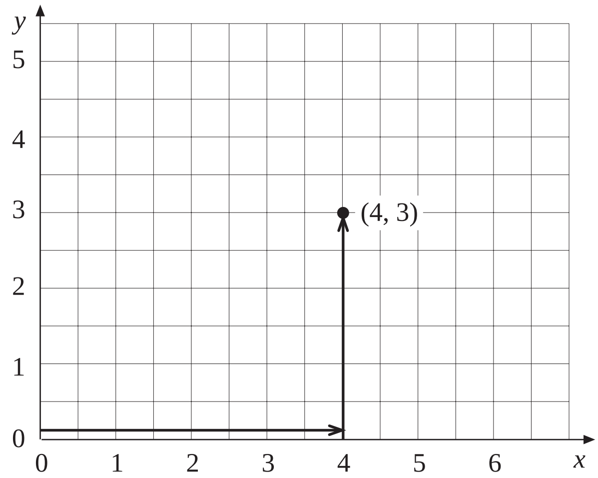
13 Graphs

13.1 Positive Coordinates

Coordinates are pairs of numbers that uniquely describe a position on a rectangular grid. The first number refers to the horizontal (x-axis) and the second the vertical (y-axis). The coordinates (4, 3) describe a point that is 4 units across and 3 units up on a grid from the origin (0, 0).



Worked Example 1

Plot the points with coordinates

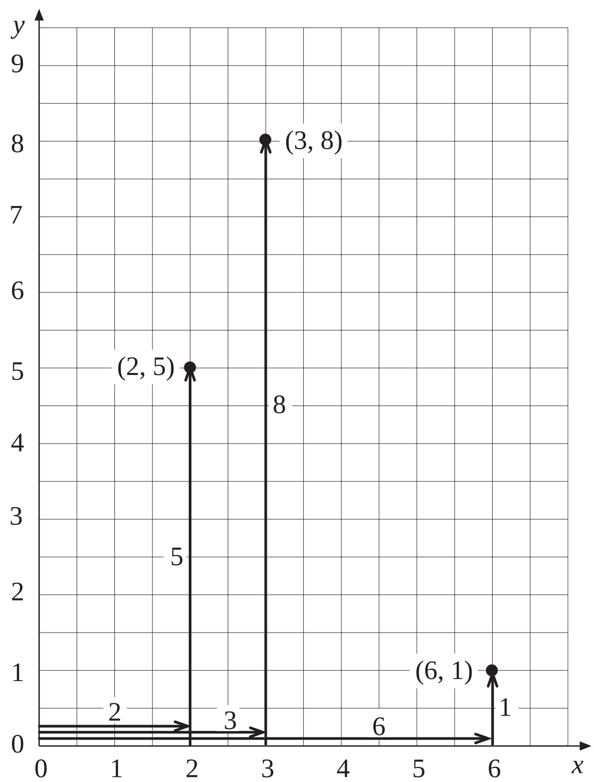
(3, 8), (6, 1) and (2, 5)

Solution

For (3, 8) move 3 across and 8 up.

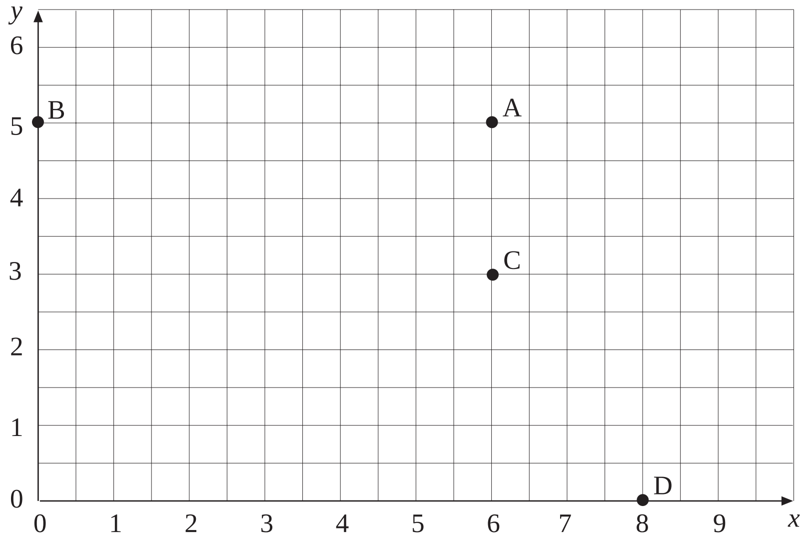
For (6, 1) move 6 across and 1 up.

For (2, 5) move 2 across and 5 up.



Worked Example 2

Write down the coordinates of each point in the diagram below.



Solution

A is 6 across and 5 up, so the coordinates are (6, 5).

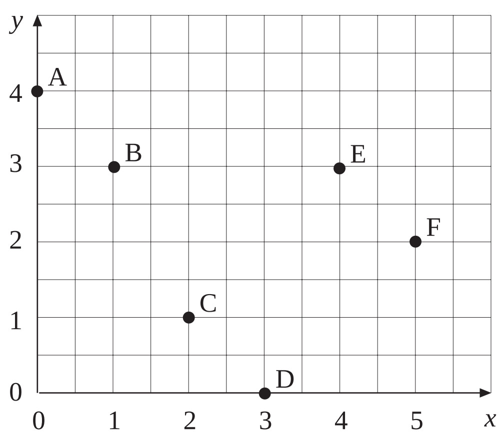
B has no movement across and is straight up 5, so the coordinates are (0, 5).

C is 6 across and 3 up, so the coordinates are (6, 3).

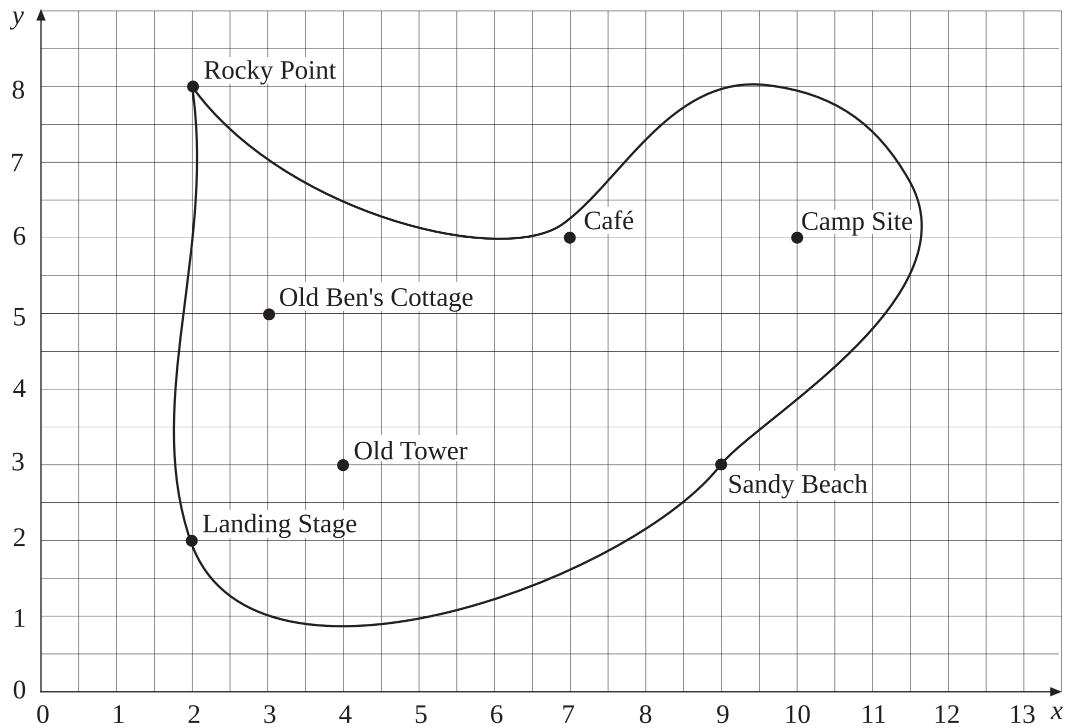
D is 8 across and no movement up, so the coordinates are (8, 0).

Exercises

1. Write down the coordinates of each point on the diagram below.



2. The map of an island has been drawn on a grid.

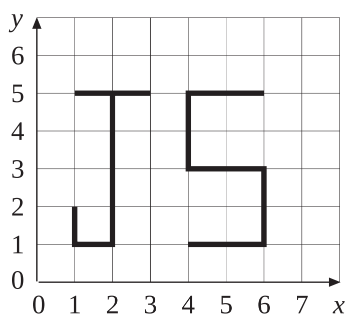


Write down the coordinates of each place marked on the map.

3. On a grid, join the points with the following coordinates and write down the name of the shape you draw.

(a)	(4, 2)	(8, 2)	(8, 5)	(4, 5)		
(b)	(2, 1)	(6, 1)	(4, 6)			
(c)	(1, 4)	(3, 7)	(5, 4)	(3, 1)		
(d)	(4, 0)	(3, 2)	(5, 4)	(7, 2)	(6, 0)	
(e)	(1, 1)	(0, 3)	(1, 5)	(3, 5)	(4, 3)	(3, 1)

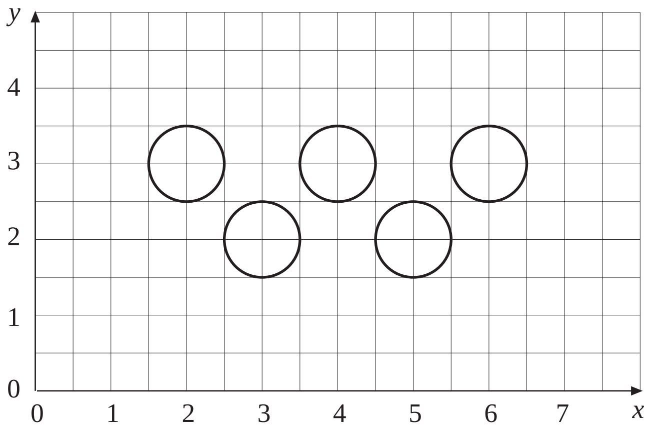
4. Jenny writes her initials on a grid.



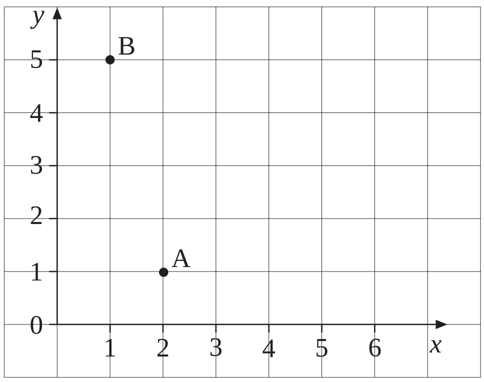
(a) Write down the coordinates of the corners of each letter.

(b) Write your initials in the same way and write down the coordinates of your initials.

5. The pattern below is made up of 5 circles. Write down the coordinates of the centre of each circle.



6.



(a) On the co-ordinate grid above plot the following points

P (3,4), Q (0,2) R (4,0)

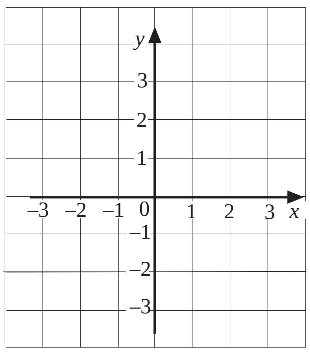
(b) Write down the co-ordinates of the points

(i) A,

(ii) B.

(LON)

13.2 Coordinates



The coordinates of a point are written as a pair of numbers, (x, y), which describe where the point is on a set of axes.

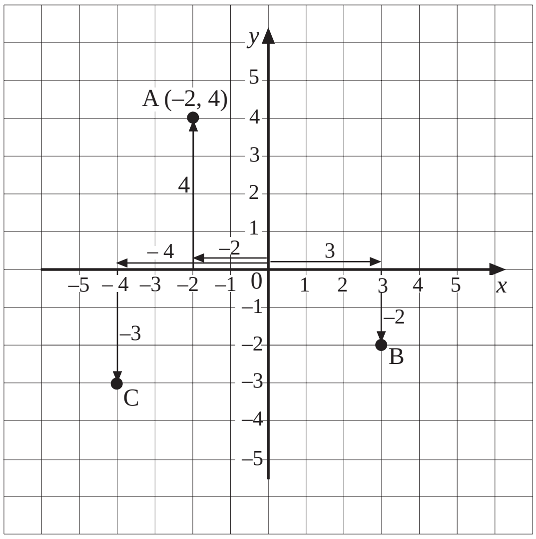
The x-axis is always horizontal (i.e. *across* the page) and the y-axis always vertical (i.e. *up* the page).

The x-coordinate is always given first and the y-coordinate second.

Worked Example 1

On a grid, plot the point A which has coordinates $(-2, 4)$, the point B with coordinates $(3, -2)$ and the point C with coordinates $(-4, -3)$.

Solution



For A, begin at $(0, 0)$, where the two axes cross.

Move -2 in the x direction.

Move 4 in the y direction.

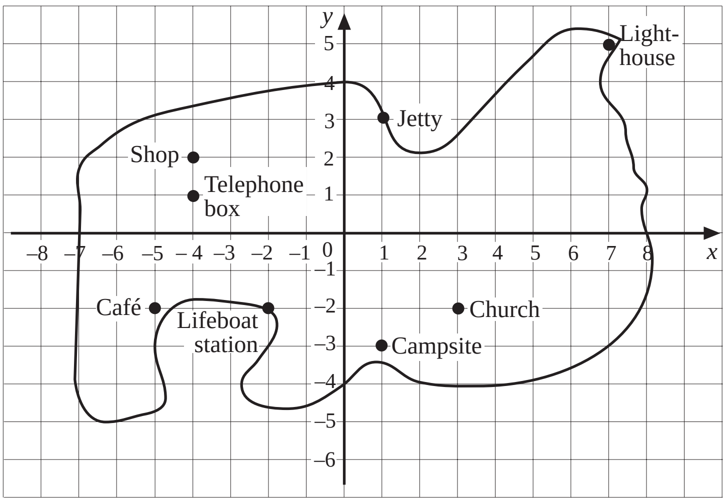
Points B and C are plotted in a similar way.

For B, move 3 in the x direction and -2 in the y direction.

For C, move -4 in the x direction and -3 in the y direction.

Worked Example 2

Write down the coordinates of each place on the map of the island.



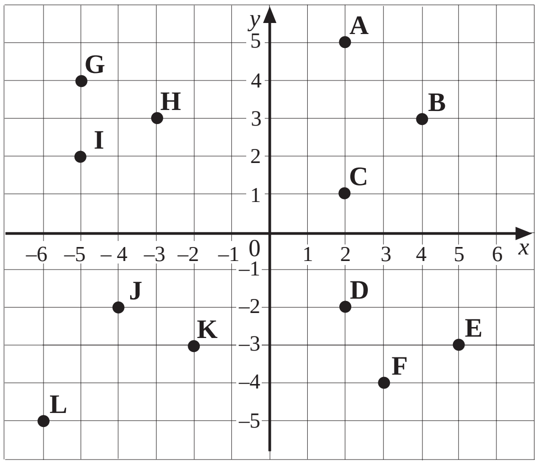
Solution

Lighthouse	$(7, 5)$	All coordinates positive
Jetty	$(1, 3)$	
Church	$(3, -2)$	Negative y-coordinates
Camp Site	$(1, -3)$	
Shop	$(-4, 2)$	Negative x-coordinates
Telephone Box	$(-4, 1)$	

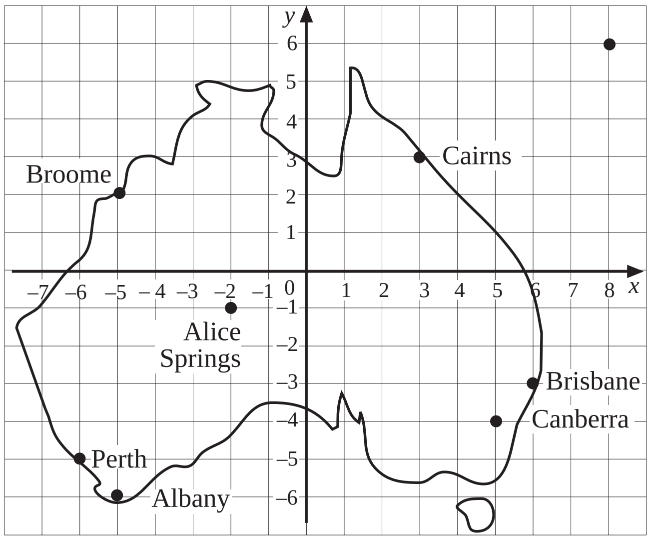
Café	$(-5,\ -2)$	All coordinates negative
Lifeboat Station	$(-2,\ -2)$	

Exercises

1. Write down the coordinates of each point marked on the grid below.



2. The map shows some Australian towns and cities.

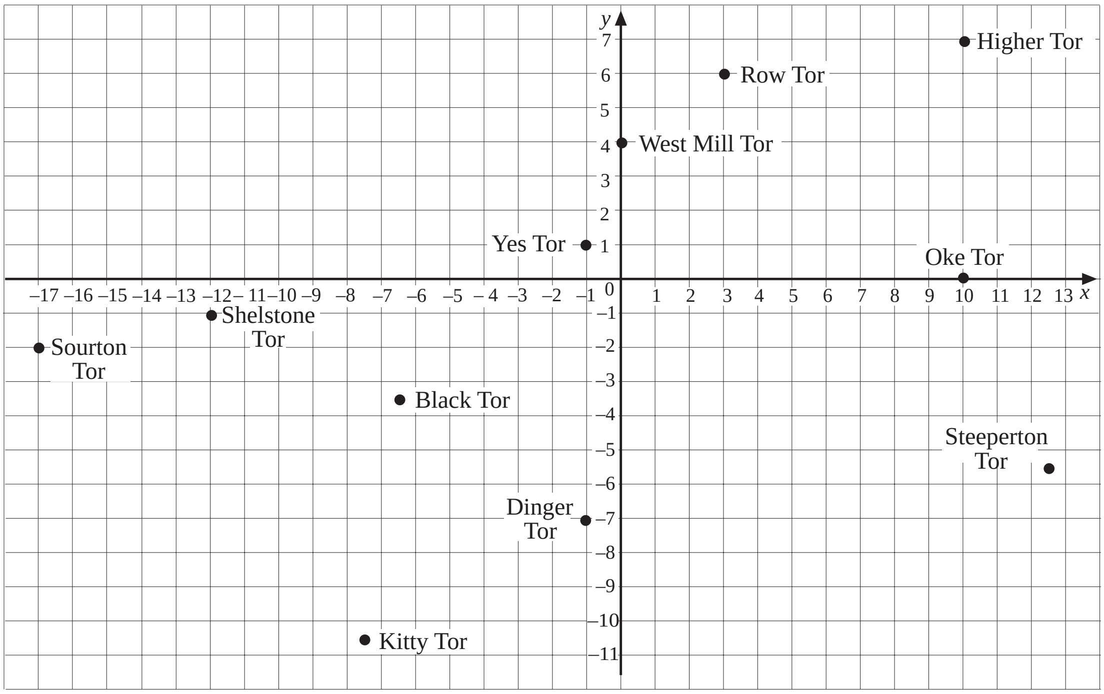


(a) Write down the coordinates of Canberra, Brisbane and Perth.

(b) A plane flies from the place with coordinates (–5, –6) and lands at the place with coordinates (–2, –1). From where does the plane take off and where does it land?

(c) A ship has coordinates (–5, 2) at the start of a voyage and coordinates (–6, –5) at the end. Where does it start and where does it finish?

3. The map shows some of the tors (rocky outcrops) on Dartmoor in the south west of England.



(a) Write down the coordinates of the following tors.

West Mill Tor

Steeperton Tor

Shelstone Tor

Black Tor

Dinger Tor

(b) The highest tor marked on this map is Yes Tor. Write down the coordinates of this tor.

(c) A boy and his dog walk from Oke Tor to Kitty Tor. Write down the coordinates of the point where they start and the point where they finish.

(d) Sourton Tor is the tor that is the farthest west on this map. What are the coordinates of this tor?

(e) Higher Tor is the tor that is farthest north. What are the coordinates of this tor?

4. Draw a set of axes with *x*-values from –5 to 5 and *y*-values from –3 to 9.

(a) Join together the points with coordinates (5, 0), (0, 9) and (–5, 0). What shape do you get?

(b) On the same diagram, join together the points (5, 6), (–5, 6) and (0, –3).

5. Draw a set of axes with x-values from – 4 to 4 and y-values from – 4 to 3.

   Join each set of points below in the order listed.

   (a) (2, 0), (1, 1), (1, 2), (2, 3), (3, 3), (4, 2), (4, 1), (3, 0).

   (b) (0, 1), (1, –1), (–1, –1), (0, 1).

   (c) (–2, 0), (–1, 1), (–1, 2), (–2, 3), (–3, 3), (– 4, 2), (– 4, 1), (–3, 0).

   (d) (3, –1), (2, –3), (0, – 4), (–2, –3), (–3, –1), (–1, –2), (1, –2), (3, –1).

6. (a) Draw a set of axes with x-values from – 4 to 4 and y-values from –5 to 4.

   (b) Plot the following points and join them in the order listed.

   (3, –5), (2, –5), (– 4, –2), (–2, –3), (0, –2), (0, 0), (3, 2), (3, 3), (4, 3), (4, 2), (3, 2), (3, 0), (–2, 2).

7. Three corners of a square have coordinates (4, 2), (–2, 2) and (4, – 4).

   (a) Draw a set of axes with x-values from –2 to 4 and y-values from – 4 to 2. Plot the three points and draw the square.

   (b) Write down the coordinates of the centre of the square.

8. Two corners of a rectangle have coordinates (–3, –1) and (–3, 3). The centre of the rectangle has coordinates (1, 1).

   (a) Plot the three points given and draw the rectangle.

   (b) Write down the coordinates of the other two corners of the rectangle.

9. A dodecagon is a twelve-sided, plane shape.

   (a) Draw a set of axes that have x-values from –7 to 1 and y-values from 0 to 8.

   (b) Plot the points listed below and join them to draw half a dodecagon.

   (–3, 0), (–2, 0), (0, 1), (1, 3), (1, 5), (0, 7), (–2, 8), (–3, 8)

   (c) Draw the other half of the dodecagon. It is symmetric about the line joining (–3, 0) to (–3, 8).

   (d) Write down the coordinates of the six new corners of the dodecagon that you have drawn.

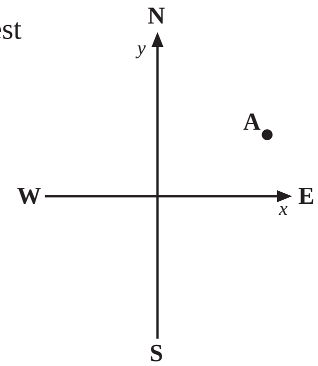
10. A set of axes are arranged so that the x-axis runs from west to east and the y-axis from south to north.

    A ship is at the point A which has coordinates (4, 2).

    (a) How far south can the ship travel before its y-coordinate becomes negative?

    (b) How far west can the ship travel before its x-coordinate becomes negative?

    (c) If the ship travels SW, how far does it travel before both coordinates become negative?



13.3 Plotting Straight Lines

By calculating values of coordinates you can find points and draw a graph for any relationship, such as $y = 2x - 5$.

Worked Example 1

(a) Copy and complete the following pairs of coordinates using the relationship $y = x - 2$.

(4, ?), (1, ?), (–1, ?)

(b) Plot the points on a set of axes.

(c) Draw a straight line through these points.

Solution

(a) For the first point $x = 4$, so

$$\begin{aligned} y &= 4 - 2 \\ &= 2 \end{aligned}$$

So the first point is (4, 2).

For the second point $x = 1$, so

$$\begin{aligned} y &= 1 - 2 \\ &= -1 \end{aligned}$$

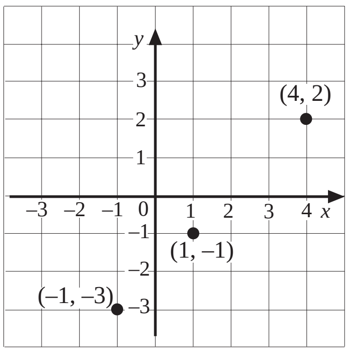
So the second point is (1, –1).

For the third point $x = -1$

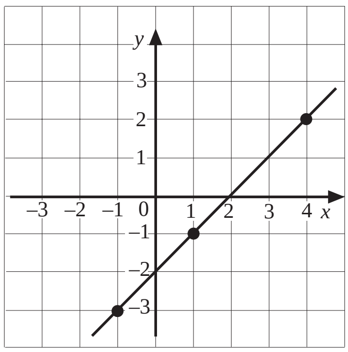
$$\begin{aligned} y &= -1 - 2 \\ &= -3 \end{aligned}$$

So the third point is (–1, –3).

(b) The points are plotted on these axes.



(c) The three points lie on a straight line as shown on these axes.



Worked Example 2

Draw the graph of $y = 2x - 1$.

Solution

The first step is to find the coordinates of three points on the line. Choose any three x-values for the coordinates. Three possible values are given below.

(4, ?), (2, ?), (–2, ?)

Now calculate the y-values using $y = 2x - 1$.

For the first point $x = 4$, so

$$\begin{aligned} y &= 2 \times 4 - 1 \\ &= 7 \end{aligned}$$

The coordinates of this point are (4, 7).

For the second point $x = 2$, so

$$\begin{aligned} y &= 2 \times 2 - 1 \\ &= 3 \end{aligned}$$

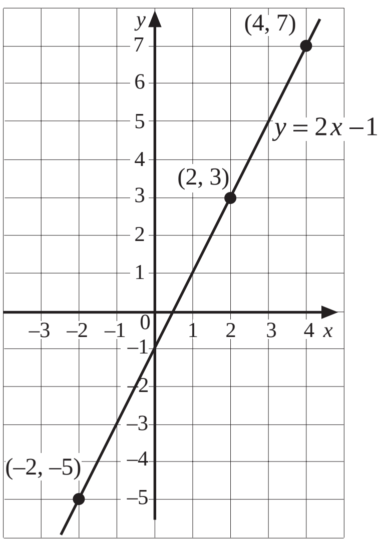
The coordinates of the second point are (2, 3).

For the third point $x = -2$, so

$$\begin{aligned} y &= 2 \times (-2) - 1 \\ &= -5 \end{aligned}$$

The third point is (–2, –5).

These points are now plotted on the axes below. Then a straight line can be drawn through them.



Note In fact, just *two* points uniquely define a straight line but it is safer to use three as a check.

Exercises

1. (a) Copy and complete the coordinates below using the relationship $y = x + 2$.

   (4, ?), (1, ?), (–3, ?)

   (b) Draw a set of axes with x-values from –3 to 4 and y-values from –1 to 6.

   (c) Plot the points with the coordinates found in (a).

   (d) Check that the points lie on a straight line and draw a straight line through the points.

   (e) Write down the coordinates of the point where the line crosses the y-axis.

2. (a) Use the relationship $y = 2x + 1$ to complete the coordinates below.

   (3, ?), (1, ?), (–2, ?)

   (b) Draw a set of x-axes with values from –2 to 3 and y-values from –3 to 7. Then plot the points with the coordinates obtained in (a).

   (c) Draw a straight line through the points in (b).

   (d) Does the line in (c) go through the point (2, 5)?

   Check if these coordinates satisfy the relationship $y = 2x + 1$.

3. (a) Use the relationship $y = 3x - 2$ to complete the coordinates below.

   (3, ?), (0, ?), (–2, ?)

   (b) Plot these points on a set of axes and draw a straight line through the points.

   (c) Write down the coordinates of two other points on the line. Check that they satisfy the relationship $y = 3x - 2$.

   (d) Does the point (2, 5) lie on the line? Check if these coordinates satisfy the relationship $y = 3x - 2$.

4. Use a different set of axes with x-values from – 5 to 5 to draw the graph of each relationship. You will need to decide what y-values to use for each graph.

   (a) $y = x + 3$

   (b) $y = x - 5$

   (c) $y = 3x$

   (d) $y = 2x + 2$

   (e) $y = 3x - 1$

5. Draw the lines with the equations listed below. You should decide what size axes you need to use to plot the points.

   (a) $y = x + 8$

   (b) $y = 2x + 4$

   (c) $y = 3x - 7$

   (d) $y = 4x + 2$

6. The relationship $d = 4t$ gives the distance walked by a person. The distance in kilometres is *d* and *t* is the time in hours for which the person has been walking.

   (a) Complete a copy of the table below.

t	0	2	4
d			

   (b) Complete and plot the coordinates below.

   (0, ?), (2, ?), (4, ?)

   (c) Draw a straight line through the points.

   (d) Which axis has the time (*t*) values?

   (e) Which axis has the distance (*d*) values?

   (f) Use the graph to find the time taken to walk 12 km.

   (g) Use the graph to find the distance walked in $3\frac{1}{2}$ hours.

7. A teacher uses the relationship $p = 4m$ to convert the marks obtained on a test to percentages. Here *m* is the mark and *p* is the percentage.

   (a) Complete a copy of the table below.

m	0	10	25
p			

   (b) Complete the coordinates below using the contents of your table.

   (0, ?), (10, ?), (25, ?)

   (c) Using *m* on the horizontal axis and *p* on the vertical axis, plot the points with the coordinates you obtained in (b).

   (d) Copy the following table and then use the graph to fill in the missing entries.

	Mark	*Percentage*
John	15	
Stuart	21	
Jenny	18	
Karen		80
Mike		60

8. The relationship $F = 32 + 1.8C$ can be used to convert temperatures in degrees Celsius, °C, to temperatures in degrees Fahrenheit, °F.

   (a) Complete a copy of the table below.

Temperature in °Celsius	0	20	100
Temperature in °Fahrenheit			

   (b) Use the information in the table to draw a graph of $F = 32 + 1.8C$.

   (c) The recommended temperature for a greenhouse is 80 °F. Use your graph to convert this to °Celsius.

   (d) The temperature at a holiday resort is 30 °C. Use the graph to convert this to °Fahrenheit.

9. (a) Complete the coordinates below for the relationship $y = 4 - x$.

   (0, ?), (2, ?), (4, ?)

   (b) Draw a set of axes with *x* and *y* values from 0 to 4 and plot the points with coordinates obtained in (a).

   (c) Write down the coordinates of the points where the line crosses the *x*-axis and the *y*-axis.

10. (a) Write down the coordinates of three points on the line $y = 6 - x$.

    (b) Plot the points you obtained on a set of axes and draw a straight line through them.

    (c) Find the area of the triangle that is formed by the axes and the line.

11. (a) Draw the lines $y = x - 2$ and $y = 10 - x$.

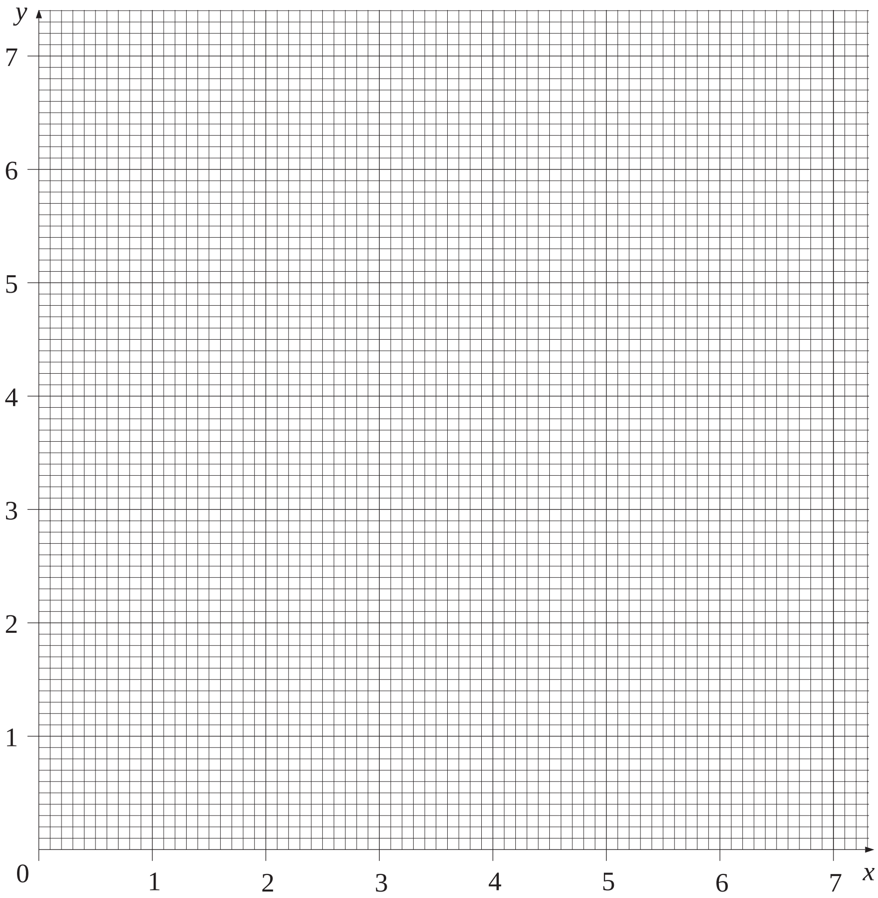
    (b) State the coordinates of the points where the lines cross.

    (c) Find the area of the triangle formed by the two lines and the *x*-axis.

    (d) Repeat (a) to (c) for $y = 2x + 2$ and $y = 10 - 2x$.

    (e) Repeat (a) to (c) for $y = x$ and $y = 2x - 4$.

12.



(a) Plot the points (0, 6), (1, 4), (2, 2) and (3, 0) on a copy of the grid above. Draw a straight line through the four points.

(b) Draw the line $y = x + 1$ on a copy of the grid above.

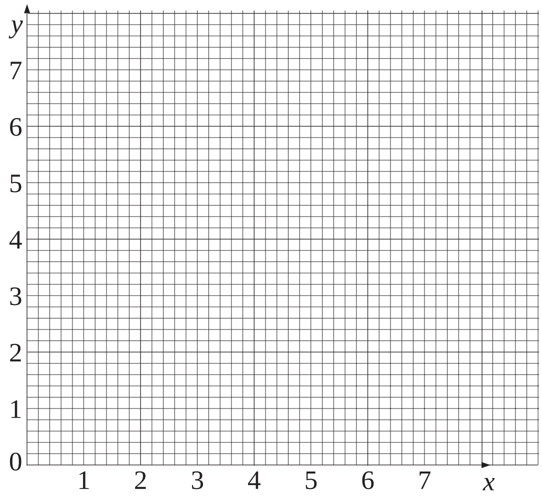
(MEG)

13. Copy and complete the following table for the rule.

(a) "To find *y*, double *x* and add 1."

x	y
0	
1	
3	

(b) Plot the values from the table onto a copy of the following coordinate grid. Join your points with a straight line.



(c) Write, in symbols, the rule

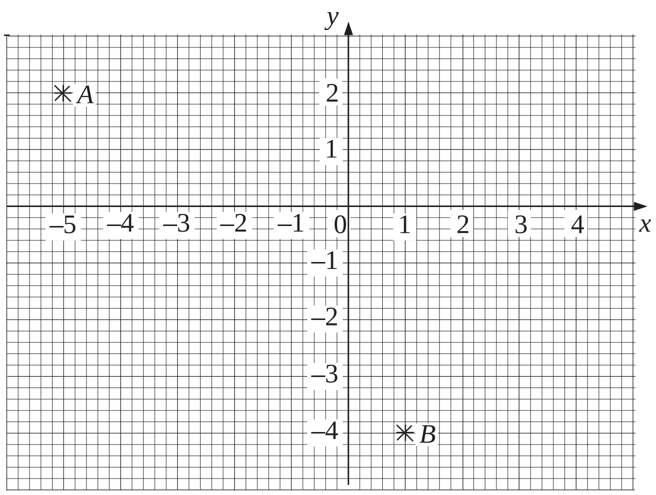
"To find *y*, double *x* and add 1."

(d) Use your rule from part (c) to calculate the value of *x* when $y = 9$.

(LON)

14. (a) Two points *A* and *B* are shown on the grid below.

Write down their coordinates.



(b) Copy and complete the table of values below for

$$y = x - 2$$

x	–2	0	2	3	4
y		–2			2

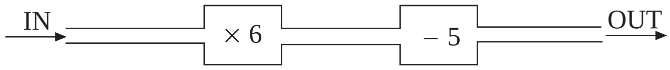
(c) Draw the graph of $y = x - 2$ on your copy of the graph above.

(d) On the grid, draw the straight line *AB*.

Write down the coordinates of the point where the graph of $y = x - 2$ cuts the line *AB*.

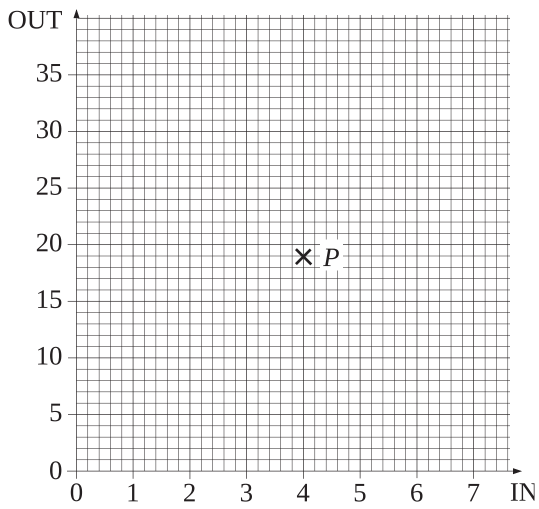
(MEG)

15. This machine multiplies all numbers by 6 and then subtracts 5.



(a) Complete a copy of the table below for this machine by putting values into the two empty boxes.

IN	1	2	3	?
OUT	1	?	13	31



(b) (i) Using the values from the table, plot four points on a copy of the grid above.

(ii) Draw the line joining the four points that you have plotted.

(iii) Write down the coordinates of the point labelled *P* on the grid.

(MEG)

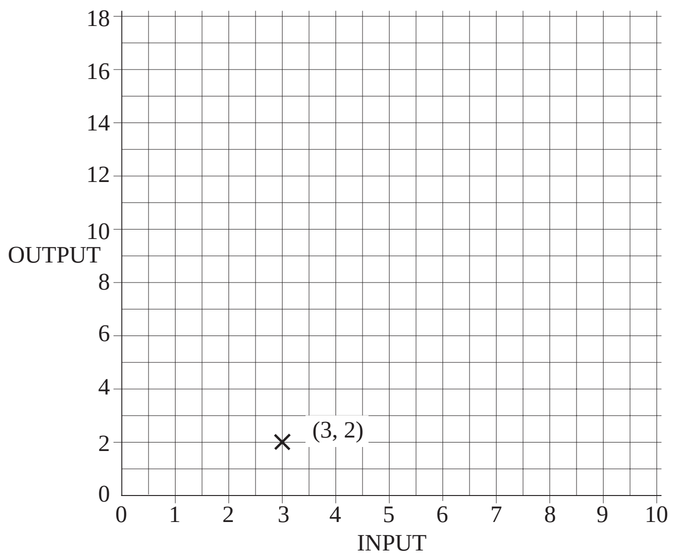
16.

When you put 3 in to this number machine, out comes 2.

(a) Fill in the missing numbers in a copy of the table below.

IN	OUT
3	2
5	
6	
	17

(b) Plot points on a copy of the following grid to represent your answers in part (a). The point (3, 2) has been plotted for you.



(c) What do you notice about the points that you have plotted?

(MEG)

13.4 Plotting Curves

Some relationships produce *curves* rather than straight lines when plotted.

Worked Example 1

(a) Complete the table below using the relationship $y = x^2 - 2$.

x	–3	–2	–1	0	1	2	3
y							

(b) Write a list of coordinates using the data in the table.

(c) Plot the points and draw a smooth curve through them.

Solution

(a) For each value of x the y value can be calculated using $y = x^2 - 2$.

For example: If $x = 3$ then
$$y = 3^2 - 2$$
$$= 9 - 2$$
$$= 7$$

If $x = 1$ then $y = 1^2 - 2$

$= 1 - 2$

$= -1$

If $x = -2$ then $y = (-2)^2 - 2$

$= 4 - 2$

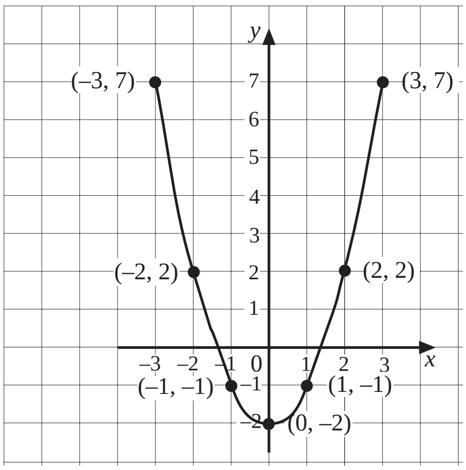
$= 2$

Calculating all the values gives the table.

x	–3	–2	–1	0	1	2	3
y	7	2	–1	–2	–1	2	7

(b) The coordinates of the points to plot are (–3, 7), (–2, 2), (–1, –1), (0, –2), (1, –1), (2, 2) and (3, 7).

(c) These points are plotted on the graph below and have been joined by a smooth curve.



Worked Example 2

Draw the graph of $y = x^3 - 4x$ for values of x from –3 to 3.

Solution

The first step is to draw up and complete a table of values using the relationship $y = x^3 - 4x$, as below.

x	–3	–2	–1	0	1	2	3
y	–15	0	3	0	–3	0	15

For example: If $x = -3$ then $y = (-3)^3 - 4 \times (-3)$

$= -27 + 12$

$= -15$

If $x = 2$ then $y = 2^3 - 4 \times 2$

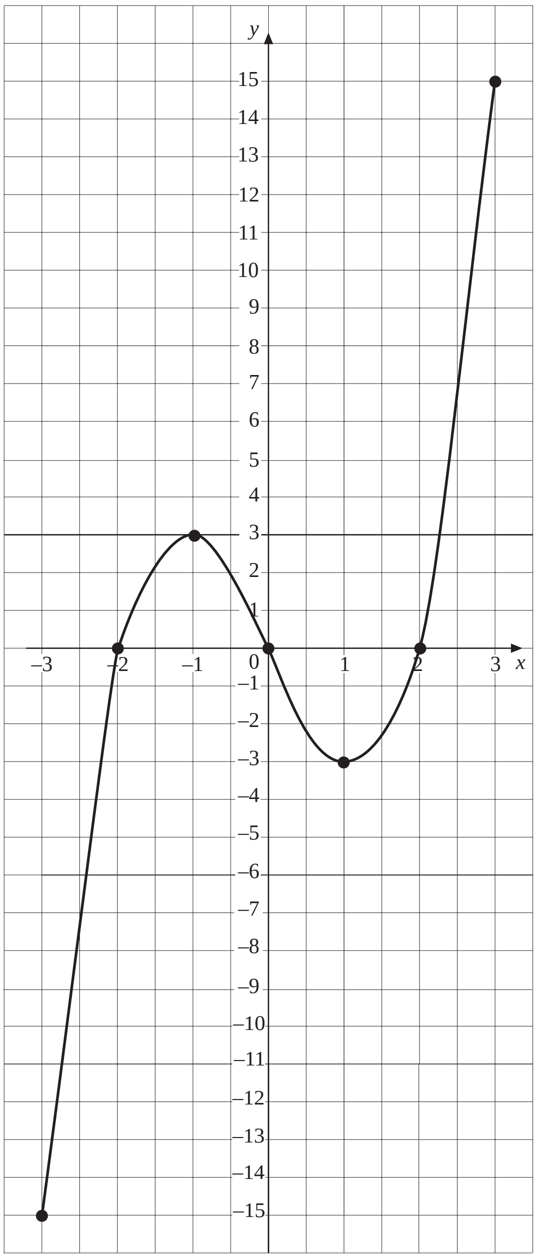
$= 8 - 8$

$= 0$

Each pair of values can be written as coordinates,

$$(-3, -15),\ (-2, 0),\ (-1, 3),\ (0, 0),\ (1, -3),\ (2, 0),\ (3, 15)$$

These can then be plotted and a smooth curve drawn through the points as shown in the following graph.



Exercises

1. (a) Complete a copy of the table below for $y = x^2 - 5$.

x	–3	–2	–1	0	1	2	3
y			–4				4

(b) Write a list of coordinates. Plot these points and draw a smooth curve through the points.

2. (a) Complete a copy of the table below using the relationship $y = x^2 - 2x$.

x	–2	–1	0	1	2	3	4
y		3					8

(b) Plot the graph of $y = x^2 - 2x$ using the data in the table.

3. Complete a copy of the table and draw the graph of $y = 6 - x^2$.

x	–3	–2	–1	0	1	2	3
y			5			2	

4. (a) Complete a copy of the table and draw the graph of $y = x^2 - x - 2$.

x	–2	–1	0	1	2	3
y		0		–2		

(b) What is the value of x at the lowest point of the curve?

(c) Use your answer to (b) to calculate the corresponding value of y.

5. (a) Complete a copy of the table using the relationship $y = x^3 - x$.

x	–2	–1	0	1	2
y		0			6

(b) Using the information in the table, sketch the graph of $y = x^3 - x$.

(c) Complete the following pairs of coordinates.

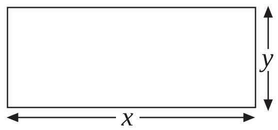
(–0.5, ?), (0.5, ?)

(d) Check that your graph passes through the points with the coordinates calculated in (c).

6. Complete a copy of the table and then draw the graph of $y = \frac{15}{x}$.

x	1	2	3	4	5	6
y	15					2.5

7. A manufacturer of postcards decides to experiment with cards of different shapes. The cards should be rectangles with an area of 120 cm^2. The height of the cards is y cm and the width is x cm.



(a) Explain why $y = \dfrac{120}{x}$.

(b) Complete a copy of the table.

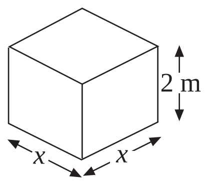
x	5	10	15	20	25	30
y		12				4

(c) Draw a graph of $y = \dfrac{120}{x}$.

(d) Use your graph to find the width of a postcard that has a height of 7 cm.

(e) If the height of a postcard must be no greater than 14 cm, what is the least width it can have?

8. Some water tanks have square bases and a height of 2 m.



(a) Explain why the volume, V, of a tank is $2x^2$.

(b) Complete a copy of the table and draw the graph of $V = 2x^2$.

x	0	0.5	1.0	1.5	2.0	2.5
V						12.5

(c) If the base of a tank is a square sheet of metal 2.2 m by 2.2 m, find from the graph the volume of the tank.

(d) What should be the size of the base in order to give a volume of

(i) 10 m^3 (ii) 5 m^3?

9. The height, h, in metres, of the distance travelled by a ball hit straight up into the air is given by

$$h = 18t - 5t^2$$

where t is the time in seconds.

(a) Complete a copy of this table and draw a graph of h against t.

t	0	1	2	3	4
h			16		

(b) Use your graph to estimate when the ball hits the ground.

(c) What is the maximum height reached by the ball?

10. (a) Complete the table below using the relationship $y = \dfrac{12}{x}$.

x	– 4	–3	–2	–1	0	1	2	3	4
y		– 4			No value	12			

(b) Draw the graph of $y = \dfrac{12}{x}$ using the points calculated in (a).

(c) Find the coordinates of extra points on the curve between $x = 0$ and $x = 1$.

(d) Describe what happens to the curve as the values of x get closer and closer to 0.

(e) Investigate what happens between –1 and 0.

11. (a) Use the equation $y = x + 3$ to complete the table of values.

x	–3	–2	–1	0	1	2	3
y	0			3		5	

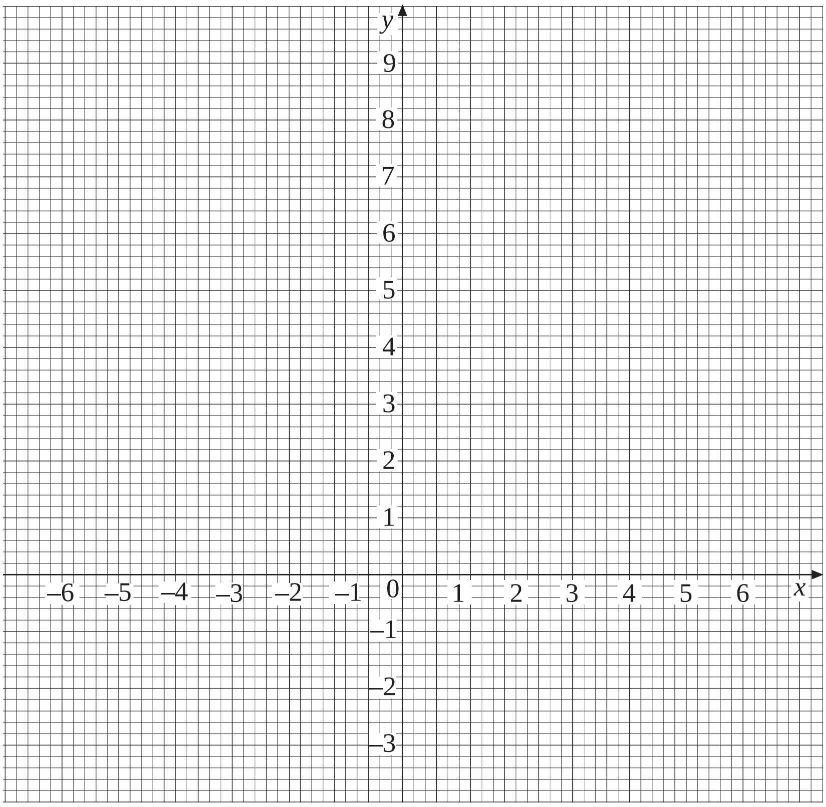
(b) Use the equation $y = x^2$ to complete the table of values.

x	–3	–2	–1	0	1	2	3
y						4	

(c) Draw the graphs of

$$y = x + 3 \text{ and } y = x^2$$

on a copy of the grid below.

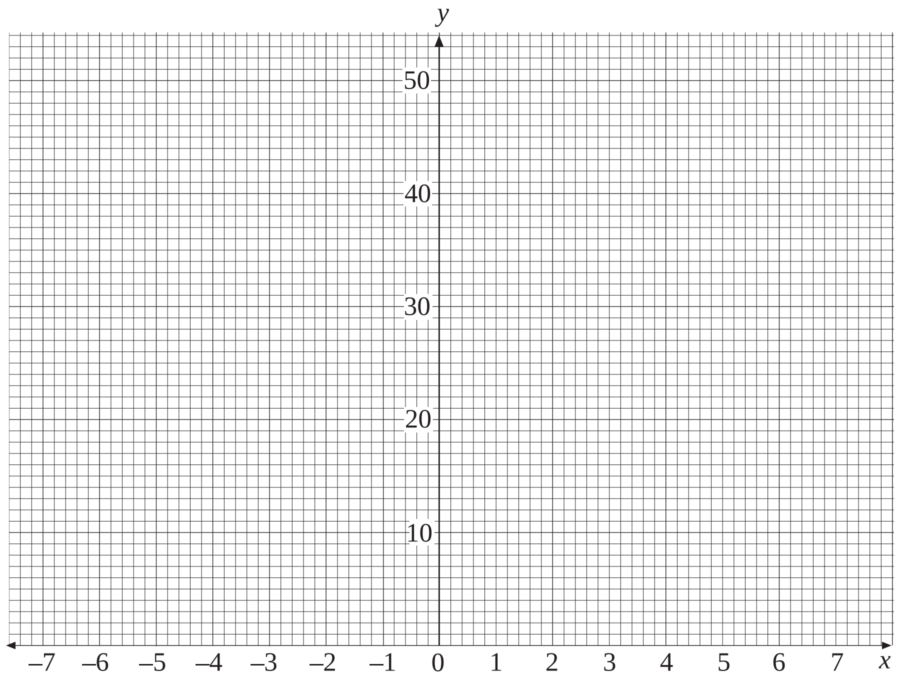


(LON)

12. (a) Use the formula $y = x^2$ to complete a copy of the table.

x	–7	–6	–5	– 4	–3	–2	–1	0	1	2	3	4	5	6	7
y	49	36		16	9	4	1	0	1	4	9	16			49

(b) Use your table of values to draw the graph of $y = x^2$.



(c) Use your graph to find the values of x when $y = 20$.

(SEG)

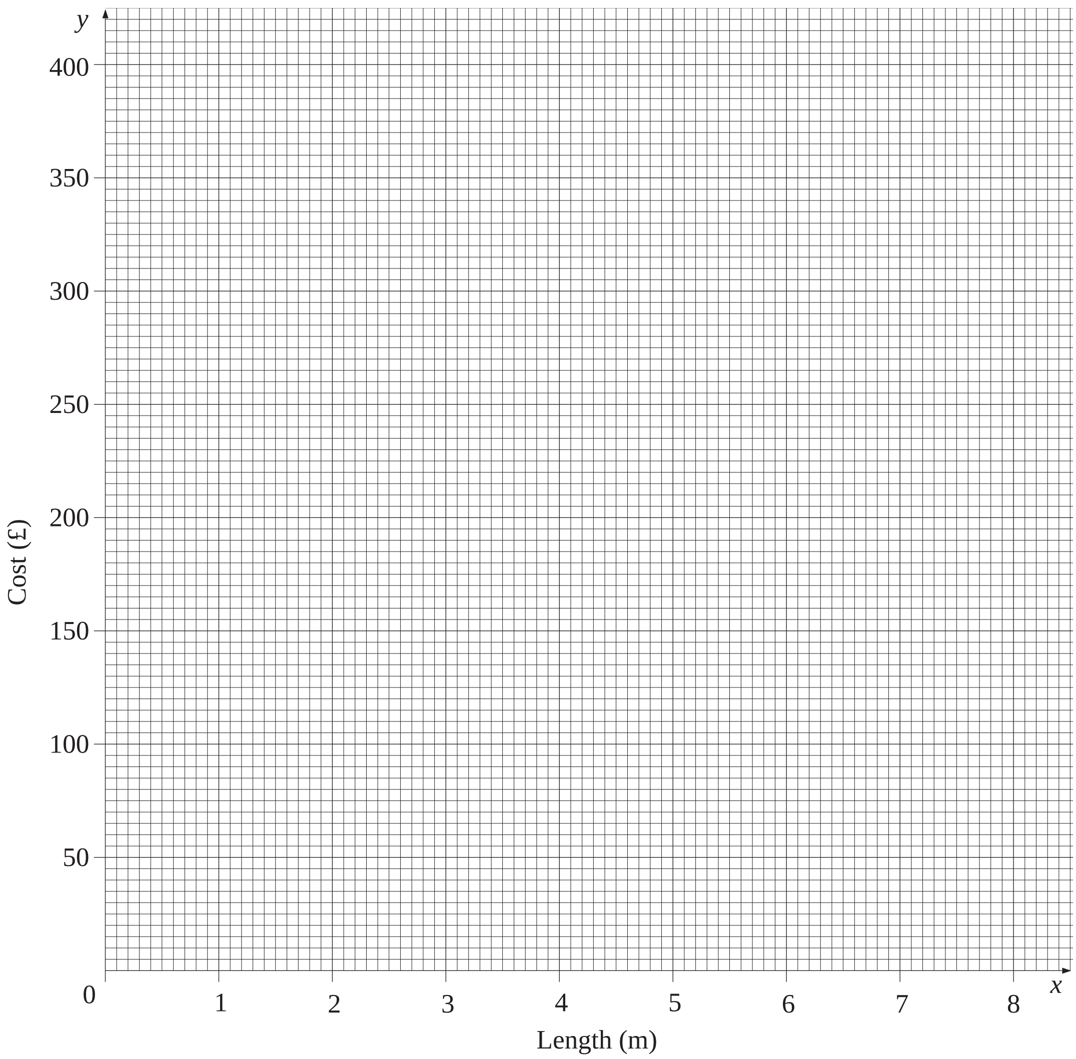
13. The annual cost of the heat lost through a wall depends on the length of the wall. When the wall is a square of length x m the annual cost, £y, is given by the equation $y = 5x^2$

(a) Calculate the cost, £y, when x is 8 m.

(b) The table shows the cost, £y, for different values of x m.

Length, x (m)	3	4	5	6	7
Cost, y (£)	45	80	125	180	245

Use the table of values to draw the graph of $y = 5x^2$ on a copy of the grid on the following page.

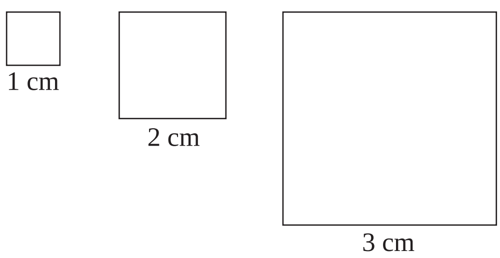


(c) The annual cost of the heat lost through a square wall is £150.

Use your graph to estimate the length of the wall

(SEG)

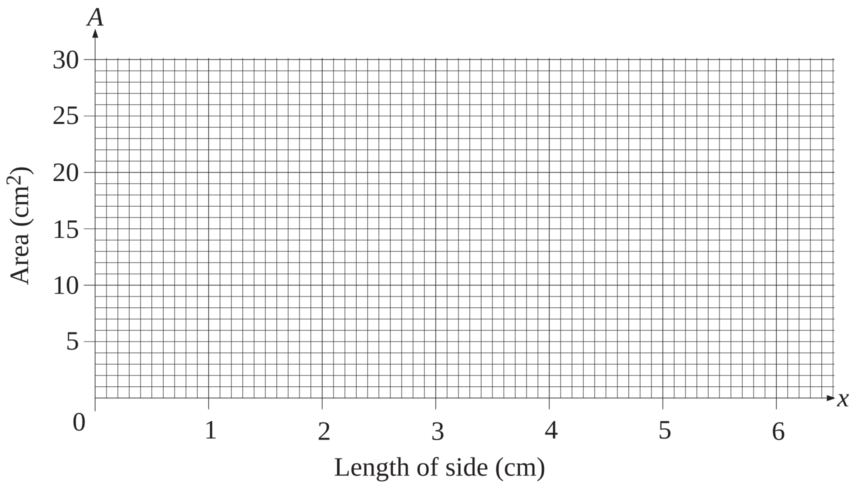
14. Angela is investigating the area of squares.



She makes a table.

Length of side (cm)	0	1	2	3	4	5
Area (cm^2)	0	1	4	9	16	25

(a) Draw a graph of area against length of side.



(b) The side of a square is x cm. The area of a square is $A\ \text{cm}^2$. Write down the formula which may be used to calculate the area from the length of the side.

(c) A square has an area of $12\ \text{cm}^2$. Angela wants to use the graph to find the length of the side of this square.

She draws a line on the graph to help her do this.

(i) Draw this line.

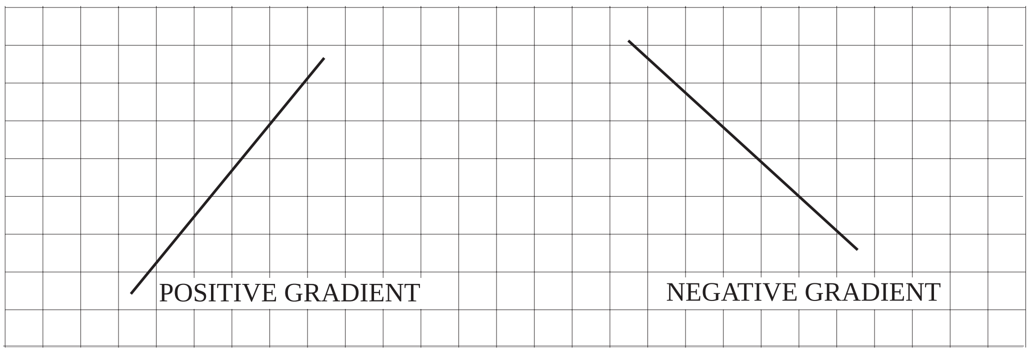
(ii) Write down the length of the side of the square whose area is $12\ \text{cm}^2$.

(SEG)

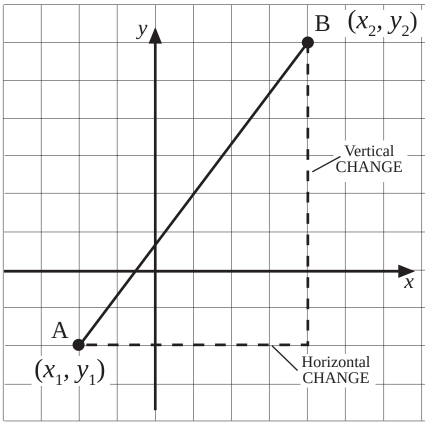
13.5 Gradient

The *gradient* of a line describes how steep it is.

The diagram below shows two lines, one with a *positive* gradient and the other with a *negative* gradient.



The gradient of a line between two points, A and B, is calculated using



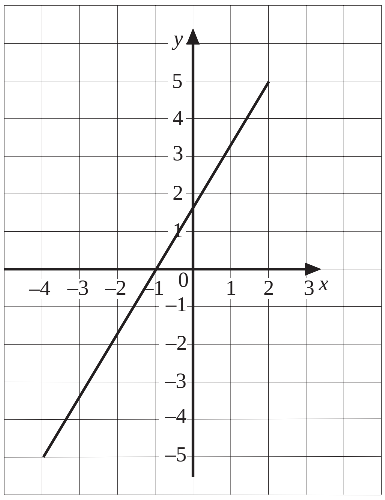
$$\text{gradient of AB} = \frac{\text{vertical change}}{\text{horizontal change}}$$

$$= \frac{(y\text{-coordinate of B}) - (y\text{-coordinate of A})}{(x\text{-coordinate of B}) - (x\text{-coordinate of A})}$$

$$= \frac{y_2 - y_1}{x_2 - x_1}$$

Worked Example 1

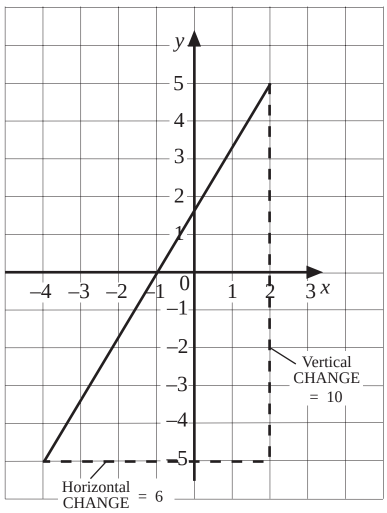
Find the gradient of the line shown in the diagram.



Solution

Draw a triangle under the line below to show the horizontal and vertical distances.

Here the vertical distance is 10 and the horizontal distance is 6.



$$\text{Gradient} = \frac{\text{vertical change}}{\text{horizontal change}}$$

$$= \frac{10}{6}$$

$$= \frac{5}{3} \quad \text{or} \quad 1\frac{2}{3}$$

Worked Example 2

Find the gradient of the line joining the point A with coordinates (2, 4) and the point B with coordinates (4, 10).

Solution

$$\text{Gradient} = \frac{(y\text{-coordinate of B}) - (y\text{-coordinate of A})}{(x\text{-coordinate of B}) - (x\text{-coordinate of A})}$$

$$= \frac{10-4}{4-2}$$

$$= \frac{6}{2}$$

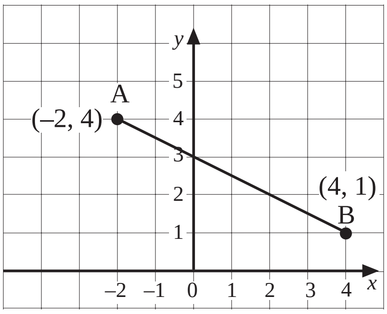
$$= 3$$

Worked Example 3

Find the gradient of the line that joins the points with coordinates (–2, 4) and (4, 1).

Solution

The diagram shows the line. It will have a negative gradient because of the way it slopes.



$$\text{Gradient} = \frac{(y\text{-coordinate of B}) - (y\text{-coordinate of A})}{(x\text{-coordinate of B}) - (x\text{-coordinate of A})}$$

$$= \frac{1-4}{4-(-2)}$$

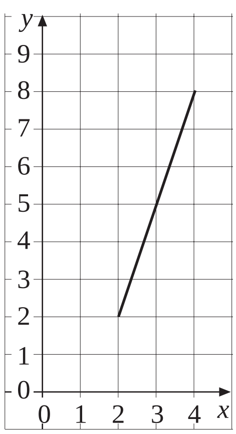
$$= \frac{-3}{6}$$

$$= -\frac{1}{2}$$

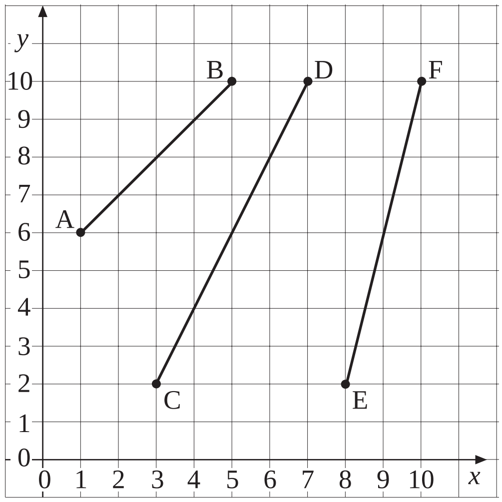
So the gradient is $-\frac{1}{2}$.

Exercises

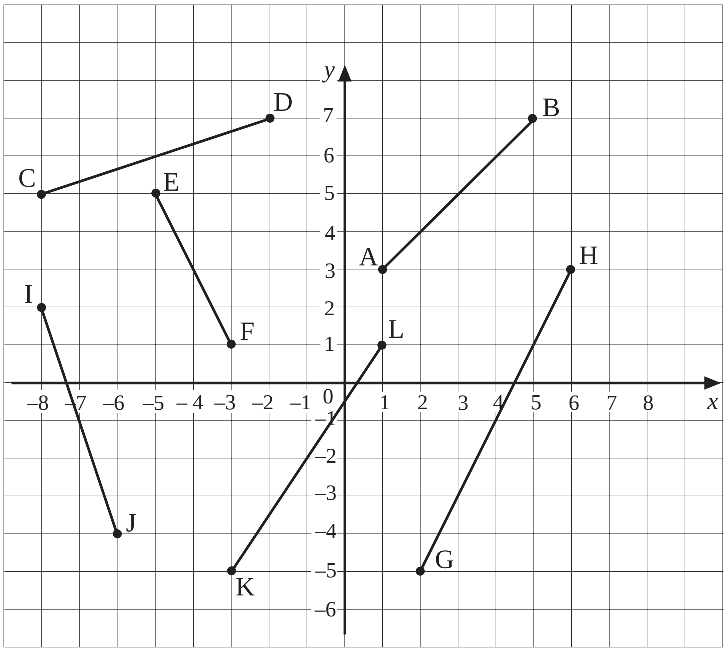
1. Find the gradient of the line shown on this graph.



2. Find the gradient of each line in the diagram below.



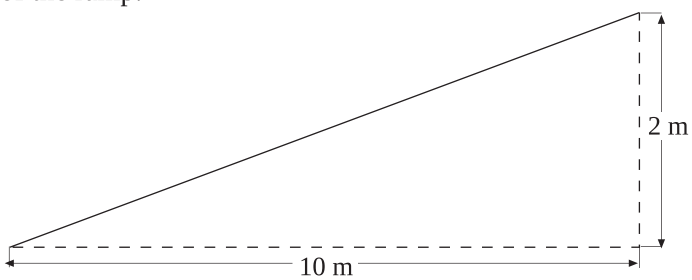
3. (a) Which of the lines in the diagram below have a positive gradient?



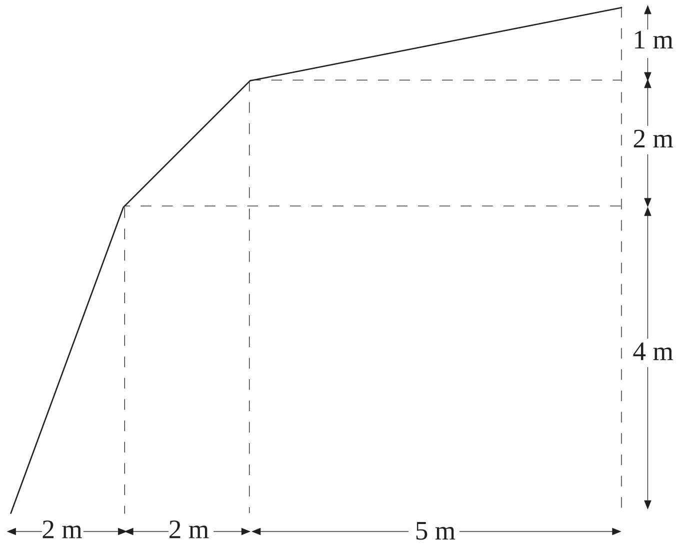
(b) Which lines have a negative gradient?

(c) Find the gradient of each line.

4. The diagram shows a side view of a ramp in a multistorey car part. Find the gradient of the ramp.



5. The diagram shows the cross-section of a roof. Find the gradient of each part of the roof.



6. Find the gradient of the line that joins the points with the coordinates:

   (a) (1, 1) and (9, 5),

   (b) (2, 1) and (3, 6),

   (c) (2, 2) and (3, 6),

   (d) (0, 4) and (10, 12),

   (e) (–1, 2) and (5, 8),

   (f) (–2, –2) and (0, 12).

7. Find the gradient of the line joining the points with the coordinates:

   (a) (– 4, 2) and (4, –2),

   (b) (3, 4) and (7, –1),

   (c) (2, 4) and (7, –1),

   (d) (–2, –3) and (2, –8),

   (e) (0, 6) and (2, –7),

   (f) (8, 1) and (14, 2).

8. A quadrilateral is formed by joining the points A, B, C and D. The coordinates of each point are:

   A (2, 4)   B (–1, 5)   C (– 4, 5)   D (1, –1)

   Find the gradient of each side of the quadrilateral.

9. (a) For the line $y = 2x + 1$, complete the coordinates

$$(2, \ ? \ ), \quad (0, \ ? \ ) \quad \text{and} \ (-2, \ ? \ )$$

(b) Plot the points in (a) and draw a line through them.

(c) What is the gradient of the line you have drawn?

(d) Repeat (a) to (c) for $y = 2x + 3$ and $y = 2x - 1$.

(e) What do you notice?

10. (a) Calculate the coordinates for three points on the line $y = 3x + 2$.

(b) Plot these points and draw a straight line through them.

(c) Find the gradient of the line that you have drawn.

(d) Repeat (a) to (c) for the lines $y = 4x - 1$ and $y = 5x + 1$.

(e) What is the connection between the equation of a line and its gradient?

(f) What do you think the gradient of the line $y = 7x + 5$ will be?

11. For each pair of coordinates below, find the gradient of the straight line that joins them.

(a) (0, 0) and (a, a)

(b) (a, –a) and (–a, a)

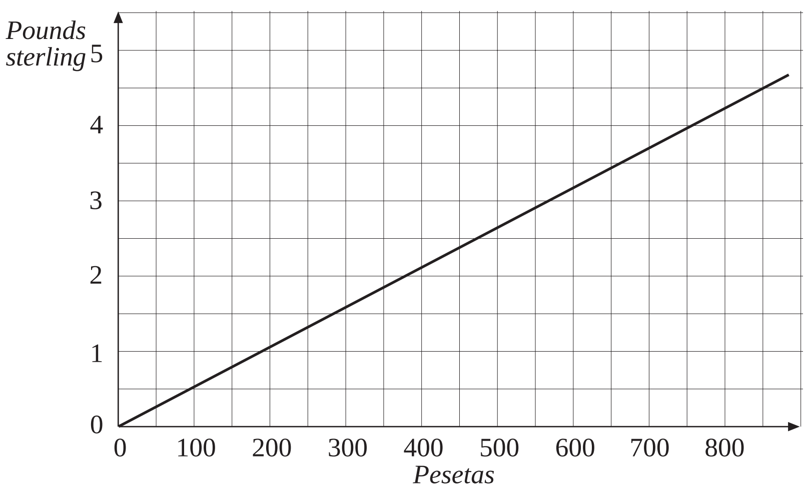
(c) (a, 2a) and (0, –2a)

(d) (–a, 4a) and (2a, –2a)

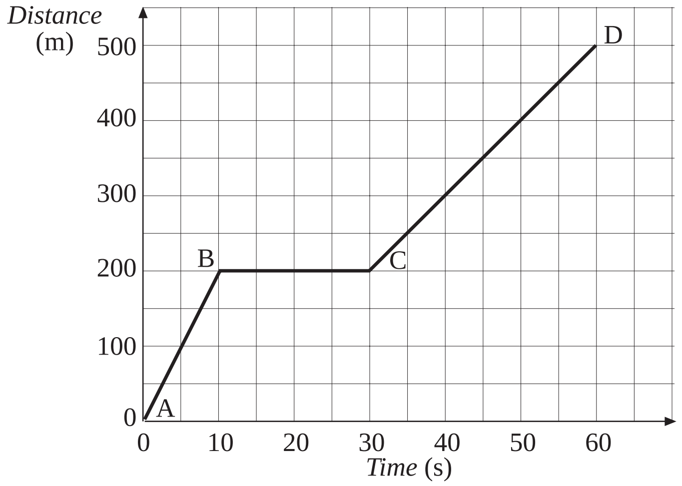
13.6 Applications of Graphs

In this section some applications of graphs are considered, particularly *conversion graphs* and graphs to describe *motion.*

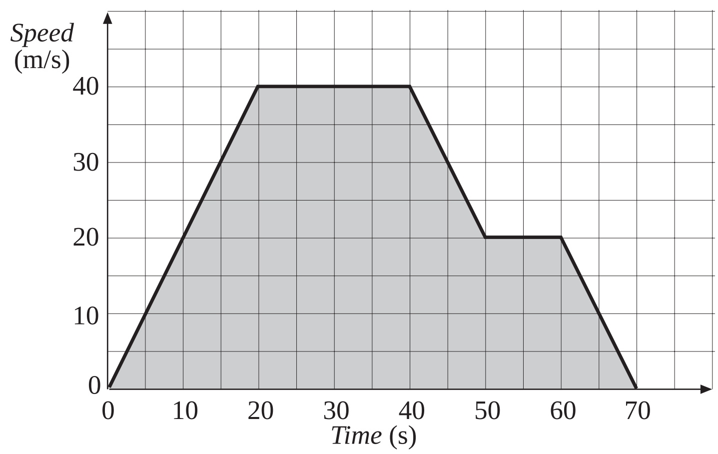
The graph below can be used for converting pounds sterling (British pounds) into and from Spanish pesetas.



A distance-time graph of a car is shown below. The gradient of this graph gives the speed of the car. The gradient is steepest from A to B, so this is when the care has the greatest speed. The gradient BC is zero, so the car is not moving.



The area under a speed-time graph gives the distance travelled. Finding the shaded area on the graph below would give the distance travelled.



Worked Example 1

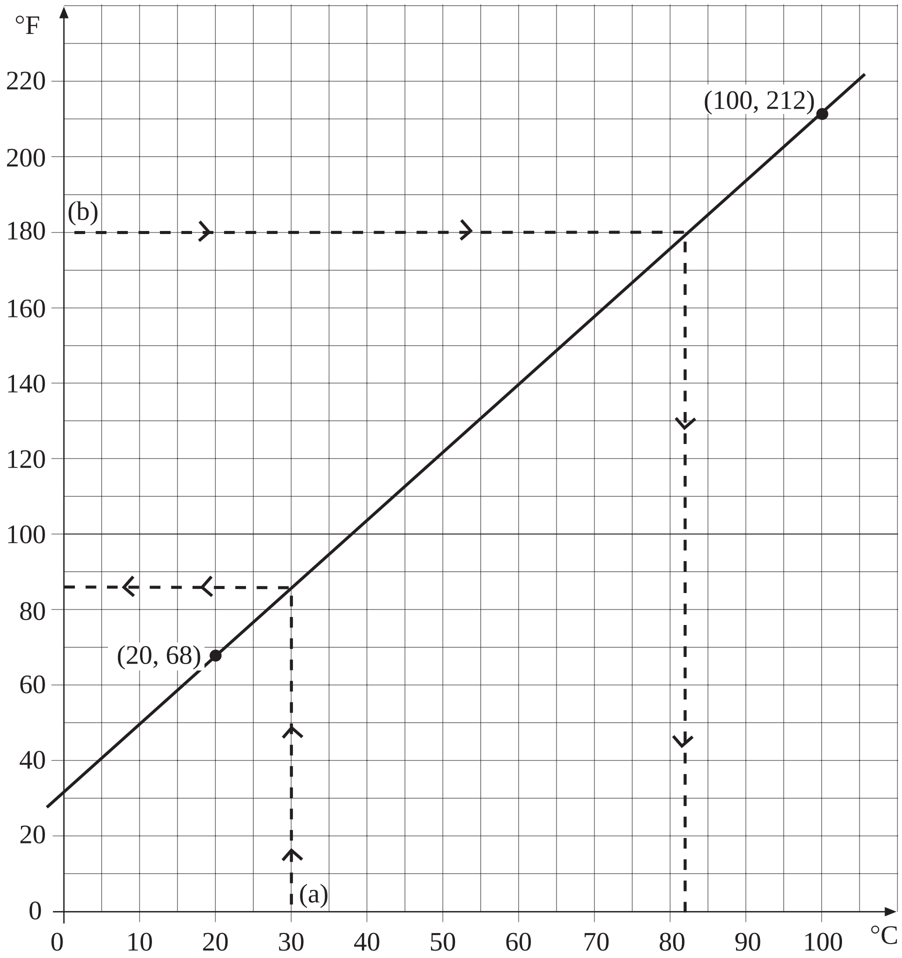
A temperature of $20\ °C$ is equivalent to $68\ °F$ and a temperature of $100\ °C$ is equivalent to a temperature of $212\ °F$. Use this information to draw a conversion graph. Use the graph to convert:

(a) $30\ °C$ to $°$Fahrenheit,

(b) $180\ °F$ to $°$Celsius.

Solution

Taking the horizontal axis as temperature in $°C$ and the vertical axis as temperature in $°F$ gives two pairs of coordinates, (20, 68) and (100, 212). These are plotted on a graph and a straight line drawn through the points.

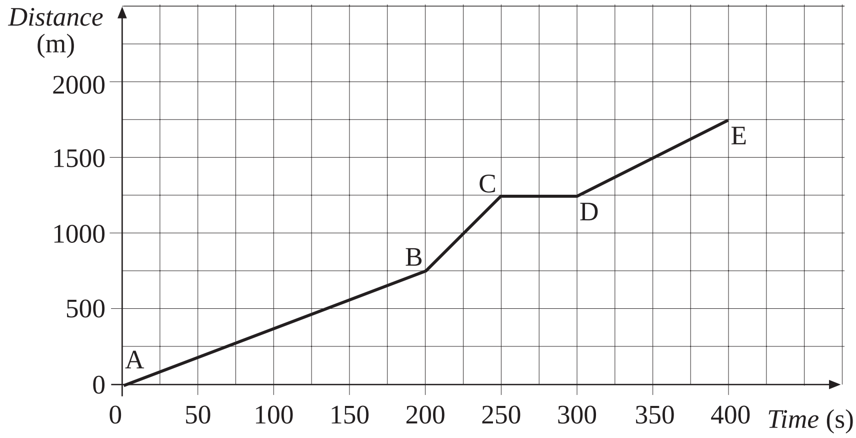


(a) Start at $30\ °C$, then move up to the line and across to the vertical axis, to give a temperature of about $86\ °F$.

(b) Start at $180\ °F$, then move across to the line and down to the horizontal axis, to give a temperature of about $82\ °C$.

Worked Example 2

The graph shows the distance travelled by a girl on a bike.



Find the speed she is travelling on each stage of the journey.

Solution

For AB the gradient $= \dfrac{750}{200}$

$= 3.75$

So the speed is 3.75 m/s.

Note

The units are m/s (metres per second), as m are the units for distance and s the units for time.

For BC the gradient $= \dfrac{500}{50}$

$= 10$

So the speed is 10 m/s.

For CD the gradient is zero and so the speed is zero

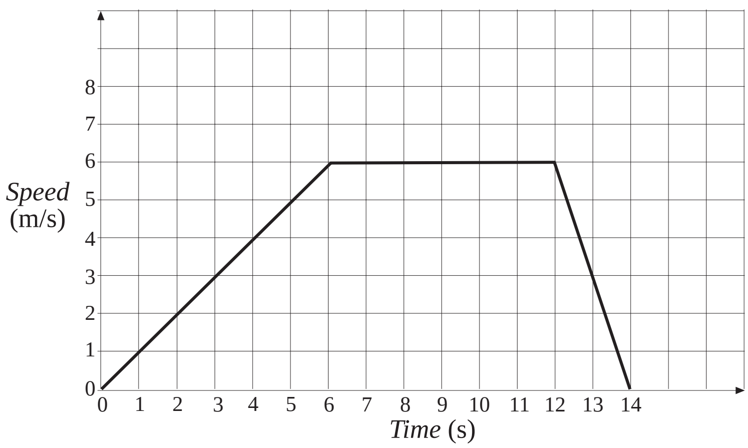
For DE the gradient is $= \dfrac{500}{200}$

$= 2.5$

So the speed is 2.5 m/s.

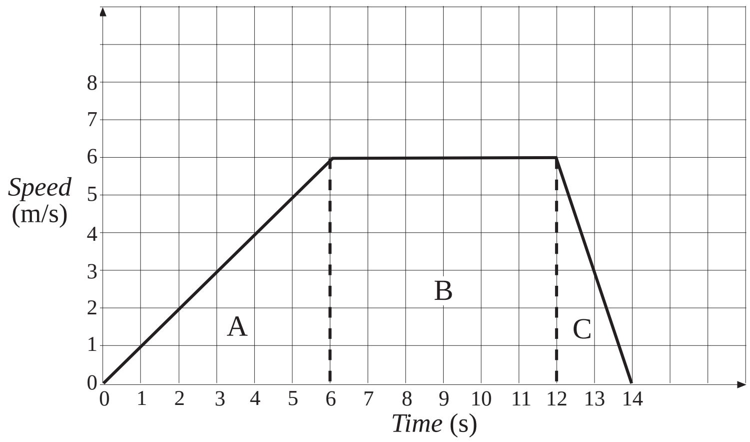
Worked Example 3

The graph shows how the speed of a bird varies as it flies between two trees. How far apart are the two trees?



Solution

The distance is given by the area under the graph. In order to find this area it has been split into three sections, A, B and C.



$$\text{Area of A} = \frac{1}{2} \times 6 \times 6$$
$$= 18$$

$$\text{Area of B} = 6 \times 6$$
$$= 36$$

$$\text{Area of C} = \frac{1}{2} \times 2 \times 6$$
$$= 6$$

$$\text{Total Area} = 18 + 36 + 6$$
$$= 60$$

So the trees are 60 m apart. Note that the units are m because the units of speed are m/s and the units of time are s.

Exercises

1. Use the approximation that 10 kg is about the same as 22 lbs to draw a graph for converting between pounds and kilograms. Use the graph to convert the following:

   (a) 6 lbs to kilograms,

   (b) 8 lbs to kilograms,

   (c) 5 kg to pounds,

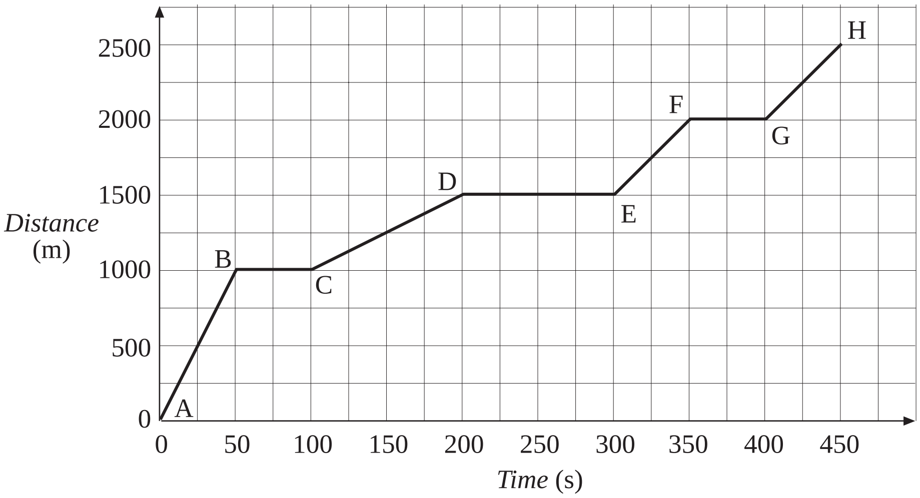
   (d) 3 kg to pounds.

2. Use the approximation that 10 gallons is about the same as 45 litres to draw a conversion graph. Use the graph to convert:

   (a) 5 gallons to litres,

   (b) 30 litres to gallons.

3. The graph shows how the distance travelled by a bus increased.

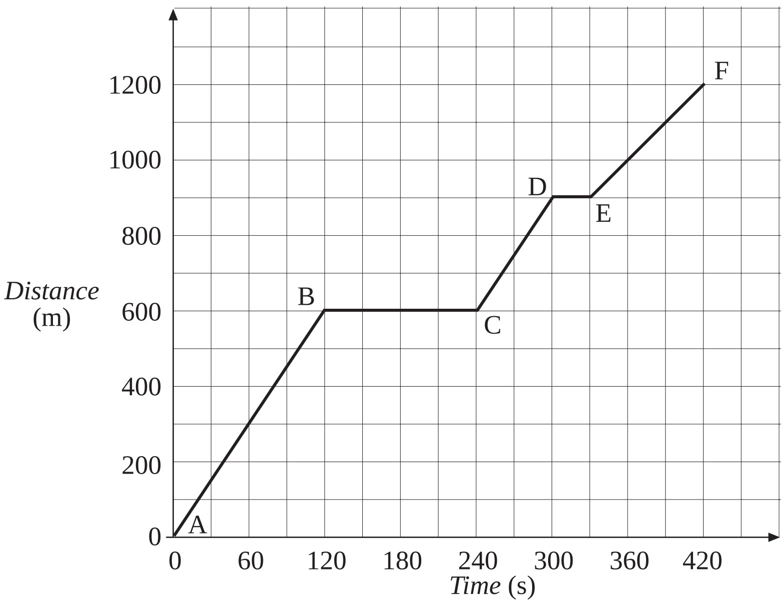


(a) How many times did the bus stop?

(b) Find the speed of the bus on each section of the journey.

(c) On which part of the journey did the bus travel fastest?

4. The distance-time graph shows the distance travelled by a car on a journey to the shops.

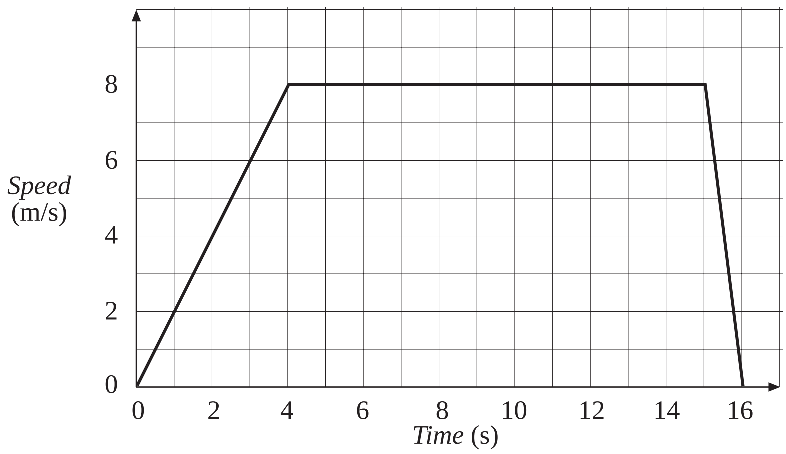


(a) The car stopped at two sets of traffic lights. How long did the car spend waiting at the traffic lights?

(b) On which part of the journey did the car travel fastest? Find its speed on this part.

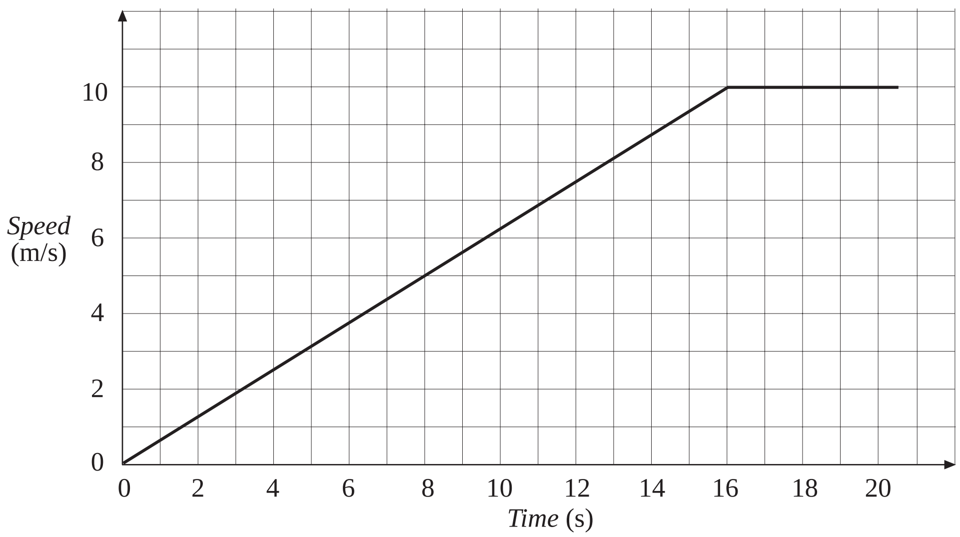
(c) On which part of the journey did the car travel at its lowest speed? What was this speed?

5. The graph below shows how the speed of an athlete varies during a race.



What was the distance of the race?

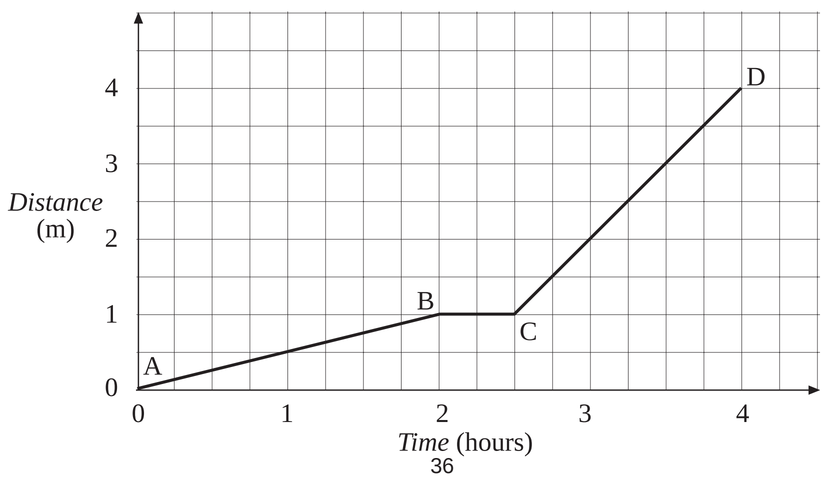
6. The graph below shows how the speed of a lorry varies as it sets off from a set of traffic lights.



Find the distance travelled by the lorry after

(a) 8 seconds, (b) 16 seconds, (c) 20 seconds.

7. The graph shows how the distance travelled by a snail increases.



Find the speed of the snail on each section in m/hour.

8. Hannah runs at 2 m s^{-1} for 5 seconds and then her speed decreases to zero at a steady rate over the next 4 seconds.

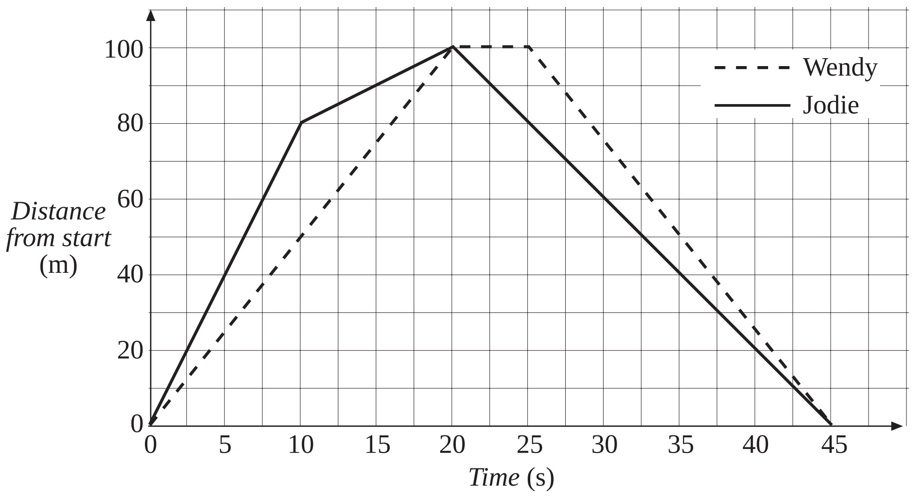
   Find the distance that Hannah runs.

9. Ian runs at a constant speed for 10 seconds. He has then travelled 70 m. He then walks at a constant speed for 8 seconds until he is 86 m from his starting point.

   (a) Find the speed at which he runs and the speed at which he walks.

   (b) If he had covered the complete distance in the same time, with a constant speed, what would that speed have been?

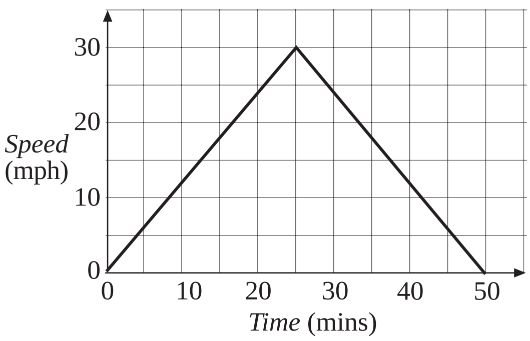
10. The graph shows how the distance travelled by Wendy and Jodie changes during a race from one end of the school field to the other end, and back.



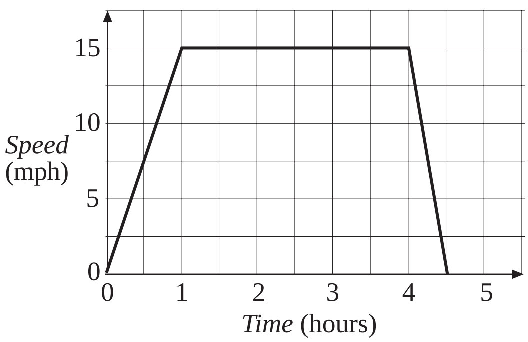
Describe what happens during the race.

11. Find the area under each graph below and state the distance that it represents.

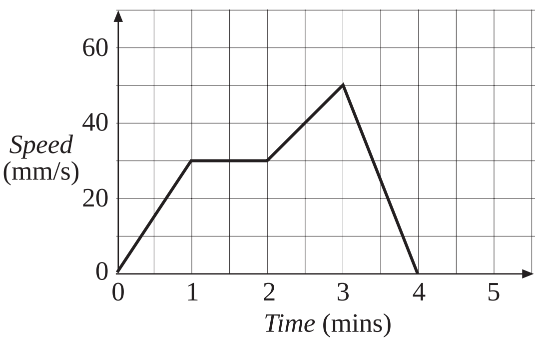
(a)



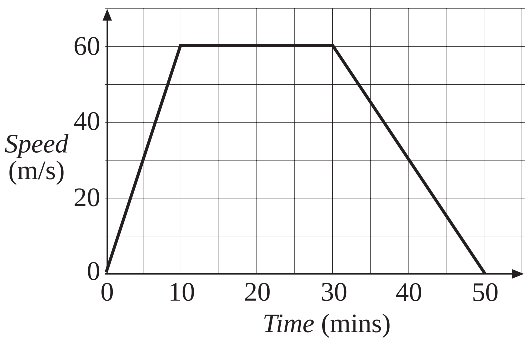
(b)



(c)

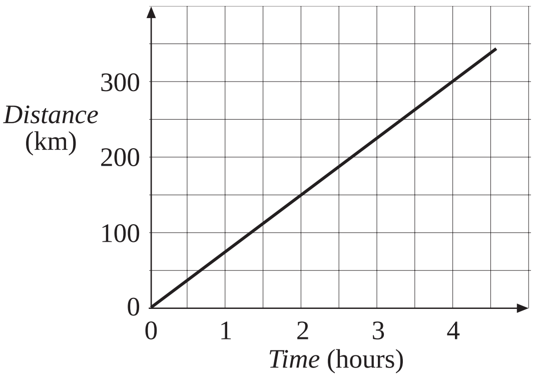


(d)

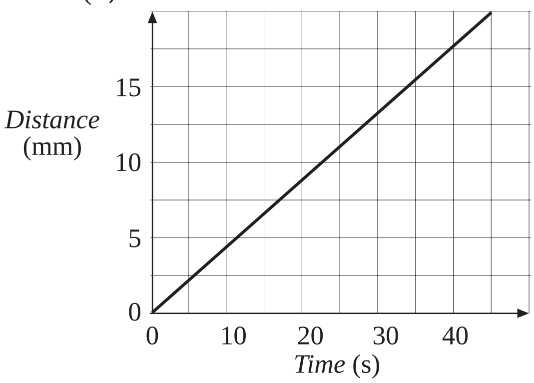


12. For each distance-time graph, find the speed in the units used on the graph and in m/s.

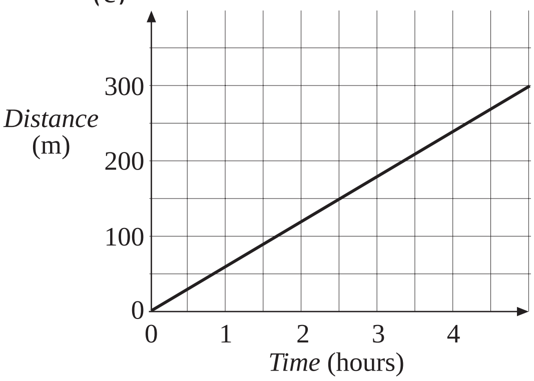
(a)



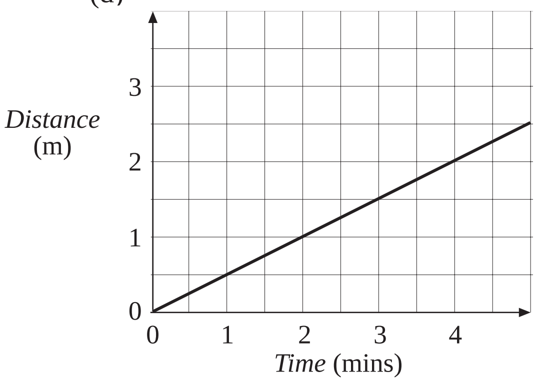
(b)



(c)



(d)



13. Jennifer walks from Corfe Castle to Wareham Forest and then returns to Corfe Castle.

The following travel graph shows her journey.

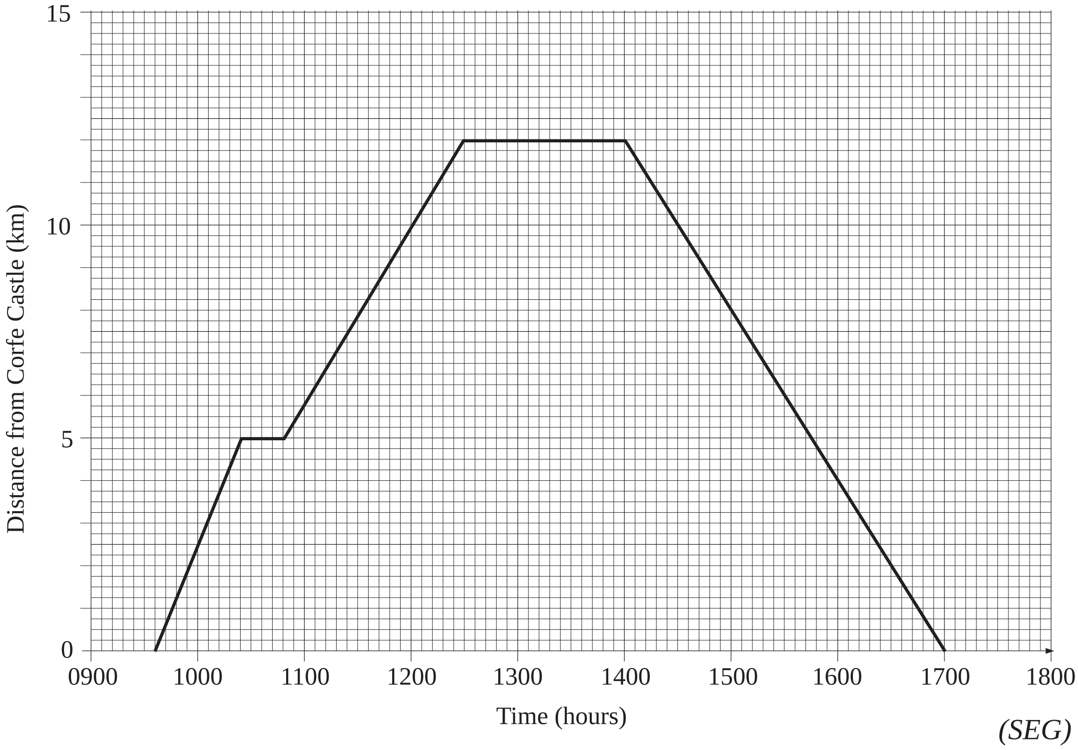
(a) At what time did Jennifer leave Corfe Castle?

(b) How far from Wareham Forest did Jennifer make her first stop?

(c) Jennifer had lunch at Wareham Forest.

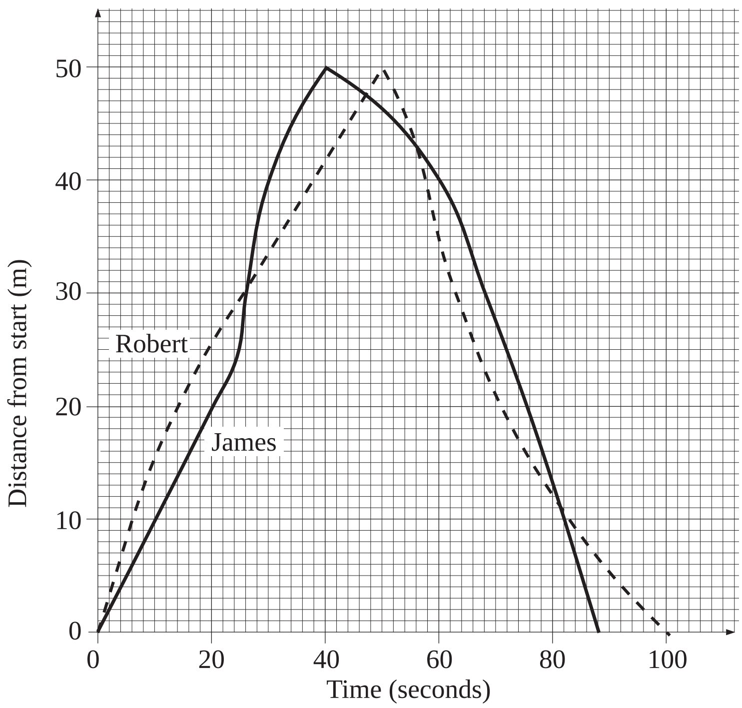
For how many minutes did she stop for lunch?

(d) At what average speed did Jennifer walk back from Wareham Forest to Corfe Castle?



(SEG)

14. The graph represents a swimming race between Robert and James.



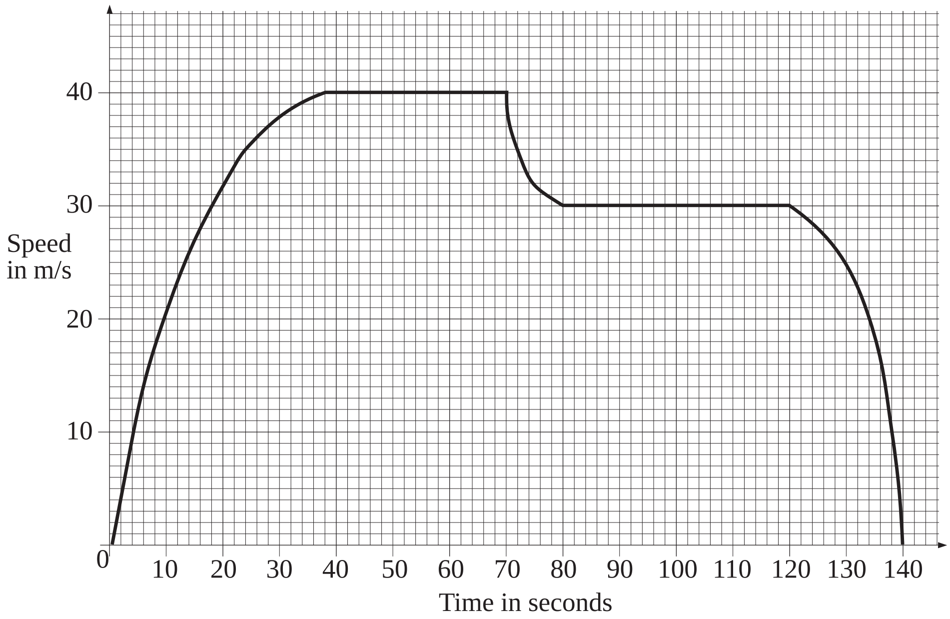
(a) At what time did James overtake Robert for the second time?

(b) What was the **maximum** distance between the swimmers during the race?

(c) Who was swimming faster at 56 seconds? How can you tell?

(SEG)

15. The graph illustrates the journey of a car.



(a) Estimate the area under the graph taking into account the scales of the graph.

(b) State the units of the quantity represented by the area under the graph.

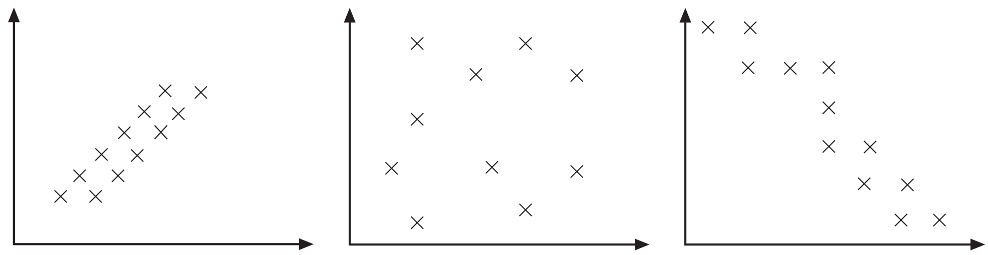
(c) Another car did the same journey in the same time at constant speed. On a copy of the grid above, draw the graph which illustrates the second car's journey.

(MEG)

13.7 Scatter Plots and Lines of Best Fit

When there might be a connection between two different quantities, a *scatter plot* can be used. If there does appear to be a connection, a *line of best fit* can be drawn.

The following diagrams show 3 different scatter plots.



Positive correlation *No correlation* *Negative correlation*

If there is a relationship between the two quantities, there is said to be a *correlation* between the two quantities. This may be *positive* or *negative*, as shown in the examples above.

Worked Example 1

A salesman records, for each working day, how much petrol his car uses and how far he travels. The table shows his figures for 10 days.

Day	1	2	3	4	5	6	7	8	9	10
Petrol used (litres)	24	13	29	19	21	35	10	40	44	18
Distance travelled (miles)	200	150	320	180	190	280	120	360	400	160

(a) Plot a scatter graph and describe any connection that is present.

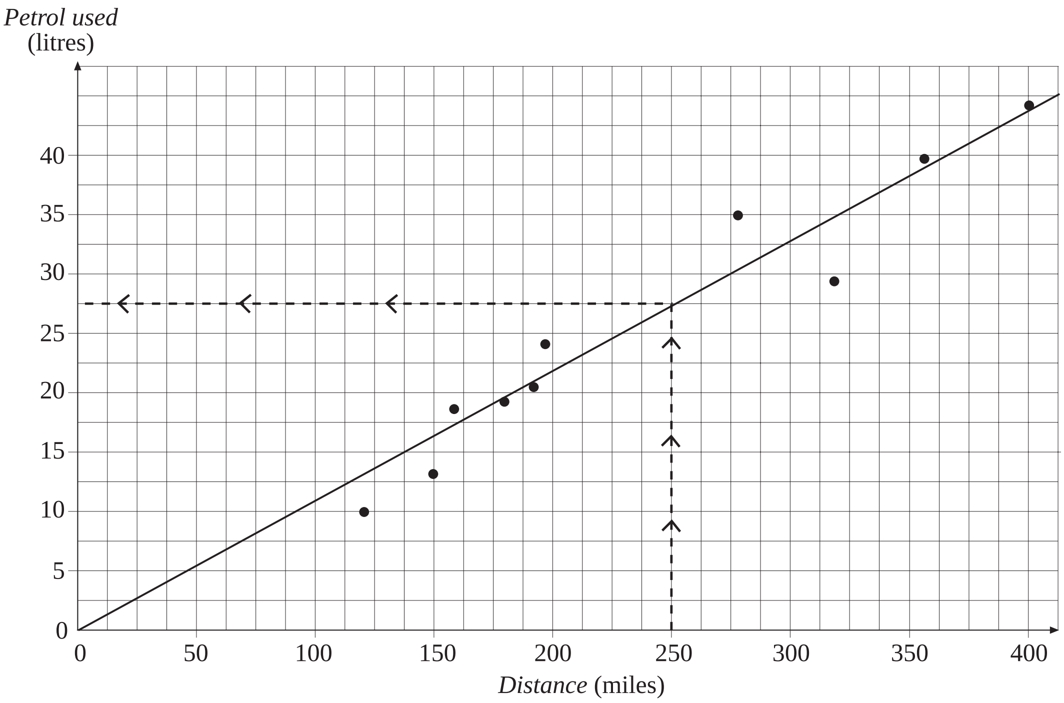
(b) Draw a line of best fit.

(c) Explain why it is sensible for the line to go through (0, 0)

(d) Estimate how much petrol would be used on journey of 250 miles.

Solution

(a) Each point has been plotted on the graph below.
This is an example of *positive correlation.*



(b) A line of best fit has been drawn.

There are roughly the same number of points above and below the line.

(c) This is sensible because a car will not use any petrol if it is not used.

(d) The dashed lines on the graph predict that 27.5 litres are needed for a journey of 250 miles.

Worked Example 2

The table shows how the Olympic record for the marathon has decreased since 1900.

Year	*Record* (minutes)
1908	175
1912	156
1920	152
1932	151
1936	149
1952	143
1960	135
1964	132
1976	130
1984	129

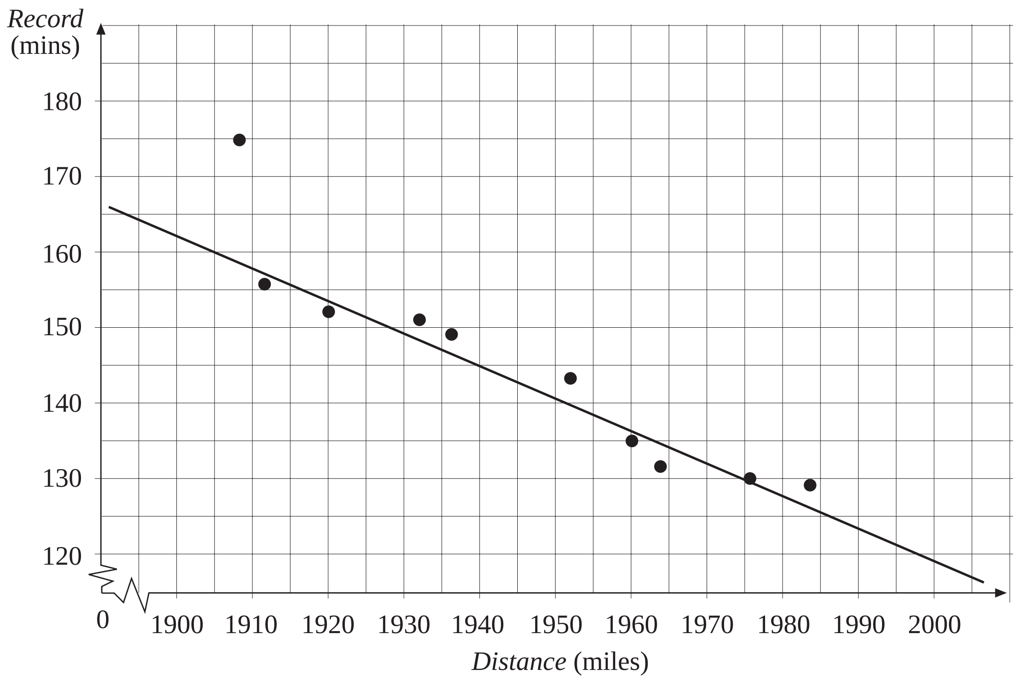
(a) Draw a scatter graph to illustrate this.

(b) Draw a line of best fit.

(c) Estimate what the Olympic record will be in the year 2000.

Solution

(a) Each point has been plotted on the scatter graph below. This is an example of *negative correlation*.



(b) A line of best fit has been added to the graph. The first point has been ignored as it would have moved the line a long way from the other points.

(c) Using the line you can predict that in the year 2000 the record will drop to just under 120 minutes.

Note The line of best fit becomes unreliable as time progresses; for example, the straight line will predict a zero-minute mile in about the year 2300!

Exercises

1. The table gives the scores obtained by 10 students on three different tests.

Maths Test	7	19	7	18	10	18	11	14	11	16
Science Test	8	19	9	17	10	17	11	13	12	13
French Test	11	10	14	16	7	13	19	5	16	10

(a) Draw a scatter graph for maths against science.

(b) Draw a scatter graph for maths against French.

(c) Which set of points lie closer to a straight line?

(d) Would it be reasonable to draw a line of best fit in both cases?

2. A firm records how long it takes a driver to make deliveries at warehouses at different distances from the factory.

Distance (miles)	155	65	80	145	100	95	50	120	90
Time taken (hours)	4.8	1.8	2.9	3.5	3.0	2.2	1.0	3.5	2.6

(a) Draw a scatter graph of time taken against distance, and describe any correlation between the two quantities.

(b) Draw a line of best fit.

(c) A delivery takes 2 hours. Use your line to estimate how far the driver has travelled.

(d) How long would you expect a delivery to take if the driver has to travel 140 miles?

3. The table shows the flying time and costs for holidays in some popular resorts.

Destination	*Flying time (hours)*	*Cost of holiday (£)*
Algarve	2.0	194
Benidorm	2.5	139
Gambia	6.0	357
Majorca	2.5	148
Morocco	3.0	237
Mombasa	8.5	523
Tenerife	4.5	238
Torremolinos	2.5	146
Tunisia	3.0	129

(a) Draw a scatter graph of cost against time.

(b) Draw a line of best fit.

(c) Estimate the cost of a holiday with a flying time of 5 hours.

(d) Estimate the flying time for a holiday that costs £400.

4. Ten children were weighed and then had their height measured. The results are in the table.

Weight (kg)	47	60	47	49	50	59	46	54	57	53
Height (cm)	82	110	95	101	88	121	79	98	105	100

(a) Draw a scatter graph of height against weight.

(b) Draw a line of best fit and comment on how well it can be applied to the data.

(c) Estimate the height of a boy who weighs 60 kg.

(d) Estimate the weight of a girl who is 110 cm tall.

5. A group of children were tested on their tables. The time in seconds taken to do a test on the 7 times table and a test on the 3 times table were recorded.

Time for 3 times table	6	12	10	13	12	14	12	9	13	10	9
Time for 7 times table	9	20	15	20	19	22	18	15	17	17	13

(a) Draw a scatter graph and a line of best fit.

(b) Ben missed the test for the 7 times table but took 11 seconds for the 3 times table test. Estimate how long he would have taken for the 7 times table test.

(c) Emma completed her 3 times table test in 5 seconds. She missed the test for the 7 times table. How long do you estimate that she would have taken for this test?

6. The table below shows how the Olympic record for the men's and women's 400 m freestyle has decreased. The times are given to the nearest second.

Year	*Men's record* (s)	*Women's record* (s)
1908	337	–
1912	324	–
1924	304	362
1928	301	342
1932	288	328
1936	284	328
1948	281	318
1952	271	312
1956	267	295
1960	258	291
1964	252	283
1968	249	272
1972	240	259
1976	232	250
1980	231	249
1984	231	247
1988	227	244

(a) On the same set of axes, plot scatter graphs for both the men's and the women's records. What type of correlation do you see in the plot?

(b) Draw a line of best fit for each sets of records against time.

(c) Use your lines to estimate what the records will be in the year 2000.

(d) Do the graphs suggest that the women's record will be less than the men's record? Is this a realistic prediction?

7. Describe two quantities that you would expect to have

(a) a positive correlation,

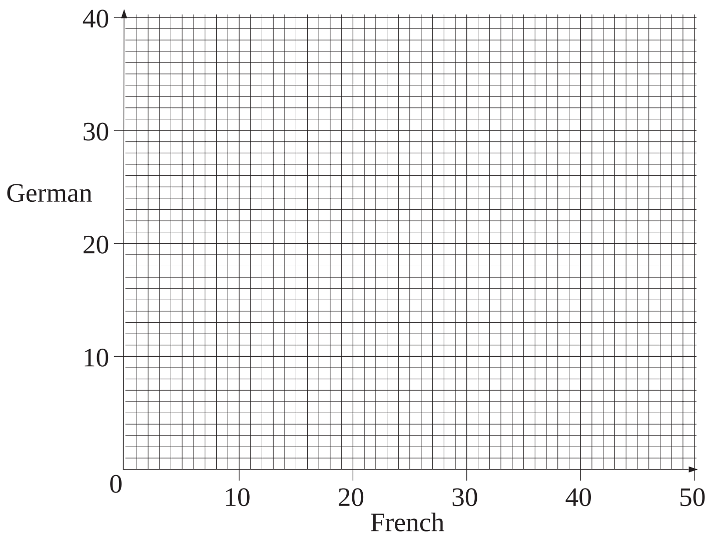
(b) no correlation,

(c) a negative correlation.

8. The table gives you the marks scored by pupils in a French test and in a German test.

French	15	35	34	23	35	27	36	34	23	24	30	40	25	35	20
German	20	37	35	25	33	30	39	36	27	20	33	35	27	32	28

(a) On a copy of the grid below, draw a scatter graph of the marks scored in the French and German tests.



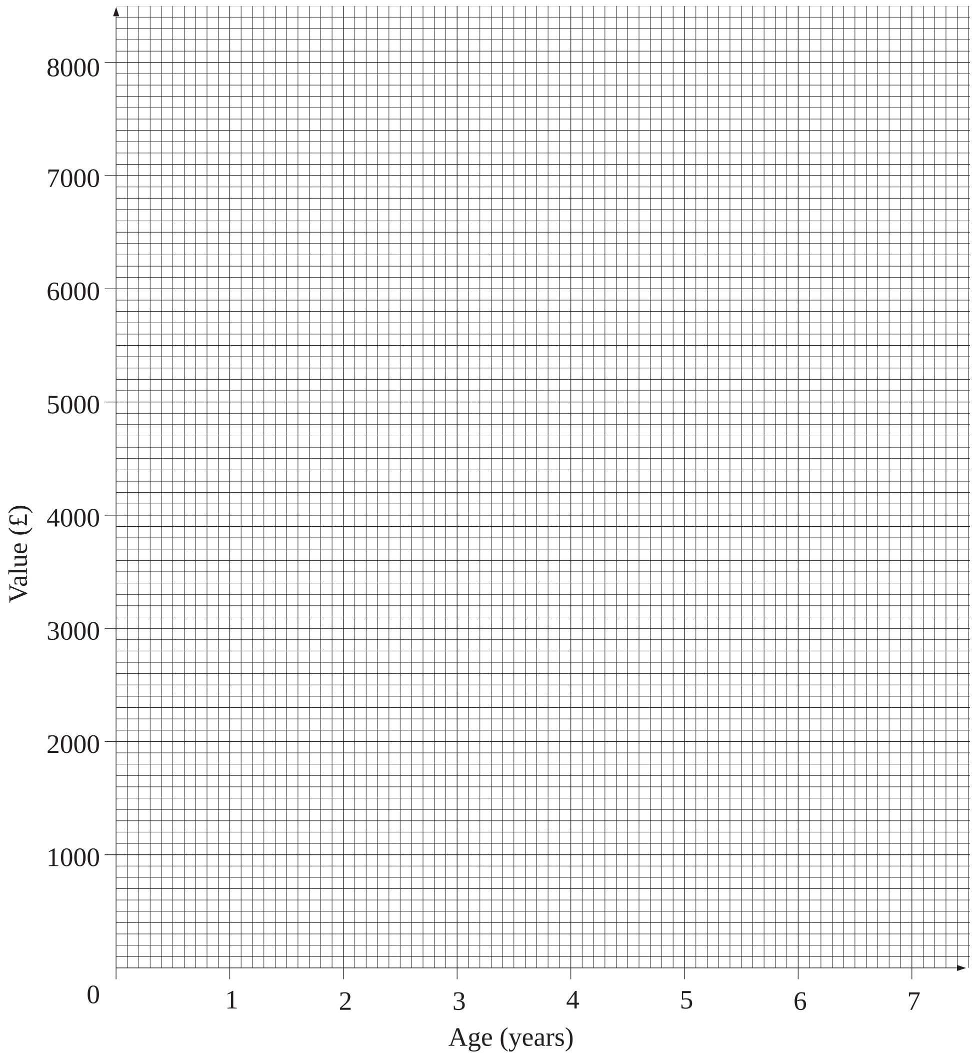
(b) Describe the correlation between the marks scored in the two tests.

(LON)

9. The table gives information about the age and value of a number of cars of the same type.

Age (years)	1	3	$4\frac{1}{2}$	6	3	5	2	$5\frac{1}{2}$	4	7
Value (£)	8200	5900	4900	3800	6200	4500	7600	2200	5200	3200

(a) Use this information to draw a scatter graph on a copy of the following grid.



(b) What does the graph tell you about the value of theses cars as they get older?

(SEG)

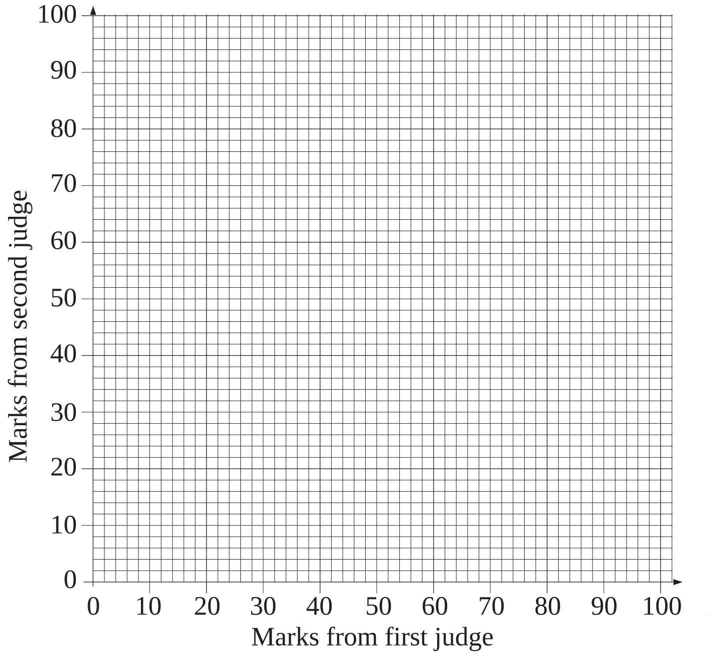
10. Ten people entered a craft competition.

Their displays of work were awarded marks by two different judges.

Competitor	*A*	*B*	*C*	*D*	*E*	*F*	*G*	*H*	*I*	*J*
First judge	90	35	60	15	95	25	5	100	70	45
Second judge	75	30	55	20	75	30	10	85	65	40

The table shows the marks that the two judges gave to each of the competitors.

(a) (i) On a copy of the following grid, draw a scatter diagram to show this information.



(ii) Draw a line of best fit.

(b) A late entry was given 75 marks by the first judge.

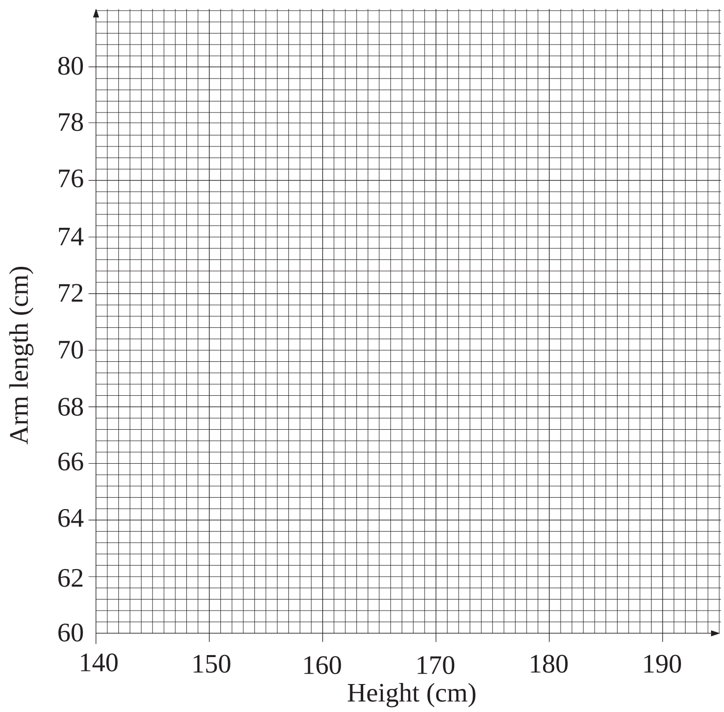
Use your scatter diagram to estimate the mark that might have been given by the second judge. (Show how you found your answer.)

(NEAB)

11. The height and arm length for each of eight pupils are shown in the table.

Height (cm)	169	176	182	157	166	188	154	190
Arm length (cm)	67	73	70	63	69	74	62	77

(a) On a copy of the grid below, plot a scatter graph for these data.



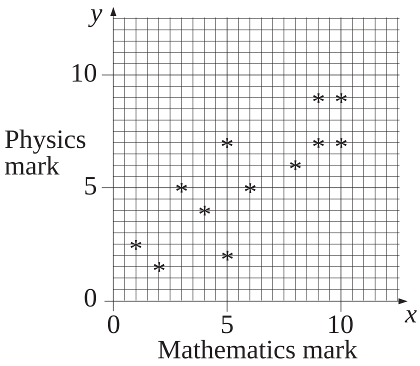
(b) (i) Peter gives his height as 171 cm.

Use the scatter graph to estimate Peter's arm length.

(ii) Explain why your answer can only be an estimate.

(SEG)

12. A group of schoolchildren took a Mathematics test and a Physics test.
The results for 12 children were plotted on a scatter diagram.



(a) Does the scatter diagram show the results you would expect? Explain your answer.

(b) (i) Add a line of best fit, by inspection, to the scatter diagram.

(ii) One pupil scored 7 marks for Mathematics but missed the Physics test.

Use the line of best fit to estimate the mark she might have score for Physics.

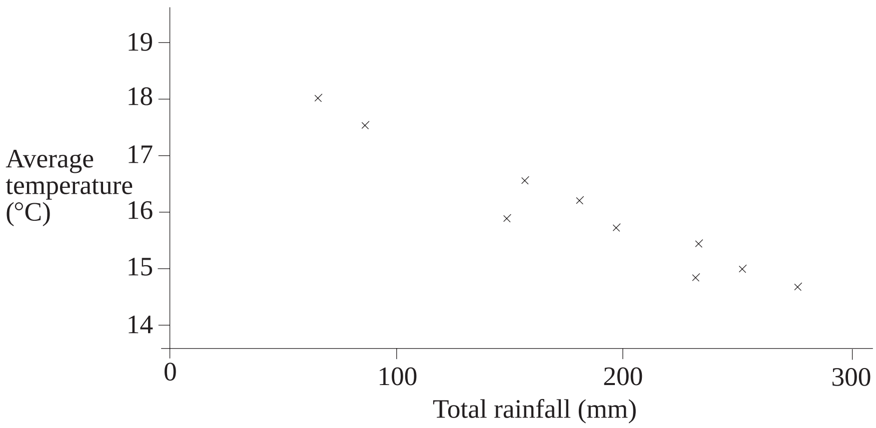
(iii) One pupil was awarded the prize for the best overall performance in Mathematics and Physics.

Put a ring around the cross representing that pupil on the scatter diagram.

(MEG)

13. Megan wanted to find out if there is a connection between the average temperature and the total rainfall in the month of August.

She obtained weather records for the last 10 years and plotted a scatter graph.



(a) What does the graph show about a possible link between temperature and rainfall in August?

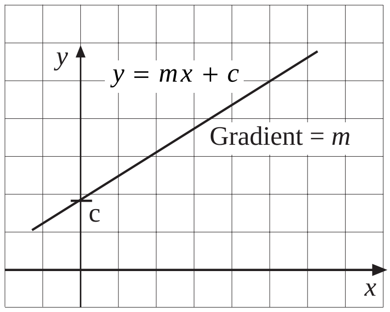
(b) Estimate the total rainfall in August when the average temperature is $17\ °C$.

13.8 The Equation of a Straight Line

The equation of a straight line is usually written in the form

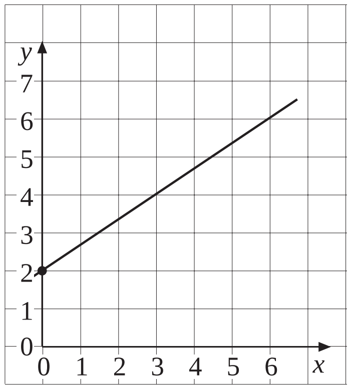
$$y = mx + c$$

where m is the gradient and c is the y intercept.



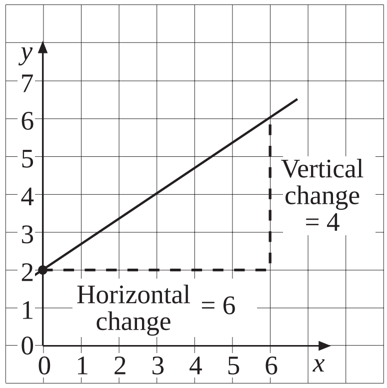
Worked Example 1

Find the equation of the line shown in the diagram.



Solution

The first step is to find the gradient of the line.
Drawing the triangle shown under the line gives



$$\text{gradient} = \frac{\text{vertical change}}{\text{horizontal change}}$$

$$= \frac{4}{6}$$

$$= \frac{2}{3}$$

So the value of m is $\frac{2}{3}$. The line crosses the y-axis at 2, so the value of c is 2. The equation of a straight line is

$$y = mx + c$$

In this case

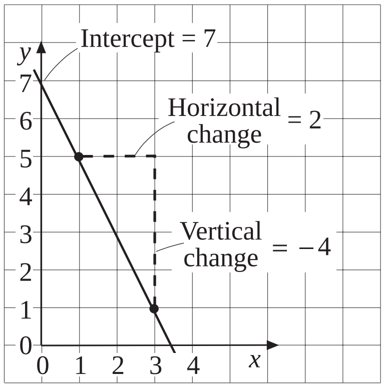
$$y = \frac{2}{3}x + 2.$$

Worked Example 2

Find the equation of the line that passes through the points (1, 5) and (3, 1).

Solution

Plotting these points gives the straight line shown. Using the triangle drawn underneath the line allows the gradient to be found.



$$\text{gradient} = \frac{\text{vertical change}}{\text{horizontal change}}$$

$$= \frac{-4}{2}$$

$$= -2$$

The y intercept is 7.

So $m = -2$ and $c = 7$ and the equation of the line is

$$y = -2x = 7 \quad \text{or} \quad y = 7 - 2x$$

Exercises

1. Find the equation of the straight line with:

(a) gradient $= 2$ and y-intercept $= 4$,

(b) gradient $= 3$ and y-intercept $= -5$,

(c) gradient $= \frac{1}{2}$ and y-intercept $= 2$,

(d) gradient $= -2$ and y-intercept $= 1$,

(e) gradient $= \frac{3}{4}$ and y-intercept $= -3$.

2. Write down the gradient and y-intercept of each line.

(a) $y = 2x + 3$

(b) $y = 4x - 2$

(c) $y = \frac{1}{2}x + 1$

(d) $y = \frac{2}{3}x - 4$

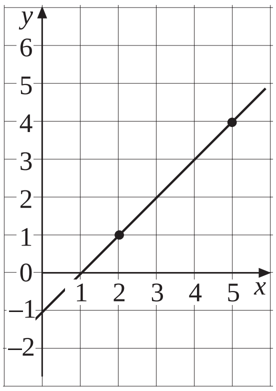
(e) $y = 4(x + 2)$

(f) $y = 3(x - 7)$

(g) $y = \frac{x + 5}{2}$

(h) $y = \frac{x - 10}{4}$

3. The diagram shows the straight line that passes through the points (2, 1) and (5, 4).

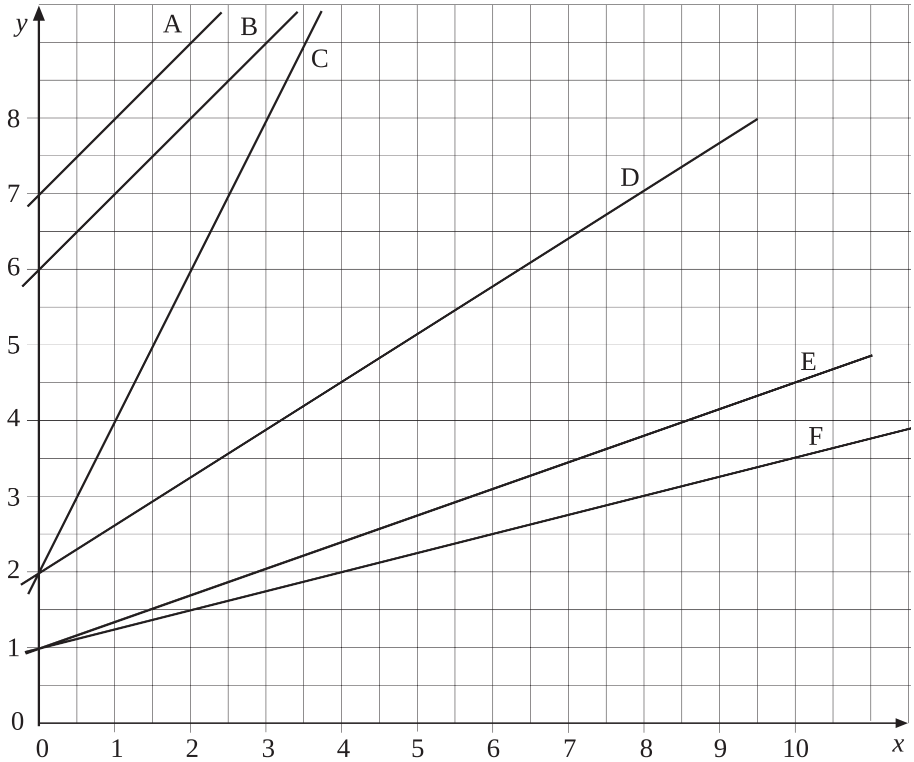


(a) Find the gradient of the line.

(b) Write down the *y*-intercept.

(c) Write down the equation of the straight line.

4. Find the equation of each line shown in the diagram below.



5. Write down the gradient of each line and the coordinates of the *y*-intercept.

(a) $y = 2x - 8$ (b) $y = -3x + 2$

(c) $y = 4x - 3$ (d) $y = \dfrac{1}{2}x + 2$

(e) $y = 8 - 2x$ (f) $y = 4 - 3x$

(g) $x + y = 8$ (h) $y = 3(5 - x)$

6. Find the equation of the line that passes through the points with the coordinates below.

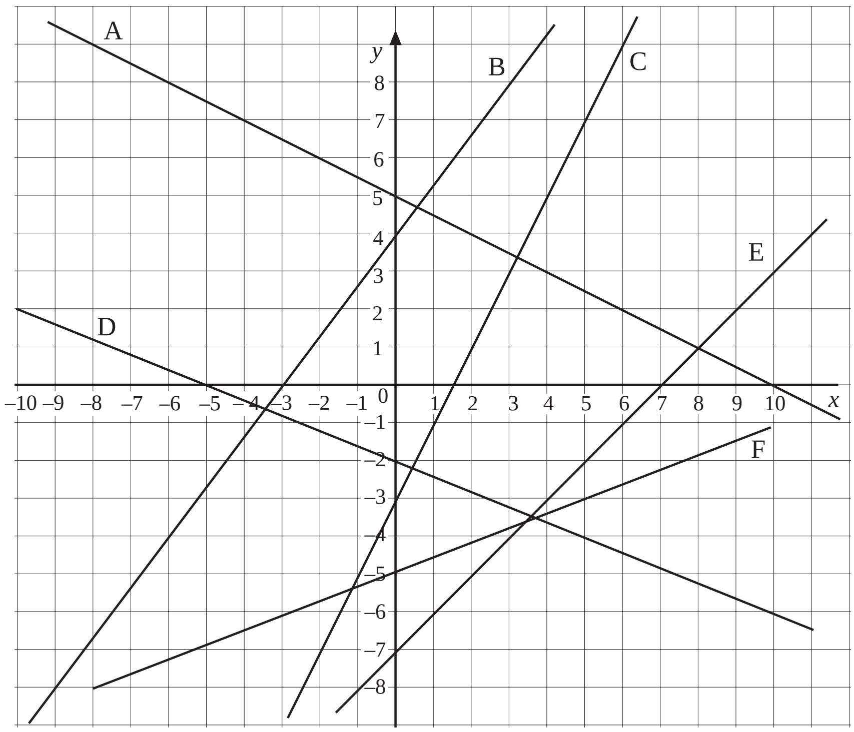
   (a) (0, 2) and (4, 10)

   (b) (4, 2) and (8, 4)

   (c) (0, 6) and (6, 4)

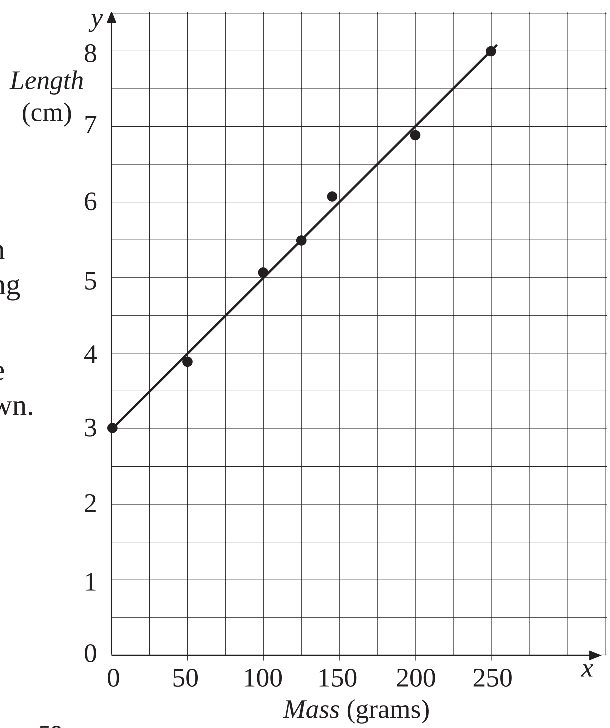
   (d) (1, 4) and (3, 0)

7. Find the equation of each line in the diagram below.



8. The graph shows the results of an experiment. Masses were hung on a spring and the length of the spring recorded.

   (a) Find the equation of the line of best fit that has been drawn.



(b) The data in the following table was collected using a different spring.

Mass (grams)	0	100	200	300	400
Length (cm)	10	14	18	22	26

Draw a graph and find the equation of the straight line that passes through these points.

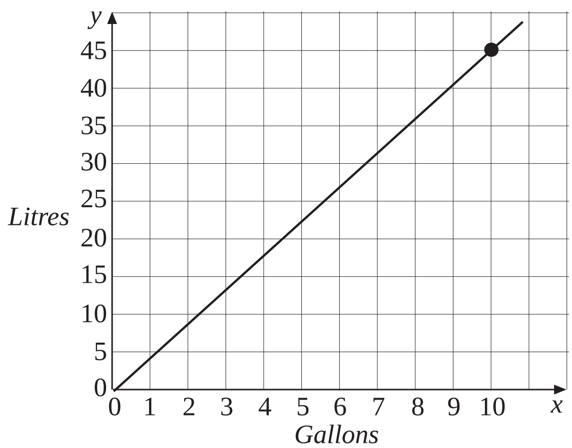
9. Some students made a simple thermometer and recorded the results below.

Temperature (°C)	0	20	40	60
Height of mercury (mm)	19	34	49	64

(a) Plot the data points with temperature on the horizontal axis.

(b) Draw a straight line through the points and find its equation.

10. The graph below can be used for converting gallons to litres.



(a) Find the equation of the line.

(b) Draw a similar graph for converting litres to pints, given that 11 litres is approximately 20 pints. Use the horizontal axis for pints.

(c) Find the equation of the line drawn in (b).

11. The velocity of a ball thrown straight up into the air was recorded at half second intervals.

Time (s)	0.5	1.0	1.5	2.0	2.5	3.0
Velocity $(\mathrm{m\,s^{-1}})$	15	10	5	0	–5	–10

(a) Plot a graph with time on the horizontal axis.

(b) Draw a line through the points and find its equation.

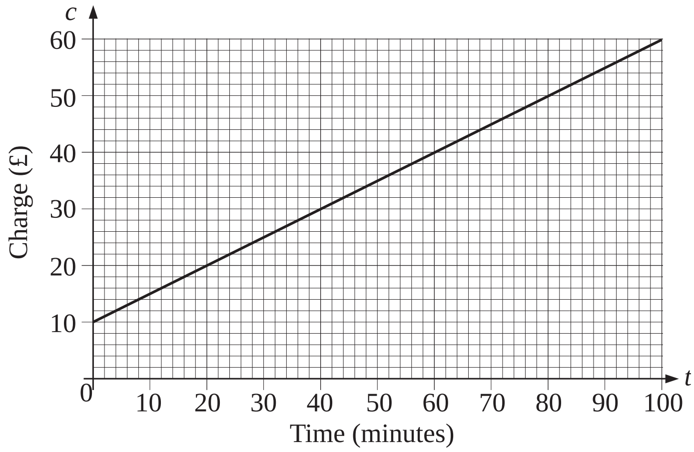
(c) What was the velocity of the ball when it was thrown upwards?

12. The line $y = x + c$ passes through the point (4, 7). Find the value of c.

13. The point (5, –2) lies on the line $y = 2x + c$. Find the value of c.

14. The line $y = mx + 2$ passes through the point (3, 17). Find the value of m.

15. Television repair charges depend on the length of time taken for the repair, as shown on the graph.



The charge is made up of a fixed amount plus an extra amount which depends on time.

(a) What is the charge for a repair which takes 45 minutes?

(b) (i) Calculate the gradient of the line.

(ii) What does the gradient represent?

(c) Write down the equation of the line.

(d) Mr Banks' repair will cost £84 or less. Calculate the maximum amount of time which can be spent on the repair.

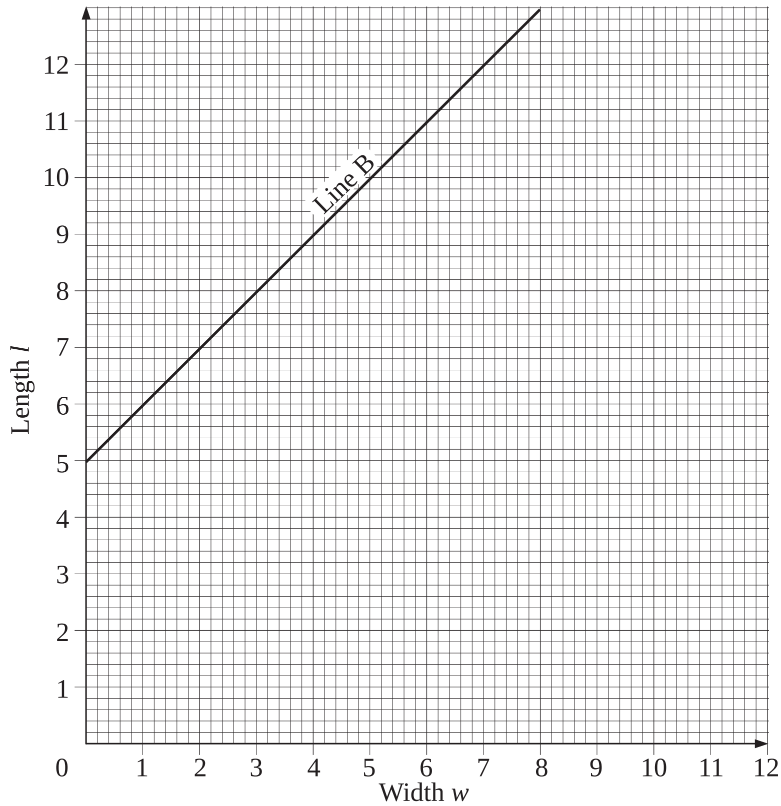
(SEG)

16. A set of rectangular tiles is made.

The length l cm, of each tile is equal to its width, w cm, plus two centimetres so $l = w + 2$.

(a) Draw the graph of $l = w + 2$ to show this rule.

Label this Line A.



(b) Line **B** shows the rule connecting the length and width of a different set of tiles.

What is this rule?

(SEG)

17. The table shows the largest quantity of salt, w grams, which can be dissolved in a beaker of water at temperature t °C.

t °C	10	20	25	30	40	50	60
w grams	54	58	60	62	66	70	74

(a) On a copy of the following grid, plot the points and draw a graph to illustrate this information.

(b) Use your graph to find

(i) the lowest temperature at which 63 g of salt will dissolve in the water.

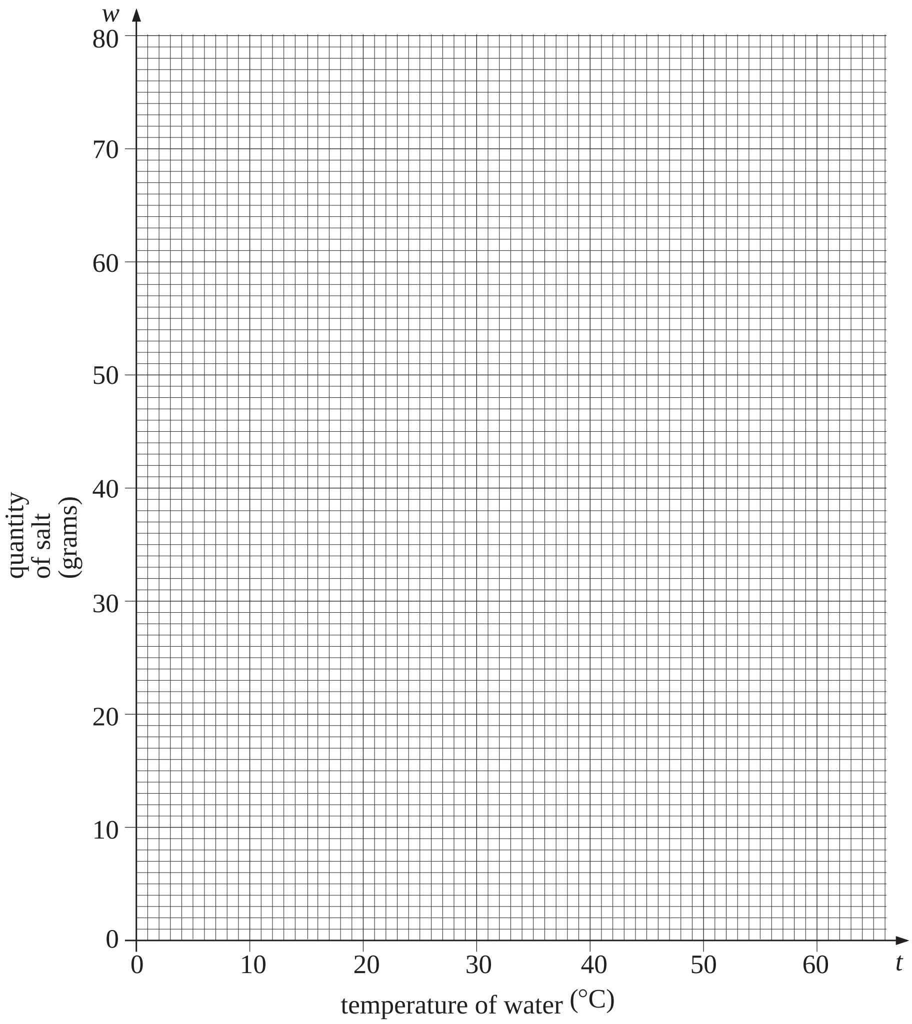
(ii) the largest amount of salt that will dissolve in the water at 44 °C.

(c) (i) The equation of the graph is of the form

$$w = at + b.$$

Use your graph to estimate the values of the constants a and b.

(ii) Use the equation to calculate the largest amount of salt which will dissolve in the water at 95 °C.

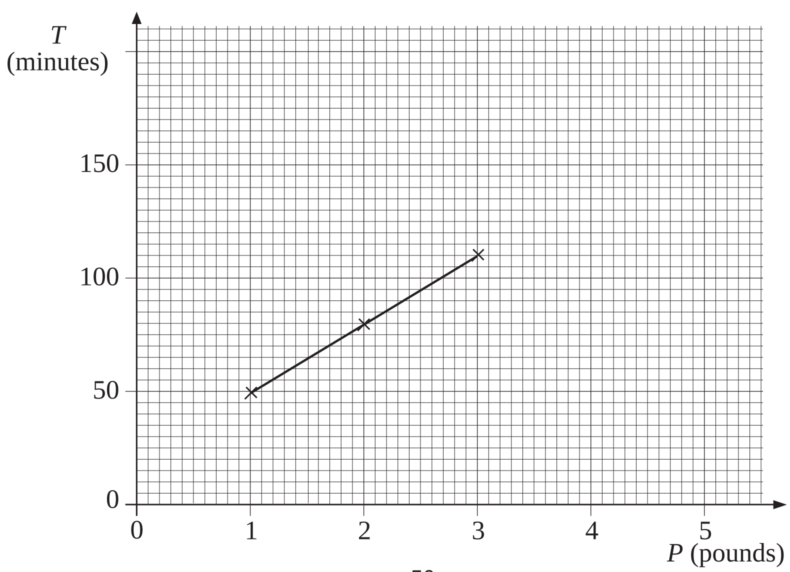


18. The time, T minutes, for cooking a piece of meat, weighing P pounds, is found using the instruction

'cook for 20 minutes then add 30 minutes for each pound'.

(a) Write down a formula for T in terms of P.

(b) The graph shows cooking times for pieces of meat which weigh from 1 to 3 pounds.

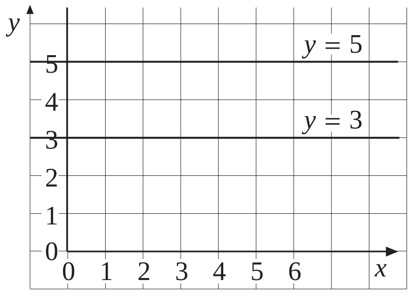


(i) Plot the point to show the cooking time for a piece which weighs 4 pounds.

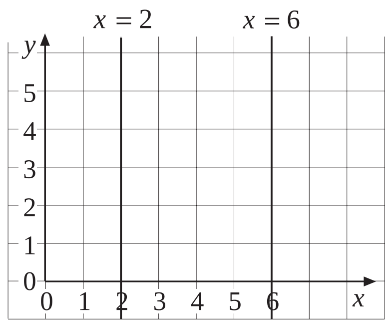
(ii) A piece of meat took 100 minutes to cook. Use the graph to estimate the weight of the meat.

13.9 Horizontal and Vertical Lines

Horizontal lines have equations like $y = 3$ or $y = 5$.



Vertical lines have equations like $x = 2$ or $x = 6$.



Worked Example 1

(a) Draw the line $x = 4$.

(b) Draw the line $y = 2$.

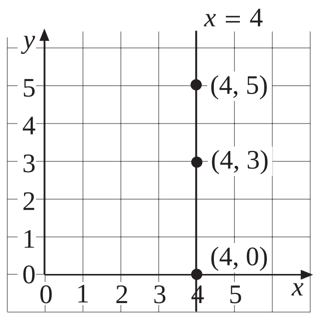
(c) Write down the coordinates of the points where these lines cross.

Solution

(a) For the line $x = 4$ the x-coordinate of every point will always be 4. So the points

(4, 0) (4, 3) (4, 5)

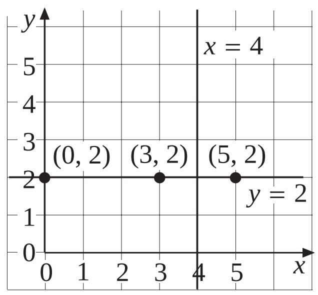
all lie on the line $x = 4$.



(b) For the line $y = 2$ the y-coordinate of every point will always be 2. So the points

(0, 2) (3, 2) and (5, 2)

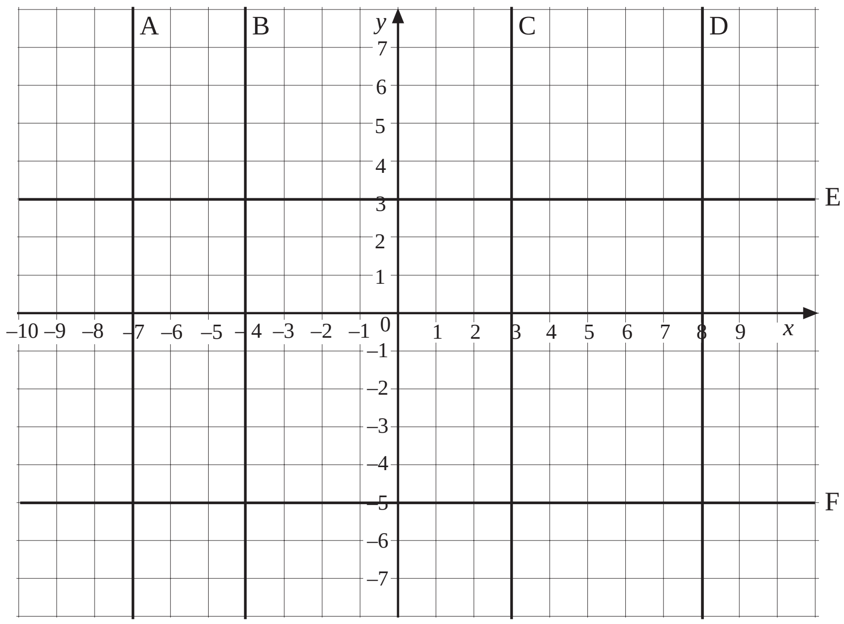
all lie on the line $y = 2$.



(c) The graph in (b) shows that the lines cross at the point with coordinates (4, 2).

Exercises

1. Write down the equation of each line in the diagram below.



2. (a) Draw the lines $x = 2$, $x = 6$, $y = 1$ and $y = 5$.

   (b) Write down the coordinates of the points where the lines cross.

   (c) Find the area of the rectangle formed by the four lines

3. (a) Draw the lines $x = -2$, $x = -4$, $y = -1$ and $y = -3$.

   (b) What are the coordinates for the centre of the square you obtain?

4. (a) Draw the rectangle which has corners at the points with coordinates (–1, 4) , (–1, 1), (3, 4) and (3, 1).

   (b) Write down the equations of the lines that form the sides of the rectangle.

5. (a) Draw the lines $y = 2x$, $x = 2$ and $y = 8$.

   (b) Find the area of the triangle that you obtain.

13.10 Solution of Simultaneous Equations by Graphs

Solutions to pairs of simultaneous equations can be found by plotting lines and finding the coordinates of the point where they cross.

Worked Example 1

Solve the pair of simultaneous equations

$$x + y = 8 \quad \text{and} \quad 2x + 3y = 21$$

Solution

First it can be helpful to write the two equations in the form $y = \ldots$.

For the first equation,

$$\begin{aligned} x + y &= 8 \\ y &= 8 - x \qquad \text{(subtracting } x\text{)} \end{aligned}$$

For the second equation,

$$\begin{aligned} 2x + 3y &= 21 \\ 3y &= 21 - 2x \qquad \text{(subtracting } 2x\text{)} \\ y &= 7 - \frac{2}{3}x \qquad \text{(dividing by 3)} \end{aligned}$$

Now three pairs of coordinates can be found for each line.

For $y = 8 - x$

If $x = 0$, $y = 8$ so $(0, 8)$ lies on the line

If $x = 4$, $y = 8 - 4$
$= 4$ so $(4, 4)$ lies on the line.

If $x = 8$, $y = 8 - 8$
$= 0$ so $(8, 0)$ lies on the line.

For $y = 7 - \frac{2}{3}x$

If $x = 0$, $y = 7 - 0$
$= 7$ so $(0, 7)$ lies on the line.

If $x = 3$, $y = 7 - \frac{2}{3} \times 3$

$= 7 - 2$

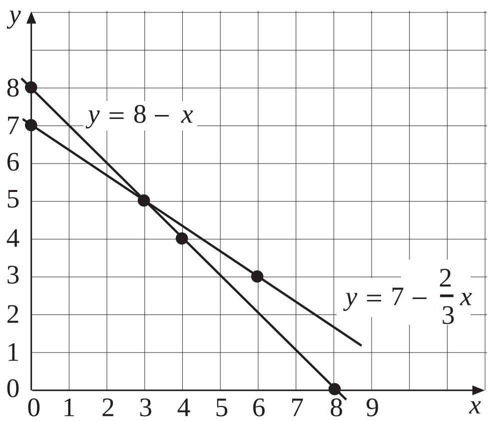
$= 5$ so $(3, 5)$ lies on the line.

If $x = 6$, $y = 7 - \frac{2}{3} \times 6$

$= 7 - 4$

$= 3$ so $(6, 3)$ lies on the line.

These points are then used to plot the lines shown below.



The two lines cross at the point (3, 5), so the solution is

$$x = 3 \text{ and } y = 5$$

Worked Example 2

Tickets for a school concert cost £3 for adults and £2 for children. If 200 tickets are sold and a total of £520 is paid, find the number of each type of ticket sold.

Solution

Let x = number of adult tickets sold

and y = number of children's tickets sold.

As 200 tickets are sold in total,

$$x + y = 200$$

As £520 is taken in payment for the tickets,

$$3x + 2y = 520$$

These equations can be written in the form $y = \ldots$.

For the first equation,

$$x + y = 200$$

$$y = 200 - x \qquad \text{(subtracting } x\text{)}$$

For the second equation,

$$3x + 2y = 520$$

$$2y = 520 - 3x \qquad \text{(subtracting } 3x\text{)}$$

$$y = 260 - \frac{3}{2}x \qquad \text{(dividing by 2)}$$

Now the coordinates of some points can be found for each equation.

For $y = 200 - x$

If $x = 0$, $y = 200 - 0$

$= 200$ so $(0, 200)$ lies on the line

If $x = 50$, $y = 200 - 50$

$= 150$ so $(50, 150)$ lies on the line.

If $x = 100$, $y = 200 - 100$

$= 100$ so $(100, 100)$ lies on the line.

For $y = 260 - \frac{3}{2}x$

If $x = 0$, $y = 260 - 0$

$= 260$ so $(0, 260)$ lies on the line

If $x = 50$, $y = 260 - \frac{3}{2} \times 50$

$= 260 - 75$

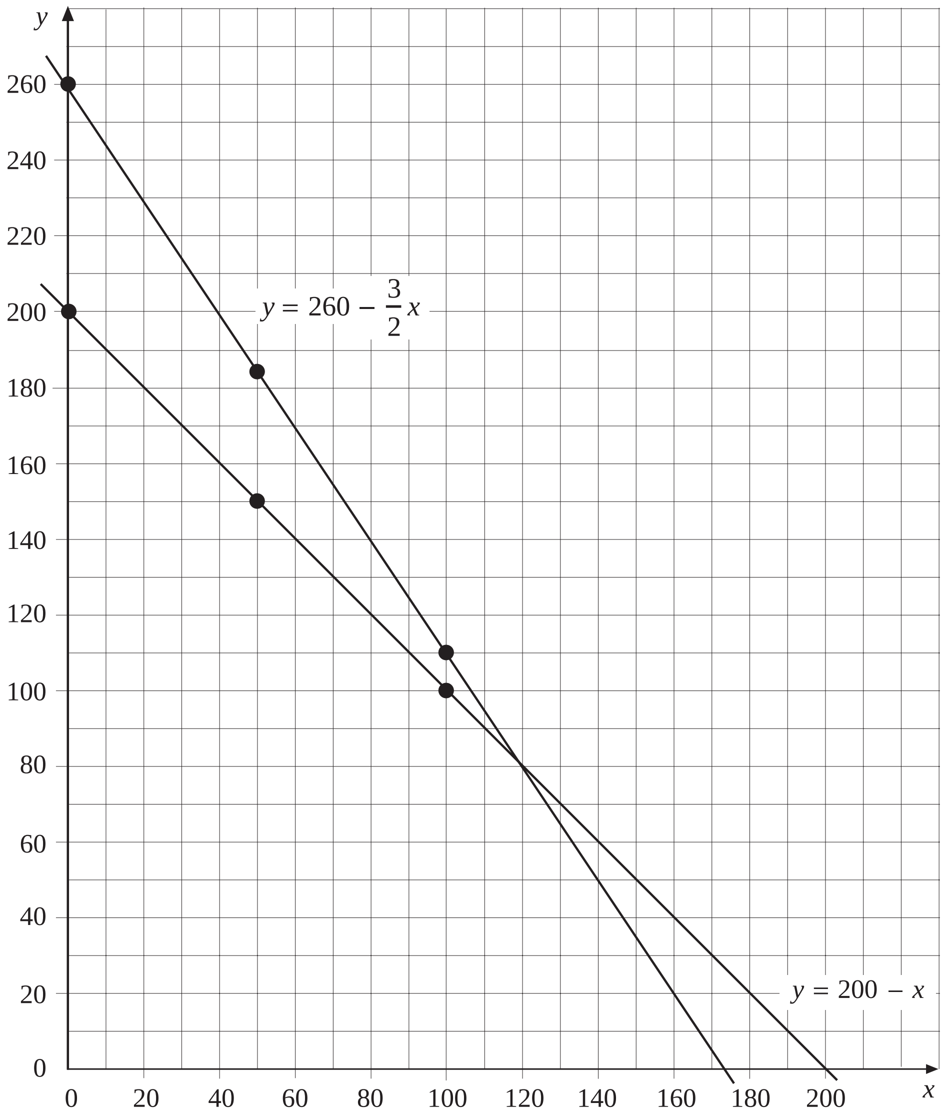
$= 185$ so $(50, 185)$ lies on the line.

If $x = 100$, $y = 260 - \frac{3}{2} \times 100$

$= 260 - 150$

$= 110$ so $(100, 110)$ lies on the line.

The graph shows these points plotted and lines drawn through them.



The point where the lines cross has coordinates (120, 80), so the solution is $x = 120$, $y = 80$. That is, 120 adult tickets and 80 children's tickets.

Exercises

1. Use the graph below to solve the simultaneous equations.

(a) $y = 8 - x$
$y = 2x - 1$

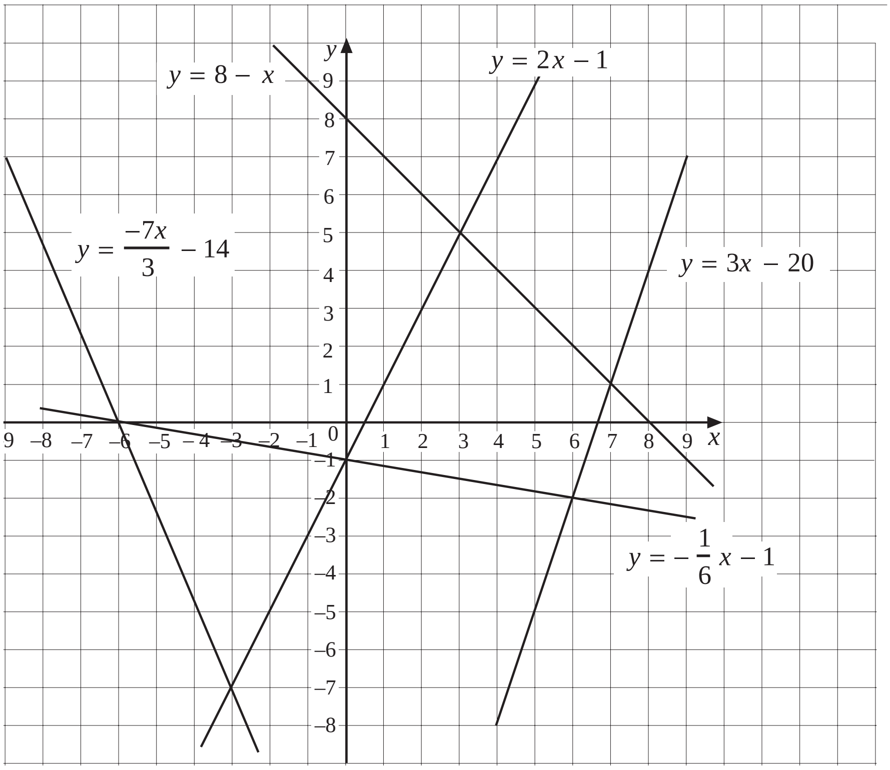
(b) $y = 8 - x$
$y = 3x - 20$

(c) $y = 2x - 1$
$y = -\frac{1}{6}x - 1$

(d) $y = 3x - 20$
$y = -\frac{1}{6}x - 1$

(e) $y = -\frac{7}{3}x - 14$
$y = 2x - 1$

(f) $y = -\frac{1}{6}x - 1$
$y = -\frac{7}{3}x - 14$



2. (a) The points with coordinates (0, –3), (3, 3) and (4, 5) lie on the line $y = 2x - 3$. Plot these on a set of axes with x-values from 0 to 4 and y-values from –3 to 7. Draw a straight line through these points.

(b) The points with coordinates (0, 7), (1, 4) and (3, –2) lie on the line $y = -3x + 7$. Plot these points using the set of axes in (a), and draw a line through them.

(c) Find the solution of the simultaneous equations

$$y = 2x - 3$$
$$y = -3x + 7$$

3. (a) Complete these coordinates for $y = x + 2$.

(0, ?), (2, ?) and (6, ?)

(b) Use these points to draw the line $y = x + 2$.

(c) Complete these coordinates for $y = 3x - 4$

(1, ?), (2, ?) and (4, ?)

(d) Use the points to draw the line $y = 3x - 4$.

(e) What is the solution of the simultaneous equations

$$y = x + 2$$
$$y = 3x - 4?$$

4. (a) Draw a set of axes with x-values from -4 to 4 and y-values from –5 to 5.

(b) Write down the coordinates of three points on the line $y = x - 1$ and use them to draw the line $y = x - 1$.

(c) On the same set of axes draw the lines $y = 5 - x$ and $y = 3x + 1$.

(d) Write down the solution of each set of simultaneous equations.

(i) $y = x - 1$, $y = 5 - x$ (ii) $y = x - 1$, $y = 3x + 1$ (iii) $y = 5 - x$, $y = 3x + 1$

5. Use a graph to solve each set of simultaneous equations below.

(a) $y = 2x$, $y = x + 4$ (b) $y = 4 - x$, $y = 3x - 4$ (c) $y = 2 - x$, $y = 6 - 2x$

6. (a) Write the equations $y - 3x = 1$ and $y + 2x = 6$ in the form $y = \ldots$.

(b) Draw the graphs of both equations.

(c) What is the solution of the simultaneous equations?

7. Solve the following sets of simultaneous equations.

(a) $x + y = 7$, $3x + y = 17$ (b) $x + 2y = 4$, $2x + y = 5$ (c) $x + y = 4$, $3x + y = 1$

8. When two numbers x and y are added together you obtain 20, and when x is subtracted from y you obtain 14.

(a) Write down two equations involving x and y.

(b) Write both equations in the form $y = \ldots$.

(c) Draw the graphs of both equations and find the values of x and y which satisfy both equations.

9. A discount store sells CDs and tapes. The price of every CD is £x and the price of every tape is £y.

(a) Jane buys 2 CDs and 4 tapes which cost a total of £40. Write down an equation involving x and y using this information.

(b) Write down an equation in the form $y = \ldots.$

(c) Christopher buys 3 CDs and 2 tapes which cost a total of £36. Write down a second equation using this information.

(d) Write this equation in the form $y = \ldots.$

(e) Draw the graphs of both equations on the same set of axes.

(f) What is the price of a CD?

(g) What is the price of a tape?

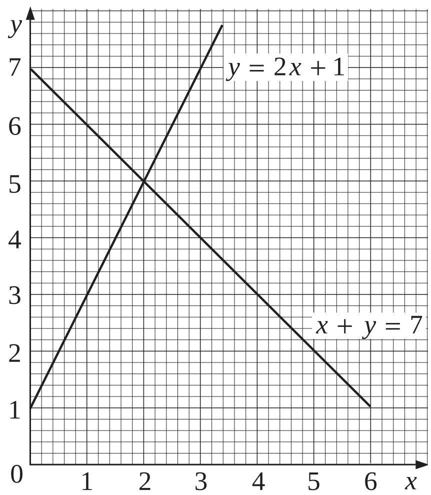
10. An enterprising schoolboy charges £2 to wash a car and £5 to wash and polish a car. One day he earns £80 and cleans 28 cars. Let x be the number of cars that he only washed, and y the number of cars washed and polished.

(a) Write down two equations involving x and y.

(b) Write these equations in the form $y = \ldots.$

(c) Draw the graphs of both equations and find out how many cars were washed and polished.

11.



The diagram shows the graphs of the equations

$$y = 2x + 1 \text{ and } x + y = 7.$$

Use the diagram to solve the simultaneous equations

$$y = 2x + 1,$$
$$x + y = 7.$$

(LON)

12. A longlife battery and a standard battery were both tested for their length of life.

The longlife battery lasted for x hours.

The standard life battery lasted for y hours.

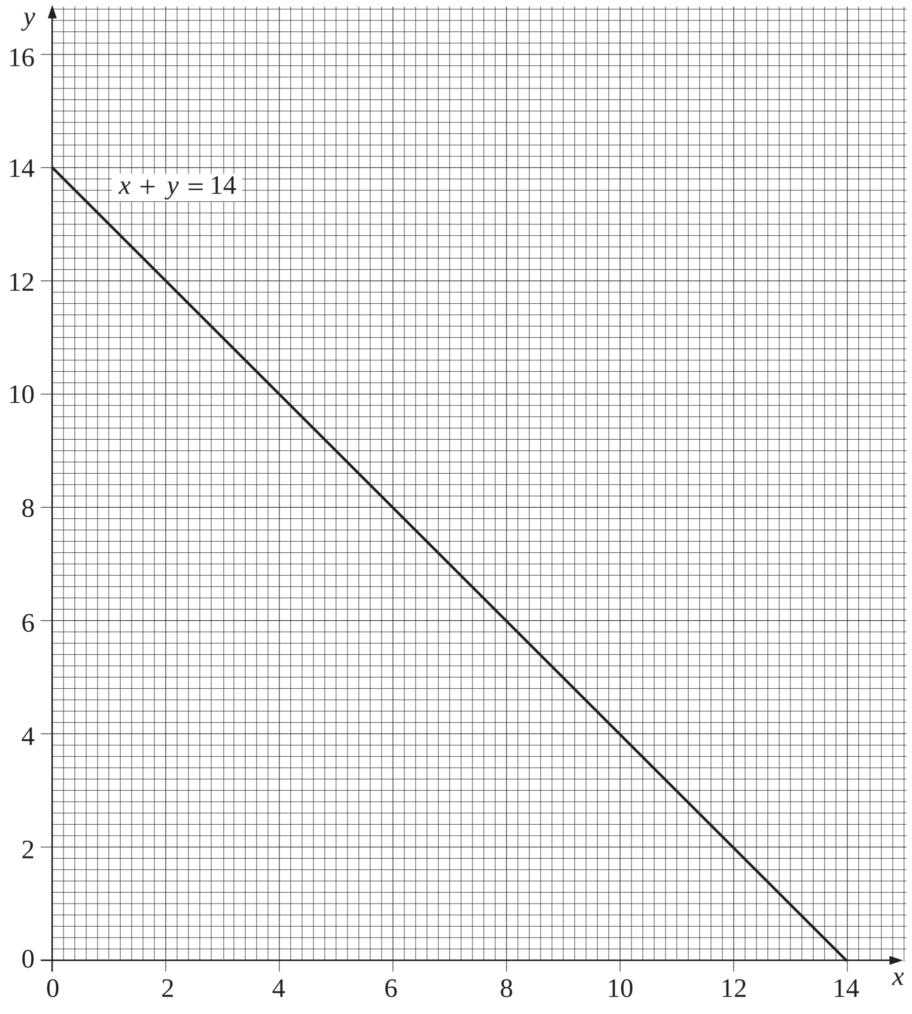
(a) The combined length of life of the two batteries was 14 hours.

Explain why $x + y = 14$

(b) The longlife battery lasted 3 hours longer than the standard battery.

Write down another equation connecting x and y.

The graph of $x + y = 14$ has been drawn below.



(c) Complete a copy of the table of values for your equation in (b) and use it to draw the graph of your equation.

x	3	7	10
y			

(d) Use your graphs to find the length of life of each type of battery.

(SEG)

13. For off-peak electricity, customers can choose to pay by *Method A* or *Method B.*

Method A: £8 per quarter plus 4p per unit.

Method B: £12 per quarter plus 3p per unit.

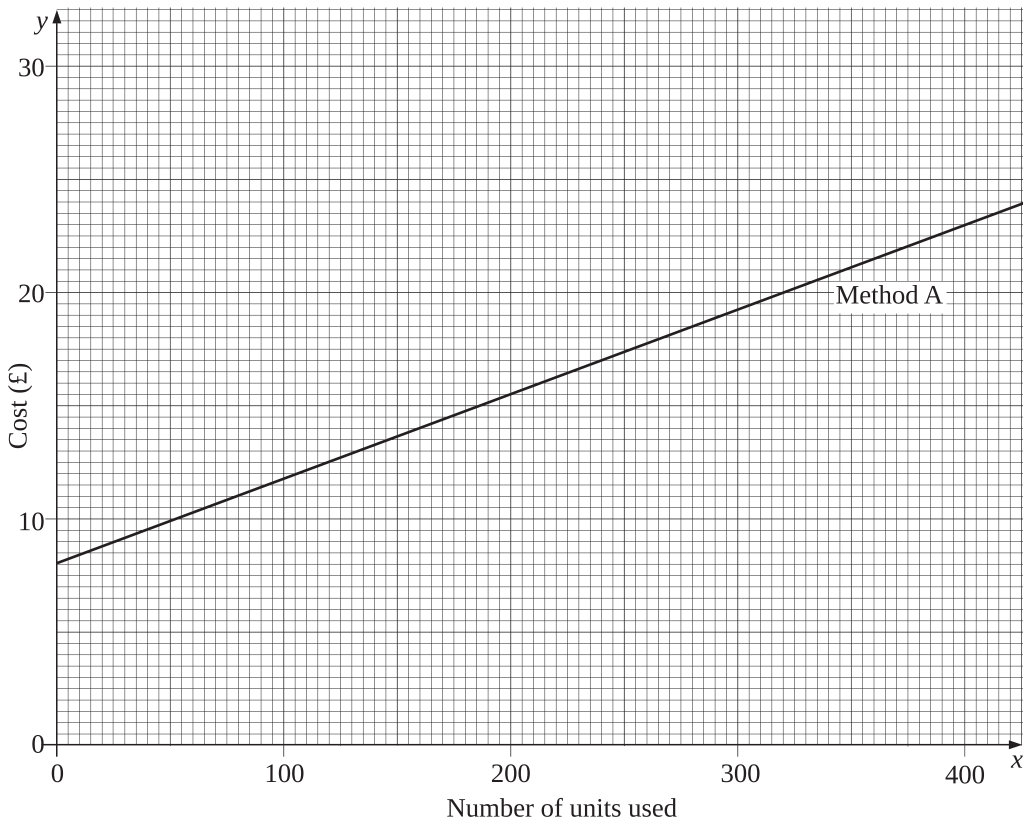
A customer uses x units. The quarterly cost is £y.

The cost, £y, by *Method B* is $y = 0.03x + 12$.

(a) (i) Complete the table of values of $y = 0.03x + 12$.

x	100	200	300
y	15		

(ii) Draw the graph of $y = 0.03x + 12$.



(b) For a certain number of units both methods give the same cost.

Use the graphs to find

(i) this number of units,

(ii) this cost.

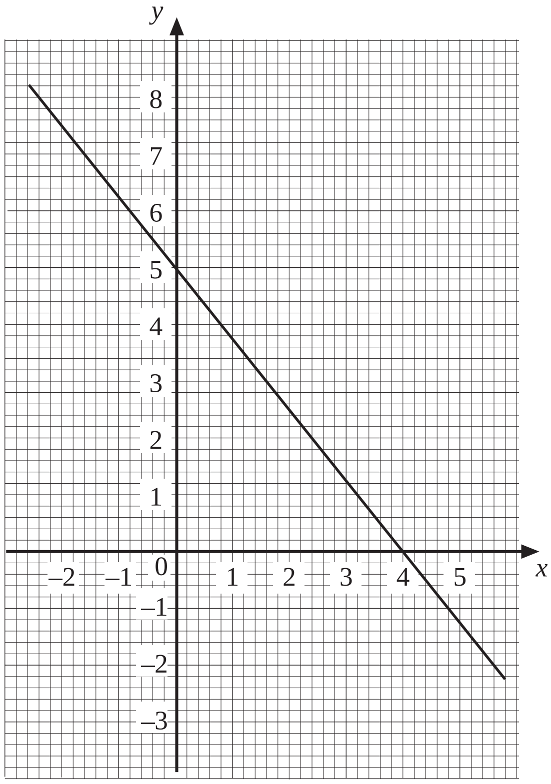
(c) Copy and complete the following statement

"Method .... is always cheaper for customers who use

more than .... units".

(SEG)

14. (a)



(i) The graph of $5x + 4y = 20$ is shown on the diagram above.

On a copy of the diagram, draw the graph of $y = 2x$.

(ii) Use the graphs to find the solution of the simultaneous equations

$$5x + 4y = 20,$$
$$y = 2x.$$

Give the value of x and the value of y to one decimal place.

(b) Calculate the **exact** solution of the simultaneous equations

$$5x + 4y = 20,$$
$$2x - y = 0.$$

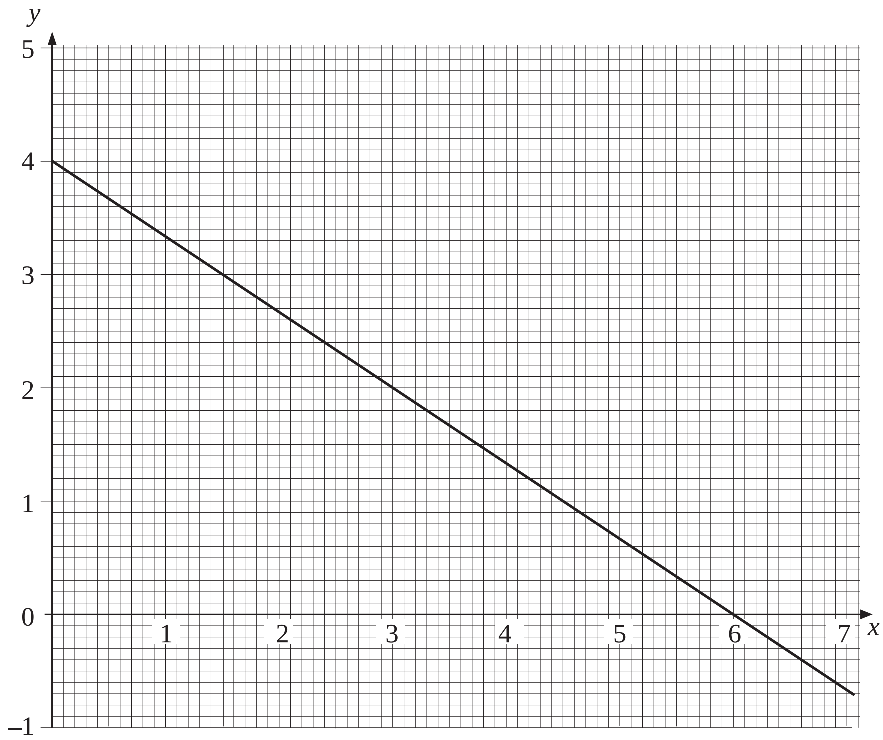
(MEG)

15. The line with equation $2x + 3y = 12$ is drawn on the following grid.

Solve the simultaneous equations

$$y = 2x - 1,$$
$$2x + 3y = 12,$$

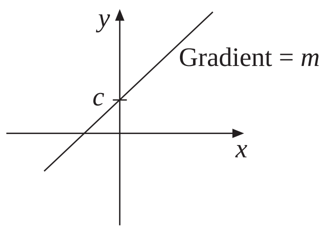
by a graphical method.



(MEG)

13.11 Graphs of Common Functions

Linear Functions



Linear functions are always straight lines and have equations which can be put in the form

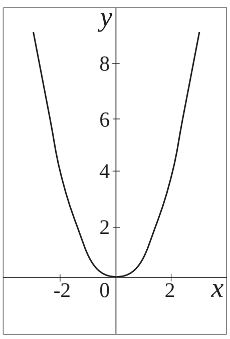
$$y = mx + c$$

Quadratic Functions

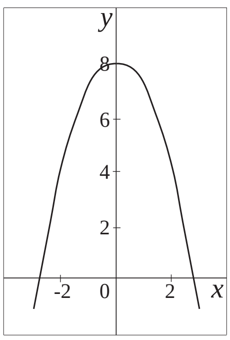
Quadratic functions contain an x^2 term as well as multiples of x and a constant. Some examples are:

$$y = 2x^2 \qquad y = x^2 - x + 5 \qquad y = 6 - x^2$$

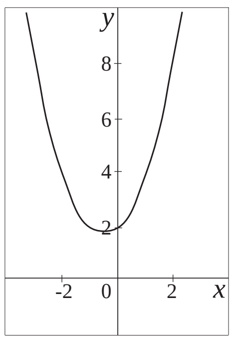
The following graphs show 3 examples.



$y = x^2$



$y = 8 - x^2$



$y = x^2 + x + 2$

Note that each curve has either a maximum or a minimum point which lies on its axis of symmetry. The curve has a maximum point when the coefficient of x^2 is negative as in the second example, or minimum if the coefficient of x^2 is positive. Also the curve can cross the x-axis twice, just touch it once or never meet the x-axis.

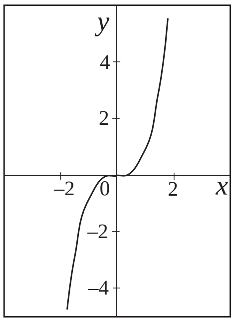
Cubic Functions

Cubic functions involve an x^3 term and possibly x^2, x and constant terms as well. Some examples are:

$$y = x^3, \qquad y = x^3 + 3x^2 + 4x - 8, \qquad y = x^3 - 5, \qquad y = x^3 - x + 1$$

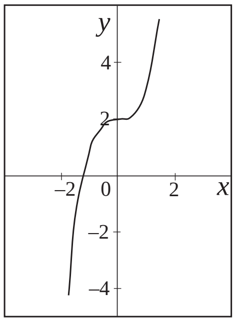
The graphs below show some examples.

(a)



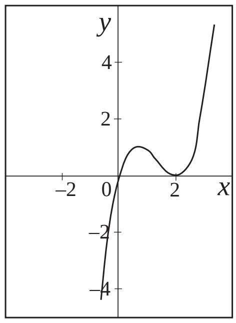
$y = x^3$

(b)



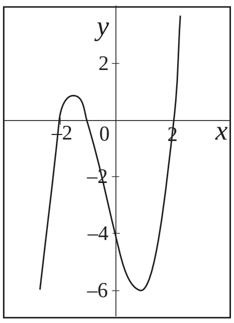
$y = x^3 + 2$

(c)



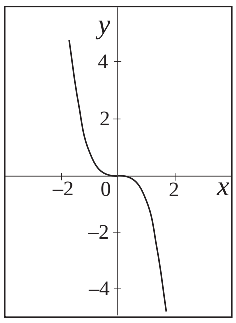
$y = x^3 - 4x^2 + 4x$

(d)



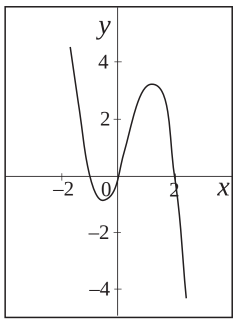
$y = x^3 + x^2 - 4x - 4$

(e)



$y = -x^3$

(f)



$y = -x^3 + x^2 + 2x$

The graph of a cubic function can cross the x-axis once as in examples (a), (b) and (e), touch the axis once and cross it once as in example (c) or cross the x-axis three times as in examples (d) and (f).

In examples (c), (d) and (f) the curve has a local minimum and a local maximum.

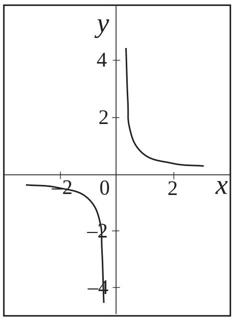
Note how the shape of the curve changes when a $-x^3$ is introduced. Compare examples (a) and (e).

Reciprocal Functions

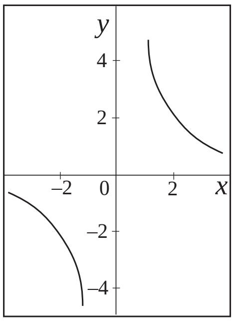
Reciprocal functions have the form of a fraction with x as the denominator. Examples of reciprocal functions are:

$$y = \frac{1}{x} \qquad y = \frac{10}{x}, \qquad y = \frac{-2}{x}, \qquad y = \frac{1}{5x}$$

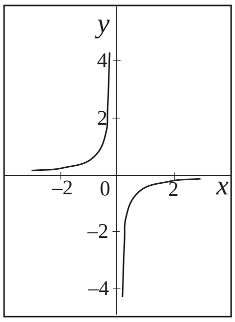
The graphs below show some examples.



$y = \dfrac{1}{x}$



$y = \dfrac{4}{x}$



$y = -\dfrac{1}{2x}$

The curves are split into two distinct parts. The curves get closer and closer to the axes as is clear in the diagrams. The curves have two lines of symmetry, $y = x$ and $y = -x$.

Exercises

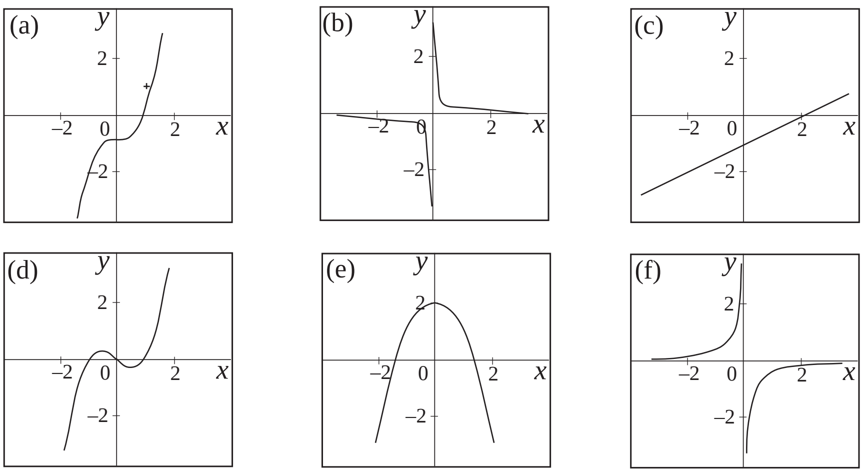
1. State whether each equation below would produce the graph of a *linear, quadratic, cubic* or *reciprocal* function.

   (a) $y = \dfrac{4}{x}$ (b) $y = x^2 - 2$

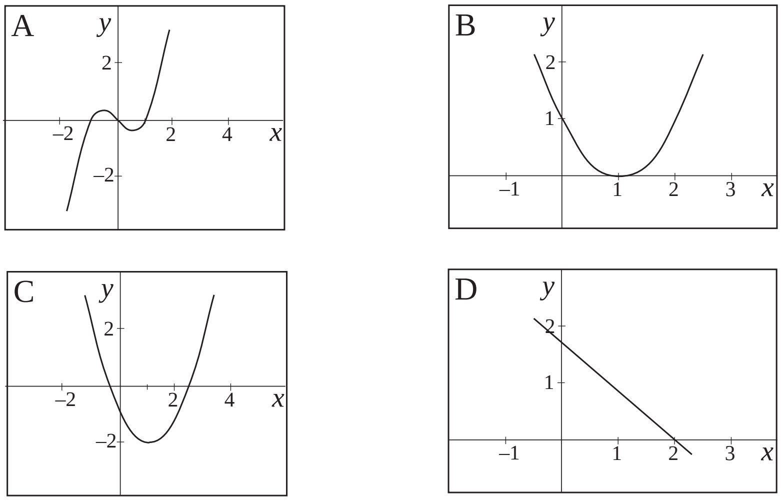
   (c) $y = x - 2$ (d) $y = x^3 + x^2 + 4$

   (e) $y = x^2 + x$ (f) $y = \dfrac{-1}{7x}$

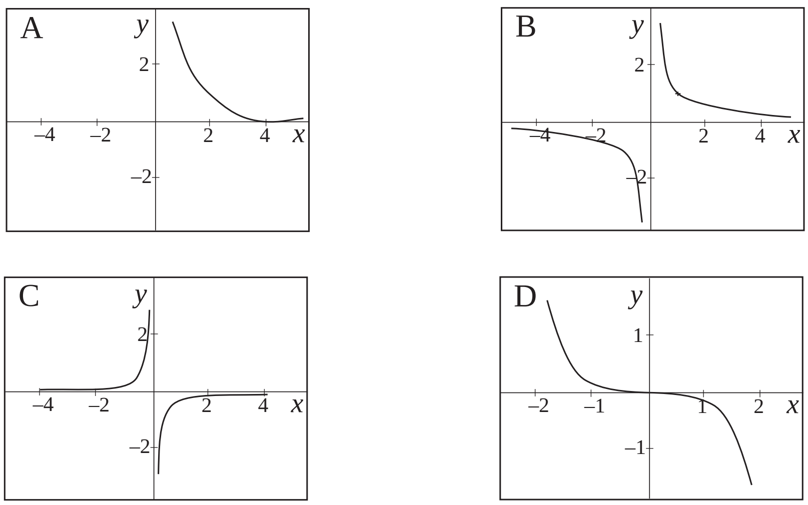
2. Each of the following graphs is produced by a *linear, quadratic, cubic* or *reciprocal* function. State which it is for each graph.



3. One of the graphs shown below is $y = x^2 - 2x + 1$. Which one?

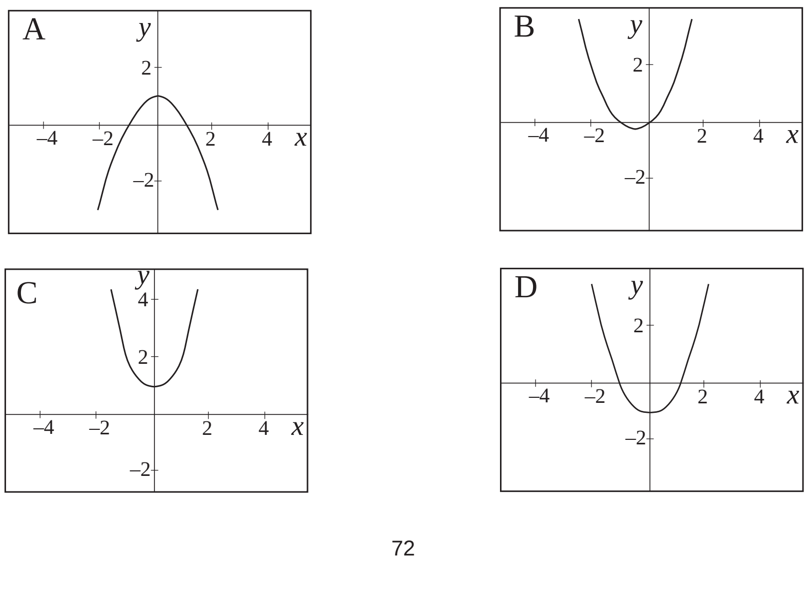


4. Which of the graphs shown below are reciprocal functions?



5. Each equation below has been plotted. Select the correct graph for each equation.

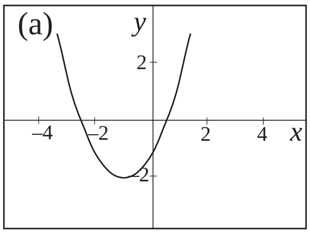
(a) $y = x^2 + 1$ (b) $y = x^2 - 1$ (c) $y = 1 - x^2$ (d) $y = x^2 + x$

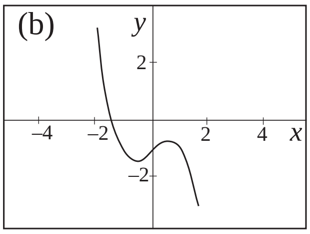


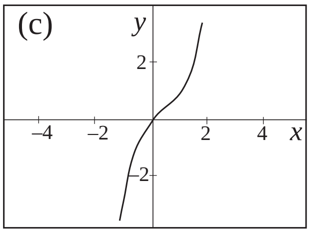
6. Match each graph below to the appropriate equation.

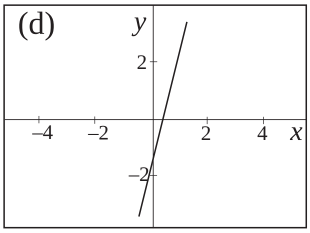
A $y = -x^3 + x - 1$ B $y = x^3 - x^2 + x$

C $y = 4x - 1$ D $y = x^2 + 2x - 1$









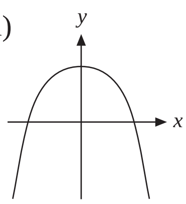
7. (a) Which of the following equations are illustrated by the graphs shown?

Write the equation illustrated beside the number of each graph.

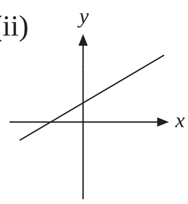
$y = -x$ $y = 2 - x$ $y = 1 - x^2$

$y = x^2$ $2y = 2 + x$ $xy = 1$

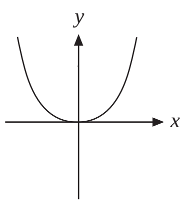
(i)



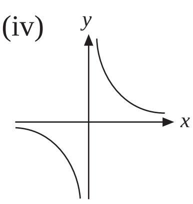
(ii)



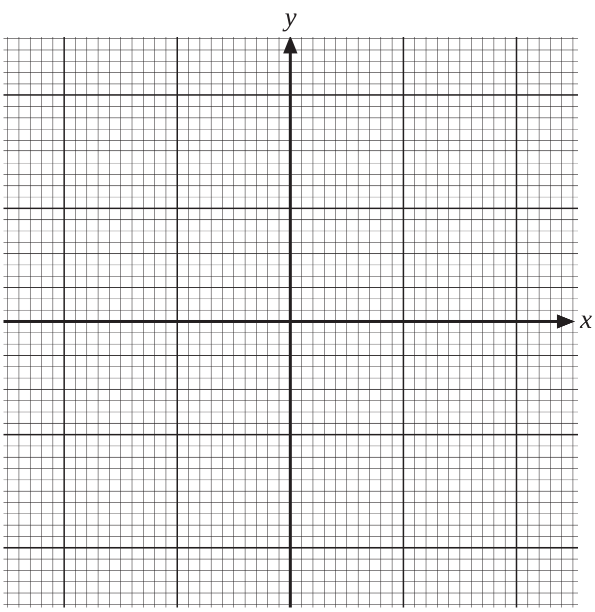
(iii)



(iv)



(b) Sketch a graph of the equation $y = x^3 + 1$ on a copy of this graph.



(SEG)

13.12 Graphical Solutions of Equations

Equations of the form $f(x) = g(x)$ can be solved graphically by plotting the graphs of $y = f(x)$ and $y = g(x)$. The solution is then given by the x-coordinate of the point where they cross.

Worked Example 1

Find any positive solutions of the equation

$$x^2 = \frac{1}{x} + x$$

by a graphical method.

Solution

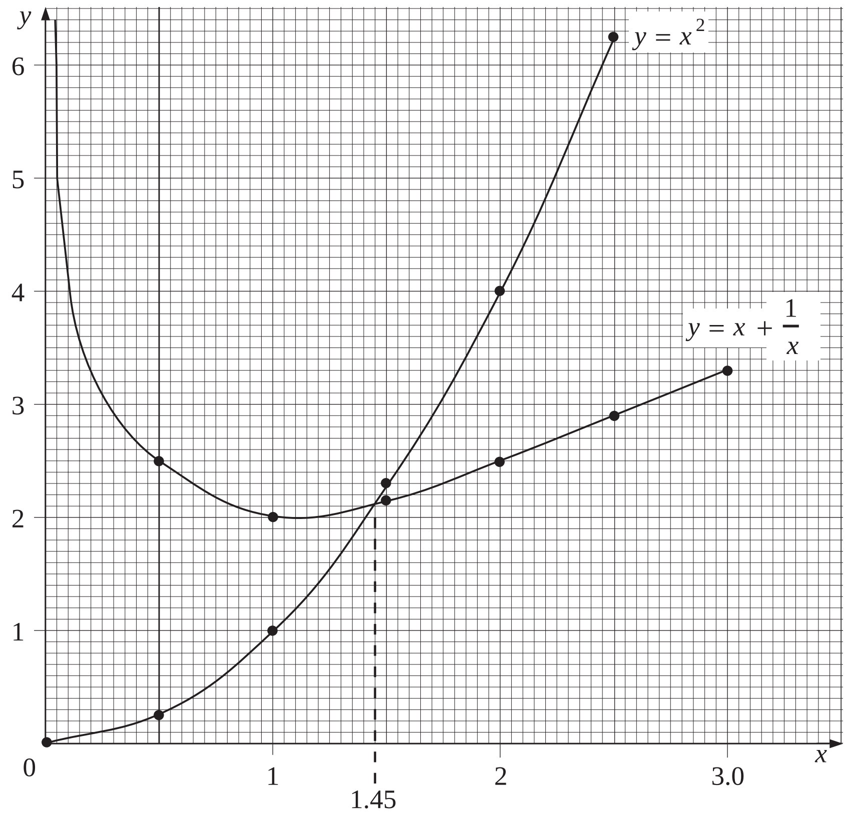
Completing the table below provides the points needed to draw the graphs $y = x^2$ and $y = \frac{1}{x} + x$.

x	0	0.5	1.0	1.5	2.0	2.5	3.0
x^2	0	0.25	1	2.25	4	6.25	9
$\frac{1}{x}+x$	Infinity	2.5	2	2.17	2.5	2.9	3.33

Where necessary the values have been rounded to 2 decimal places.

The graph below shows $y = x^2$ and $y = \frac{1}{x} + x$.

The curves cross where $x = 1.45$ and so this is the solution of the equation.



Worked Example 2

Solve the equation $2^x = 3$ by a graphical method.

Solution

To find the solution, plot the graphs $y = 2^x$ and $y = 3$.

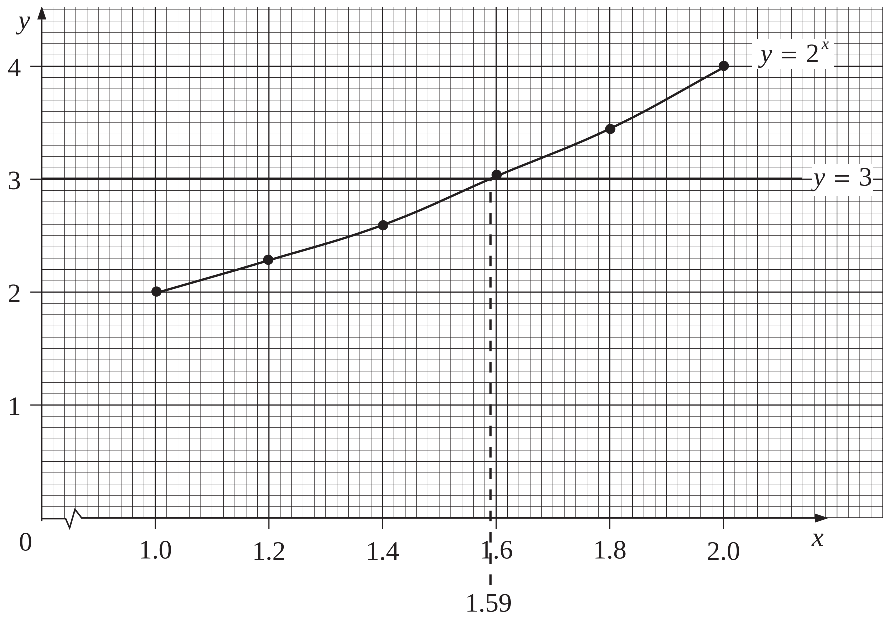
The table below can be used to find the points needed to plot $y = 2^x$.

Note that since $2^1 = 2$ and $2^2 = 4$, it is only necessary to consider values of x between 1 and 2.

x	1.0	1.2	1.4	1.6	1.8	2.0
2^x	2	2.30	2.64	3.03	3.48	4

The two graphs are plotted below.

The line and the curve cross when $x \approx 1.59$ and so this is the solution of the equation.



Worked Example 3

Describe 3 different ways to solve the equation

$$x^2 + \frac{1}{x} - 6x = 4$$

using a graphical method.

Solution

The equation can be rearranged in a number of ways.

One possibility is

$$x^2 + \frac{1}{x} = 4 + 6x$$

Solutions could then be found by plotting $y = x^2 + \dfrac{1}{x}$ and $y = 4 + 6x$.

A second possibility is

$$x^2 - 4 = 6x - \frac{1}{x}$$

Graphs of $y = x^2 - 4$ and $y = 6x - \dfrac{1}{x}$ would give the solutions.

A third possibility is

$$x^2 - 6x = 4 - \frac{1}{x}$$

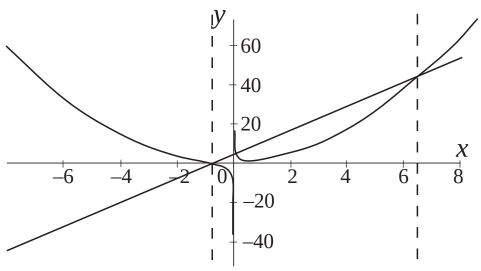
The graphs of $y = x^2 - 6x$ and $y = 4 - \dfrac{1}{x}$ could then be plotted.

The graphs in the diagram show that each pair of graphs gives the same three solutions, including one solution that is small and positive and, in consequence, difficult to show on the scale used for the graph.

$y = x^2 + \dfrac{1}{x}$

and

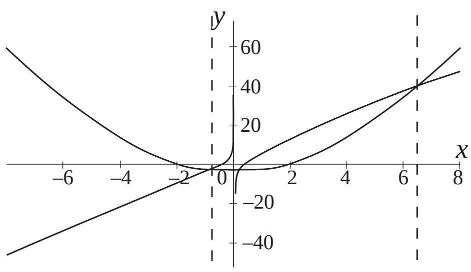
$y = 4 + 6x$



$y = x^2 - 4$

and

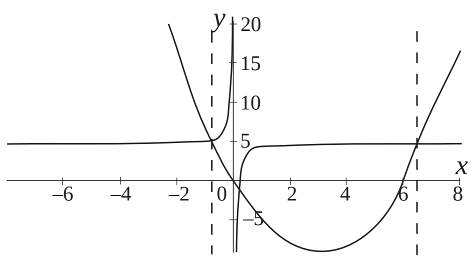
$y = 6x - \dfrac{1}{x}$



$y = x^2 - 6x$

and

$y = 4 - \dfrac{1}{x}$



Exercises

1. Draw the graph of $y = 3^x$ for $0 \le x \le 2$. Use the graph to solve the equations $4 = 3^x$ and $5 = 3^x$.

2. Solve the quadratic equation $x^2 - x - 2 = 0$ by plotting the graphs $y = x^2$ and $y = x + 2$.

3. Find the x-coordinates of the two points where the lines $y = x^2 - 2$ and $y = x + 4$ intersect. Write down the quadratic equation which has the two solutions you found from the points of intersection.

4. Find the solutions of the following equations

(a) $2x - x^2 = x^3$ (b) $3^x = x - 2$

(c) $x^4 = 8 - x^2$ (d) $\frac{1}{x^2} = \frac{x^3}{8} - 1$

5. Describe 3 different ways to find solutions of the equation

$$x^3 - 8x + 5 - \frac{3}{x} = 0$$

6. Draw the graph $y = x^2 + 2x$ for $-3 \le x \le 2$. Use the graph to solve the following equation,

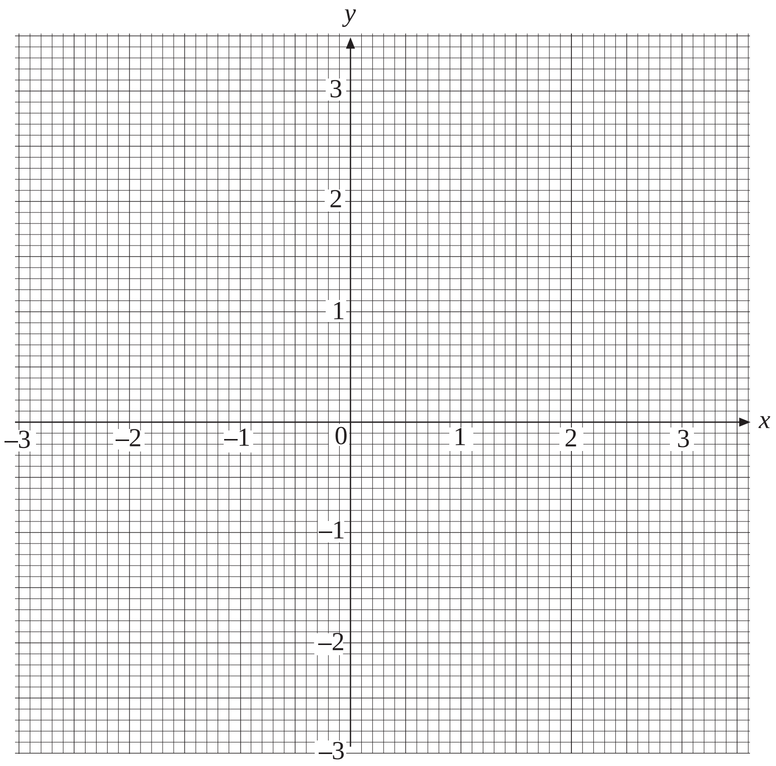
$$x^2 + 2x - a = 0$$

if (a) $a = 3$ (b) $a = 2$ (c) $a = -\frac{1}{2}$

For what value of a is there only one solution?

For what value of a are there no real solutions to the equation?

7.



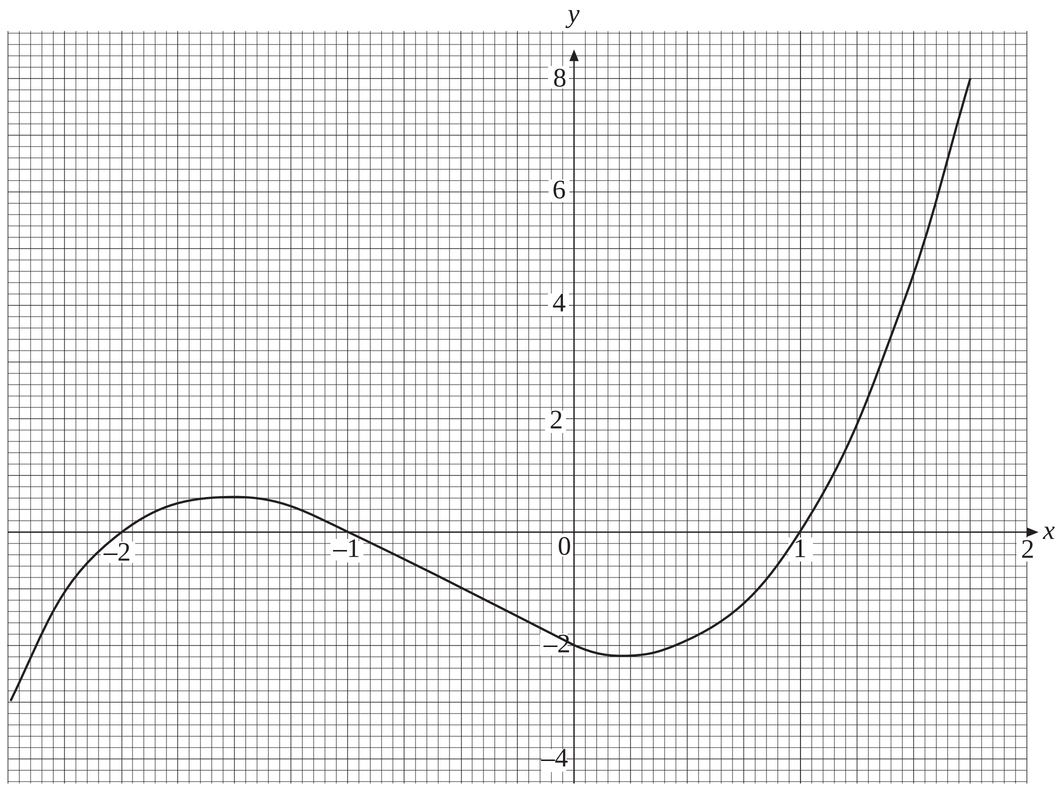
Use a graphical method to solve the equation

$$\frac{1}{x} = x^2 - 1$$

You **must** show all your working.

(SEG)

8.



The diagram above shows the graph of $y = x^3 + 2x^2 - x - 2$.

(a) Use the graph to find the solutions of the equation

$$x^3 + 2x^2 - x - 2 = 0.$$

(b) By drawing the graph of $y = 2x^2$ on a copy of the diagram, find the solution of the equation

$$x^3 - x - 2 = 0.$$

(c) Use the graph to find solutions of the equation

$$x^3 + 2x^2 - x - 1 = 0.$$

(MEG)

Just for Fun

There are 10 bank notes altogether. They consist of £10, £20 and £50 notes. If the total value of the notes is £180, find the number of each type of notes.

Can the problem be solved by a system of simultaneous equations?