UNIT 6 Arithmetic: Multiplication of Decimals Activities

Activities

6.1 Secret Sums
6.2 Egyptian Multiplication
6.3 Russian Multiplication
6.4 Multiplying using Roman Numerals
6.5 Napier's Bones (or Rods)
6.5 Resource Sheet
    Notes and Solutions (2 pages)
1. Find the missing numbers, marked by * in the following sums:

   (a) \[ 2 * 3 \]
   \[ + 1 7 * \]
   \[ * 6 5 \]

   (b) \[ 4 * 7 \]
   \[ - * 1 8 \]
   \[ 1 7 * \]

   (c) \[ 7 * 0 \]
   \[ - 3 6 * \]
   \[ * 3 8 \]

   (d) \[ 3 7 \]
   \[ \times * \]
   \[ * 8 \]

   (e) \[ 1 * 4 \]
   \[ \times * \]
   \[ 4 0 2 \]

2. Find two possible solutions of:

   \[ * 2 * \]
   \[ \times 2 \]
   \[ 8 * 6 \]

3. Find the missing numbers in the sums:

   (a) \[ * 2 * \]
   \[ \times * 7 \]
   \[ 8 * 9 \]

   (b) \[ * 3 \]
   \[ 5 * 1 * \]
   \[ 2 0 \]

   (c) \[ 2 * 6 \]
   \[ \times * 4 \]
   \[ \times * 6 * \]

   (There are 2 possible solutions to this sum!)

4. The 10 letters A to K, leaving out I, stand for the 10 digits 0 – 9, but not necessarily in that order. Find which letter stands for each digit, if the following sums hold:

   \[ A \times B = B \]
   \[ F \times H = CJ \]

   \[ B \times C = AC \]
   \[ H \times J = KJ \]

   \[ C \times D = BC \]
   \[ J \times K = E \]

   \[ D \times E = CH \]
   \[ K \times G = G \]

   \[ E \times F = DK \]
   \[ A \times G = G \]

**Extension**

Find the missing numbers for the sum:

\[ *** \]
\[ * 9 *** \]
\[ ** \]
\[ * * * \]
\[ 2 * * \]
\[ * * * \]
ACTIVITY 6.2  

Egyptian Multiplication

The Egyptian method for multiplication was based simply on a continual doubling process. For example, to multiply 27 by 137, follow these instructions:

1. In two columns write down 1 and 137 (always choose the larger number)
   
   | Line 1 |
   | 1 137 |
   |
   | Line 2 |
   | 2 274 |
   |
   | Line 3 |
   | 4 548 |
   |
   | Line 4 |
   | 8 1096 |
   |
   | Line 5 |
   | 16 2192 |

2. Double both sides until the number 27 will be passed on the left hand side on the next double (e.g. $16 \times 2 = 32$ so go no further than Line 5)

3. Select on the left hand side the numbers that add up to 27

4. Delete any number not used in the addition to 27 (i.e. 4), and the corresponding number on the right hand side (see Line 3)

5. Add up the numbers remaining on the right hand side

6. This is the answer, i.e.

   $27 \times 137 = 3699$

Problems

Use Egyptian multiplication to find:

1. $13 \times 250$
2. $16 \times 135$
3. $25 \times 49$

Extension

Analyse the method to see why it works.

(Hint: Write $27 \times 137 = (16 + 8 + 2 + 1) \times 137$

$= (16 \times 137) + (8 \times 37) + (2 \times 137) + (1 \times 137)$)
ACTIVITY 6.3  

Russian Multiplication

One upon a time, so legend has it, Russian peasants could only add and multiply or divide by 2. So they developed a clever method of multiplying any two numbers.

For example, to multiply 27 by 137, they followed this method:

1. In two columns write down the numbers  
   | 27 | 137  | Line 1 |

2. Divide the left hand column by 2 ignoring any remainders, and multiply the right hand column by 2  
   | 13 | 274  | Line 2 |
   | even 6 | 548 | Line 3 |

3. Repeat this process until the number 1 is reached in the left hand column (Line 5)  
   | 3   | 1096 | Line 4 |
   | 1   | 2192 | Line 5 |

4. Delete any row which has an even number in the left hand column (Line 3)

5. Add up the numbers remaining in the right hand column (Lines 1, 2, 4 and 5)  
   3699

6. Check the answer – it should be 27 × 137

Problems

Use Russian multiplication to find:

1. 13 × 250
2. 16 × 135
3. 25 × 49

Extension  

Analyse the method to see why it works.

(Hint: Write 27 × 137 = (26 + 1) × 137

= 26 × 137 + 137
= 13 × 2 × 137 + 137
= 13 × 274 + 137
= (12 + 1) × 274 + 137, etc.)
ACTIVITY 6.4  Multiplication Using Roman Numerals

You may be familiar with Roman numerals, but, in case not, the first twenty numbers are shown on the right. Note that:

'IV' means 1 before 5, i.e. 4

and that the system is based on '5' rather than '10'.

The next symbols used are:

<table>
<thead>
<tr>
<th>Number</th>
<th>Roman Numeral</th>
</tr>
</thead>
<tbody>
<tr>
<td>100</td>
<td>C</td>
</tr>
<tr>
<td>500</td>
<td>D</td>
</tr>
<tr>
<td>1000</td>
<td>M</td>
</tr>
</tbody>
</table>

Problems

1. Write out Roman numerals for 21 to 50 inclusive.

2. What is 137 in Roman numerals?

We will now see how to multiply two numbers, expressed in Roman numerals, together. But first, some important multiplication.

3. What is:
   (a) I × V   (b) V × V   (c) V × X   (d) V × L   (e) I × X
   (f) X × X   (g) L × X   (h) I × C   (i) V × C   (j) X × C

4. For 27 × 137, copy and complete this long multiplication calculation:

   C X X X V I I
   X X V I I
   C X X X V I I × I →
   C X X X V I I × I →
   C X X X V I I × V →
   C X X X V I I × X →
   C X X X V I I × X →
   Add →

   (Check that your answer is correct.)

5. Calculate 16 × 135 using Roman numerals. Check your answer.

You probably know now why this system is no longer in everyday use for calculations of this type!
ACTIVITY 6.5  
Napier's Bones (or Rods)

You will need a copy of the Resource Sheet in order to first obtain a set of Napier's Bones. Note how they are constructed. The first row is the whole numbers 1 to 9, and the following rows are $2 \times$, $3 \times$, $4 \times$, \ldots, $9 \times$ the first row, but note that two-digit numbers are placed either side of the diagonals. From your copy of the sheet, cut out each column – these are the bones or rods!

To multiply, for example, $137 \times 27$:

1. Pick out the rods which start with 1, 3 and 7

2. Pick out the 2nd row, and add up along the diagonals

   
   \[ \begin{array}{ccc}
   2 & 6 & 1 \\
   2 & 7 & 4
   \end{array} \]

   This shows that $2 \times 137 = 274$

3. Pick out the 7th row, and add up along the diagonals

   
   \[ \begin{array}{ccc}
   7 & 2 & 1 \\
   9 & 5 & 9
   \end{array} \]

   This shows that $7 \times 137 = 959$

4. As we want $27 \times 137$, the final calculation is

   
   \[
   27 \times 137 = (20 \times 137) + (7 \times 137) \\
   = 2740 + 959 \\
   = 3699
   \]

Problems

Use your Napier's Rods to find:

1. $16 \times 135$
2. $25 \times 49$
3. $13 \times 250$

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Napier's Bones (or Rods)
**ACTIVITIES 6.1 - 6.3**

*Notes and Solutions*

*Notes and solutions are given only where appropriate.*

6.1

1. (a) 293 + 172 = 465
   (b) 497 – 318 = 179
   (c) 700 – 362 = 338
   (d) 37 × 4 = 148
   (e) 134 × 3 = 402

2. 423 × 2 = 846
   428 × 2 = 856

3. (a) 127 × 27 = 3429
   (b) 43 × 52 = 2215
   (c) 216 × 74 or 216 × 24
       15984
   5184

4. A = 1, B = 3, C = 5, D = 7, E = 8,
   F = 9, G = 0, H = 6, J = 4, K = 2

**Extension** 19 107 divided by 99

6.2

1. 3250
2. 2160
3. 1225

6.3

1. 3250
2. 2160
3. 1225
ACTIVITIES 6.4 - 6.5

Notes and Solutions

6.4

<p>| | | |</p>
<table>
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<td>37</td>
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<td>23</td>
<td>XXIII</td>
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<tr>
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<td>XXXIV</td>
<td>49</td>
</tr>
<tr>
<td>35</td>
<td>XXXV</td>
<td>50</td>
</tr>
</tbody>
</table>

2. C X X X V I I

3. (a) V (b) X X V (c) L (d) C C L (e) X
   (f) C (g) D (h) C (i) D (j) M

4. C X X X V I I

   C X X X V I I \times I \rightarrow C X X X V I I
   C X X X V I I \times I \rightarrow C X X X V I I
   C X X X V I I \times V \rightarrow D L L L (X X V) V V
   C X X X V I I \times X \rightarrow M C C C L X X
   C X X X V I I \times X \rightarrow M C C C L X X
   M M M D C I C (after adding and simplifying!)

5. M M C L X

6.5

1. 2160
2. 1225
3. 3250