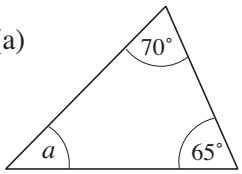
3 Angle Geometry

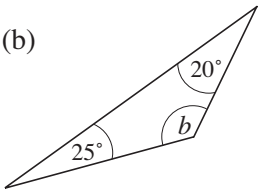
3.3 Angle Geometry

1. Calculate the size of the angles marked with a letter in each diagram. *None to scale*

(a)



(b)



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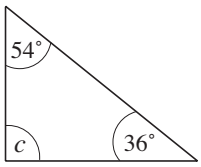

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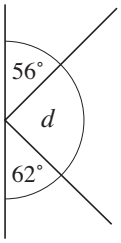

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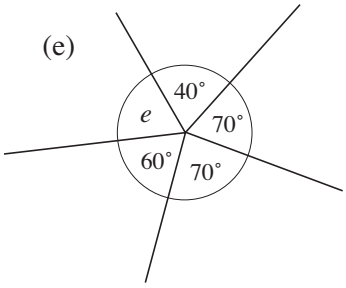

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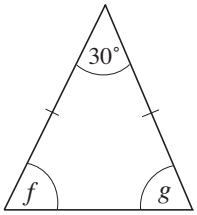

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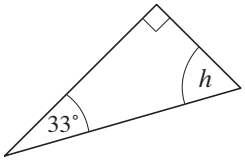

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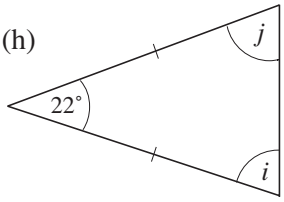
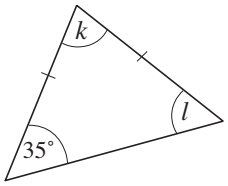


(i)



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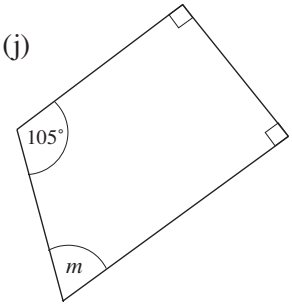

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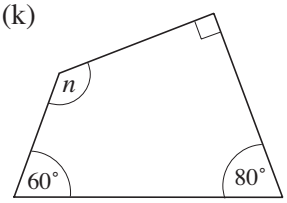

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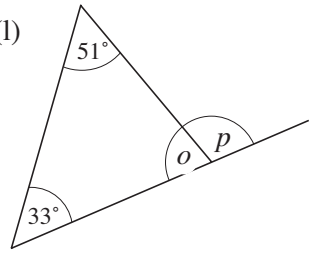
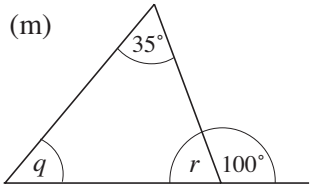
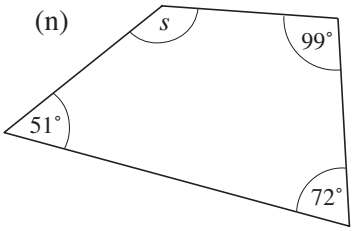


(m)



(n)



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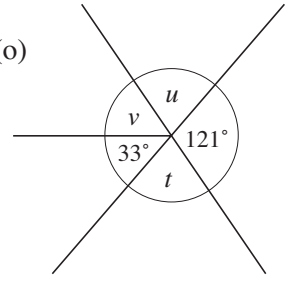
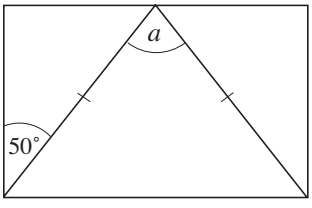
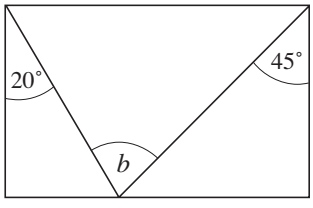


2. Find the angles marked with a letter in each rectangle below.

(a)



(b)



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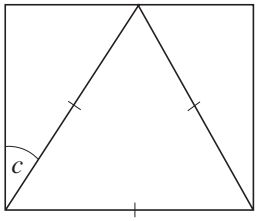
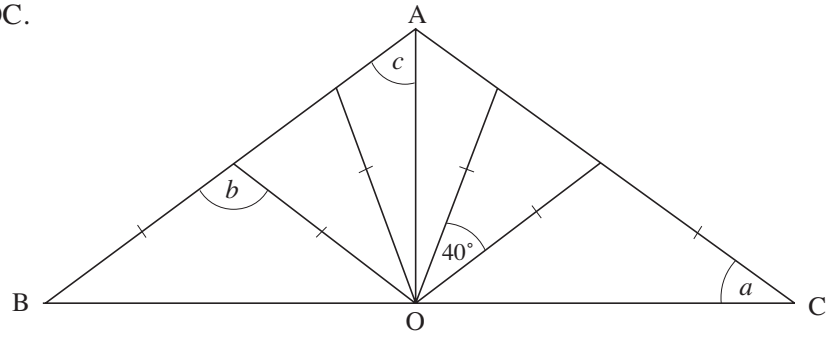


3. The framework of a symmetrical roof is illustrated below. OA is perpendicular to BOC.

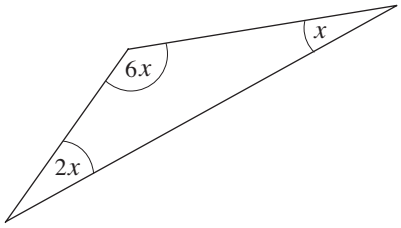


Find the size of the angles marked a, b and c.

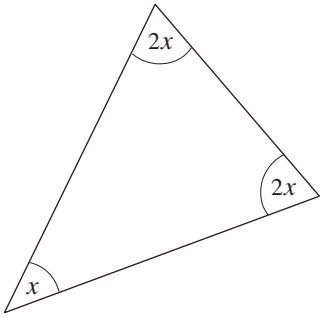
4. Write down an equation that is satisfied in each of the following diagrams.

In each case, solve the equation for x.

(a)



(b)



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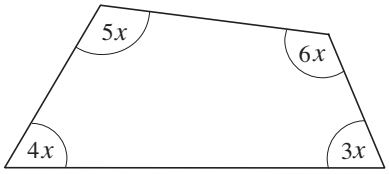

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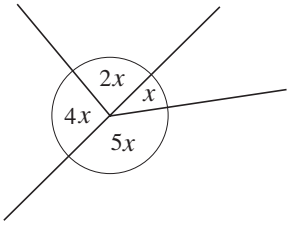
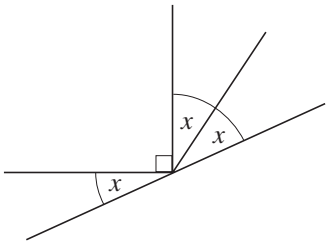


(e)



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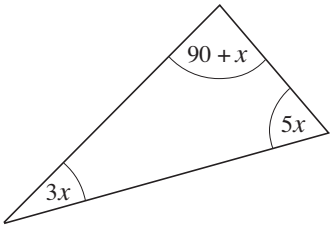
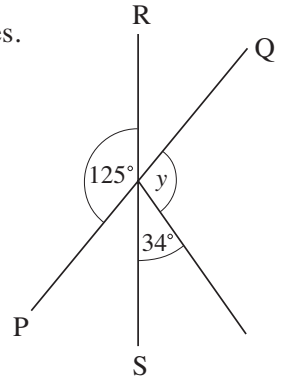


Q.5 PQ and RS are straight lines.

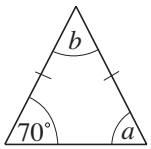
Work out the value of y.



Not drawn accurately

(AQA)

6. This triangle has two equal sides.



Not drawn accurately

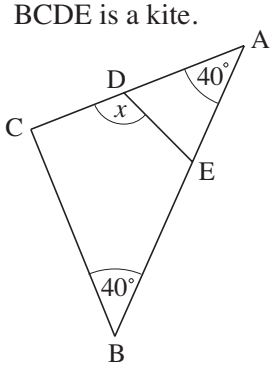
(a) What name is given to this type of triangle?

(b) Find the values of a and b.

(AQA)

7. ABC is an isosceles triangle. BCDE is a kite.

Work out the value of x.



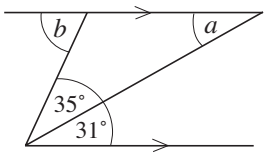
Not drawn accurately

(AQA)

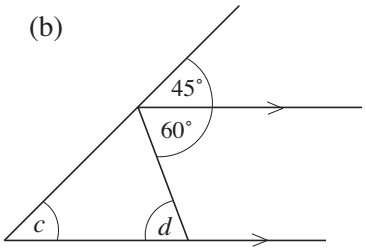
3.4 Angles with Parallel and Intersecting Lines

1. Calculate the unknown angles in the following diagrams.

(a)



(b)



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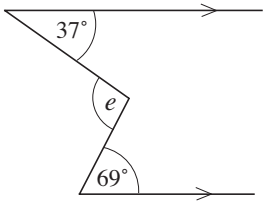
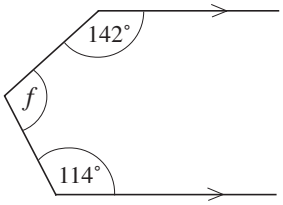


(d)



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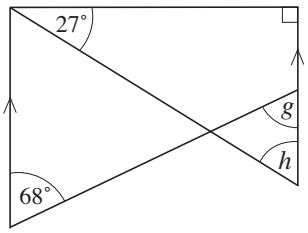

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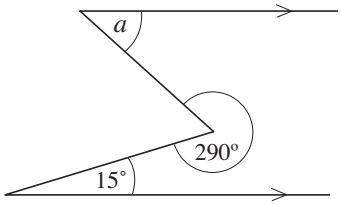

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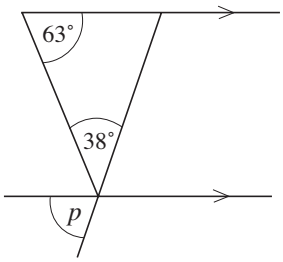

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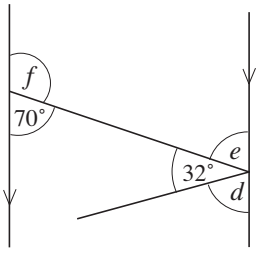

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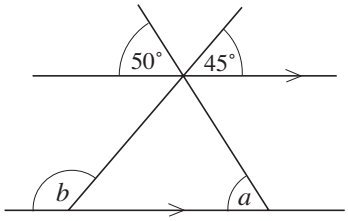

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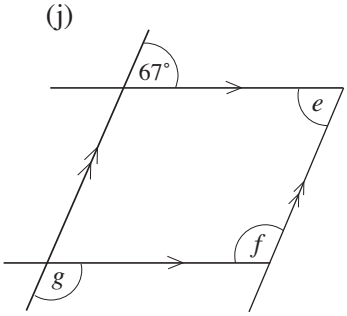

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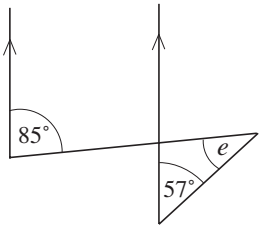

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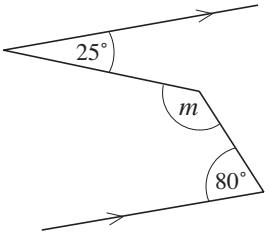

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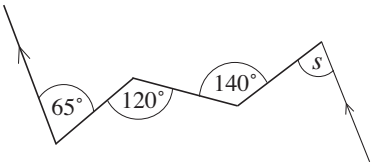

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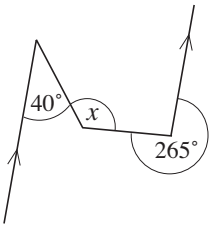

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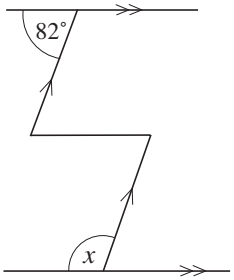

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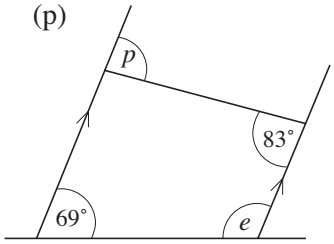
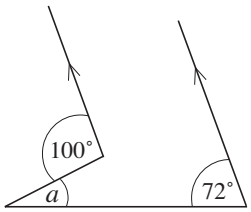


(q)



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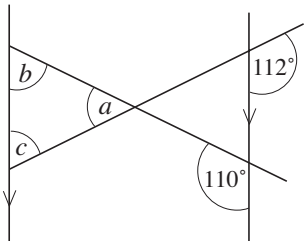

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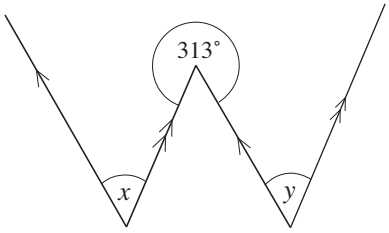

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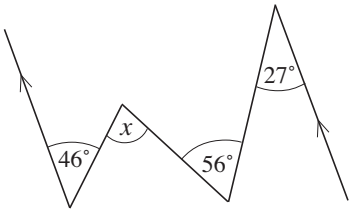
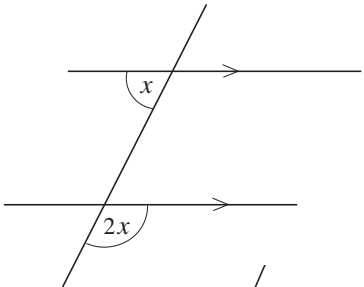
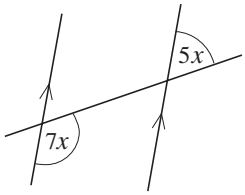


2. For each diagram, find an equation in x, and hence solve for x.

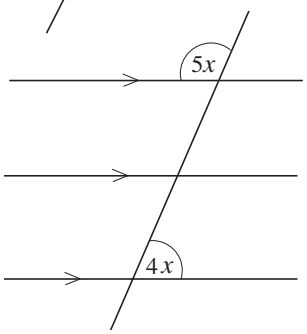
(a)



(b)

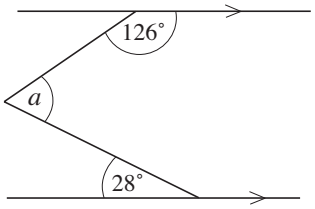


(c)



3. Find the values of the unknown angles in each of the following.

(a)



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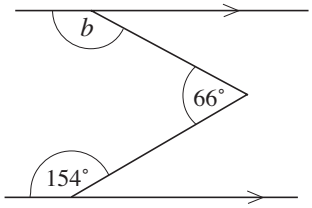

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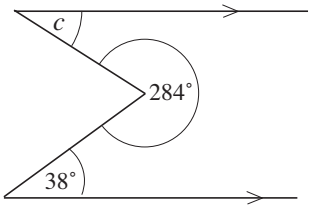
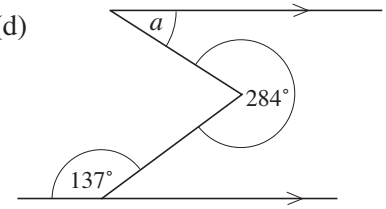
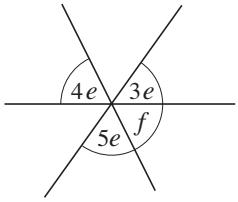


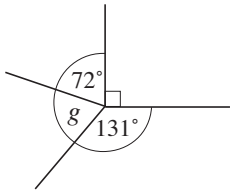
(d)



(e)



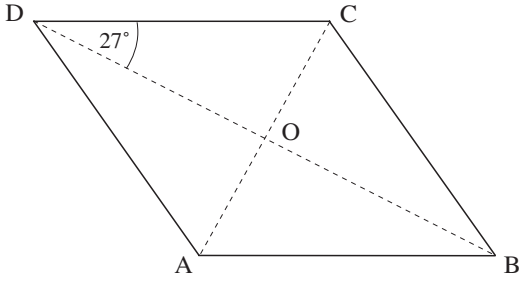
(f)



4. ABCD is a rhombus.

Angle BDC = 27°
The diagonals AC and BD cross at O.

Calculate the size of the angle ADC.

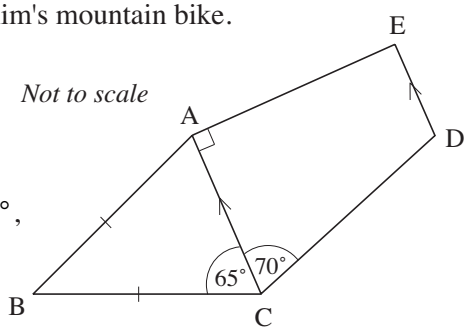


5. The pentagon ABCDE is the frame for Ibrahim's mountain bike.

ABC is an isosceles triangle in which
AB = BC and angle BCA = 65°.

In the quadrilateral ACDE angle ACD = 70°,
angle CAE = 90° and AC is parallel to ED.

Not to scale



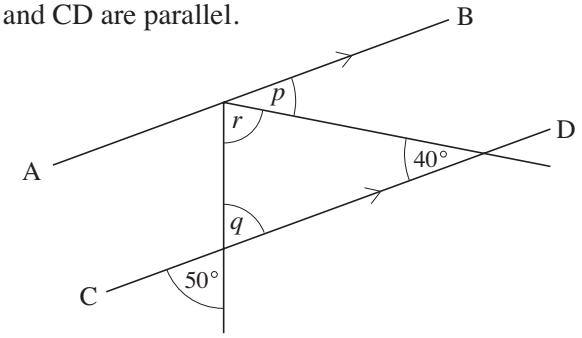
(a) (i) Calculate the size of angle ABC.

(ii) What facts about the angles of a triangle did you use in your calculation?

(b) Calculate the size of the angle CDE.

(MEG)

6. The lines AB and CD are parallel.



Not drawn accurately

(a) Write down the value of p. Give a reason for your answer.

(b) Write down the value of q. Give a reason for your answer.

(c) Work out the value of r.

(AQA)

7. PQR and STUV are parallel straight lines.

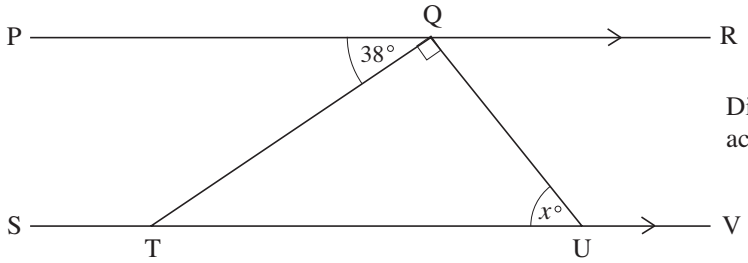


Diagram not accurately drawn

Work out the value of the angle marked $x°$. Give reasons for your answer.

(Edexcel)

3.5 Angle Symmetry in Polygons

1. Find the sum of the interior angles of

   (a) a quadrilateral (b) a pentagon.

2. Find the size of each interior angle of

   (a) a regular hexagon (b) a regular nonagon.

3. Find the number of sides of a polygon if the sum of its interior angles is

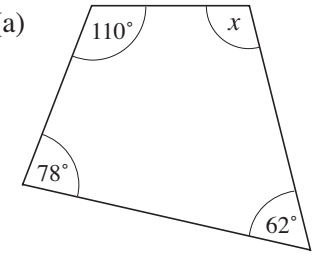
   (a) $1800°$ (b) $1080°$.

4. Each interior angle of a regular polygon is $140°$. Find the number of sides of the polygon.

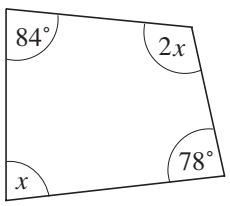
5. Each interior angle of a regular n-gon is $168°$. What is the value of n?

6. Find the value of x in each of the following diagrams.

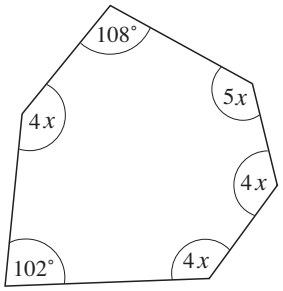
(a)



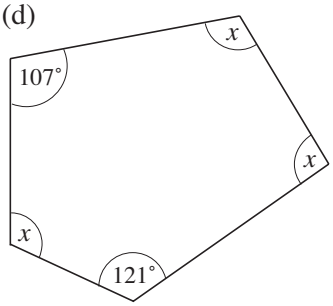
(b)



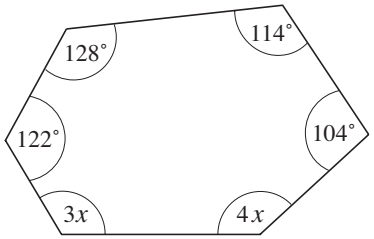
(c)



(d)



(e)



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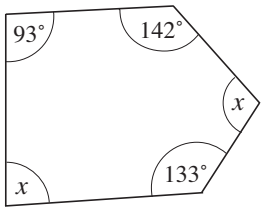
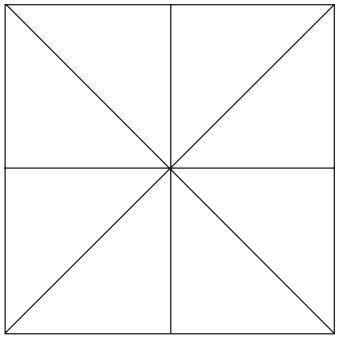
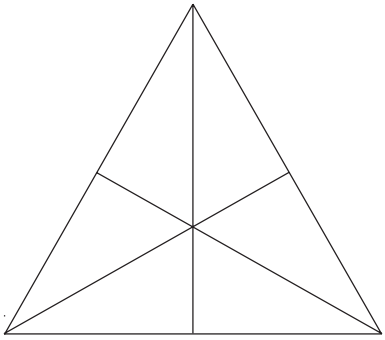


7. The angles of a quadrilaterial are $3x$, $4x$, $5x$ and $6x$.

(a) Find x. (b) What are the angles in degrees?

8.





(a) For each diagram above, show three different ways of shading parts of the shapes so that they have line symmetry but no rotational symmetry.

(b) Shade sections of one shape so that it has rotational symmetry of order 2 but no lines of symmetry. Is it possible to do this for both shapes?

(c) Repeat (b) for rotational symmetry of order 3.

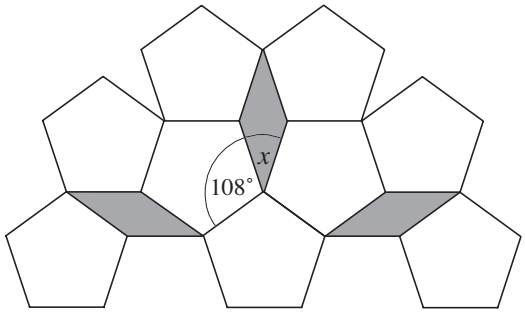
(d) Repeat (b) for rotational symmetry of order 4.

9. (a) A regular polygon has an interior angle of $175°$.
How many sides does it have?

(b) A second regular polygon has an interior angle which is $1°$ smaller.
How many sides does it have?

(c) Is it possible for a regular polygon to have an interior angle of $173°$?

10. (a) The diagram shows part of a tiling pattern of regular pentagons and another shape.

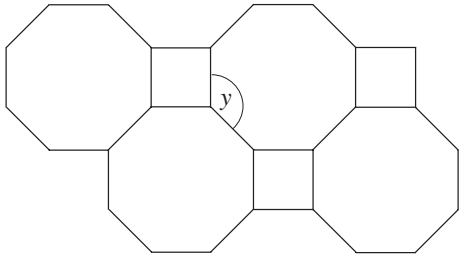


(i) Which of the following correctly describes the shaded shape:
square, rhombus, trapezium, rectangle, parallelogram, kite?

(ii) Calculate the size of the angle marked x.

(iii) A regular pentagon has rotational symmetry. What is the order of rotational symmetry of a regular pentagon?

(b) Another tiling pattern is formed using regular octagons and squares, as shown.

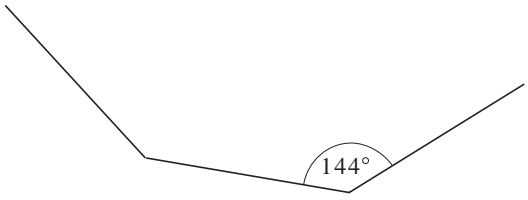


Calculate the size of the angle marked y.

(c) Draw a tiling pattern using regular hexagons only. You must draw at least five hexagons.

(SEG

11. The diagram shows part of a regular polygon. Each interior angle is $144°$.



Not drawn accurately

Calculate the size of the exterior angle of the polygon.

(AQA)

12. The diagram shows a 6-sided shape, ABCDEF. All the sides of the shape are equal in length.

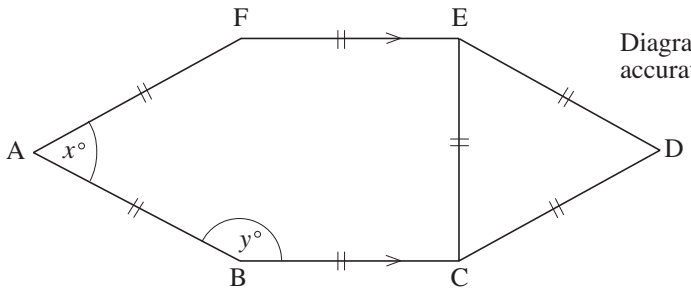


Diagram not accurately drawn

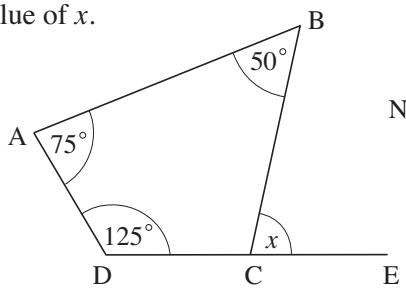
(a) (i) Find the value of x. (ii) Give a reason for your answer.

(b) Work out the value of y.

(Edexcel)

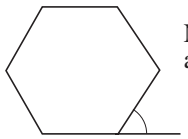
13. (a) ABCD is a quadrilateral. The side DC is extended to E.

Work out the value of x.



Not drawn accurately

(b) Calculate the size of the exterior angle of a regular hexagon.



Not drawn accurately

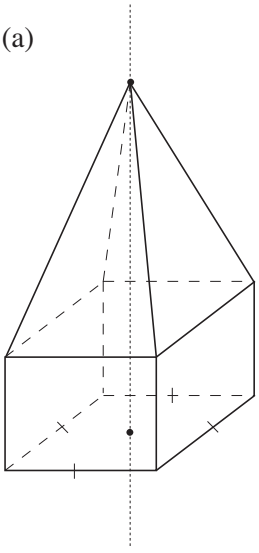
(AQA)

3.6 Symmetry Properties of 3D Shapes

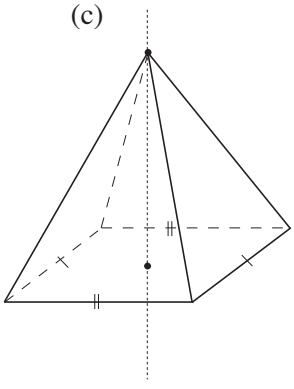
1. The following solids have rotational symmetry.

For each of them, state the order of rotational symmetry about the given axis.

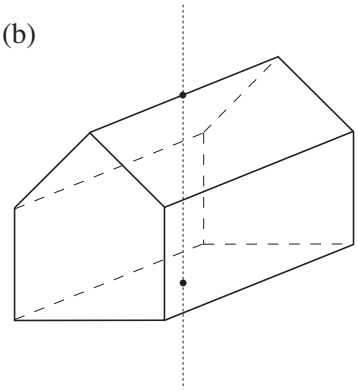
(a)



(c)

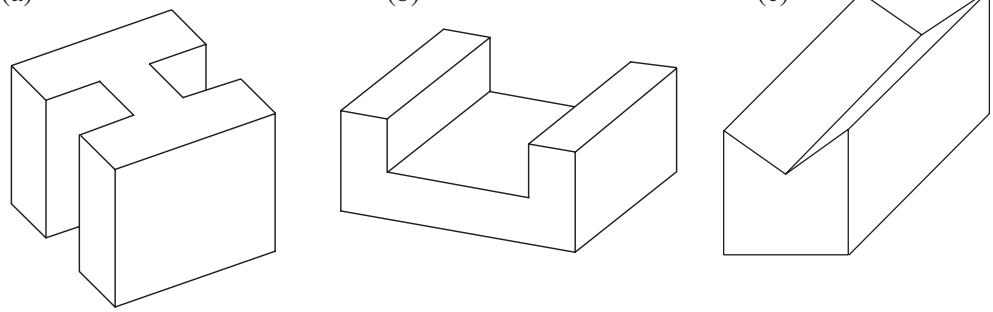


(b)

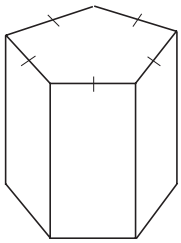


2. For each of the following prisms, draw an axis so that the order of rotational symmetry about that axis is 2.

(a) (b) (c)



3. In the given prism, the cross-section is in the shape of a regular pentagon. Draw

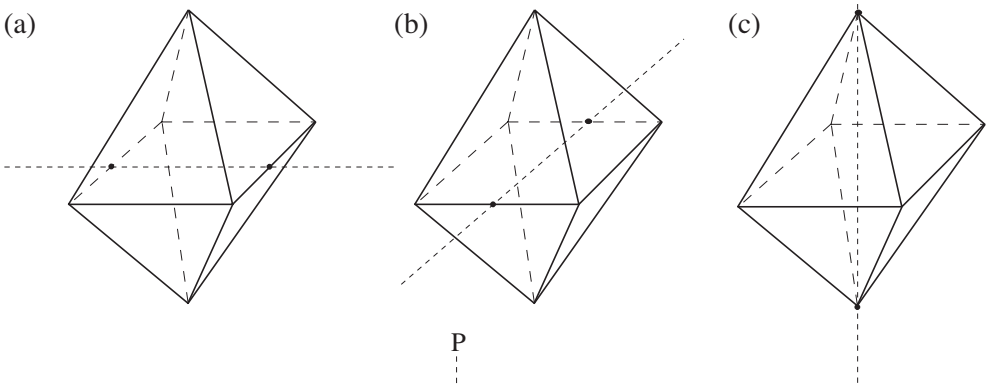


(a) an axis ST so that the order of rotational symmetry about ST is 2;

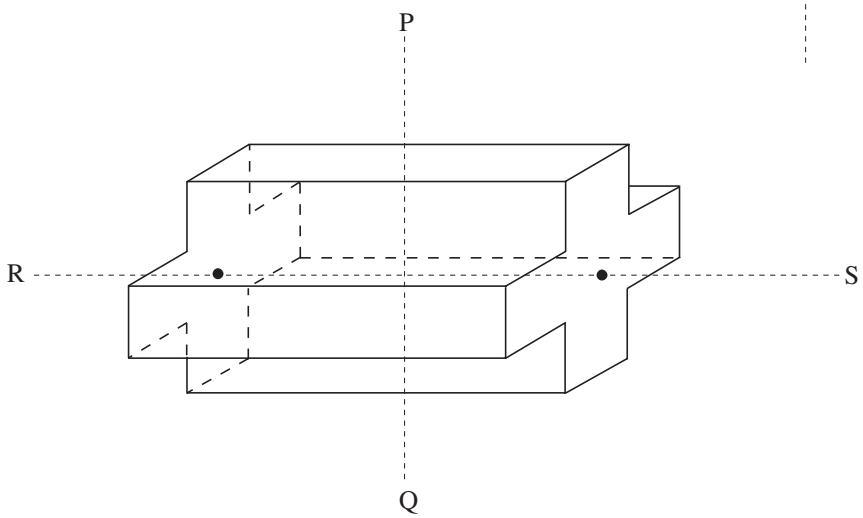
(b) an axis XY so that the order of rotational symmetry about XY is 5.

4. State the order of rotational symmetry about each of the axes shown. All the 12 edges of the solid are equal in length.

(a) (b) (c)



5.



For the solid above, find the order of its rotational symmetry about

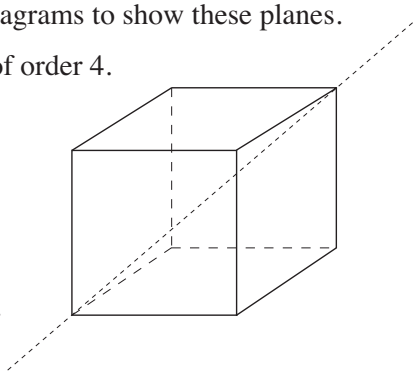
(a) PQ (b) RS.

6. (a) A cube has 9 planes of symmetry. Draw diagrams to show these planes.

(b) A cube has 3 axes of rotational symmetry of order 4. Draw diagrams to show these axes.

(c) The diagram of a cube opposite shows one axis of rotational symmetry of order 3. There are 3 other axes with the same order. Draw diagrams to show these axes.

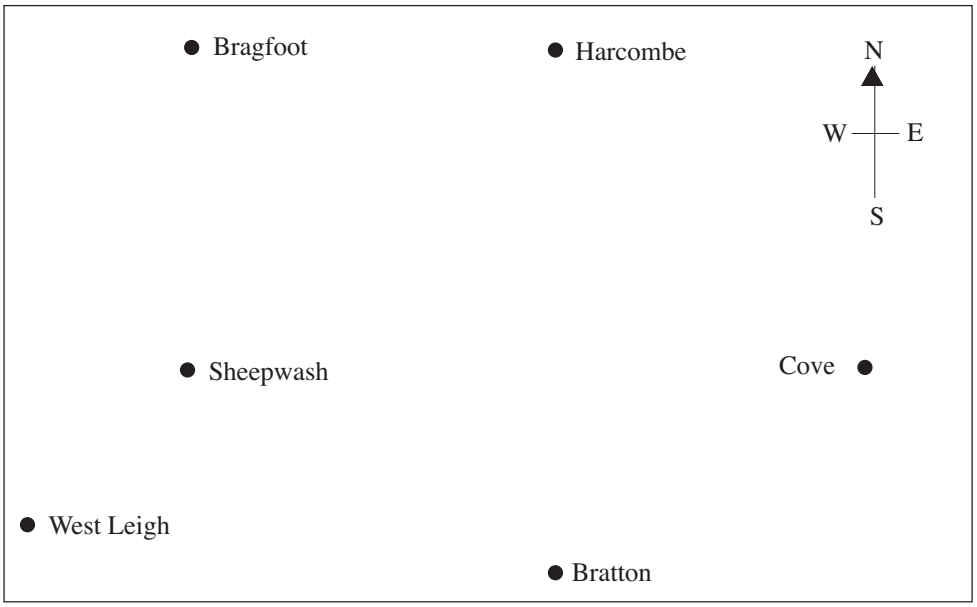
(d) There are 6 axes with symmetry of order 2. Draw diagrams to show these axes.



7. Draw a solid that has one axis of symmetry and rotational symmetry of order 5 about the axis.

3.7 Compass Bearings

1. The map below shows the positions of some villages.



Scale: 2 miles to 1 cm

(a) Which village is due north of *Sheepwash*?

(b) Which village is due west of *Cove*?

(c) What is the compass direction of *Sheepwash* from *West Leigh*?

(d) How many miles is

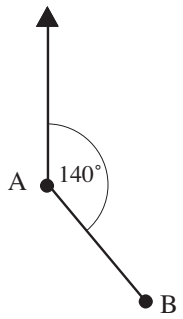
(i) *Bratton* from *Cove* (ii) *Harcombe* from *Bragfoot*?

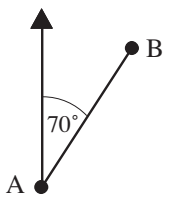
(e) Make a tracing of the map and mark the positions of

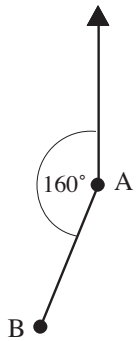
(i) *Darley*, which is 3 miles due south of *Harcombe*,

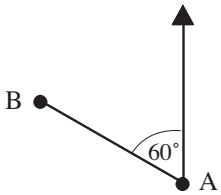
(ii) *Lee*, which is 4 miles south east of *Bragfoot*.

2. For each of the following, write down the bearing of B from A.

(a)


(b)


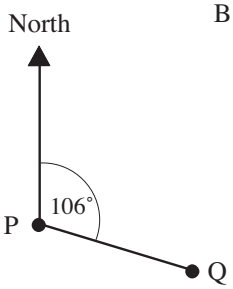
(c)


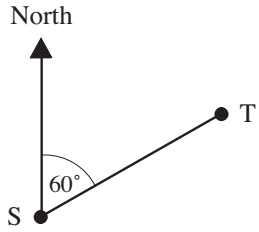
(d)


3. What is the bearing of

(a) Q from P

(b) P from Q?



4.


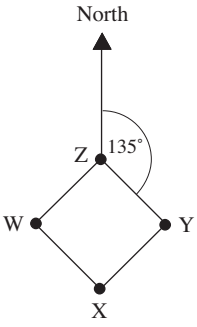
What is the bearing of

(a) T from S

(b) S from T?

5. Draw a diagram with 4 towns marked, so that that three of the towns are equidistant from the fourth town, P, and have bearings from P of

(a) 036° (b) 132° (c) 265°.

6.


A field is in the shape of a square, with corners W, X, Y and Z.

The bearing of Y from Z is 135°.

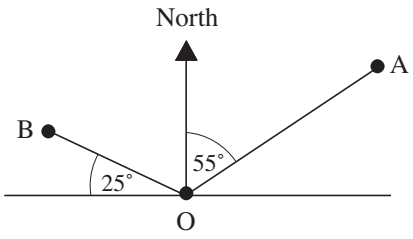
Find the bearing of

(a) Y from X (b) W from Z.

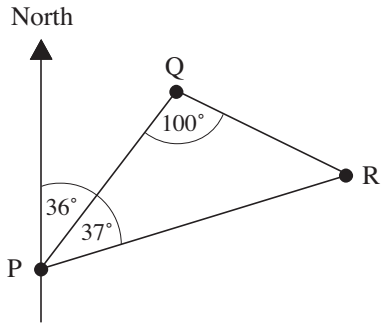
7. What is the bearing of

(a) Q from O (b) B from O

(c) O from A (d) O from B?



8. The figure shows the positions of P, Q and R.



What is the bearing of

(a) Q from P (b) P from Q

(c) R from P (d) P from R

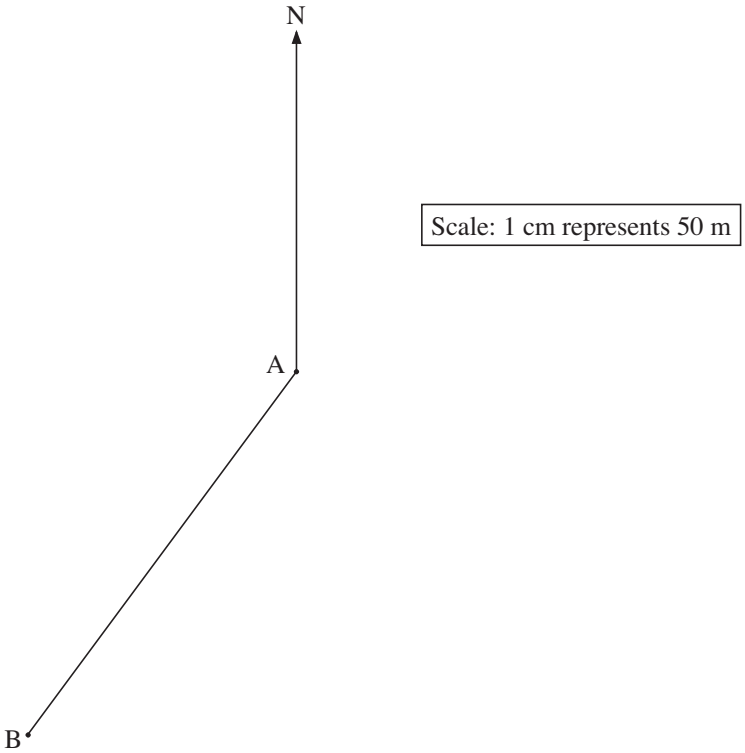
(e) Q from R (f) R from Q?

9. A point B is 280 m due North of the point A.
A man walks from A in the direction $050°$.
Calculate how far he walks before he is

(a) equidistant from A and B, (b) as close as possible to B,

(c) due east of B.

10. The diagram shows a scale drawing of one side, AB, of a triangular field, ABC.



(a) Use the diagram to calculate the actual distance fron A to B.

(b) Measure and write down the three figure bearing of B from A.

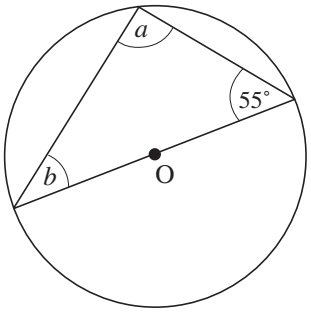
(c) The bearing of C from A is $130°$.
The actual distance from A to C is 350 metres.
Mark the point C on a copy of the diagram.

(AQA)

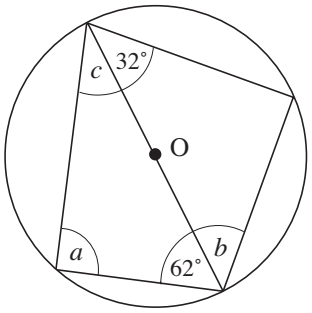
3.8 Angles and Circles 1

1. Find the angles marked with a letter in each of the following diagrams. (In each case O is the centre of the circle.)

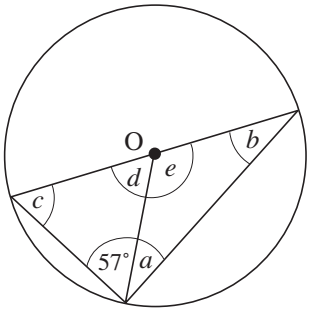
(a)



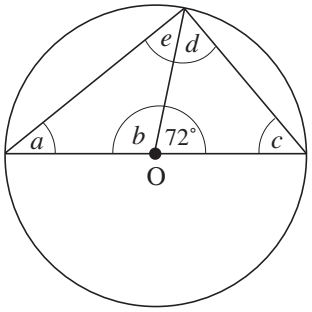
(b)



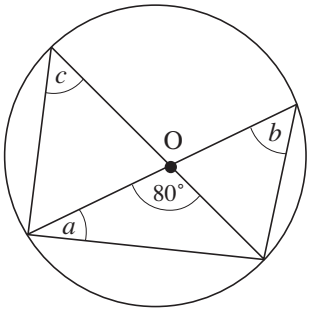
(c)



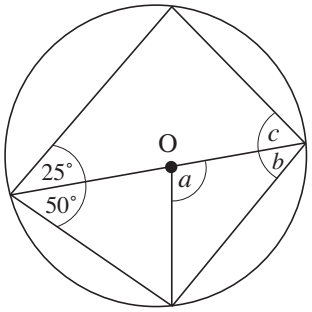
(d)



(e)

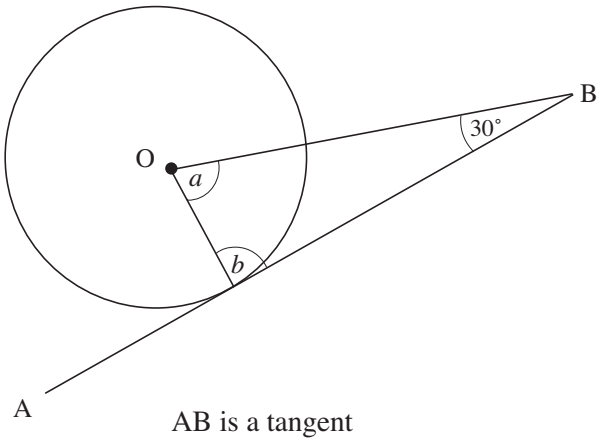


(f)



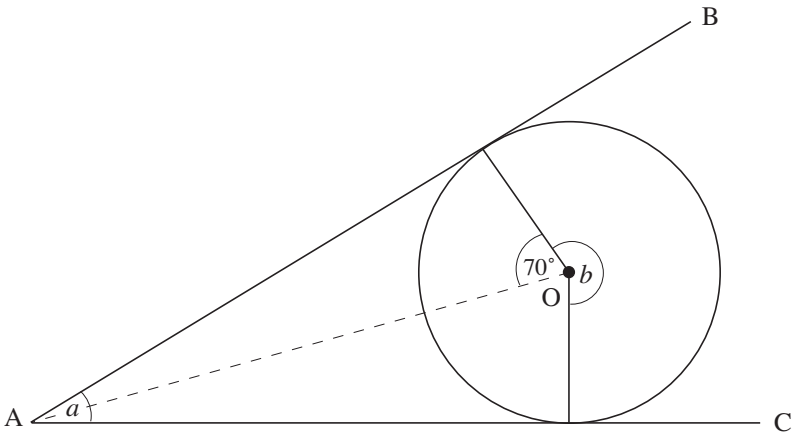
2. Find the angles marked with a letter in each diagram below. (In each case O is the centre of the circle.)

(a)



AB is a tangent

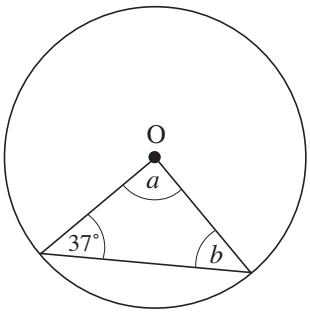
(b)



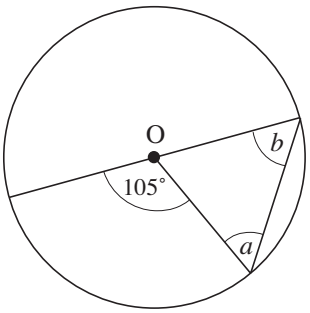
AB and AC are tangents

3. Find the angles marked with letters in each of the following diagrams. (In each case O is the centre of the circle.)

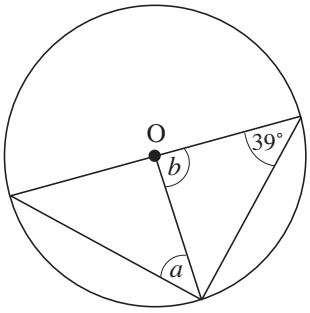
(a)



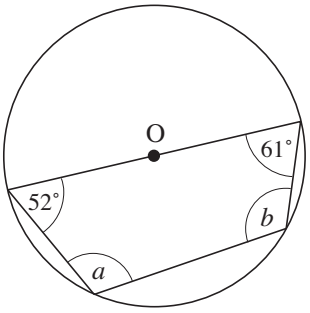
(b)



(c)

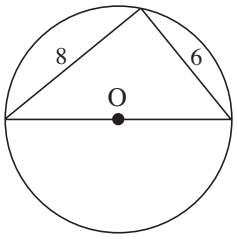


(d)

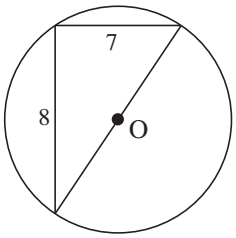


4. Find the diameter of each circle below. (In each case O is the centre of the circle.

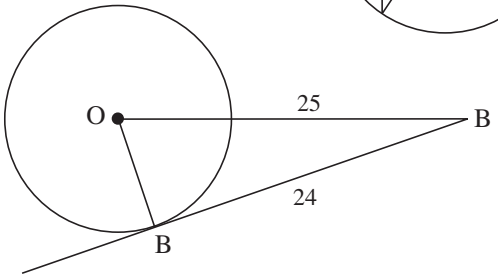
(a)



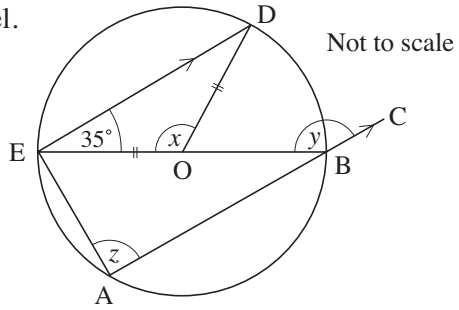
(b)



(c)



5. In the diagram, lines ABC and ED are parallel.
EOB is a diameter of the circle, centre O.
Angle OED = 35°



(a) Find the size of
(i) angle x
ii) angle y

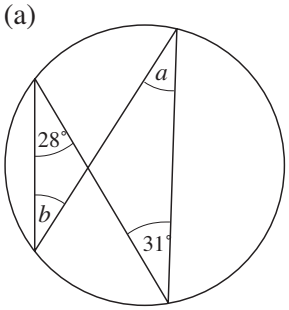
(b) Write down the size of angle z. Give a reason for your answer.

(OCR)

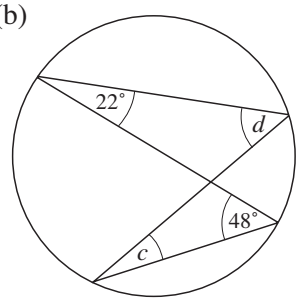
3.9 Angles and Circles 2

1. In each of the following circles, find the angles marked with letters.

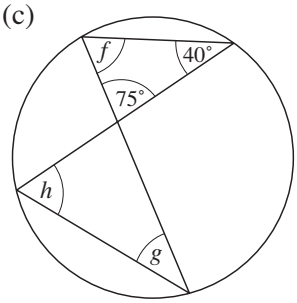
(a)



(b)

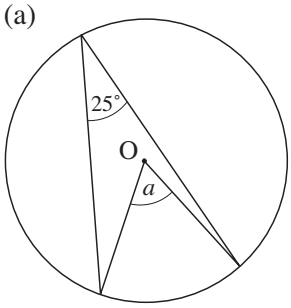


(c)

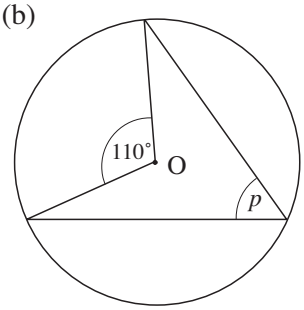


2. In each of the following circles, O is the centre. Find the angles marked with a letter.

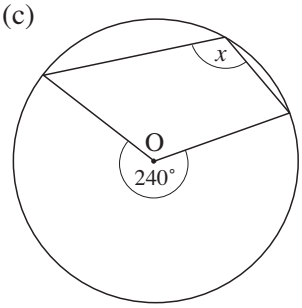
(a)



(b)

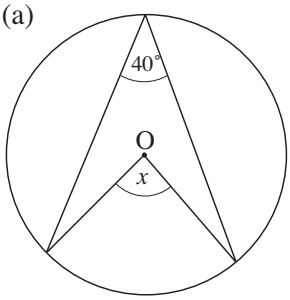


(c)

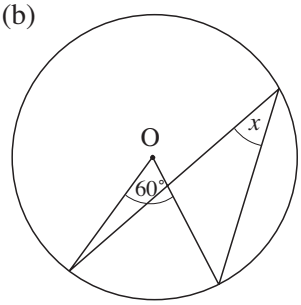


3. In each of the following figures, find the value of x. In each case, O is the centre of the circle .

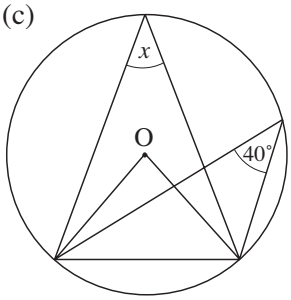
(a)



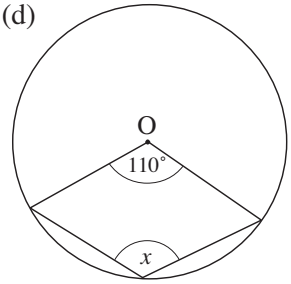
(b)



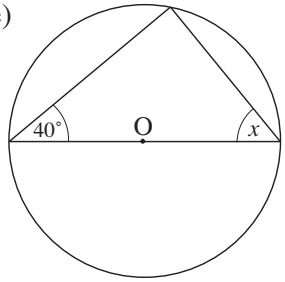
(c)



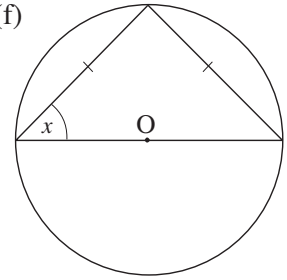
(d)



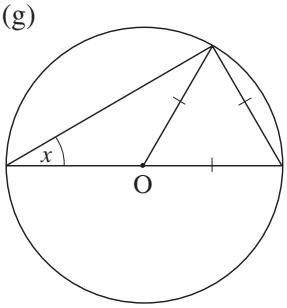
(e)



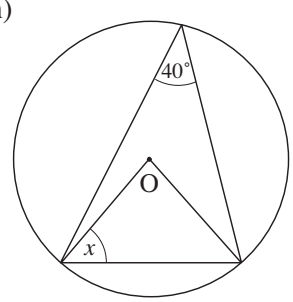
(f)



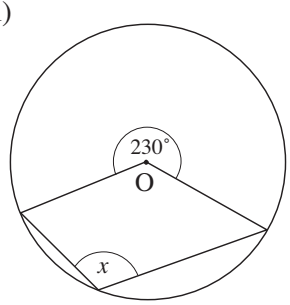
(g)



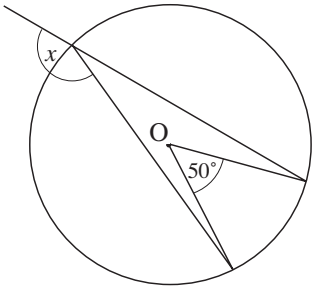
(h)



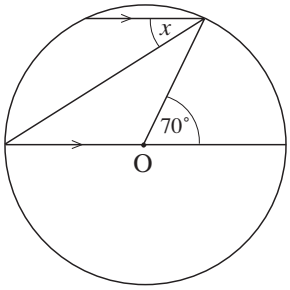
(i)



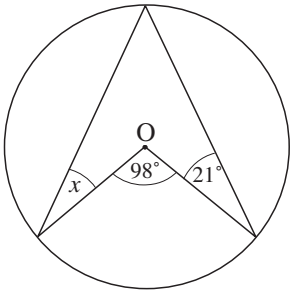
(j)



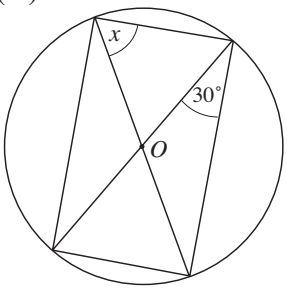
(k)



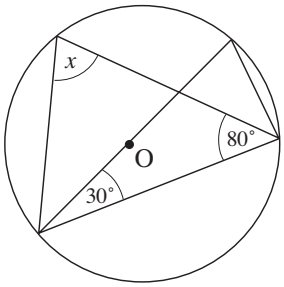
(l)



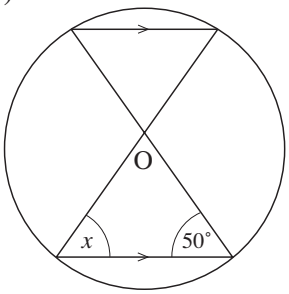
(m)



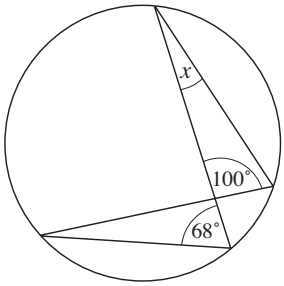
(n)



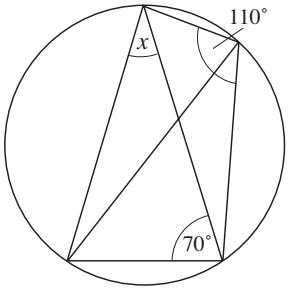
(o)



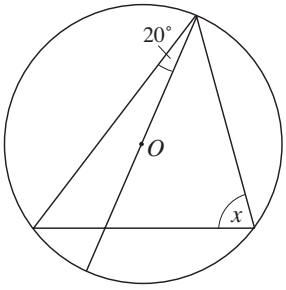
(p)



(q)



(r)

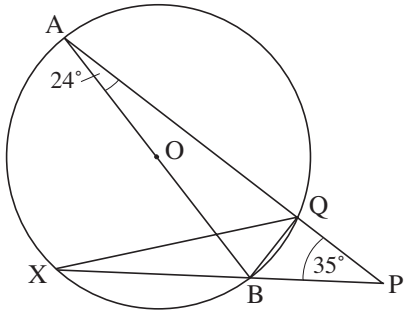


4. In the diagram, AB is a diameter of the circle.
Given that

angle BAP = 24°

and angle BPA = 35°,

find angle BQX.

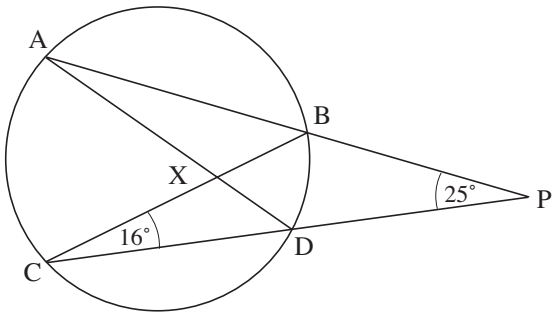


5. In the diagram,

$A\hat{P}C = 25°$

and $B\hat{C}D = 16°$.

Find $A\hat{X}B$.



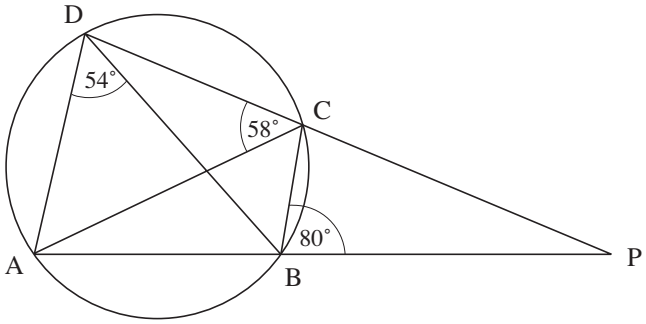
6. In the diagram,

$A\hat{D}B = 54°$

$A\hat{C}D = 58°$

and $C\hat{B}P = 80°$.

Find $A\hat{P}D$.



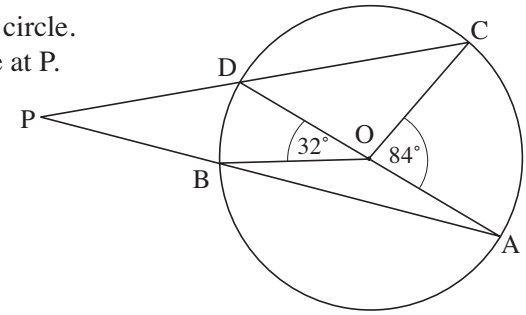
7. In the diagram, O is the centre of the circle.
AB ad CD intersect outside the circle at P.

Given that

$A\hat{O}C = 84°$

and $B\hat{O}D = 32°$,

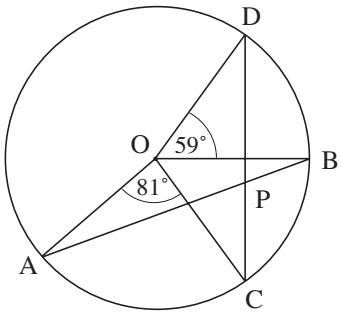
find $A\hat{P}C$.



8. In the diagram, O is the centre of the circle.
Chords AB and CD intersect inside the circle at P.
OB is perpendicular to CD.
Given that

$A\hat{O}C = 81°$ and $B\hat{O}D = 59°$,

find $A\hat{P}C$.



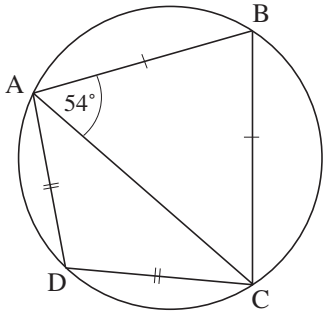
9. In the diagram,

$AB = BC$

$CD = DA$

and $B\hat{A}C = 54°$.

Find the value of $A\hat{C}D$.



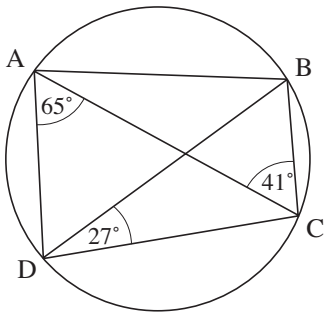
10. In the diagram,

$D\hat{A}C = 65°$

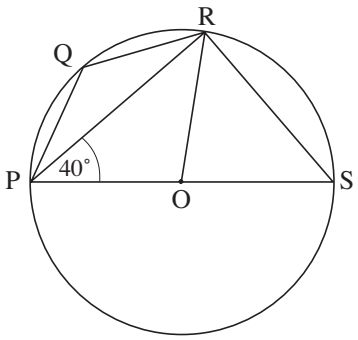
$A\hat{C}B = 41°$

and $B\hat{D}C = 27°$.

Find $A\hat{B}D$.



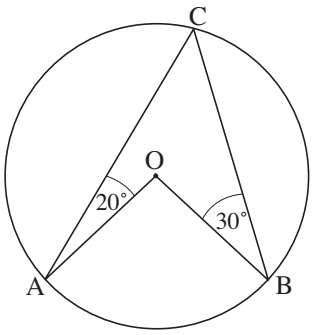
11.



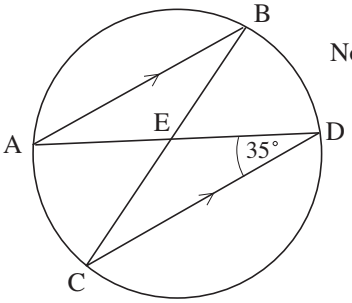
In the diagram, O is the centre of the circle and $R\hat{P}S = 40°$.

Calculate $P\hat{Q}R$ and $O\hat{R}S$.

12. In the figure, O is the centre of the circle, ABC.
Given that $C\hat{A}O = 20°$ and $C\hat{B}O = 30°$,
find $A\hat{C}B$.



13. A, B, C and D are points on a circle. AB is equal in length and parallel to CD. Lines AD and BC intersect at E. Angle EDC = 35°



Not to scale

(a) Write down the size of angle ABE.
Give a reason for your answer.

(b) (i) Find the size of angle AEC.
Show all your working clearly.

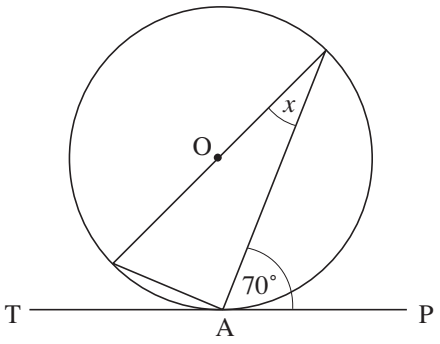
(ii) What does this tell you about point E?
Give a reason for your answer.

(OCR)

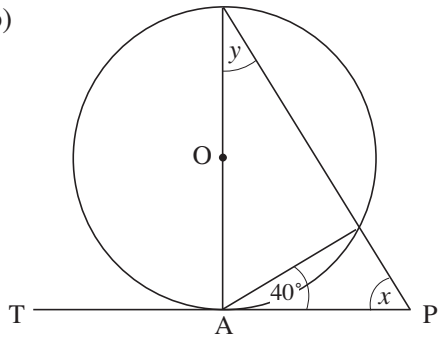
3.10 Circles and Tangents

1. Given that PAT is a tangent to the circle with centre O, find the values of x, y and z.

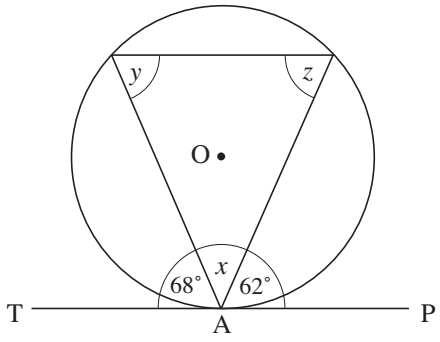
(a)



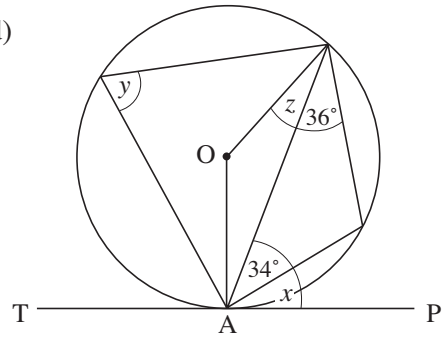
(b)



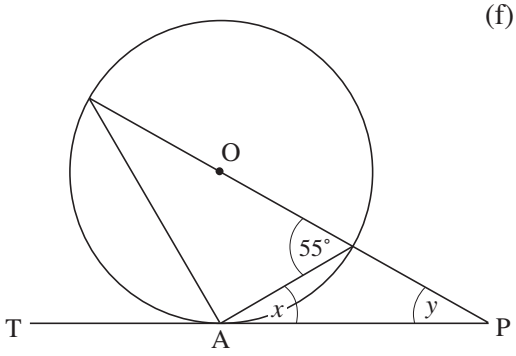
(c)



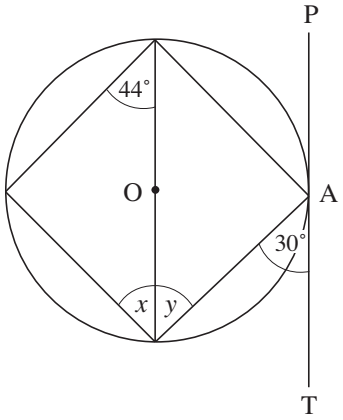
(d)



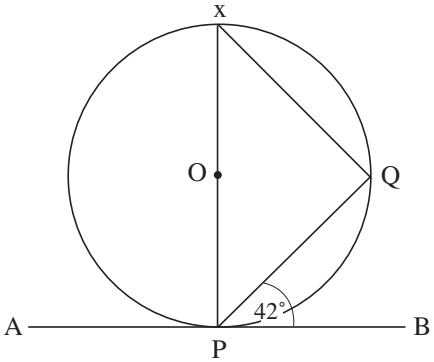
(e)



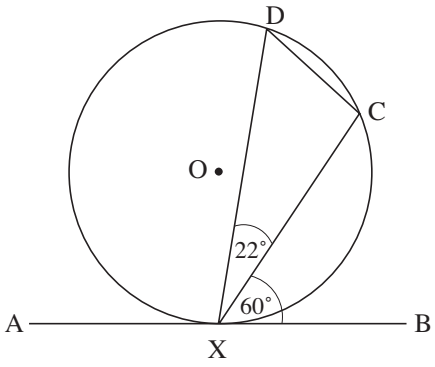
(f)



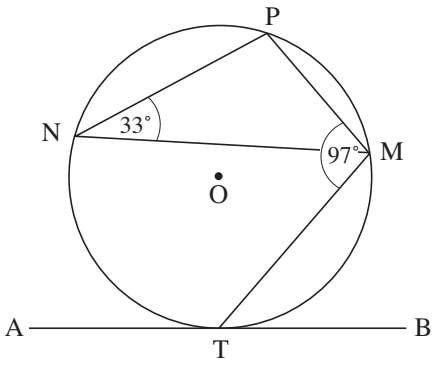
2. In the diagram, AB is the tangent to the circle at P and PX is a diameter.
Given that $B\hat{P}Q = 42°$, find $P\hat{Q}X$, $P\hat{X}Q$ and $X\hat{P}Q$.



3. In the diagram, O is the centre of the circle. AB is the tangent to the circle at X, $C\hat{X}B = 60°$ and $C\hat{X}D = 22°$. What is the size of $X\hat{C}D$?

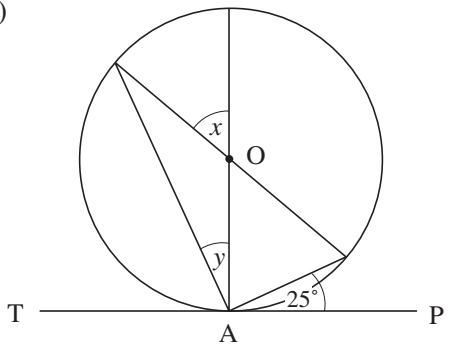


4. In the diagram, ATB is the tangent to the circle at point T. Given that $P\hat{N}M = 30°$ and $T\hat{M}P = 97°$, find $M\hat{T}B$.

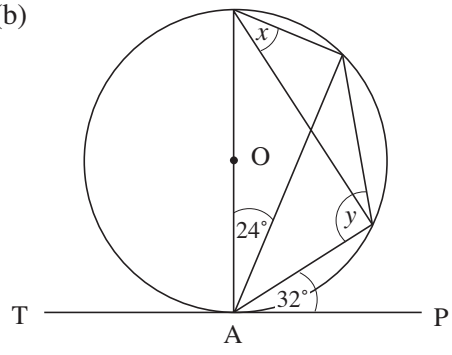


5. Given that PAT is a tangent at A to the circle with the centre O, find the value of x and of y in each case.

(a)



(b)



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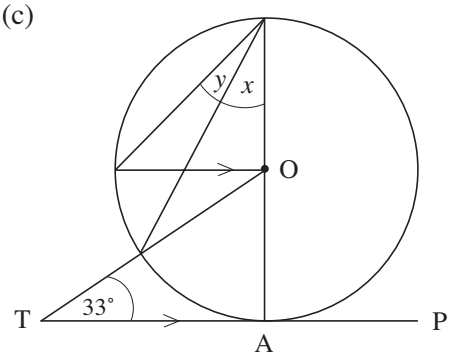
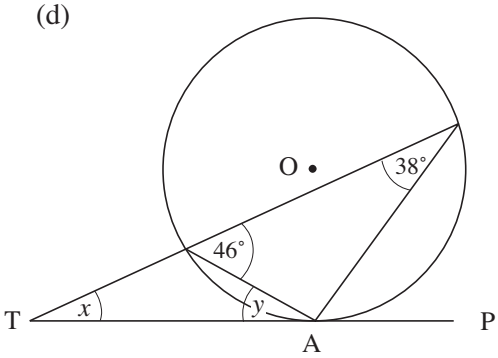


(d)



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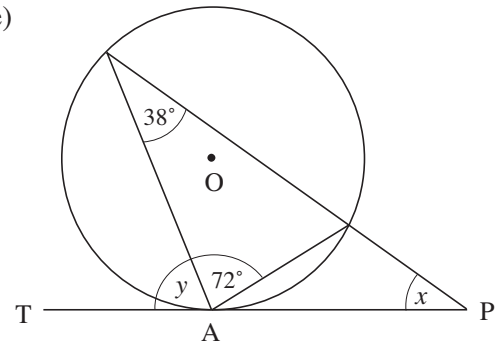

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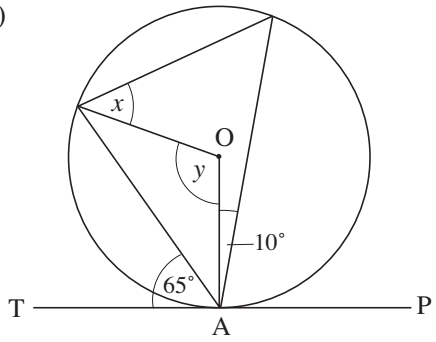

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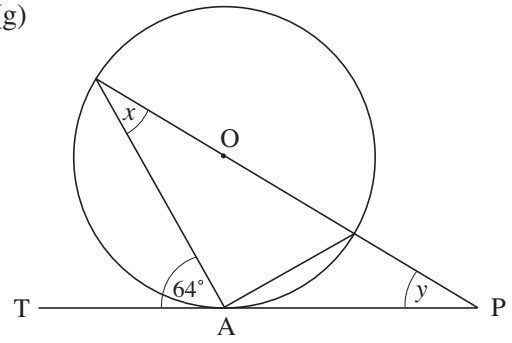

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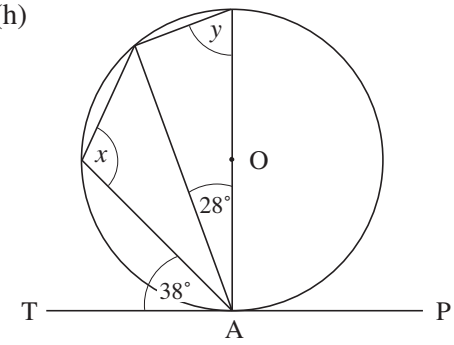
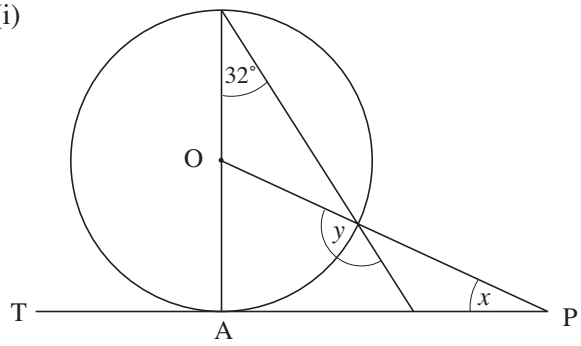
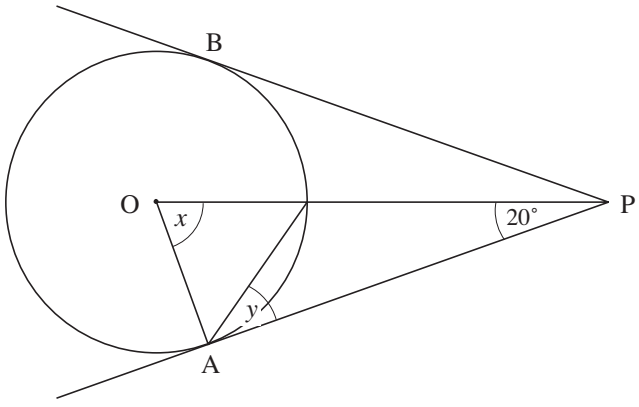


(i)

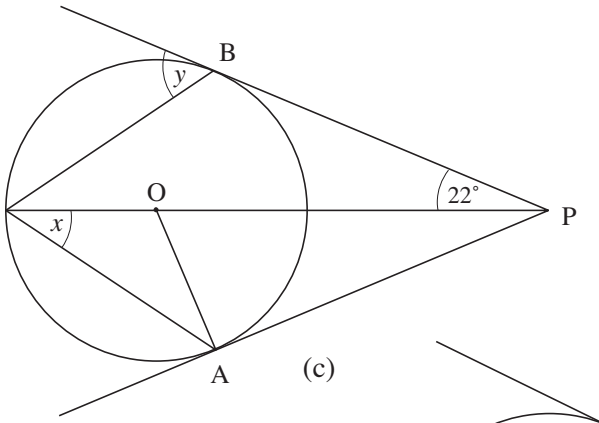


6. Given that PA and PB are tangents to the circle with centre O, find the value of x and of y in each case.

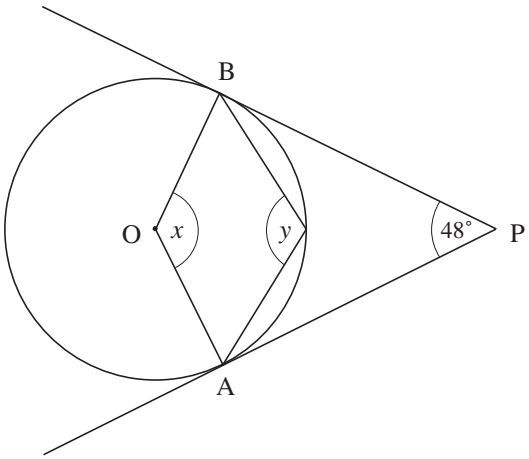
(a)



(b)



(c)



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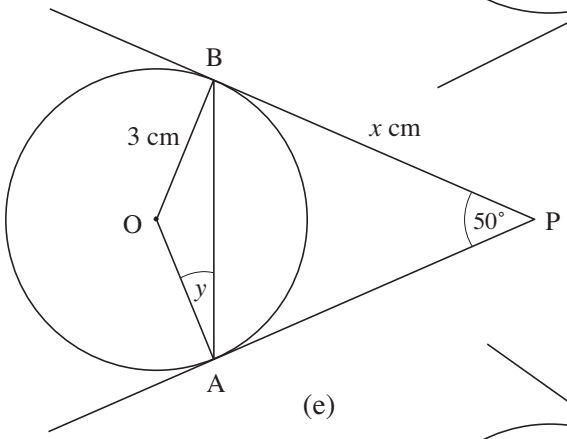

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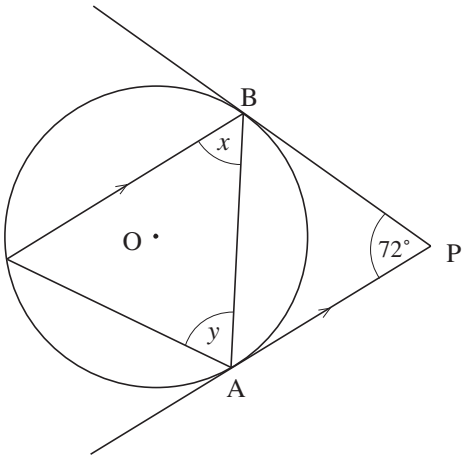

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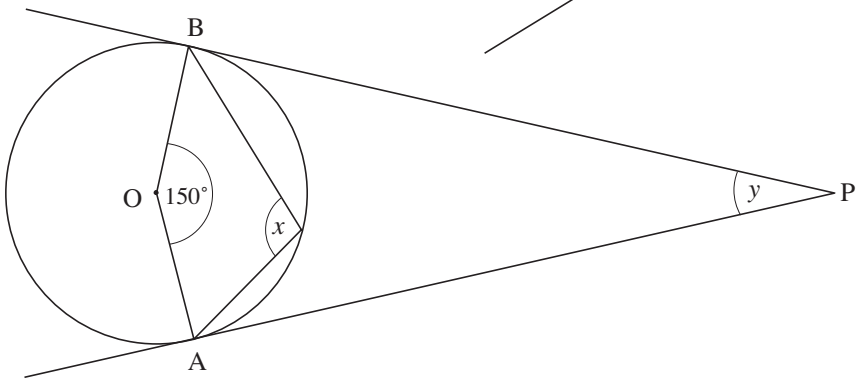
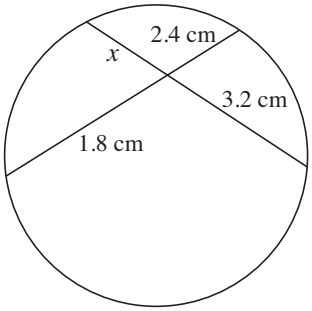
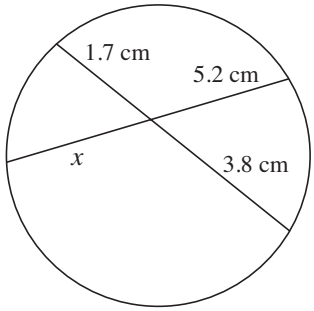


7. Find the length x in each case.

(a)



(b)



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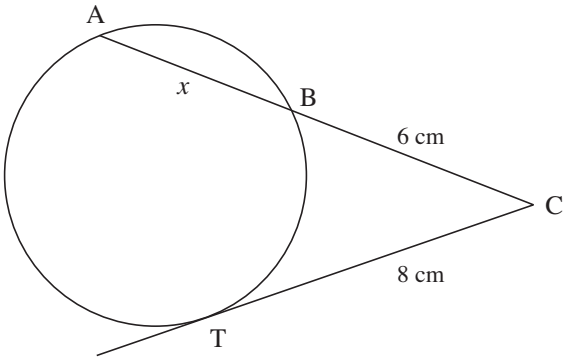
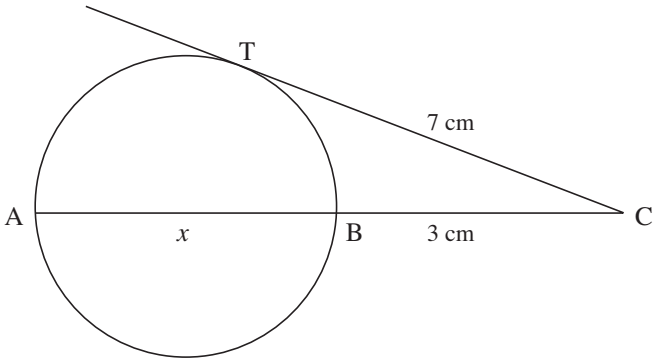


TC is a tangent.

(d)

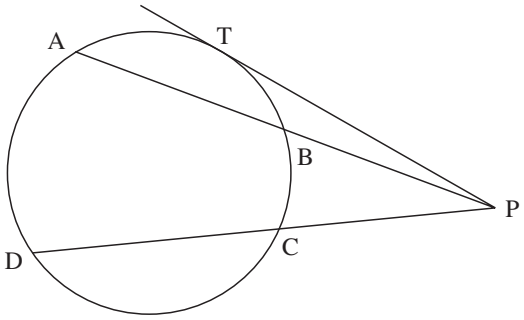


TC is a tangent.

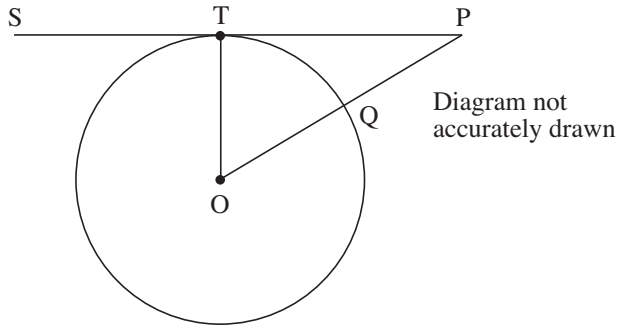
8. If BP = 8 cm, DC = 7 cm and CP = 9 cm, calculate the lengths of

(a) chord AB

(b) tangent PT.



9. STP is a tangent to the circle, centre O. Q is a point on the circumference of the circle. OQP is a straight line. OP = 26 cm and TP = 24 cm.



(a) Angle OTP = 90° Give a reason why.

(b) Work out the radius OQ of the circle.

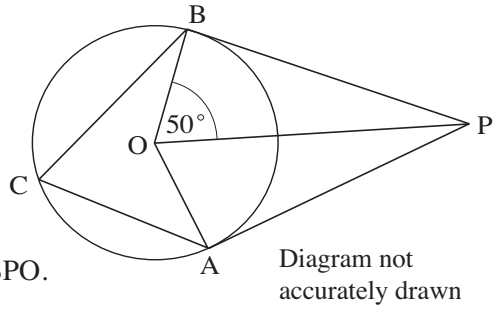
(c) Work out the area of the circle. Give your answer correct to 3 significant figures.

(Edexcel)

10. In the diagram, A, B and C are points on the circumference of a circle, centre O.

PA and PB are tangents to the circle.

Angle POB = 50°.



(a) (i) Work out the size of angle BPO.

(ii) Give a reason for your answer.

(b) (i) Work out the size of angle ACB.

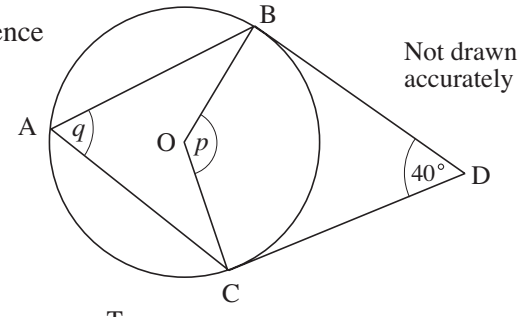
(ii) Give a reason for your answer.

(Edexcel)

11. A, B and C are points on the circumference of a circle with centre O.

BD and CD are tangents.

Angle BDC = 40°.



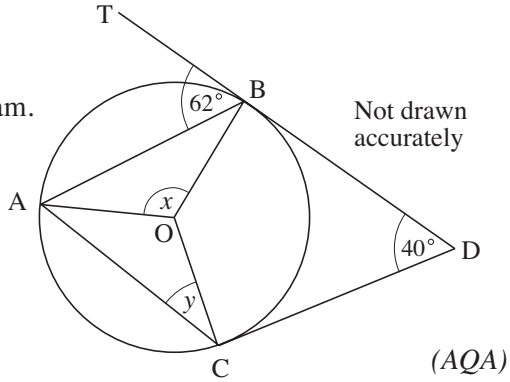
(a) (i) Work out the value of p.

(ii) Hence write down the value of q.

(b) The tangent DB is extended to T.

The line AO is added to the diagram.

Angle TBA = 62°.

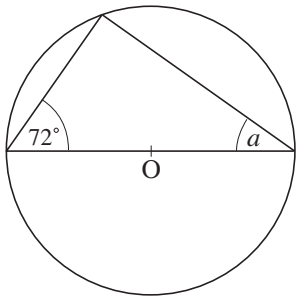


(i) Work out the value of x.

(ii) Work out the value of y.

(AQA)

12. (a)



Not drawn accurately

O is the centre of the circle.

Calculate the value of a.

(b) O is the centre of the circle.

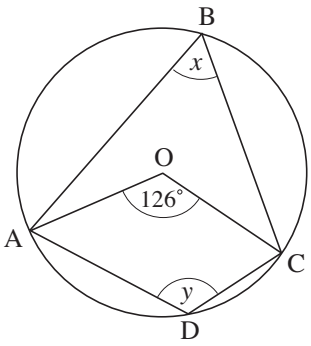
A, B, C and D are points on the circumference.

Angle AOC = 126°

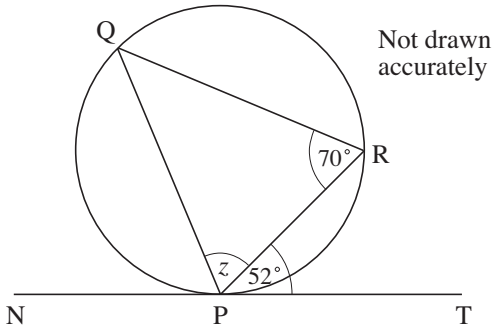
(i) Calculate the value of x.

(ii) Calculate the value of y.

Not drawn accurately



(c) P, Q and R are points on the circumference of the circle.
NPT is the tangent to the circle at P.



Calculate the value of z. Give a reason for each step of your working.

(AQA)