UNIT 14 *Loci and Transformations* Activities

Activities

ACTIVITY 14.1 *Equal Area Polygons*

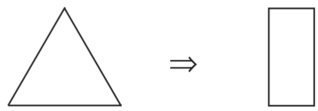
The American mathematician, *David Hilbert* (1862–1942), was the first person to prove that

> *Any polygon can be changed into any other polygon of equal area by cutting it into a finite number of pieces and rearranging.*

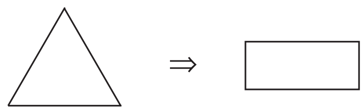
Unfortunately, the proof of this result does not tell you how to do it, just that it can be done!

Here are some easy examples.

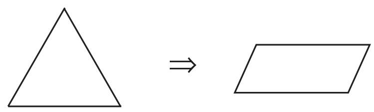
1. How can an equilateral triangle be changed into a rectangle which has one of its sides equal to the *height* of the triangle?



2. How can an equilateral triangle be changed into a rectangle which has one of its sides equal to the *length of a side* of the triangle?



3. How can you change the equilateral triangle into a parellogram whch has a base height equal to *three quarters* of the length of a side of the triangle?



A more difficult problem is to change an equilateral triangle into a *square* of the same area.

4. (a) Start with any equilateral triangle, ABC, and follow accurately these construction steps:

Step 1	Bisect AB at D and BC at E.
Step 2	Extend line AE to F, making EF equal to EB.
Step 3	Bisect AF at G.
Step 4	With G as centre and radius AG, draw an arc which cuts EB extended at H. (EH is the length of the square).
Step 5	With centre E, radius EH, draw the arc of the circle until it cuts AC at J.
Step 6	K is on AC between J and C so that JK = BE.
Step 7	Drop perpendiculars from D and K to EJ, cutting at L and M respectively.
Step 8	Cut out the pieces BELD, MECK, JADL, JMK and mix them up.

(b) (i) Put the pieces back together to make your original *equilateral triangle*.

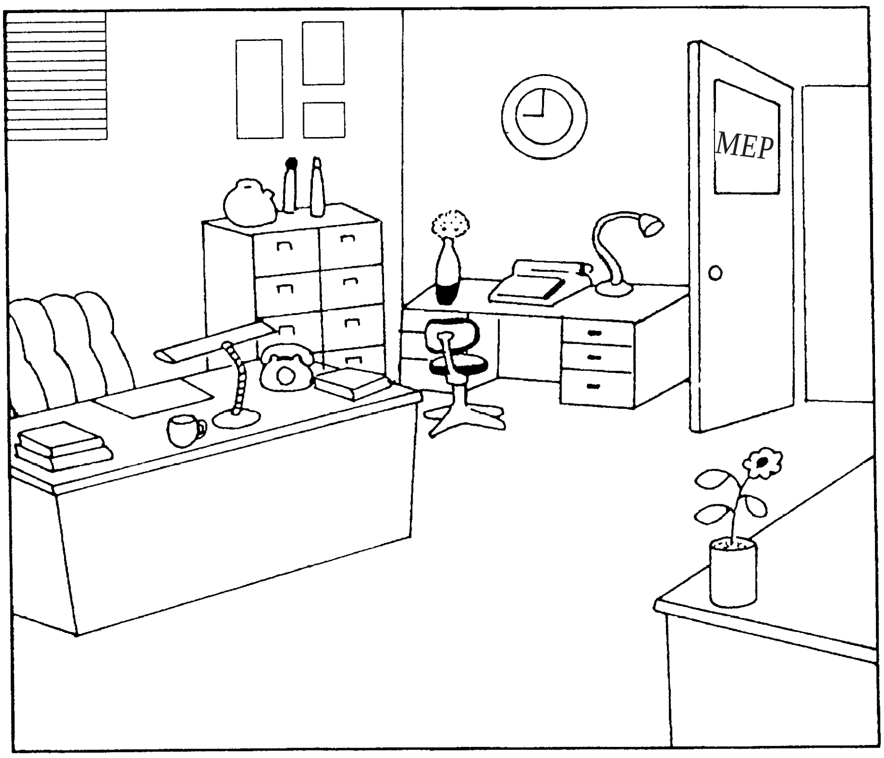
(ii) Make a *square* from the pieces.

ACTIVITY 14.2 *Reflections*

Picture 2 should be the reflection of *Picture 1* in the line AB. Circle the errors.

Picture 1

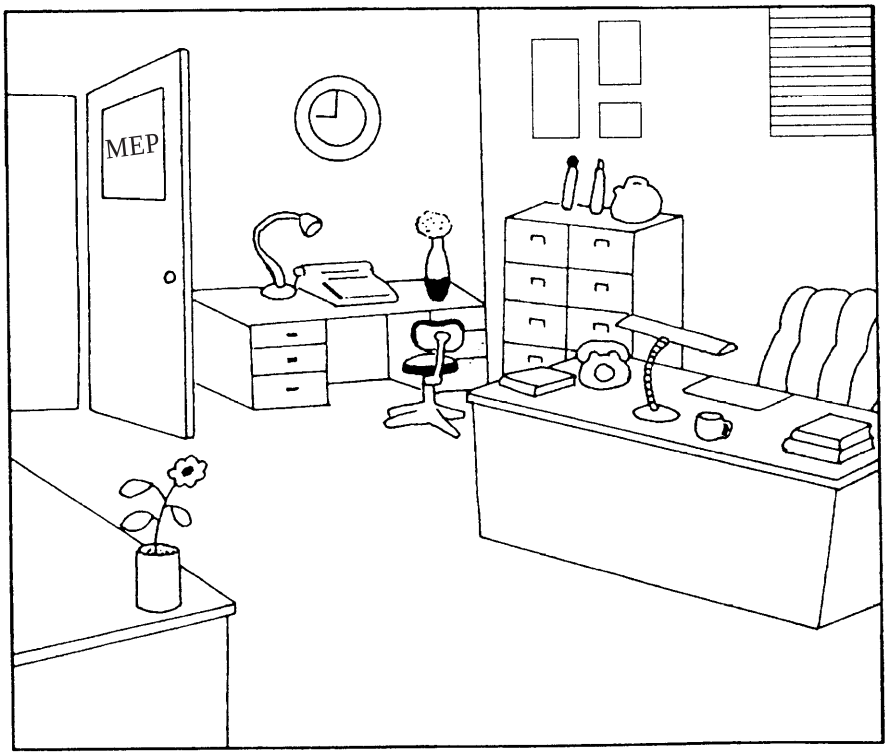
A



B

Picture 2

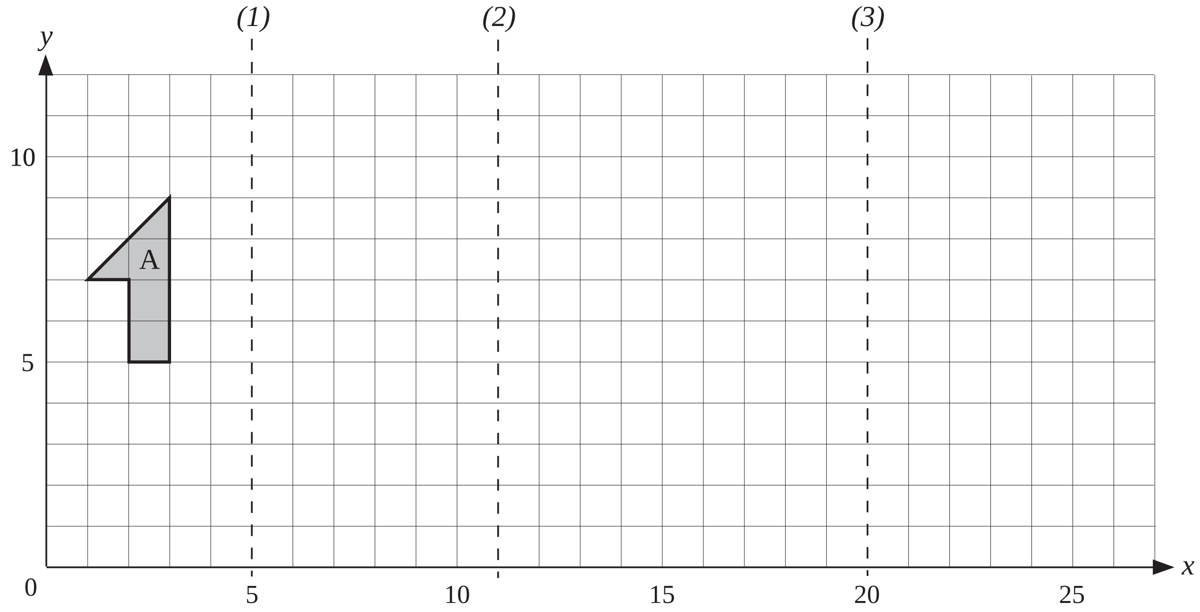
A



B

ACTIVITY 14.3 *Repeated Reflections*

The diagram below shows a shape, A, and mirror lines, 1, 2 and 3. Copy the diagram and then carry out the reflections and answer the questions below.



1. Reflect *shape A* in *mirror line 1* and label it B.

2. Reflect *shape B* in *mirror line 2* and label it C.

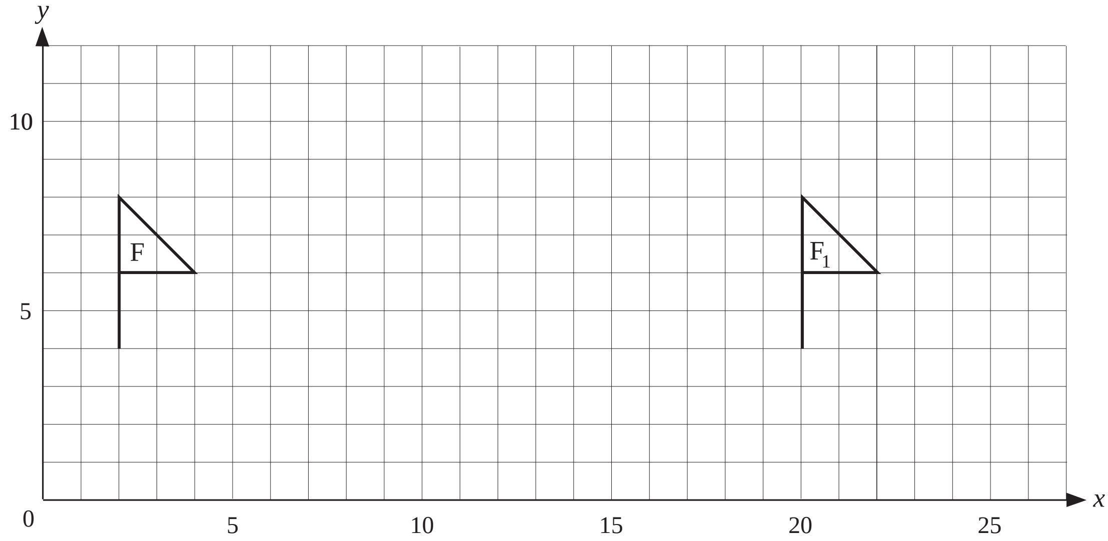
3. Reflect *shape C* in *mirror line 3* and label it D.

4. *Shape D* can be obtained from *shape A* by just one reflection.
   Draw the required mirror line on your diagram and label it 4.
   What is the equation of this line?

5. Repeat steps 1–4 on a new diagram using your own shape.
   Does your answer to Question 4 remain the same?

ACTIVITY 14.4 *Transformations*

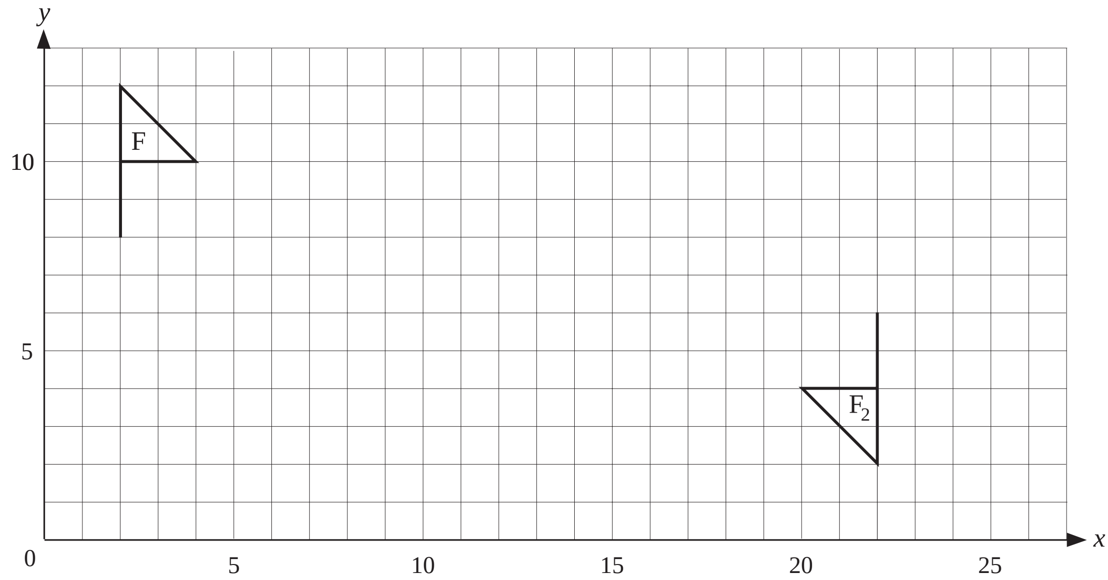
1 (a) F is mapped onto F_1 under a translation. What is the translation?



(b) (i) Describe how F can be mapped onto F_1 using two successive reflections.

(ii) In how many different ways can this be done?

2 (a) F is mapped onto F_2 under a rotation. Describe this rotation.



(b) (i) Describe how F can be mapped onto F_2 using two successive reflections.

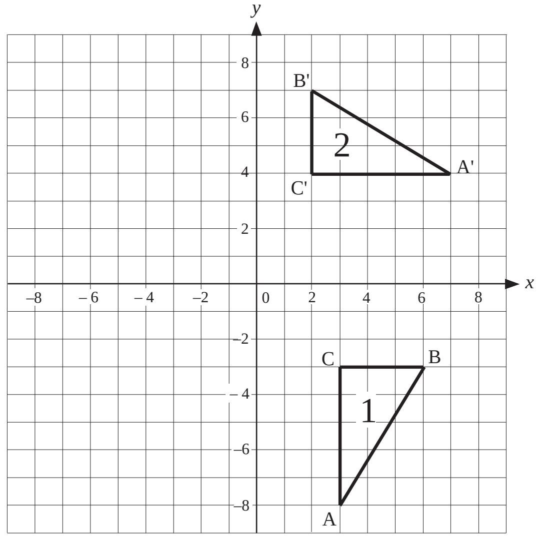
(ii) Is your answer the only way that it can be done?

ACTIVITY 14.5 *Finding the Centre of Rotation*

Problem

Shape 1 has been transformed to *Shape 2* by a rotation.

Where is the centre of rotation?



Solution

The centre of rotation can often be seen by inspection (i.e. trial and error) but this is not satisfying from a mathematical viewpoint.

Here is a procedure for determining the centre of rotation, which we will put into practice and then see why it works.

Method

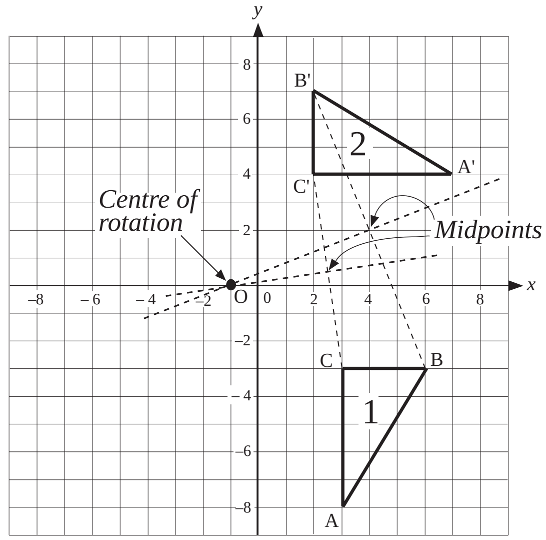
1. Join two (or more) pairs of corresponding points together,

   e.g. B to B and C to C'.

2. Find the midpoint of BB', CC', etc.

3. Draw lines through each midpoint, perpendicular to each line.

4. Where these lines cross is the centre of rotation (–1, 0).



Why does it work?

CC' is a chord of the circle, radius OC (= OC'), so a perpendicular bisector of CC' will pass through the centre of this circle, O.

Similarly for BB' – a different circle but the same centre, O.

Hence the intersection of the perpendicular bisectors gives the centre of rotation.

Extension

Draw two identical shapes, A and B, so that B is a $90°$ rotation from A and also translated to a new position. Use the method above to find the centre of rotation.