) $\frac{1}{36}$ (ii) $\frac{5}{18}$ (iii) $\frac{25}{36}$

3. (a) $\frac{1}{4}$ (b) $\frac{1}{4}$ (c) $\frac{1}{2}$

4. (a) $\frac{81}{100}$ (b) $\frac{99}{100}$ (c) $\frac{1}{100}$

5. (a) $\frac{1}{4}$ (c) (i) $\frac{9}{16}$ (ii) $\frac{3}{8}$ (iii) $\frac{1}{16}$

6. (b) (i) 0.54 (ii) 0.36 (iii) 0.04

7. (a) (i) 0.2 (ii) 0.4 (b) (i) 0.48 (ii) 0.08

8. (a) (i) 0.2704 (ii) 0.2304 (iii) 0.4992 (b) (iii)

9. (a) 0.56 (b) 0.38 (c) 0.04 (d) 0.64

10. (a) $\frac{7}{18}$ (b) $\frac{1}{36}$ (c) $\frac{3}{4}$

11. (b) (i) $\frac{1}{4}$ (ii) $\frac{1}{2}$ (iii) $\frac{3}{8}$

12. (b) 0.52

13. (a) $\frac{7}{13}$ (b) $\frac{43}{91}$

14. (b) $\frac{3}{14}$

15. (a) 0.6 (b) 0.16

16. (a) $\frac{1}{216}$ (b) $\frac{5}{216}$ (c) $\frac{5}{72}$ (d) $\frac{2}{27}$

5.8 Multiplication for Independent Events

1. NI - not independent I - independent (a) NI (b) I (c) I
   (d) NI (e) NI (f) NI

2. (a) (i) $\frac{5}{8}$ (ii) $\frac{3}{8}$ (iii) $\frac{25}{64}$ (iv) $\frac{9}{64}$ (v) $\frac{15}{64}$ (vi) $\frac{15}{64}$
   (b) (i) $\frac{9}{64}$ (ii) $\frac{15}{32}$ (iii) $\frac{17}{32}$

3. (a) (i) $\frac{7}{10}$ (ii) $\frac{3}{10}$ (iii) $\frac{9}{100}$ (iv) $\frac{49}{100}$ (v) $\frac{21}{100}$
   (vi) $\frac{21}{100}$ (b) (i) $\frac{49}{100}$ (ii) $\frac{21}{50}$ (iii) $\frac{29}{50}$
4. (a) $\frac{1}{4}$ (b) $\frac{1}{6}$
5. (a) 0.06 (b) No
6. (a) 0.72 (b) 0.02 (c) 0.08
7. (a) 0.1 (b) 0.4 (c) 0.4 (d) 0.1
8. (a) 0.42 (b) 0.16
9. (a) $\frac{1}{49}$ (b) $\frac{36}{49}$ (c) $\frac{12}{49}$ (d) $\frac{1}{343}$
10. (a) No (b) $\frac{171}{250}$ (c) $\frac{283}{1000}$
11. (a) 0.003 (b) 0.833
12. (a) $\frac{1}{25}$ (b) $\frac{8}{25}$
13. (a) 0.58 (b) 0.44
14. 0.04
15. (a) (i) $\frac{1}{6}$ (ii) $\frac{1}{36}$ (b) $\frac{13}{36}$

5.9 Mutually Exclusive Events

1. B ; C
2. (a) Yes (b) No (c) No (d) Yes (e) No
3. 0.3
4. $\frac{1}{6}$
5. (a) $\frac{3}{7}$ (b) 12 (c) 14
6. (a) $\frac{7}{20}$ (b) Not mutually exclusive
7. (a) $\frac{2}{3}$ (b) $\frac{11}{15}$ (c) $\frac{3}{5}$ (d) $\frac{3}{5}$
8. (a) $\frac{5}{8}$ (b) $\frac{11}{20}$ (c) $\frac{7}{40}$
9. (a) 0.1 (b) 0.7
10. Pink : $\frac{2}{3}$ Yellow : $\frac{2}{7}$ Black : $\frac{11}{35}$
11. (a) $\frac{13}{25}$ (b) $\frac{22}{25}$ (c) No (d) No (e) Yes : $\frac{12}{25}$
12. (a) 0.2 (b) 0.7 (c) 20
13. (a) 0.7 (b) 0.7 (c) 0.6 (d) not mutually exclusive
14. (a) (i) 0.25 (ii) 0.75 (b) (i) 0.4 (ii) not mutually exclusive

5.10 Tree Diagrams and Conditional Probability

1. (a) $\frac{1}{7}$ (b) $\frac{3}{7}$ (c) $\frac{4}{7}$

2. (a) (i) $\frac{1}{17}$ (ii) $\frac{13}{34}$ (iii) $\frac{19}{34}$ (b) (i) $\frac{1}{221}$ (ii) $\frac{188}{221}$ (iii) $\frac{32}{221}$

3. $\frac{43}{91}$

4. (a) 0.24 (b) 0.09

5. (a) (i) 0.16 (ii) 0.06 (iii) 0.78 (b) 0.01

6. (a) $\frac{1}{12}$ (b) $\frac{1}{4}$ (c) $\frac{3}{4}$

7. (a) $\frac{11}{46}$ (b) $\frac{7}{69}$ (c) $\frac{1}{46}$ (d) $\frac{25}{69}$ (e) $\frac{8}{69}$

8. (a) $\frac{585}{812}$ (b) $\frac{801}{4060}$ (c) $\frac{34}{203}$

9. 0.52

10. (a) $\frac{1}{4}$ (b) $\frac{1}{2}$ (c) $\frac{1}{2}$; $\frac{47}{108}$

11. (b) 0.15

12. (b) 0.42 (c) 0.4998

13. (a) $\frac{7}{75}$ (b) $\frac{3}{76}$ (c) $\frac{7}{15}$

14. (b) 0.384 (c) 0.388

15. (a) $\frac{7}{11}$ (c) PP , CPP , PCP (d) $\frac{126}{165} \approx 0.764$

5.11 Using Venn Diagrams to Find Probabilities

1. (i) (a) $\frac{7}{20}$ (b) $\frac{4}{20}$ (c) 1 (d) $\frac{9}{20}$

   (ii) (a) $\frac{4}{17}$ (b) $\frac{6}{17}$ (c) 1 (d) $\frac{7}{17}$

   (iii) (a) 0 (b) $\frac{1}{5}$ (c) $\frac{9}{10}$ (d) $\frac{7}{10}$

   (iv) (a) $\frac{1}{8}$ (b) $\frac{2}{5}$ (c) $\frac{3}{4}$ (d) $\frac{9}{40}$

2. (a) $\frac{1}{26}$ (b) $\frac{7}{13}$ (c) $\frac{6}{13}$

3. (a) 0.4 (b) 0 (c) 0.4

4. (a) $\frac{5}{6}$ (b) $\frac{1}{6}$ (c) $\frac{1}{3}$ (d) $\frac{1}{6}$

5. (a) $\frac{2}{5}$ (b) $\frac{1}{6}$ (c) $\frac{1}{2}$

6. (i) (a) $\frac{3}{40}$ (b) $\frac{29}{40}$ (c) $\frac{21}{40}$ (d) $\frac{1}{8}$ (e) $\frac{3}{5}$ (f) $\frac{1}{4}$

(ii) (a) $\frac{1}{12}$ (b) $\frac{47}{60}$ (c) $\frac{7}{12}$ (d) $\frac{7}{60}$ (e) $\frac{13}{20}$ (f) $\frac{11}{30}$

(iii) (a) 0 (b) $\frac{4}{5}$ (c) $\frac{31}{50}$ (d) $\frac{7}{50}$ (e) $\frac{16}{25}$ (f) $\frac{11}{50}$

(iv) (a) 0 (b) $\frac{9}{10}$ (c) $\frac{23}{50}$ (d) $\frac{1}{5}$ (e) $\frac{2}{3}$ (f) $\frac{13}{30}$

7. 0.558

6 Number system

6.1 Decimals

1. (a) 0.7 (b) 0.8 (c) 0.3 (d) 0.05 (e) 0.21 (f) 0.42
   (g) 0.005 (h) 0.151 (i) 0.022 (j) 0.08 (k) 0.13
   (l) 0.016 (m) 0.5 (n) 0.04 (o) 0.321

2. (a) $\frac{4}{10}$ (b) $\frac{3}{10}$ (c) $\frac{4}{100}$ (d) $\frac{32}{100}$ (e) $\frac{45}{100}$
   (f) $\frac{6}{100}$ (g) $\frac{8}{100}$ (h) $\frac{14}{100}$ (i) $\frac{8}{1000}$ (j) $\frac{147}{1000}$
   (k) $\frac{36}{1000}$ (l) $\frac{4}{100}$ (m) $\frac{1}{10}$ (n) $\frac{9}{1000}$ (o) $\frac{107}{1000}$

3. (a) 5.6 (b) 3.3 (c) 7.8 (d) 6.42 (e) 7.17 (f) 3.73
   (g) 4.6 (h) 4.8 (i) 3.16 (j) 3.94 (k) 10.2 (l) 1.4

5. (a) 1.51 (b) 0.424 (c) 0.282 (d) 0.839 (e) 1.102
   (f) 0.281 (g) 0.858 (h) 0.738 (i) 0.372 (j) 11.87
   (k) 12.291 (l) 17.48 (m) 8.73 (n) 130.65 (o) 50.006

6. (a) hundredths (b) tenths (c) hundredths (d) tenths
   (e) thousandths (f) thousandths

7. (a) £5.16, £3.08, £4.56, £5.50 (b) £9.15 (c) £2.11

8. (a) £3.28 (b) £1.52 (c) £8.42 (d) £11.21 (e) £0.48
   (f) £1.27 (g) £0.64 (h) £320.11 (i) £84.21

9. (a) £1.78 (b) £3.22

10. (a) £2.40 (b) £3.50

11. 1.87

12. 76 cm

13. 0.8 kg

14. (a) 5 (b) 5p

15. (a) 10.85 kg (b) 26.55 kg (c) 105 dollars

6.2 Multiplying and Dividing with Decimals

1. (a) 47.4 (b) 632 (c) 4.16 (d) 1274 (e) 0.1658
   (f) 3.24 (g) 630 (h) 4700 (i) 32000 (j) 47000
   (k) 0.0068 (l) 0.82 (m) 0.192 (n) 0.014 (o) 180

2. (a) 36 (b) 1410 (c) 10500 (d) 132000 (e) 6000
   (f) 10400 (g) 3.3 (h) 0.37 (i) 0.007 (j) 0.007
   (k) 0.171 (l) 0.13 (m) 10860 (n) 23600 (o) 0.099
   (p) 0.06 (q) 0.6 (r) 0.0035

3. (a) 40 (b) 2500 (c) 80 (d) 600 (e) 13 200
(f) 3100 (g) 5200 (h) 4000 (i) 700

4. (a) (i) 360 p (ii) 60 000 p (iii) 4800 p (b) (i) £3.60 (ii) £600
(iii) £48 (c) 300 000

5. £124.87 , £413 , £107 , £122.40

6. (a) 0.012 miles $(\approx$ 21 yards $\approx$ 63 ft $\approx$ 760 inches ≈ 19 m$)$
(b) (i) 5000 hours (ii) 90 hours

7. (a) (i) £108 (ii) £459 (iii) £810 (b) (i) £20 (ii) £88 (iii) £120

8. (a) 1500 (b) 500

9. 21 120 litres

10. (a) (i) £350 000 (ii) £560 000 (iii) £770 000 (b) 2000

11. (a) 540 (b) 50

12. (a) 6500 (b) 1037 (c) 7537

13. (a) $72 \times 100 = 7200$ (b) $60 \times 30 = 1800$

14. (a) (i) 18 (ii)81 (b) 261

6.3 Fractions and Decimals

1. (a) (i) 18.64 (ii) 19 (b) (i) 1024.84 (ii) 1000 (c) (i) 16.04 (ii) 16
(d) (i) 181.44 (ii) 180 (e) (i) 16.82 (ii) 17 (f) (i) 0.08 (ii) 0.084
(g) (i) 0.01 (ii) 0.0096 (h) (i) 4.84 (ii) 4.8 (i) (i) 3.86 (ii) 3.9

2. (a) 48 600 (b) 48 637.01 (c) 48 637.0125 (d) 48 640
(e) 48 637.012 (f) 49 000

3. (a) 0.0047 (b) 48.2 (c) 20 (d) 4.86 (e) 18.42
(f) 21.80 (g) 15 000 (h) 0.005 (i) 0.00418
(j) 15 700 (k) 55 000 (l) 31.4 (m) 14.18 (n) 0.82
(o) 1.841 (p) 15.0 (q) 14.170 (r) 201

4. (a) 40 000 , 45 000 , 44 900 , 44 850 (b) (i) 2 s.f. (ii) 1 s.f.

5. (a) 0.5 (b) 0.75 (c) 0.4 (d) 0.6 (e) 0.125 (f) 0.625
(g) 0.375 (h) 0.875 (i) 0.2

6. (a) 0.3333 (b) 0.1667 (c) 0.5714 (d) 0.1429 (e) 0.7143
(f) 0.8333

7. (a) 0.11111 , 0.22222 , 0.44444 , 0.55556
(b) Recurring decimal which is the same as the numerator
(c) 0.7777... , 0.8888...

8. (a) 0.09091 , 0.18182 , 0.27273 , 0.36364
   (b) 0.45455 , 0.54545 , 0.63636 , 0.72727 , 0.81818 , 0.90909

10. (a) 0.8 (b) 0.096 , $\frac{4}{5}$, 0.805 , 0.85

6.4 Long Multiplication and Division

1. (a) 345 (b) 684 (c) 513 (d) 9088 (e) 7308 (f) 15408
   (g) 2548 (h) 1920 (i) 23 328 (j) 10 164 (k) 2352
   (l) 5586 (m) 88 192 (n) 134 096 (o) 56 616

2. (a) 152 (b) 254 (c) 173 (d) 251 (e) 452 (f) 428
   (g) 123 (h) 35 (i) 12 (j) 32 (k) 24 (l) 153
   (m) 134 (n) 214 (o) 13

3. £2112

4. 700

5. 13

6. 350 kg

7. £333.33

8. (a) 13 (b) 7 ; 240

9. £154

10. (a) 35616 (b) 34132

11. 39

12. (a) 770 (b) (i) 19 (ii) 9

6.5 Estimating Answers

1. (a) 50 (b) 20 (c) 30 (d) 10 (e) 20 (f) 100
   (g) 60 (h) 0.2 (i) 0.04 (j) 2 (k) 20 (l) 2

2. Approximate answers are (a) 56 (b) 48 (c) 960 (d) 51
   (e) 600 (f) 540 (g) 10 (h) 7 (i) 20 (j) 120
   (k) 5 (l) 45
   Actual answers are (a) 56.01 (b) 54.20 (c) 1020 (d) 53.51
   (e) 623.4 (f) 545.5 (g) 11.11 (h) 7.634 (i) 18.59
   (j) 113.8 (k) 4.446 (l) 46.10

3. Approximate answers are (a) 2 (b) 0.5 (c) 3 (d) 7
   (e) 40 (f) 20 (g) 15 (h) 70 (i) 150
4. (a) 200 m (b) 233.28 m (c) 264.06 m

5. (a) Estimate 1200 , Actual 1286 (b) Estimate 250 s , Actual 229 s

6. (a) About 40 ms^{-1} (b) 40.04 , 39.67 , 39.02

7. (a) 10 or 11 , 11.08 km (b) about 480 km , 360 km

8. (a) Estimate £70 , Actual £63.06 (b) Estimate £30 , Actual £24.86
   (c) Estimate £40 , Actual £38.19

9. (a) Estimate 72 , Actual 91.25 (b) Estimate 48 , Actual 48.5
(c) Estimate 80 , Actual 86.02

10. (a) 60×30 (b) 1800 (c) 88

11. No ; it should be £7.02 (accurate answer: £7.03)

6.6 Using Brackets and Memory on a Calculator

1. (a) 97.2 (b) 27.40 (c) 3.55 (d) 2.69 (e) 0.767
(f) 1.54 (g) 0.348 (h) 1.28 (i) 1.65 (j) 0.372
(k) 0.0587 (l) 0.726

3. (a) 17.3 (b) 25.75 (c) 124

4. (a) 5.5 , 3.6591 , 3.1960 , 3.1625 , 3.1623 , 3.1623
(b) 3.5 , 3.1786 , 3.1623 , 3.1623 , 3.1623 , 3.1623
(c) Both sequences are converging to 3.1623, which is $\sqrt{10}$, but using 2 converges more quickly.

5. (a) (i) 141 , 141 (ii) 468 , 57 (iii) 3.2 , 28.8 , 3.2
(b) (i) $3 \times 6 + 5 \times 51 + 15 \times 2 =$ (ii) $(3+6) \times 5 \times 2 =$
(iii) & (iv) need all the brackets (v) $3 \times 4 \div (5+2) =$ (vi) $3 \times 2 \div (4 \times 6) =$

6. (a) 603.2 cm^2 (b) 258.1 cm^2

7. (a) (i) 2120575 mm^3 (ii) 5089 cm^3 (b) (i) 53.6 cm (ii) 88.4

8. 11.1

9. 7.00

10. (a) 4.29 (b) 4.652 (c) 50 (d) 2.5

11. (a) 36.6025 (b) 7.35

12. (a) Question 3 (b) Question 2

13. (a) (i) $v = 120$, $u = 20$, $t = 5$ (ii) $a = 20$ (b) $\boxed{=}$ needed after $\boxed{6}$

14. Last $\boxed{\times}$ should be replaced by $\boxed{\div}$, or insert brackets around 3.2×0.47

6.7 Upper and Lower Bounds

1. (a) $4.65 \le x < 4.75$ (b) $41.5 \le l < 42.5$ (c) $15.615 \le A < 15.625$
(d) $16.15 \le d < 16.25$ (e) $11.675 \le r < 11.685$ (f) $14.235 \le m < 14.245$
(g) $217.5 \le w < 218.5$ (h) $15.195 \le l < 15.205$ (i) $4.995 \le w < 5.005$
(j) $19.95 \le v < 20.05$ (k) $18.085 \le A < 18.095$ (l) $31.4505 \le v < 31.4515$

2. (a) $41.5 \le l < 42.5$, $31.5 \le w < 32.5$
(b) (i) $146 \le \text{perimeter} < 150$ (ii) $1307.25 \le \text{area} < 1381.25$

3. (a) $11.5 \le \text{radius} < 12.5$
(b) (i) $23 \le \text{diameter} < 25$ (ii) $72.3 \le \text{circumference} < 78.5$
(iii) $415.5 \le \text{area} < 490.9$

4. (a) 50 (b) 10 (c) 525 (d) 0.333... (e) 1.4

5. (a) 7.743 to 7.882 (b) 7.782 to 7.844

6. 1.595 m^2 to 2.605 m^2

7. (a) 116 865.25 cm^2, 116 181.25 cm^2 (b) 20989 cm^2

8. 0.0044 to 0.0064

9. 68.5 miles, 71.5 miles

10. (a) $54.11 \le \text{area} < 56.75$, $26.08 \le \text{circumference} < 26.70$,
(b) $5.383 \le \text{radius} < 5.397$ (c) $2.417 \le \text{radius} < 2.423$

11. (a) $2.4735 \le \text{mass} < 2.4745$ (b) 1.62805, 1.62795

12. (a) 251 kg (b) 2 kg

13. (a) 29.5 cm (b) $18.35 \le \text{length} < 18.45$

14. 210 g, 200 g

15. 2 m 97 cm

16. (a) 3.65 cm, 3.65 cm (b) 15 cm, 14.6 cm (c) (i) Two
(d) (i) No ; one significant figure

17. (a) 3.75 hours; 195 miles, 205 miles (b) 63.1 mph, 52.0 mph

6.8 Number System

1. Rational (terminating), Rational (reccurring), Irrational, Rational (recurring), Rational (terminating), Irrational, Irrational, Rational (terminating), Irrational, Rational (recurring), Irrational, Rational (terminating)

2. (a) $\frac{49}{100}$ (b) $\frac{1}{3}$ (c) $\frac{7}{4}$ (d) - (e) $\frac{417}{1000}$ (f) $\frac{1}{9}$
(g) $\frac{1}{11}$ (h) $\frac{6}{11}$ (i) $\frac{1}{8}$ (j) $\frac{481}{500}$

3. (a) $\frac{41}{99}$ (b) $\frac{67}{1665}$ from $\frac{402}{9990}$ (c) $\frac{1}{7}$ (d) $\frac{8}{9}$ (e) $\frac{812}{999}$
(f) $\frac{5}{9}$ (g) $\frac{101}{111}$

5. Irrational, Irrational, Rational, Irrational, Irrational, Rational

10. $p + q$ can be rational or irrational

12. recurring, non-recurring, recurring, non-recurring, recurring

13. (a) 13, Rational (b) $\sqrt{61}$, Irrational (c) 3, Rational
(d) $\frac{5}{7}$, Rational

14. (b) $2^{-2},\ 4^{\frac{1}{2}},\ 4^{-2}$

15. (a) (i) any recurring decimal (ii) "and does not repeat itself"

(b) Irrational , Rational $\left(\frac{5}{2}\right)$, Irrational , Rational $\left(\frac{1}{3}\right)$

16. (a) e.g. any square root larger than 16 and less than 25

17. (a) (i) 237 (ii) any n between $225 < n < 256$ is such that $\sqrt{n}$ is irrational

(b) 10. 24 which has 3.2 or $\frac{16}{5}$ as a square root

6.9 Surds

1. (a) $1+\sqrt{2}+\sqrt{3}+\sqrt{6}$ (b) $4-2\sqrt{3}+2\sqrt{5}-\sqrt{15}$ (c) $\sqrt{3}-3$
(d) $-14+2\sqrt{11}+7\sqrt{3}-\sqrt{33}$ (e) 11 (f) $30+4\sqrt{2}$ (g) $-16\sqrt{17}$
(h) –7 (i) $1-\pi$ (j) –4 (k) 4 (l) $\pi^2-\pi$
(m) $5+\sqrt{2}+\sqrt{3}+2\sqrt{6}$ (n) $-6-2\sqrt{10}$ (o) $7+4\sqrt{3}$
(p) $8-2\sqrt{7}$ (q) $14-6\sqrt{5}$ (r) $7+5\sqrt{2}$

2. (a) $\frac{\sqrt{2}}{2}$ (b) $\frac{2}{5}\sqrt{5}+1$ (c) $\frac{1}{2}\left(\sqrt{2}-\sqrt{6}\right)$ (d) $-6+5\sqrt{2}$
(e) $-\frac{1}{2}\left(5+\sqrt{7}-5\sqrt{3}-\sqrt{21}\right)$ (f) $\frac{1}{22}\left(15-5\sqrt{2}-3\sqrt{3}+\sqrt{6}\right)$
(g) $-\frac{1}{2}-\frac{1}{2}\sqrt{3}$ (h) $3-2\sqrt{2}$ (i) $\frac{4-\sqrt{7}}{3}$ (j) $1+\sqrt{22}+\sqrt{11}+\sqrt{2}$
(k) $-\left(\sqrt{2}+\sqrt{3}\right)$ (l) $\sqrt{3}+\sqrt{5}-\frac{1}{2}\sqrt{15}-\frac{5}{2}$

6. (a) Yes / No ; No / Yes ; No / Yes
(b) 2π etc. or the square root of any number between 36 and 49
(c) (i) Irrational (ii) Irrational

7. (a) (i) $x = 4$ etc. (ii) $y = 27$ etc.
(b) (i) $\sqrt{2}, \sqrt{3}$ etc. (ii) $1+\sqrt{2}, 1-\sqrt{2}$ etc.

8. (a) (i) Rational , $\frac{3}{11}$ (ii) Irrational (iii) Rational , $\frac{21}{16}$ (b) $b = 8$ etc.
(c) a