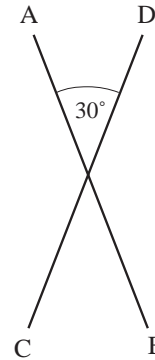# 14 Loci and Transformations

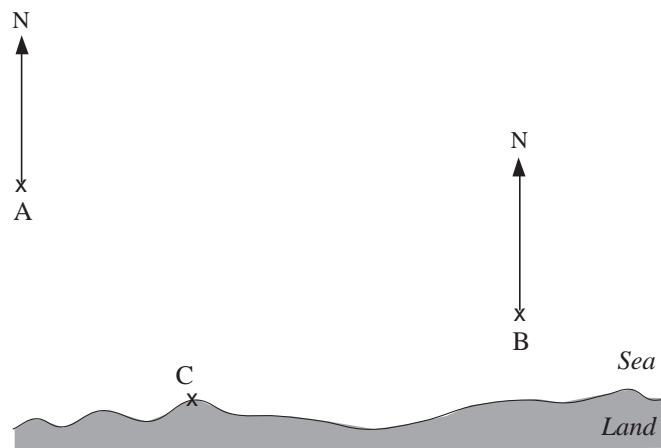## 14.6 Construction of Loci

1. The line AB is 5 cm long. Draw the locus of a point which is always 2 cm from the line AB.

2. (a) Copy the diagram opposite.

   (b) Construct the locus of a point which is equidistant from both lines.

3. Two oil rigs, A and B, are in the sea near the port of C.
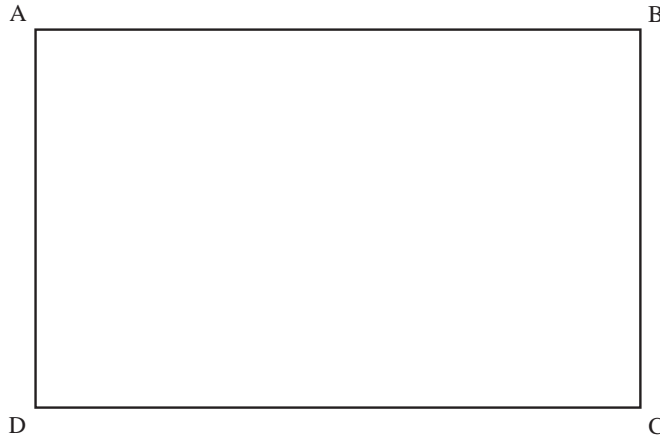
   (a) A third oil rig is the same distance from A and from B.
   Copy the diagram and construct the locus of possible positions of the third oil rig on the diagram.

   (b) The third oil rig, D, is also the same distance from C as it is from A and B.
   Mark with a cross the position of D.

   (SEG)

4.   The diagram shows a field, ABCD, drawn to a scale of 1 cm to 10 m. Treasure is hidden in the field.

A   B

D   C

(a)   The treasure is at equal distances from the sides, AB and AD.
        Copy the diagram and construct the locus of points for which this is true.

(b)   The treasure is also 60 m from the corner, C. Construct the locus of points for which this is true.

(c)   Mark with an X the position of the treasure.

(SEG)

5.   The diagram below is a scale drawing of the floor of a room.

In the diagram, 2 cm represents 1 m.

X marks the position of an electric socket.
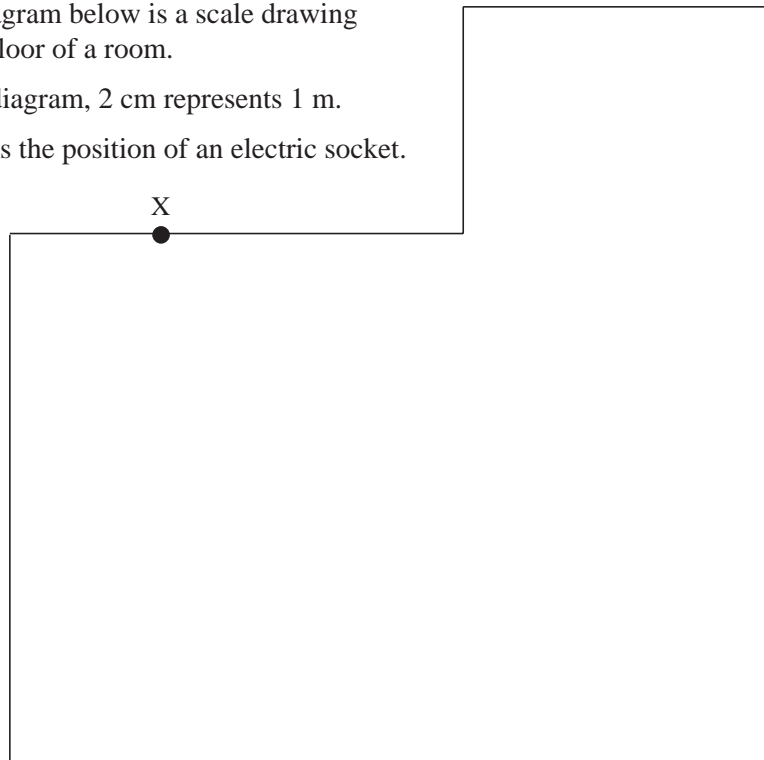
A vacuum cleaner is attached by a cable to the socket and can clean the floor up to 3 metres from the socket.

Copy the diagram and shade the part of the floor which can be cleaned by the vacuum cleaner.
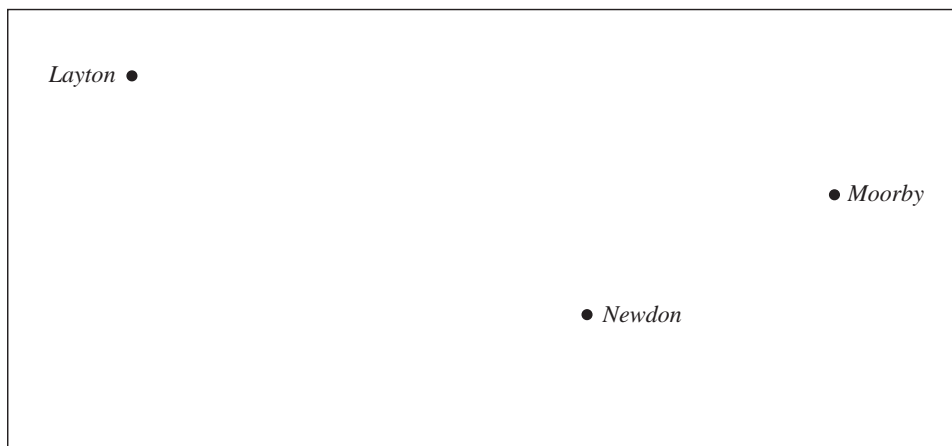
(MEG)

6. Jason has to sail his ship between two rocks so that his ship is always the same distance from Point A on the first rock and Point B on the second rock.

The diagram below shows the rocks.

Rock

A

The Symphleglades Rocks

B
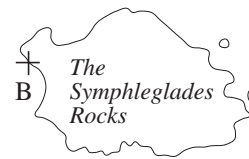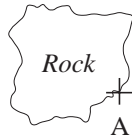
On a copy of the diagram, construct accurately the path along which Jason must sail his ship.

7.

Layton •

• *Moorby*

• *Newdon*

The map above, drawn to a scale of 4 cm to represent 1 km, shows the positions of three villages, *Layton*, *Moorby* and *Newdon*.

Simon's house is the same distance from *Moorby* as it is from *Layton*.

The house is also less than $\frac{3}{4}$ km from *Newdon*.

Draw a copy of the map and mark on your drawing the possible positions of Simon's house. Show your construction lines clearly.

(MEG)

8. Signals from a radio mast, M, can be received up to a distance of 100 km. Use a scale drawing of 1 cm to represent 20 km to answer the following questions.

(a) Shade the region in which signals from the radio mast can be received.

The distance of a helicopter from the radio mast is 70 km, correct to the nearest kilometre.

(b) Write down

(i) the maximum distance the helicopter could be from the radio mast

(ii) the minimum distance the helicopter could be from the radio mast.

(LON)

9.

The diagram represents a box which is to be moved across a floor XY.
AD = 30 cm and AB = 20 cm.

First the box is rotated about the point A so that BC becomes vertical. Then the box is rotated about the new position of the point B so that CD becomes vertical.

(a) Copy the diagram and draw the locus of the point C.

(b) Calculate the maximum height of C above the floor. Give your answer correct to one decimal place.
(A measurement from the scale drawing is unacceptable.)

(LON)

10. (a) (i) Draw a straight line, AB, 8 cm long.

(ii) Draw the locus of points, P, which lie above the line AB such that the area of triangle ABP is 12 cm^2.

(b) On the same diagram, construct the locus of points, Q, which lie above the line AB such that angle AQB is $90°$.

(c) Hence draw all triangles ABC which have C above AB, an area of 12 cm^2 and an angle of $90°$.

(MEG)

11. The diagram shows the wall of a house drawn to a scale of 2 cm to 1 m.
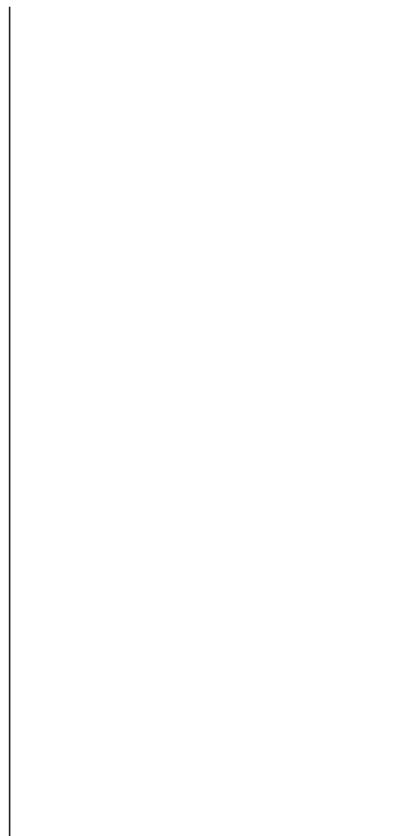A dog is fastened by a lead 3 m long to a point X on a wall.

On a copy of the diagram shade the area that the dog can reach.

Scale: 2 cm to 1 m
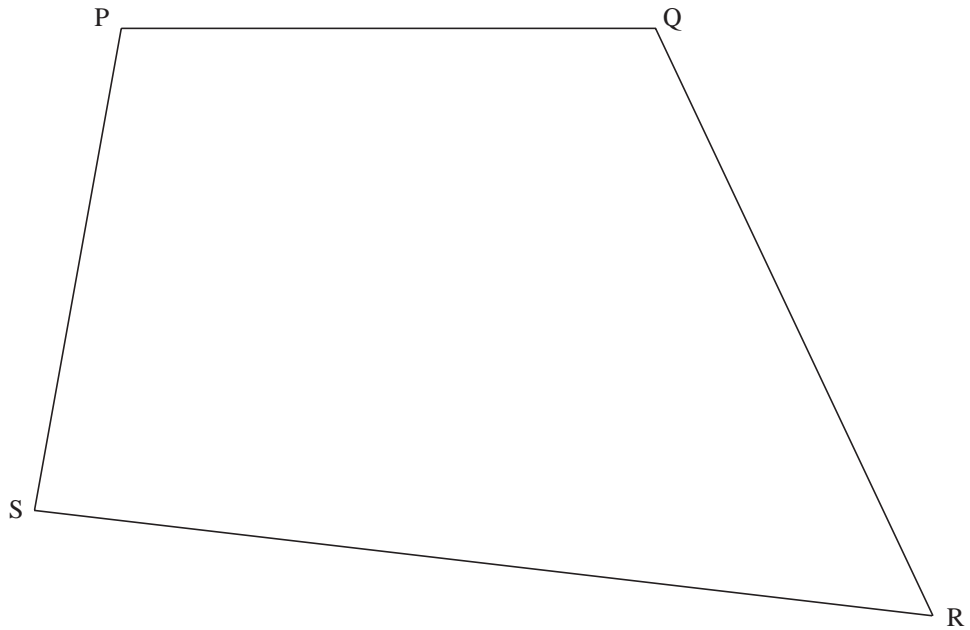
House

X
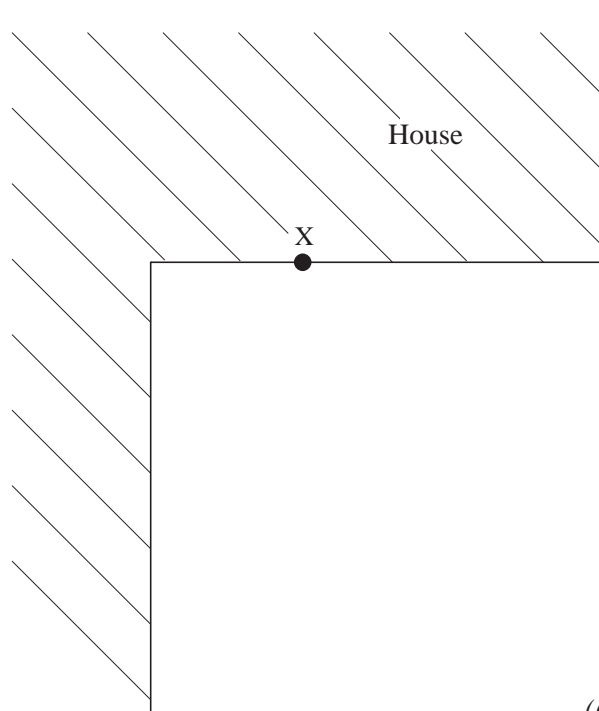
(OCR)

12. The diagram shows an L shape.

On a copy of the diagram, draw the locus of all points 2 cm from the L shape.

(AQA)

13.   The diagram shows a quadrilateral PQRS.

(a)   On a copy of the diagram, draw the locus of points that are the same distance from P as from Q

(b)   Shade the region inside the quadrilateral which is less than 7 cm from S and nearer to Q than to P.

*(AQA)*

14.7 Enlargements which Reduce

1.   For each pair of objects, state the scale factor of the enlargement which produces the smaller image from the larger one.

(a)

(b)

(c)

(d)
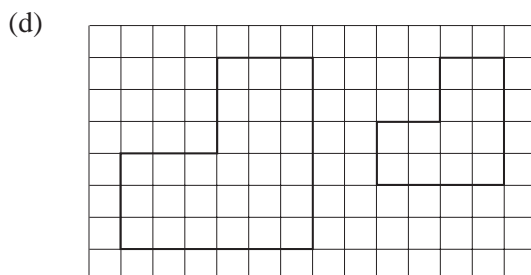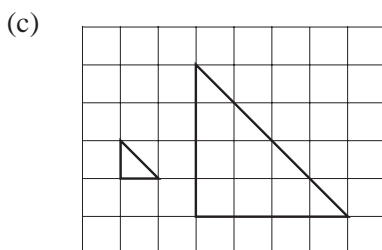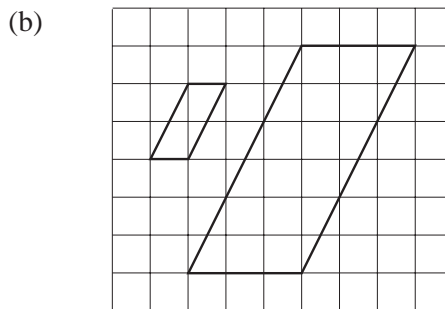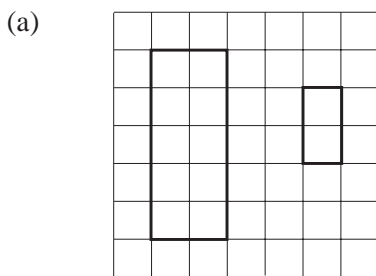
2. For each pair of objects below, the smaller shape has been obtained from the larger shape by an enlargement. For each example, state the scale factor and give the coordinates of the centre of enlargement.
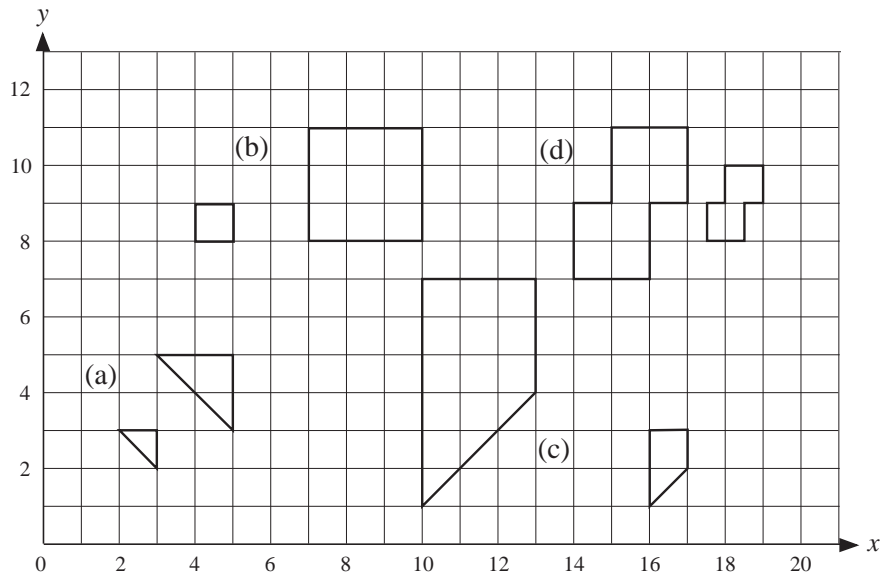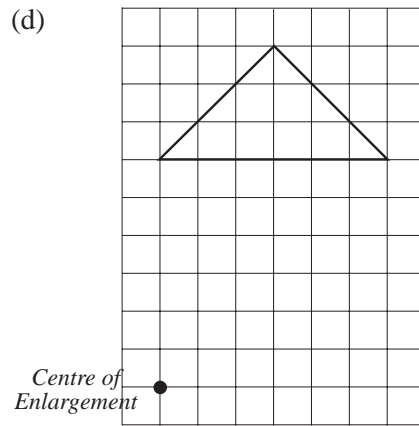

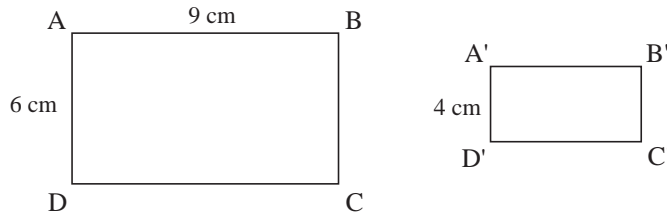3. Copy each shape and enlarge using the given centre of enlargement and the specified scale factor.

(a)

Centre of
Enlargement

Scale factor $\frac{1}{2}$

(b)

Centre of
Enlargement

Scale factor $\frac{1}{3}$

(c)

Centre of Enlargement

(d)
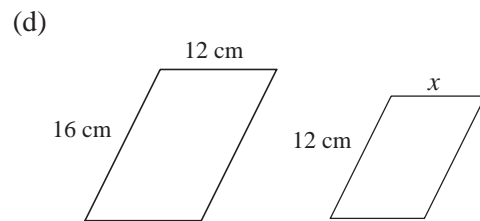

Centre of Enlargement
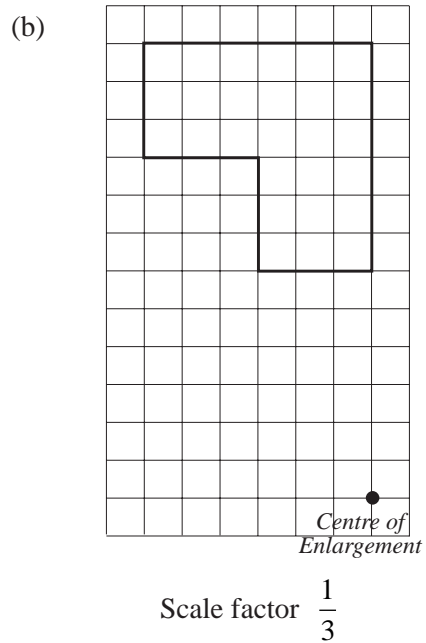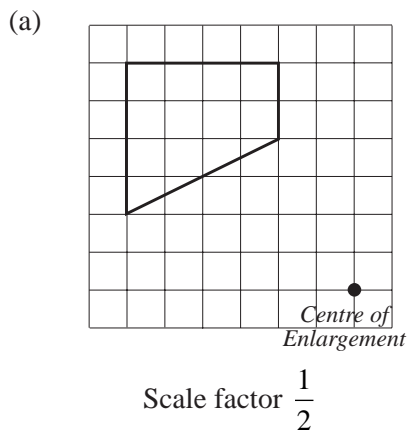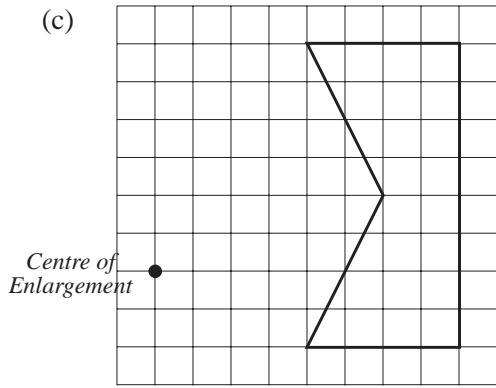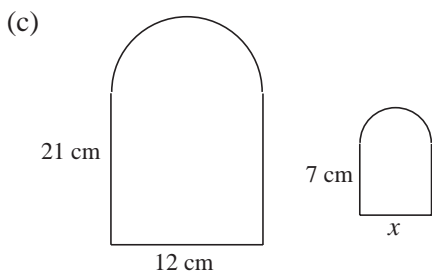
Scale factor $\dfrac{1}{2}$

Scale factor $\dfrac{2}{3}$

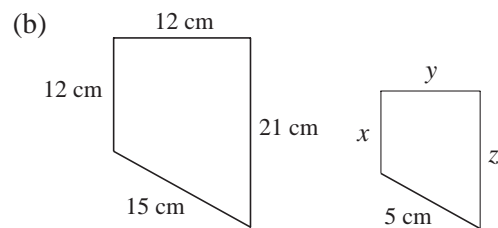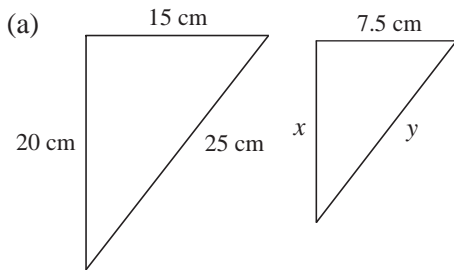4.   The larger rectangle is reduced in size to the smaller rectangle.

A   9 cm   B

6 cm

D   C

A'   B'

4 cm

D'   C'
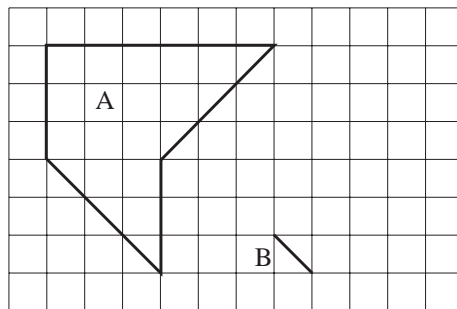
(a)   What is the scale factor of the enlargement?

(b)   What is the length of A′B′?

5.   Each diagram below shows a shape and its image after enlargement. In each case, state the scale factor and find the unknown lengths in the image.

(a)

15 cm

20 cm   25 cm

7.5 cm

x y

(b)

12 cm

12 cm

21 cm

15 cm

y

x z

5 cm

(c)

21 cm

12 cm

7 cm

x

(d)

12 cm

16 cm

12 cm

x

6.   Shape A is shown in the diagram. Shape A is enlarged to obtain the shape B.
      One side of shape B has been drawn.

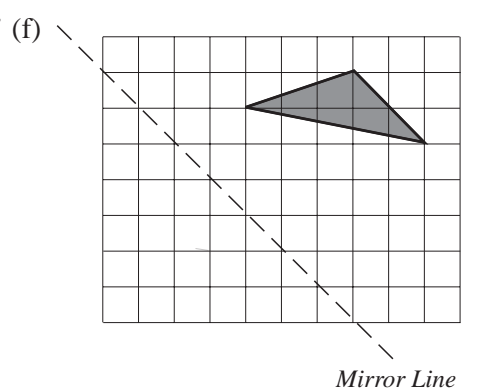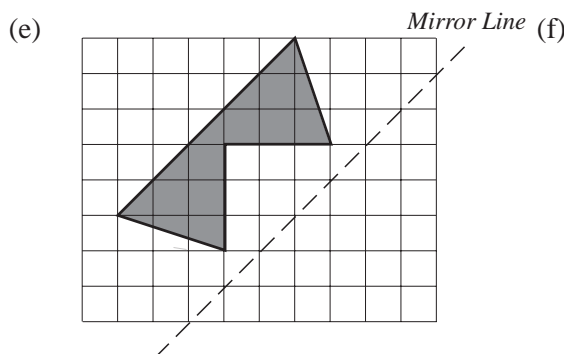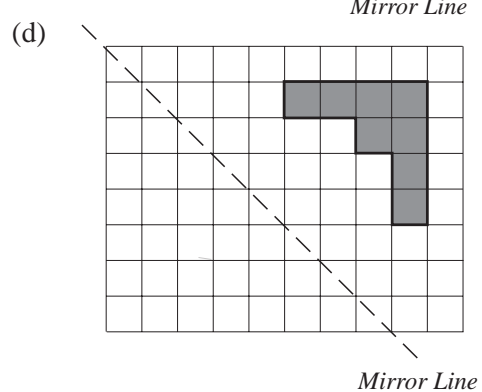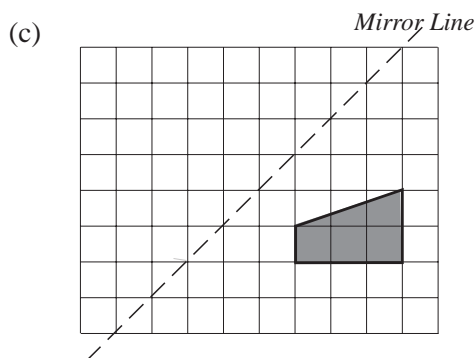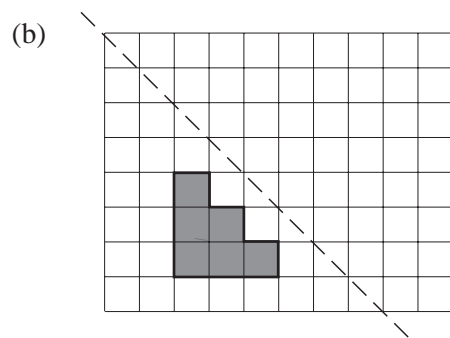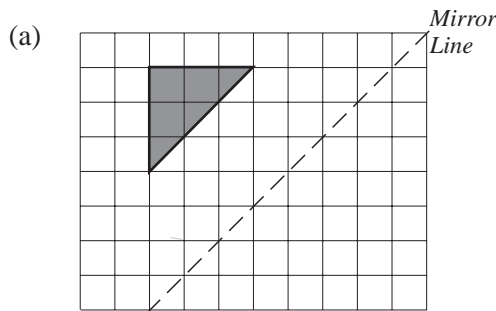   (a)   Write down the scale factor of the enlargement.

    (b)   Copy the drawing and complete the shape B on your diagram.

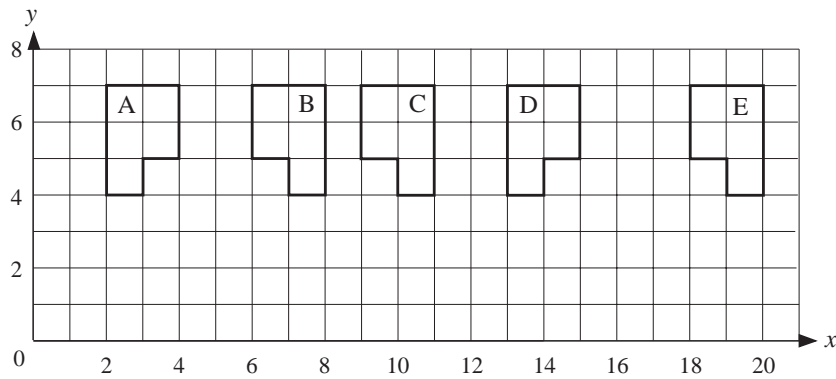   *(LON)*

14.8 Further Reflections

1.   Copy the diagrams below and draw the reflection of each shape in the mirror line.

(a)

Mirror Line

(b)

Mirror Line

(c)   *Mirror Line*

(d)

Mirror Line
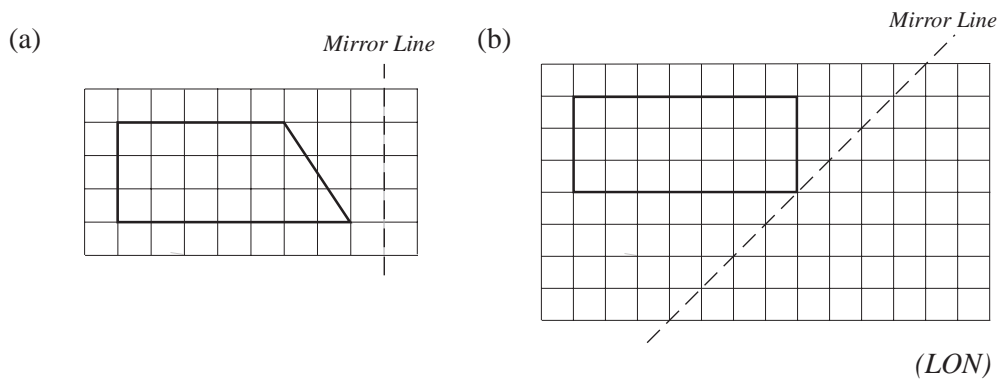
(e)   *Mirror Line*

(f)

Mirror Line

2.   The diagram shows the positions of the shapes A, B, C, D and E.
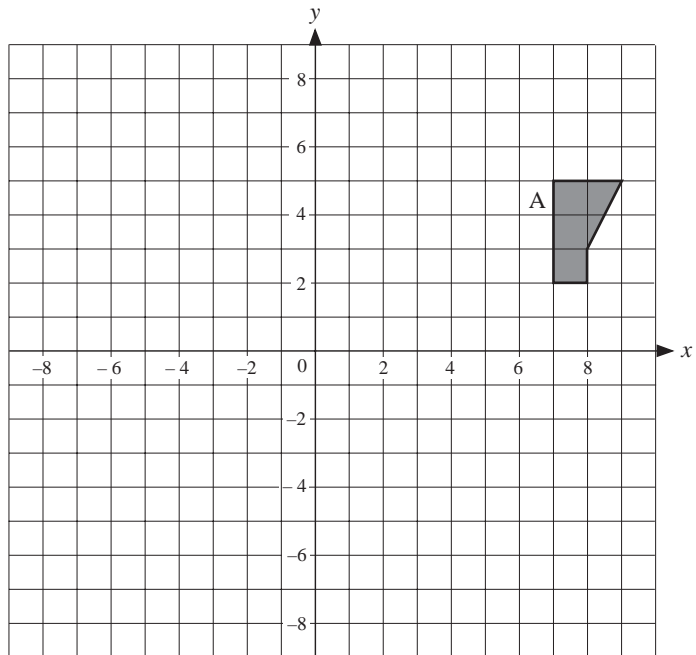

What is the equation of the line of reflection required to transform from:

(a)   A to B   (b)   B to D   (c)   D to C   (d)   C to A

(e)   A to E?

3.   Copy the diagrams and reflect each of the shapes in the mirror lines given.

(a)   *Mirror Line*   (b)   *Mirror Line*
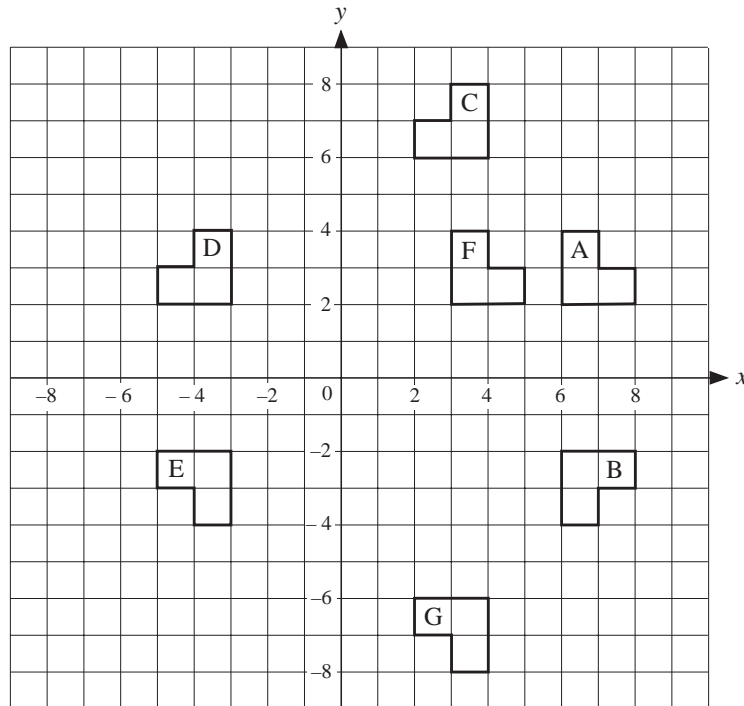

(LON)

4.   Copy the set of axes opposite and the shape labelled A.

(a)   Reflect A in the line $x = 5$ to obtain B.

(b)   Reflect B in the line $y = -2$ to obtain C.

(c)   Reflect C in the line $x = -3$ to obtain D.

(d)   Reflect D in the line $y = -2$ to obtain E.

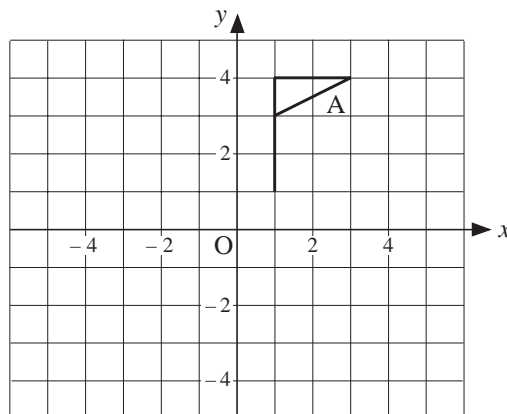(e)   Name two reflections of E which would bring the shape back to A.

5.   The diagram shows a number of shapes, some of which have been reflected in various lines.

State whether the mapping is a reflection and if so, give the equation of the mirror line:

(a)   A to B   (b)   A to D   (c)   A to C

(d)   D to E   (e)   E to F   (f)   B to G

(g)   C to G   (h)   F to D   (i)   B to E.

6.

On a copy of the diagram:

(a)   reflect flag A in the line $y = x$. Label the image B.

(b)   enlarge flag A with scale factor $\frac{1}{2}$ and centre (–3, 2). Label the image C.

(OCR)

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7. Triangle A and triangle B have been drawn on the grid.

    (a) On a copy of the diagram, reflect triangle B in the line $y = 2$. Label this image C.

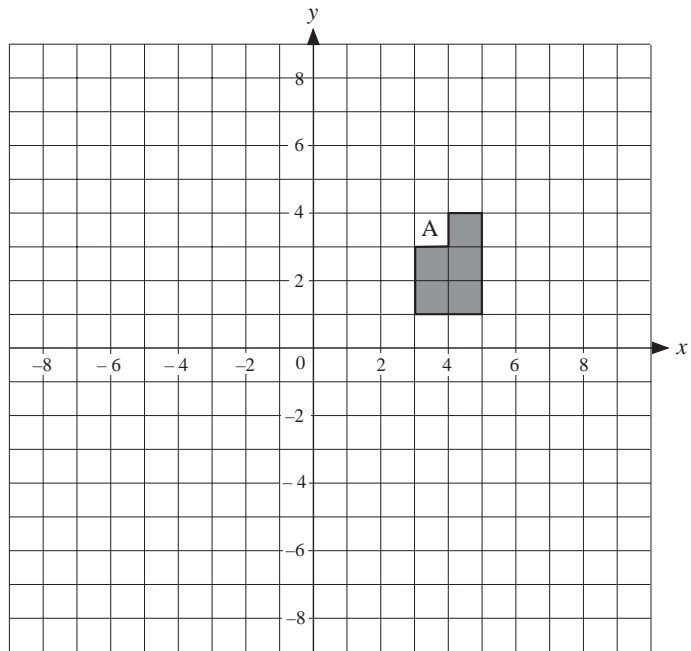   (b) Describe fully the single transformation which will map triangle B onto triangle A.
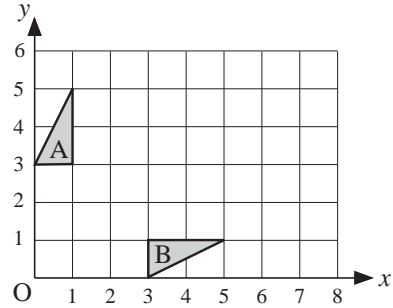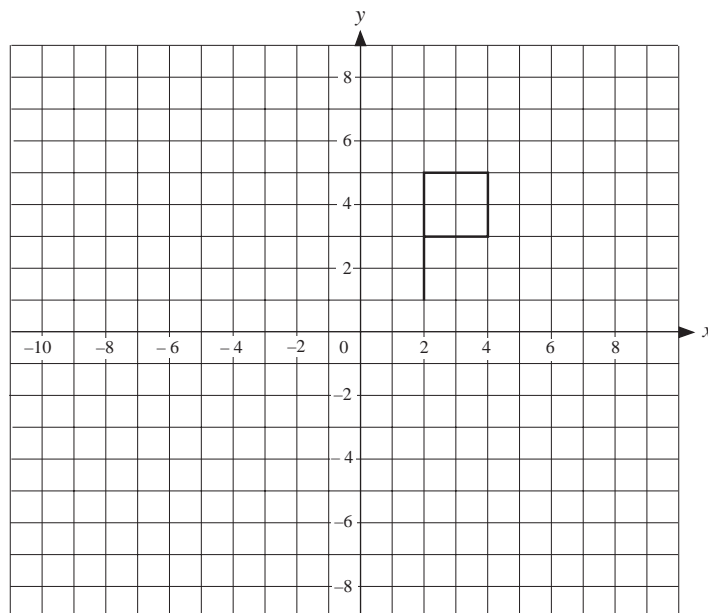

(Edexcel)

14.9 Rotations

1. Copy the axes and shape A shown.

    (a) Rotate the shape A through $90°$ anticlockwise around $(0, 0)$ to obtain B.

    (b) Rotate the shape A through $180°$ clockwise around $(0, 0)$ to obtain C.


    (c) What rotation is needed to obtain the shape C from B?

2. Copy the axes and shape shown below.

(a)   Rotate the original shape through $90°$ clockwise about the point $(1, 0)$.

(b)   Rotate the original shape through $180°$ about the point $(5, 2)$.

(c)   Describe the rotation that takes the shape in (b) to the shape in (a).

(d)   Rotate the original shape through $90°$ anti-clockwise about the point $(-2, -1)$.

3.   The diagram shows the position of a shape labelled A and other shapes which were obtained by rotating A.
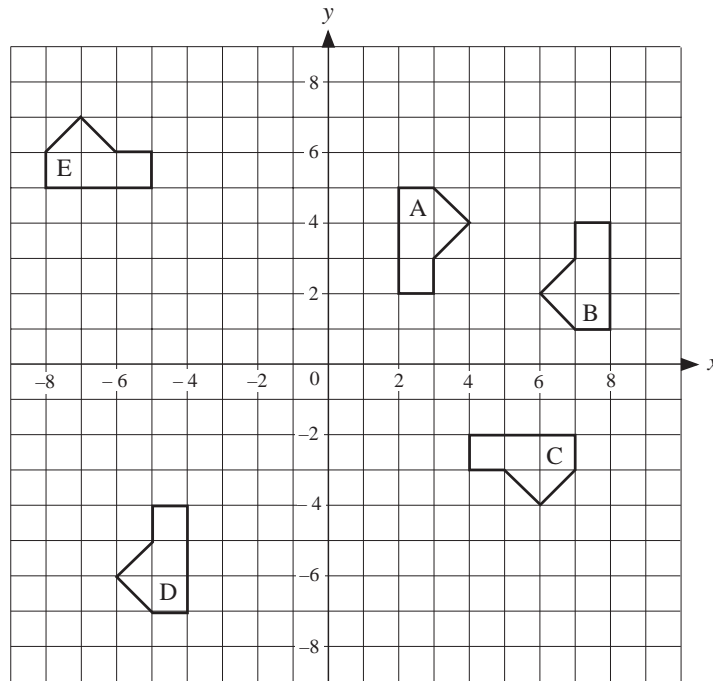
(a)   Describe how each shape can be obtained from A by a rotation.

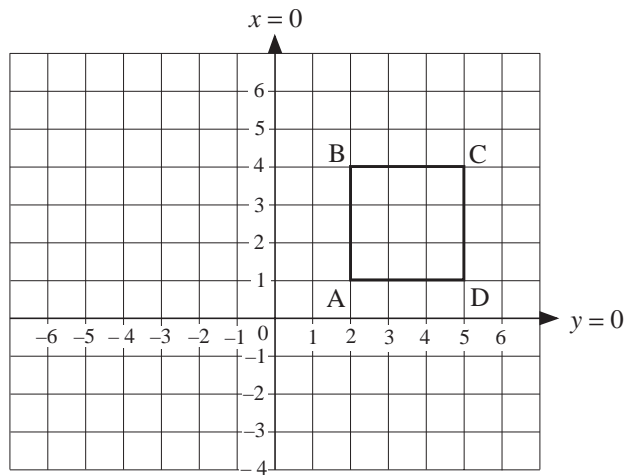(b)   Which shapes can be obtained by rotating the shape B?

4.
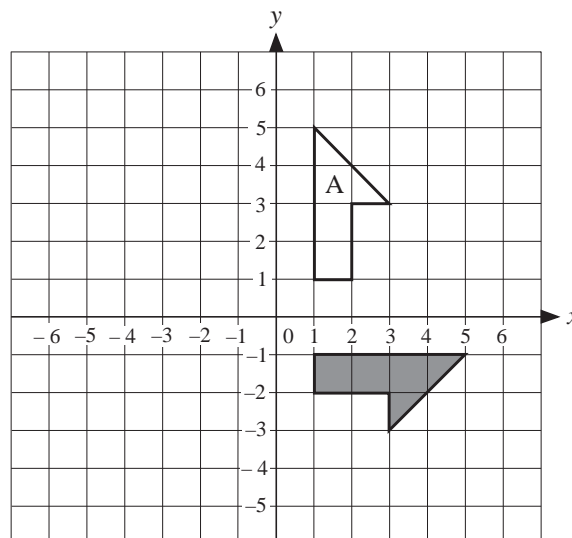
(a)   Describe fully the single transformation which will transform the shape labelled A to the shaded shape.

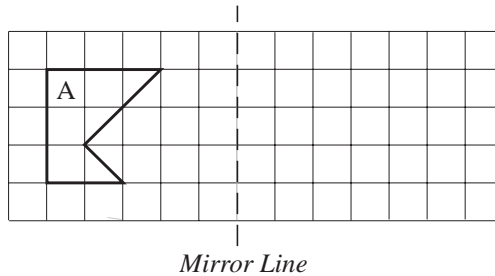(b)   On a copy of the grid, draw the shaded shape after it has been reflected in the line $y = x$.

(NEAB)

5.

(a)   The square ABCD is reflected in the line $x = 1$.
What are the new coordinates of C?

(b)   The square ABCD is rotated through 180° about A.
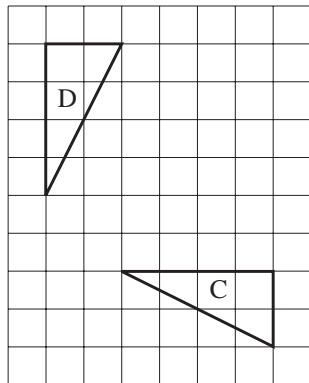What are the new coordinates of C?

(SEG)

6.   (a)   Copy the diagram below.

Mirror Line

Reflect the shape A in the mirror line. Label the reflection B.
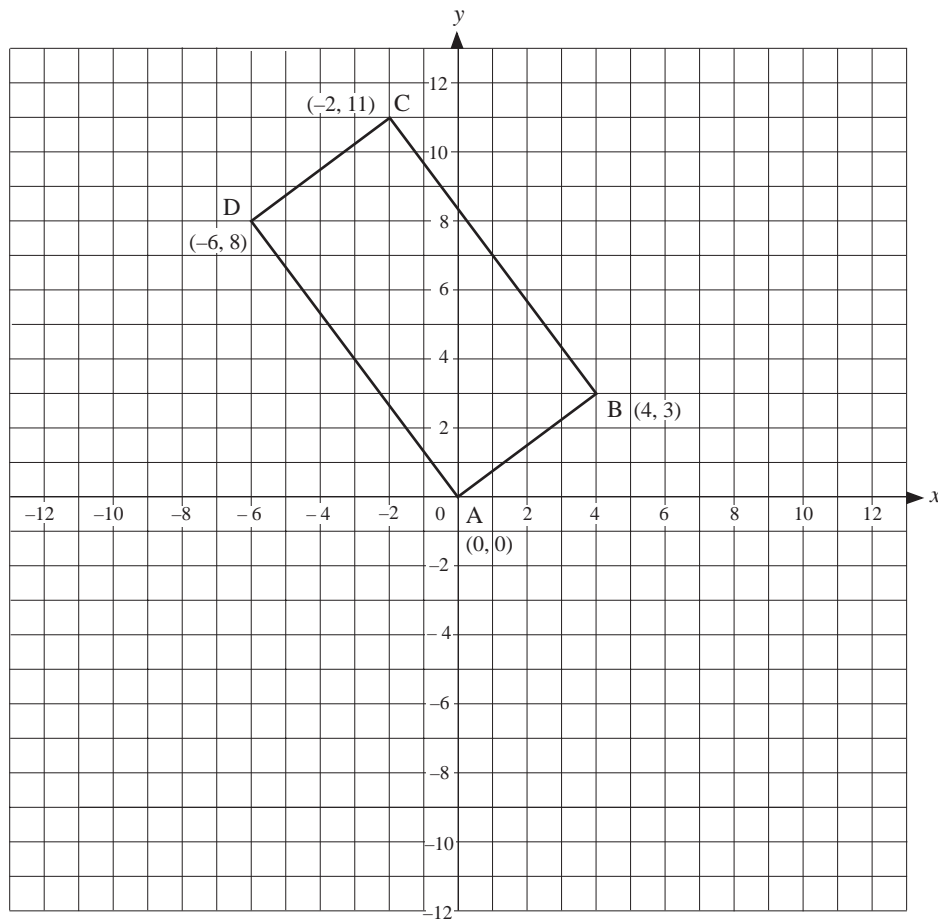
(b)

Describe fully the transformation which maps the triangle C onto the triangle D.

(LON)

7. The sketch shows the position of a rectangle ABCD.

(a) The rectangle ABCD is reflected in the line $x = 4$ to give rectangle $A_1 B_1 C_1 D_1$. What are the coordinates of C_1?

(b) The rectangle ABCD is rotated about A anticlockwise through $90°$ to give $A_2 B_2 C_2 D_2$. What are the coordinates of B_2?

(c) The rectangle ABCD is enlarged by scale factor 2, centre A. What are the coordinates of the new position of B?
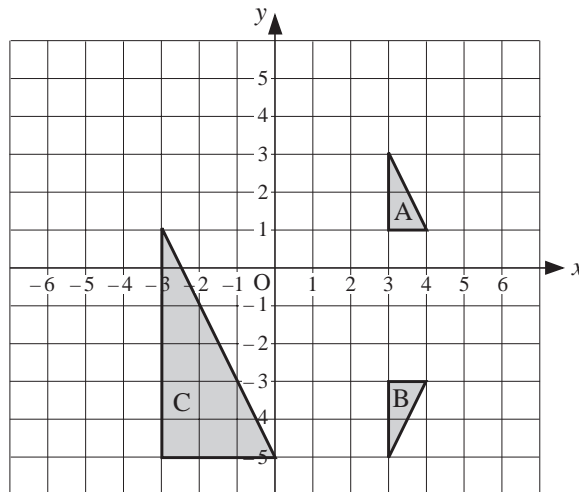
(SEG)

8.
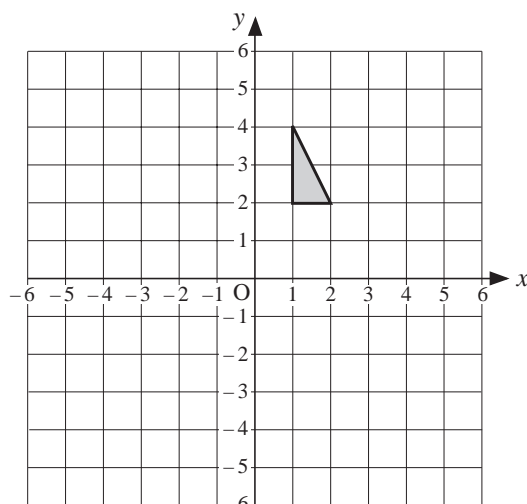
(a) On a copy of the diagram, reflect the shaded triangle in the line $y = -x$.

Label this new triangle with the letter A.

(b) Rotate the original shaded triangle by a quarter-turn anticlockwise about $(0, 2)$.

Label this new triangle with the letter B.

(OCR)

9.

(a)   Describe the transformation that maps triangle A to triangle B.

(b)   Triangle A is rotated $90°$ anti-clockwise about $(0, -1)$.
       On a copy of the diagram, draw the image of A after this transformation.

(c)   Triangle C is an enlargement of triangle A.

       (i)   Write down the scale factor of the enlargement.

       (ii)   Write down the coordinates of the centre of the enlargement.

(AQA)

10.   (a)
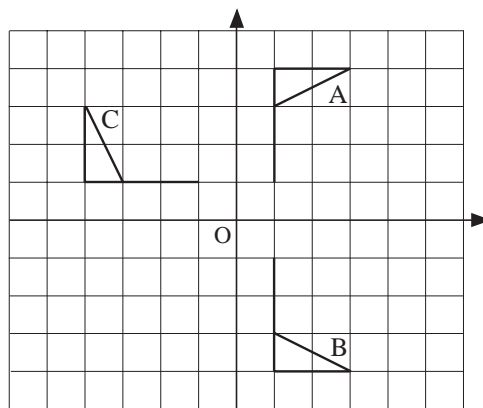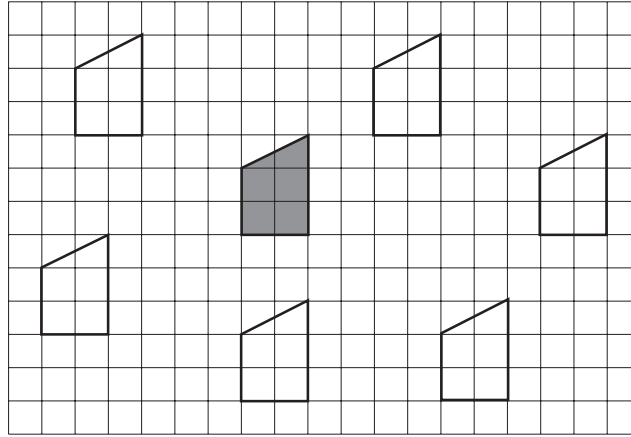

       (i)   Describe fully the single transformation that maps flag A onto flag B.

       (ii)   Describe fully the single transformation that maps flag A onto flag C.

       (b)   Calculate the area of this flag.
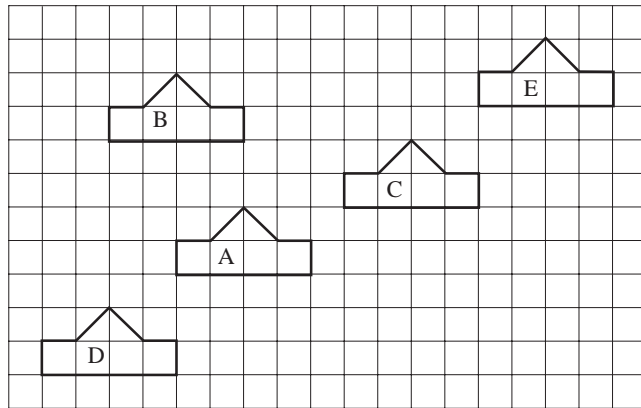

15.6 cm

←—13.2 cm—→
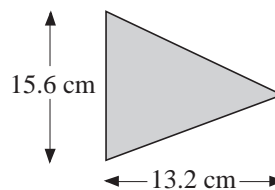
(OCR)

14.10 Translations

1.   The shaded shape has been moved to each of the other positions by a translation.
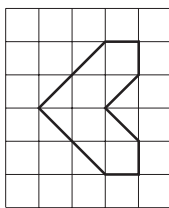   Give the vector used for each translation.

2.   For the shapes shown,
      describe the translation
      which maps:

      (a)   A → B

      (b)   A → C

      (c)   B → C

      (d)   E → A

      (e)   E → B

      (f)   B → A.

3.   Copy this diagram. Draw the image of this shape when
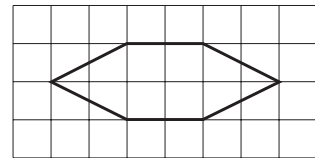   translated using:

      (a) $\begin{pmatrix} 5 \\ 0 \end{pmatrix}$ (b) $\begin{pmatrix} -3 \\ 3 \end{pmatrix}$ (c) $\begin{pmatrix} 7 \\ 2 \end{pmatrix}$.
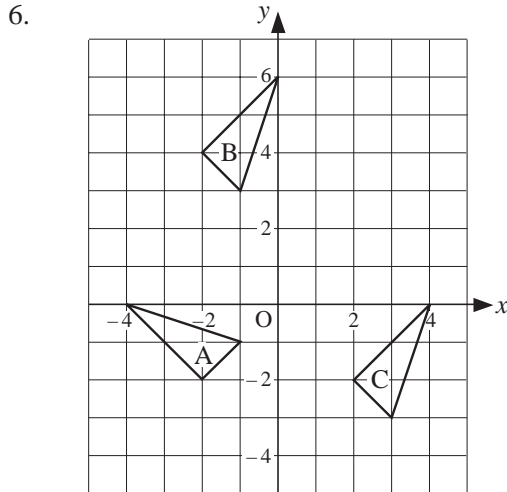
4.   Copy the shape opposite.

      (a)   Translate the shape using the vector $\begin{pmatrix} 7 \\ 0 \end{pmatrix}$.

      (b)   Translate the new shape using the vector $\begin{pmatrix} 0 \\ -3 \end{pmatrix}$.

      (c)   What translation is needed to translate the original shape to the final shape?

5. The points A, B and C have coordinates (3, 2), $(-2, 5)$ and $(-3, -1)$ respectively. Find the vector needed to translate:

    (a)   A to B   (b)   B to C   (c)   A to C.

6.

    (a)   Find the centre of the rotation which maps triangle A on to triangle B.

    (b)   Describe the single transformation which maps triangle B on to triangle C.

(OCR)

7.   The diagram shows a shaded flag. On a copy of the diagram:

    (a)   rotate the shaded flag 90° anti-clockwise about the origin. Label this new flag with the letter A.

    (b)   translate the original shaded flag 2 units to the right and 3 units down. Label this new flag with the letter B.

   (c)   reflect the original shaded flag in the line $y = 1$. Label this new flag with the letter C.
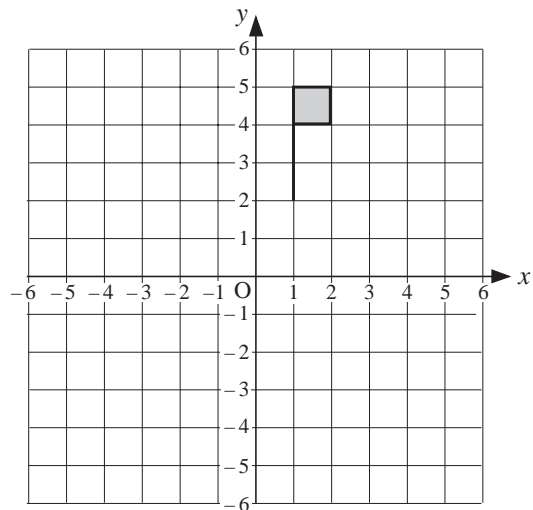
(AQA)

14.11 Combined Translations

1. Copy the set of axes and shape shown below.
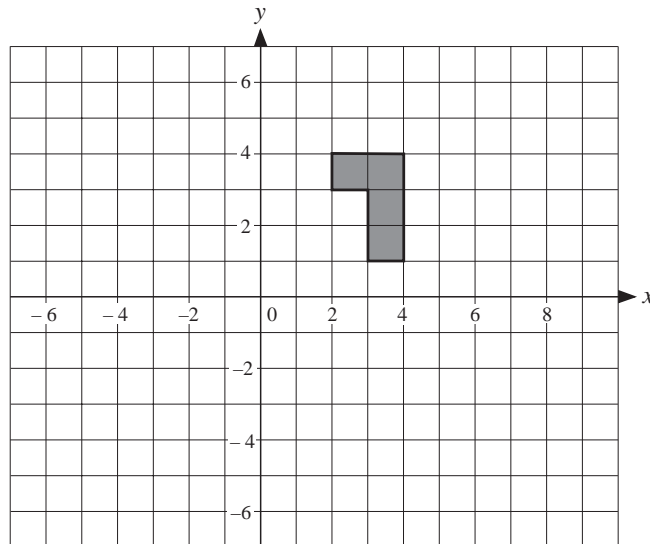
(a) Reflect this shape in the line $x = 5$.

(b) Rotate the original shape $90°$ clockwise about $(5, 0)$.

(c) Reflect the shape in (b) about the line $x = 2$.

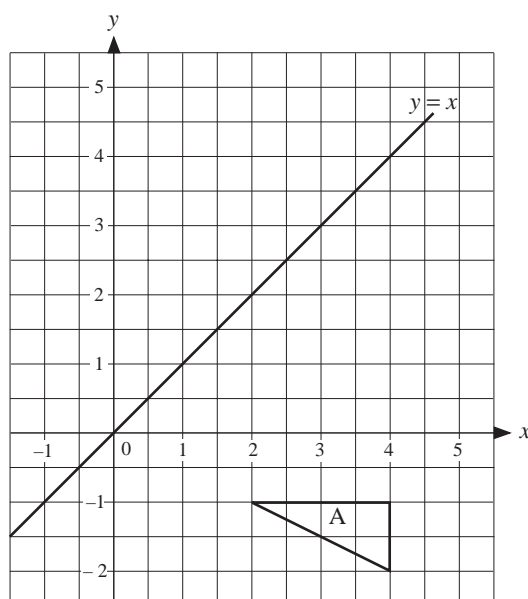(d) Describe a single transformation of the shape in part (a) to the shape formed in part (c).

2.

(a) Describe a single transfomation from

    (i) A to B   (ii) F to B   (iii) A to D   (iv) E to A.
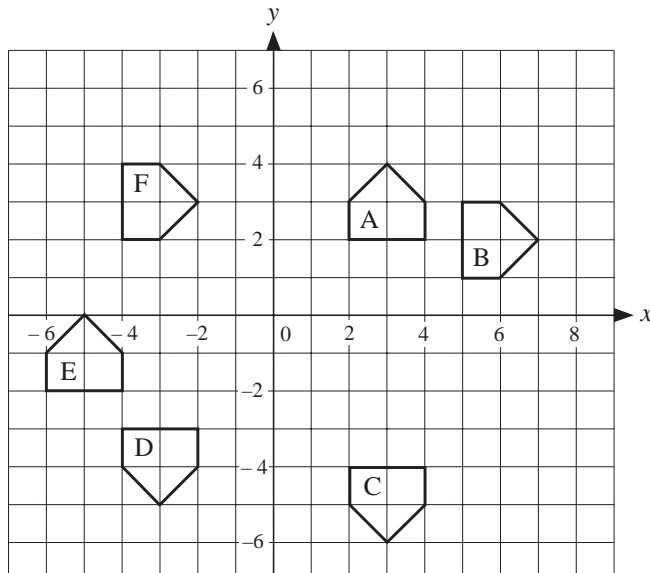
(b) Describe two combined transformations from

    (i) B to C   (ii) C to E   (iii) C to F.

3. (a) The point A is reflected in the *y*-axis.
The image is the point B.
Write down the coordinates of B.

(b) The point A is rotated through 90°
anticlockwise about O.
The image is the point C.
Write down the coordinates of C.

(c) The point B can be mapped onto
point C by a translation.
Write down the column vector of
this translation.
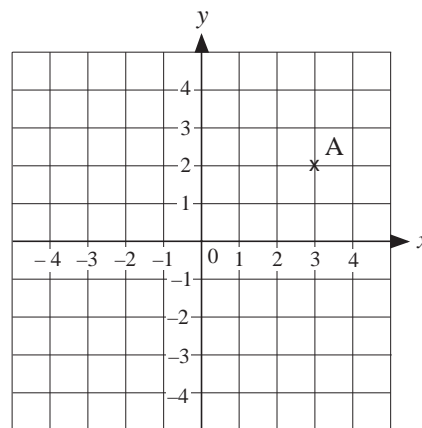
(MEG)

4.

(a) Copy the diagram and reflect the triangle A in the *x*-axis. Label the
reflection B.

(b) Reflect the triangle B in the line *y* = *x*. Label the reflection C.

(c) Describe fully the single transformation which maps triangle A onto
triangle C.

(d) Write down the equation of the line which is parallel to *y* = *x* and which
passes through the point (0, 8).

This line crosses the *x*-axis at the point P.

(e) Calculate the coordinates of P.

(LON)

5.   The parallelogram ABCD has vertices at $(6, 3)$, $(9, 3)$, $(12, 9)$ and $(9, 9)$ respectively.
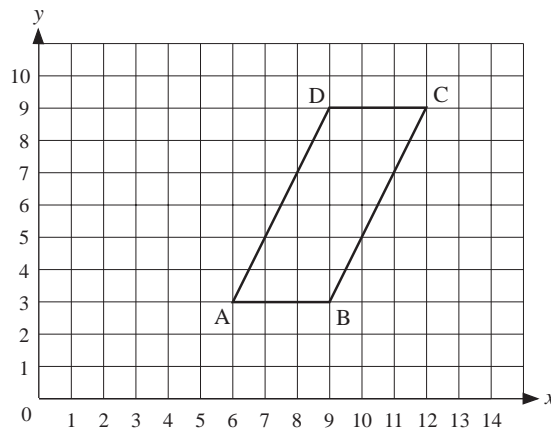
(a)   An enlargement, scale factor $\dfrac{1}{3}$ and centre $(0, 0)$, transforms parallelogram ABCD onto $A_1 B_1 C_1 D_1$. Copy the diagram and draw the parallelogram $A_1 B_1 C_1 D_1$.

(b)   The parallelogram $A_1 B_1 C_1 D_1$ is translated by the vector $\begin{pmatrix} 0 \\ 2 \end{pmatrix}$ onto $A_2 B_2 C_2 D_2$. What are the coordinates of C_2?

(c)   The parallelogram $A_2 B_2 C_2 D_2$ can be transformed onto ABCD by an enlargement. Give the scale factor and centre of this enlargement.

(SEG)

6.   The diagram shows triangles T, S and U.

S is the image of T under a reflection in the line $x = 3$.

U is the image of S under reflection in the line $x = 6$.

A reflection in the line $x = 3n$, where n is an integer, is denoted by R_n,


So S is the image of T under R_1 and U is the image of T under R_1 followed by R_2. V is the image of T under the successive transformations R_1 followed by R_2 followed by R_3.

(a)   Draw V on a copy of the diagram above.

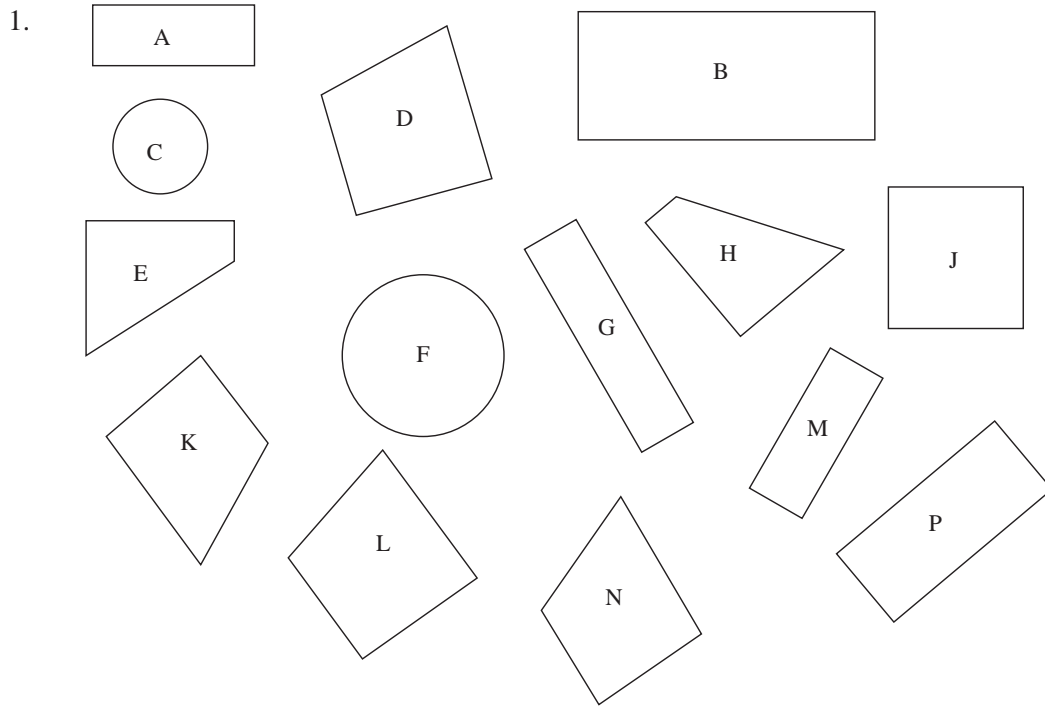(b)   Describe fully the single transformation that will map T to V.

W is the image of T under the successive transformation R_1, followed by R_2, followed by R_3 and so on to R_n.

(c)   Describe fully the single transformation that will map T to W:

(i)   when n is even   (ii)   when n is odd.
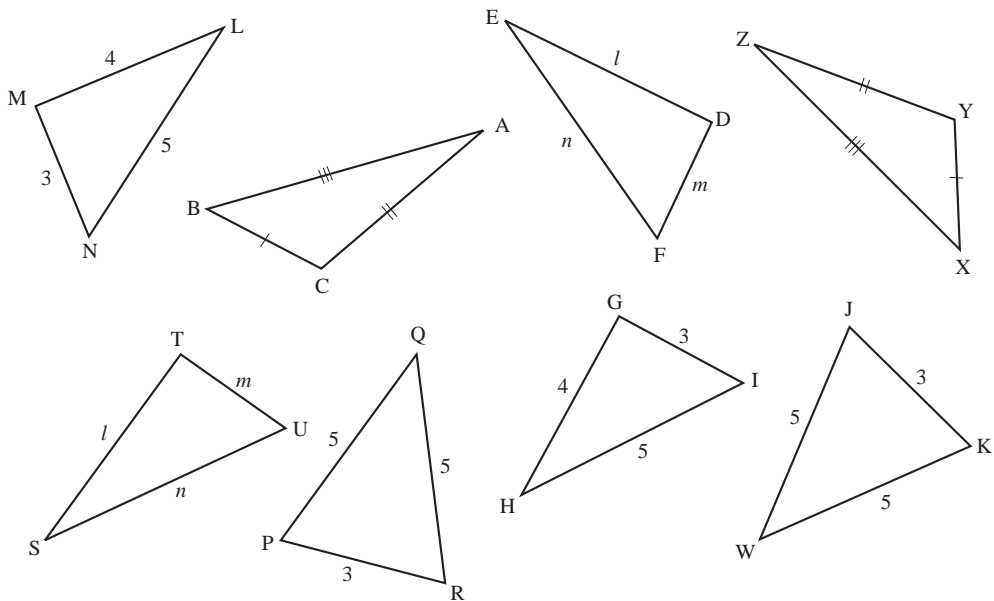
(LON)

14.12 Congruence

1.
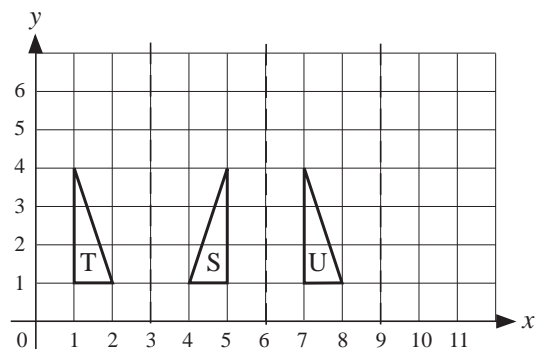
From the shapes in the diagram, write down the letters of three pairs of congruent shapes.
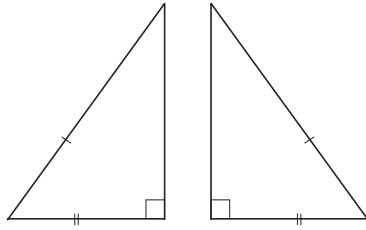
(LON)

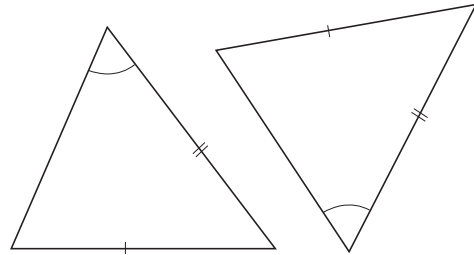2.   State which pairs of triangles are congruent.

3. For each question below, determine whether the triangles are congruent.
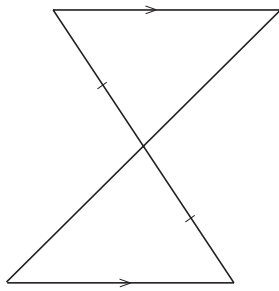   If the triangles *are* congruent, justify your answer.
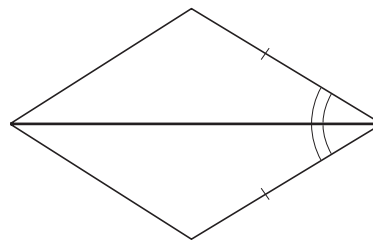
    (a)

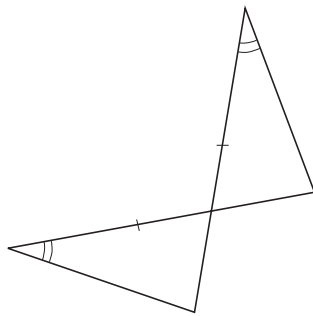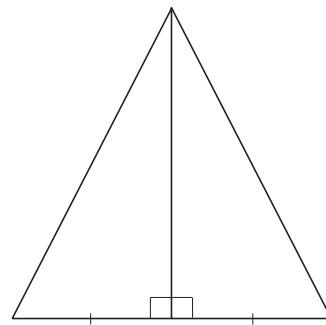   (b)

    (c)

    (d)

    (e)

    (f)

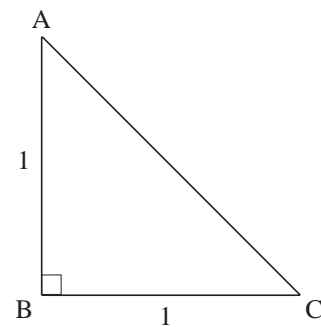4. Pythagoras wrote about a triangle where

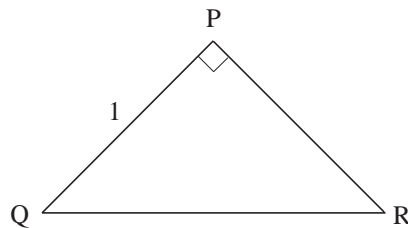   AB = 1 unit,

    BC = 1 unit , and

    $\hat{ABC} = 90°$.

    (a) Pythagoras claimed that the length of AC
         is a rational number.

         Was he correct? Explain your answer.
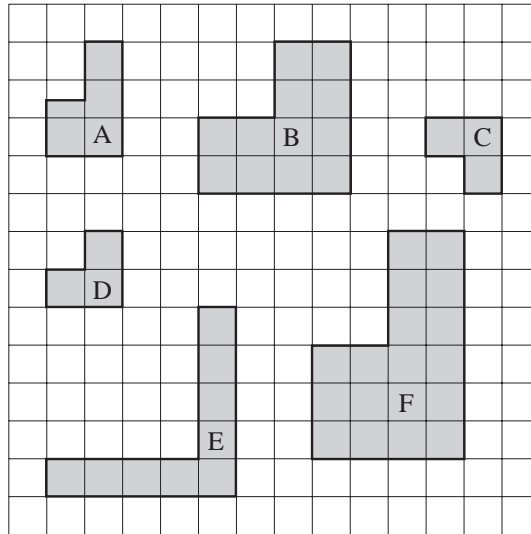
    (b) The triangle PQR has PQ = 1 unit, QR = $\sqrt{3}$ units and $\hat{QPR} = 90°$, as
         shown.

         Is the triangle PQR congruent to triangle ABC? Explain your answer.
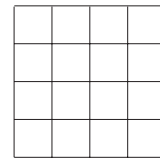
         (SEG)

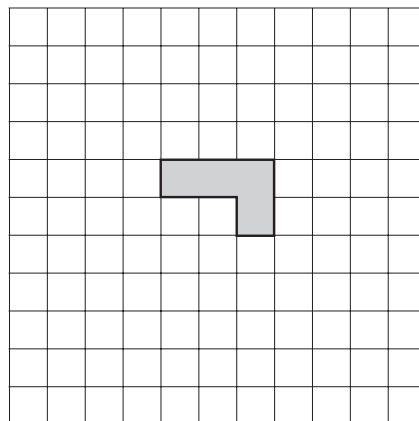5.   (a)   Here are six L-shapes on a unit square grid.

   (i)   Which two L-shapes are congruent?

   (ii)   Which L-shape is an enlargement of A?

(b)   Here is a different L-shape.

   Show on a copy of the grid opposite how four of
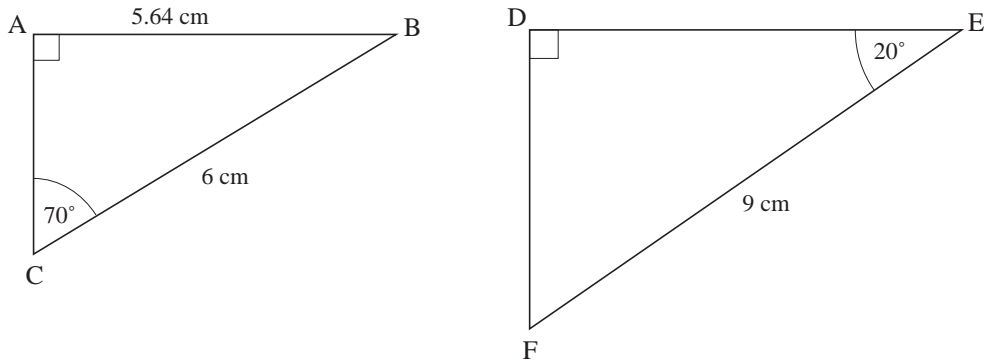   these L-shapes can be fitted together to make a square.

(c)   On a copy of the grid below, draw the L-shape after a rotation of 180°.

(AQA)

14.13 Similarity

1. The diagram shows two similar triangles.

What is:

(a)   the size of angle DFE   (b)   the length of DE

(c)   the ratio AC : DF?

2. A cylinder has a height of 10 cm and has volume 300 cm^3. Find the volumes of similar cylinders of heights:

(a)   5 cm   (b)   20 cm.

3. Two cubes have volumes 729 cm^3 and 1331 cm^3. What is the ratio of:

(a)   the side length of the cubes

(b)   the surface areas of the cubes?

4. In the diagram,   CD = 4 metres,
   CE = 3 metres and   BC = 5 metres.
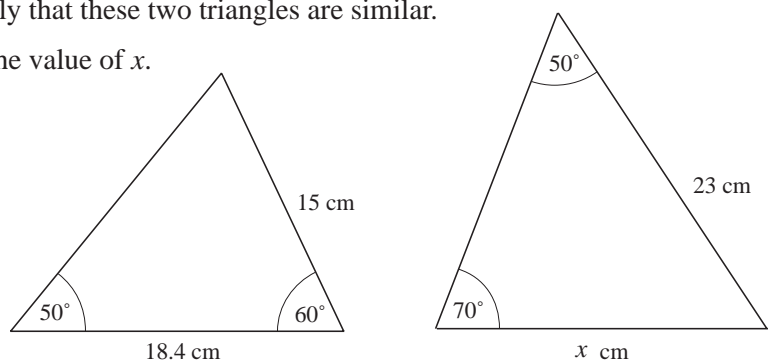
   AB is parallel to DE.
   ACE and BCD are straight lines.

(a)   Explain why triangle ABC is similar
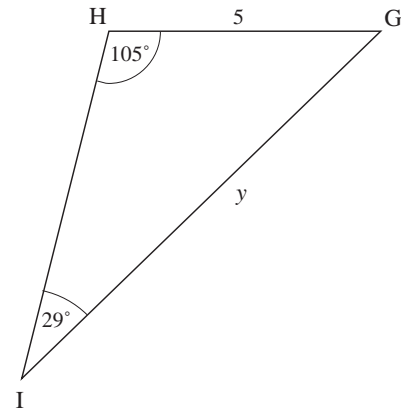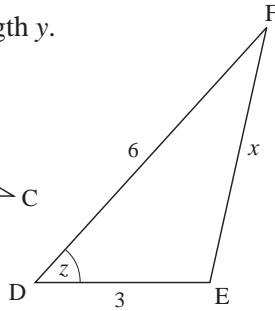   to triangle EDC.

(b)   Calculate the length of AC.
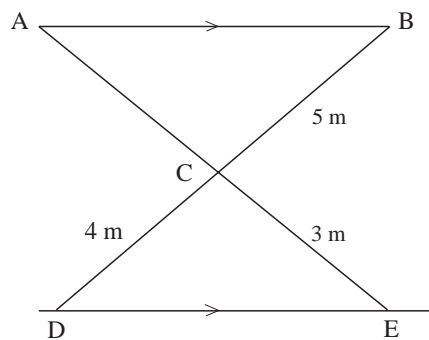
(LON)
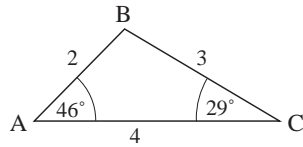
5. (a)   Show clearly that these two triangles are similar.

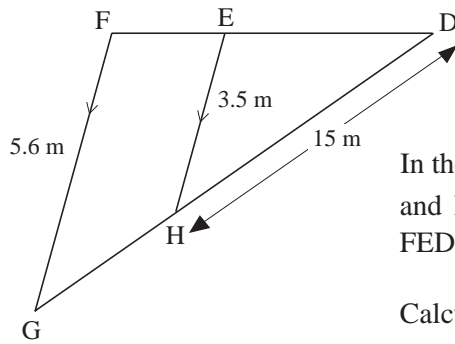   (b)   Calculate the value of *x*.

(NEAB)

14.13

6. All these triangles are similar.

   (a) Calculate the length x.
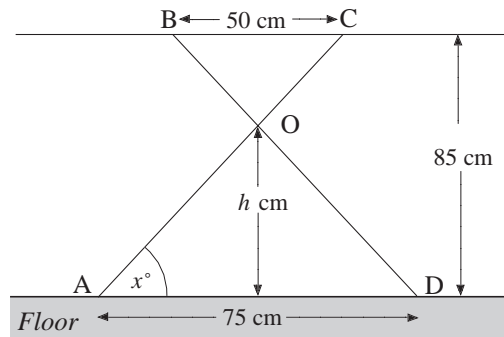
   (b) Calculate the length y.

(SEG)

7.

In the diagram, FG = 5.6 metres, EH = 3.5 metres and DH = 15 metres. EH is parallel to FG. FED and DHG are straight lines.

Calculate the length of DG.

(LON)

8.

The diagram shows a simplified ironing board. The feet, A and D, are 75 cm apart. The supports at B and C are 50 cm apart. The legs, AC and BD, are equal in length and are pivoted at O. BC is parallel to AD.

The height of the ironing board above the floor is 85 cm.

   (a) Use similar triangles to calculate the height, h cm, of O above the floor.

   (b) Calculate the value of x, the angle between AC and the floor.
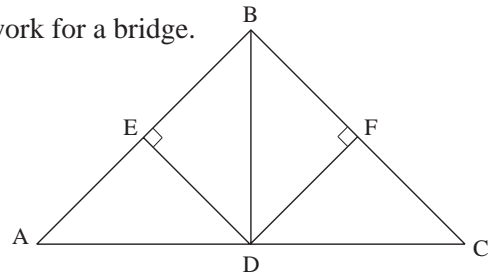
   (c) Calculate the length of AC.

(NEAB)

9.   The diagram shows a symmetrical framework for a bridge.

   AC = 100 m
   AB = BC = 70 m

   (a)   (i)   Calculate the angle BAD.
         (ii)   Calculate the length ED.

A similar framework is made with the length corresponding to   AC = 180 m .

   (b)   (i)   Calculate the length corresponding to AB.

         (ii)   What is the size of the angle corresponding to angle BAD?
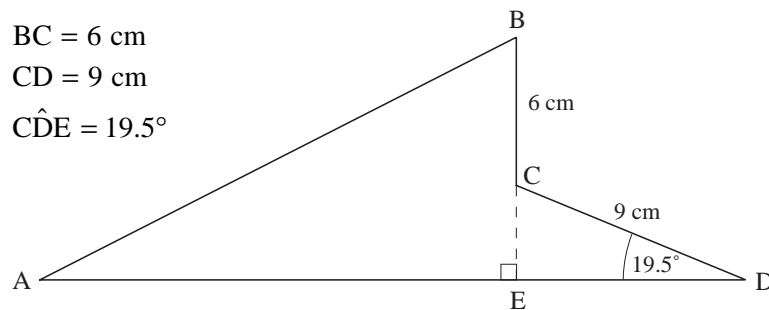
(SEG)

10.   (a)   The model of the cross-section of a roof is illustrated below.

      BC = 6 cm
      CD = 9 cm
      $\hat{CDE} = 19.5°$

      (i)   Calculate the length of CE.

      (ii)   Triangles ABE and DCE are similar triangles with angle BAE
             equal to angle CDE. Calculate the length of AB.

   (b)   When the roof is constructed, the actual length of BC is 4.5 m.
         Calculate the area of the cross-section of the actual roof space.
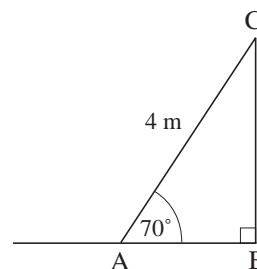
(SEG)

11.   (a)   Paul's ladder is 4 m long.

      (i)   Paul leans his ladder against a vertical wall,
             with the end, A on horizontal ground.

             The angle between the ladder and the
             ground is 70°.

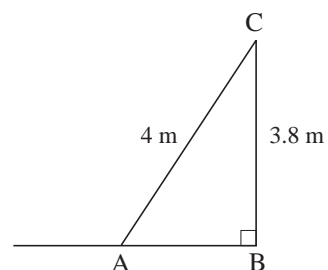   Calculate the distance of A from the wall.

      (ii)   Pamela moves the ladder and uses it to reach a windowsill which is
             3.8 m above the gournd.

             For safety, the angle between the ladder
             and the ground should be within 2° of 70°.

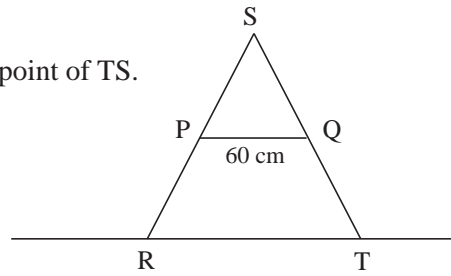   Is the ladder safely placed?
             (You must show some calculation to
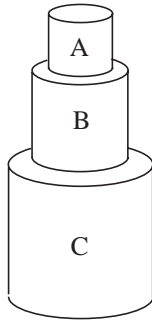   explain your answer.)

(b)   Ranjit has a stepladder.

P is the midpoint of RS. Q is the midpoint of TS.
The length of PQ is 60 cm.

(i)   Explain why triangles SPQ
and SRT are similar.

(ii)   Calculate the length of RT.

(SEG)

12.   A child builds a tower from three similar cylindrical blocks.
The smallest block, A, has radius 2.5 cm and height 6 cm.

(a)   Find the volume of the smallest block.

(b)   Block B is an enlargement of A and block C is an

enlargement of B, each with a scale factor of $1\frac{3}{4}$.
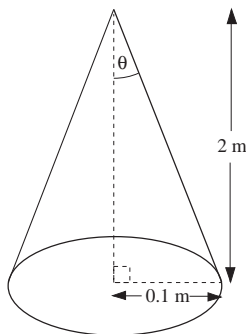
Find the total height of the tower.

(MEG)

13.   Two similar solid shapes are made. The height of the smaller shape is 7 cm.

The width of the smaller shape is 6 cm. The width of the larger shape is 9.6 cm.

Not to scale
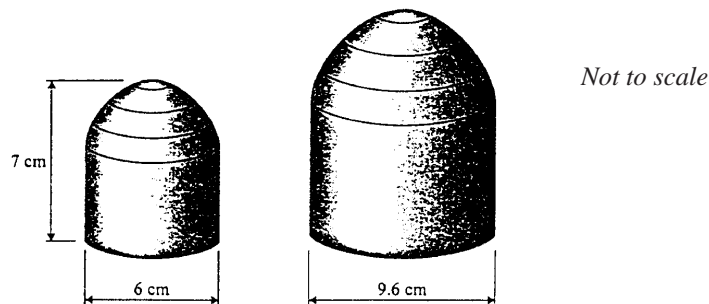
(a)   Calculate the height of the larger shape.

(b)   The volume of the larger shape is 695 cm^3. Find the volume of the smaller
shape.   *(SEG)*

14.   This is a scale model of the proposed Millennium Tower.
The model is a cone of height 2 m and base radius 0.1 m.
The angle between the slanting side and the vertical is θ.

(a)   What is the value of θ, to the nearest degree?

(b)   The proposed Millennium Tower is 1000 m high.
What is its base radius?

(c)   Calculate, in cubic metres, the volume of the model.

(d)   How many times larger than the scale model is the volume of the proposed
Millennium Tower?

(SEG)

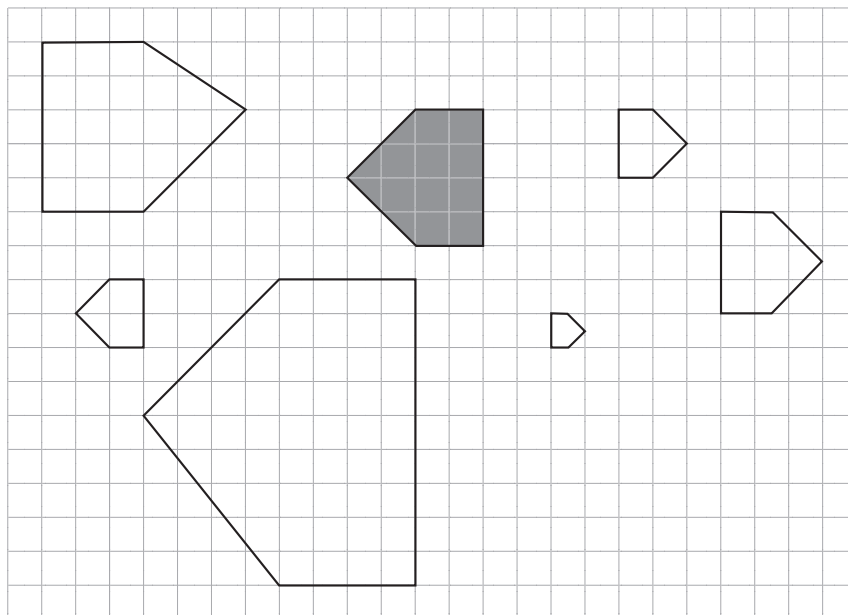15.   ABC and PQR are similar triangles.
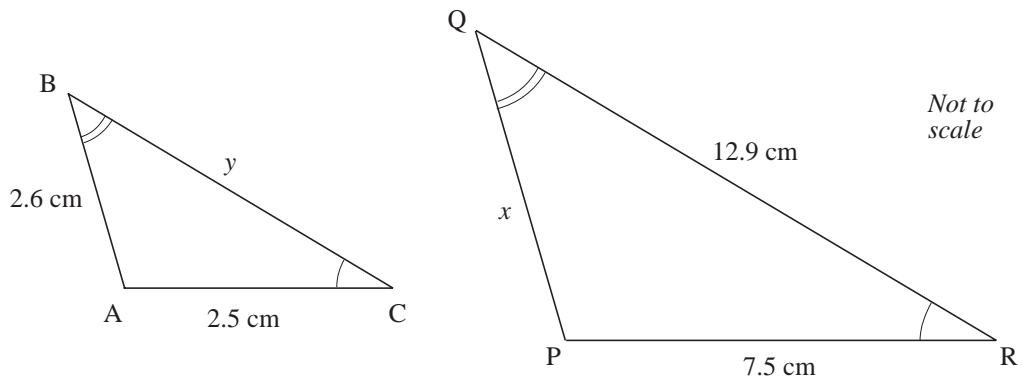
(a)   Find the length marked

(i) x

(ii) y

(b)   (Area of triangle PQR) $= n \times$ (Area of triangle ABC)

Find the value of n.

(OCR)

14.14   Enlargements with Negative Scale Factors

1.   The diagram below shows the original shape (shaded) and the images obtained by enlargements with different scale factors.
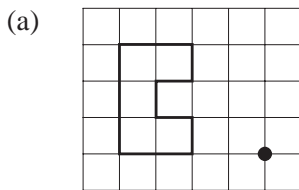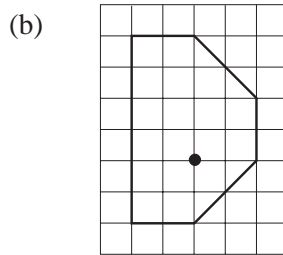
State the scale factor for each enlargement.

2. The shaded shape has been enlarged to give the other images. For each image, find the scale factor and the coordinates of the centre of enlargement.
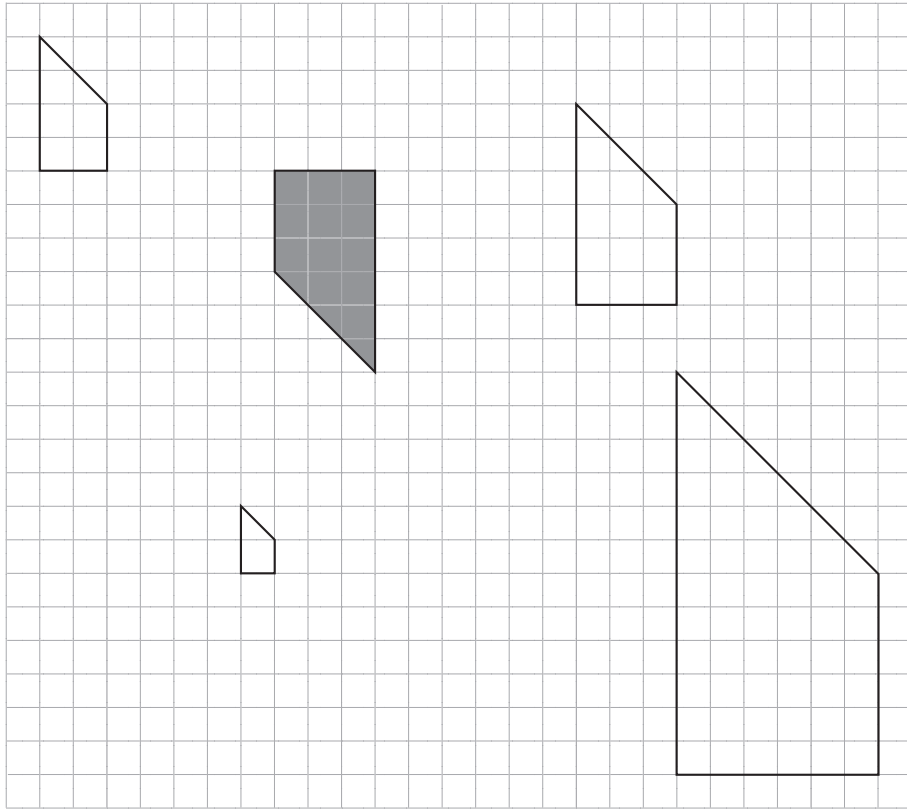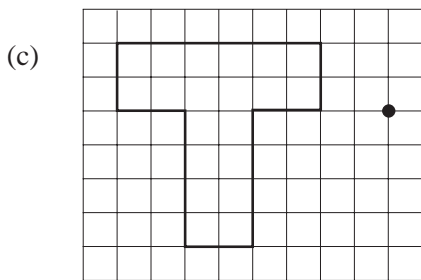

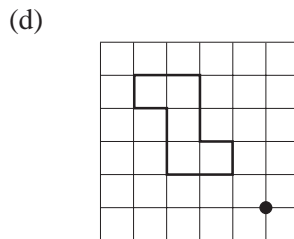3. Copy each diagram below. Enlarge each shape using the scale factor and centre of enlargement given.

(a)

Scale factor -2

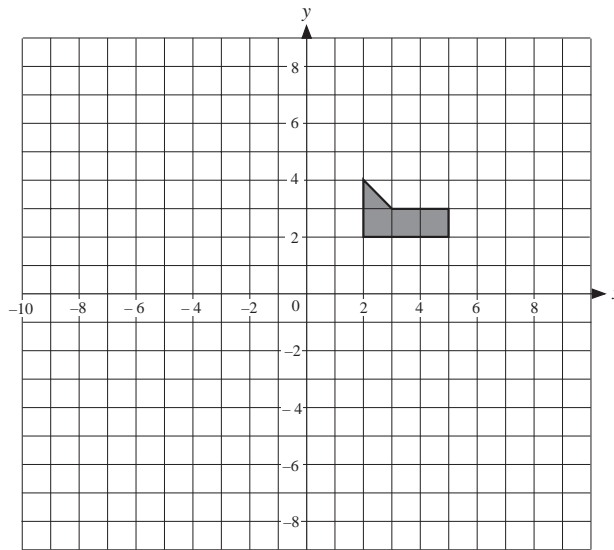(b)

Scale factor $-\dfrac{1}{2}$

(c)

Scale factor $-\dfrac{1}{2}$

(d)

Scale factor -3

4.   Copy the axes and shape shown below.

Draw the image obtained when the original shape is enlarged with:

(a)   scale factor -2, centre of enlargement $(0, 0)$

(b)   scale factor -1, centre of enlargement $(0, 2)$

(c)   scale factor -2, centre of enlargement $(4, 0)$.

5.   Triangle P is mapped to triangle Q by an enlargement of scale factor -0.4.

If BC is of length 5 cm, what is the length of DF?