UNIT 12 *Formulae* Activities

Activities

ACTIVITY 12.1 *Codebreaking*

In the code shown on the right,

if

$$a = 1,\ \ b = 2,\ \ c = 3 \text{ and } d = 4,$$

then

$$a + b + c + d = 1 + 2 + 3 + 4$$
$$= 10$$

So $a + b + c + d$ is code for the letter J, which corresponds with the number 10 in the code.

1	A
2	B
3	C
4	D
5	E
6	F
7	G
8	H
9	I
10	J
11	K
12	L
13	M
14	N
15	O
16	P
17	Q
18	R
19	S
20	T
21	U
22	V
23	W
24	X
25	Y
26	Z

1. Using the code shown above, decode the following message:

bd

$a + d$

$d(a + b)$

cd

$cd + b + a$

$a + c + d$

$c - b$

$cd + b$

$b(c + d)$

$(a + c) \div d$

$d(c - a)$

2. Write a message using this code, and ask a friend to decode it.

Extension

Use a different set of values for a, b, c and d, and see if you can still code each letter of the alphabet.

ACTIVITY 12.2 *One to Twenty Eight*

If $a = 1,\ b = 2,\ c = 7$ and $d = 9$, it is possible to make each of the numbers from 1 to 40 by using a sum that contains *all four* letters.

For example,

$$\frac{b+c}{ad} = \frac{2+7}{1\times 9} = \frac{9}{9} = 1$$

and

$$\frac{d-c}{b-a} = \frac{9-7}{2-1} = \frac{2}{1} = 2$$

and

$$\frac{c(d-a)}{b} = \frac{7\times(9-1)}{2} = \frac{7\times 8}{2} = 28$$

Find formulae for the other whole numbers from 3 to 27.

Extension

Try the same problem, but with different values for a, b, c and d.

ACTIVITY 12.3 *Odds and Evens*

General Instructions for Games 1 – 3.

(It might be useful to use overhead slides of the grids for this activity.)

Two teams are selected and are given the names 'Odds' (O) and 'Evens' (E).

Each team chooses and solves an equation in the table.

When it is solved the letter O or E is written over the equation.

The first team to get 4 of their letters in a line is the winner.

ACTIVITY 12.3. Game 1 *Odds and Evens*

Game 1

$x + 7 = 10$	$x + 1 = 14$	$x + 2 = 5$	$x - 3 = 2$	$x + 6 = 8$
$x - 3 = 12$	$x + 3 = 9$	$x + 4 = 11$	$x - 2 = 12$	$x + 8 = 10$
$x + 7 = 14$	$x + 9 = 14$	$x - 9 = 11$	$x - 3 = 18$	$x - 4 = 21$
$6 - x = 1$	$x + 2 = 14$	$x + 9 = 100$	$x + 20 = 30$	$x - 30 = 21$
$3 + x = 12$	$x + 8 = 40$	$x - 3 = 11$	$x + 9 = 21$	$x - 7 = 15$

ACTIVITY 12.3. Game 2 *Odds and Evens*

Game 2

$x + 5 = 6$	$2x + 1 = 15$	$7 + x = 14$	$7x = 21$	$x - 2 = 8$
$8x = 48$	$2x + 4 = 12$	$3x + 10 = 19$	$4x = 40$	$2x + 4 = 10$
$x - 3 = 8$	$4x = 32$	$3x + 4 = 10$	$3x + 5 = 14$	$7x = 35$
$3x = 18$	$x + 3 = 19$	$9x = 36$	$5x = 30$	$x - 7 = 8$
$x + 7 = 8$	$x + 2 = 20$	$7x = 49$	$x + 12 = 20$	$5x + 2 = 27$

ACTIVITY 12.3. Game 3 *Odds and Evens*

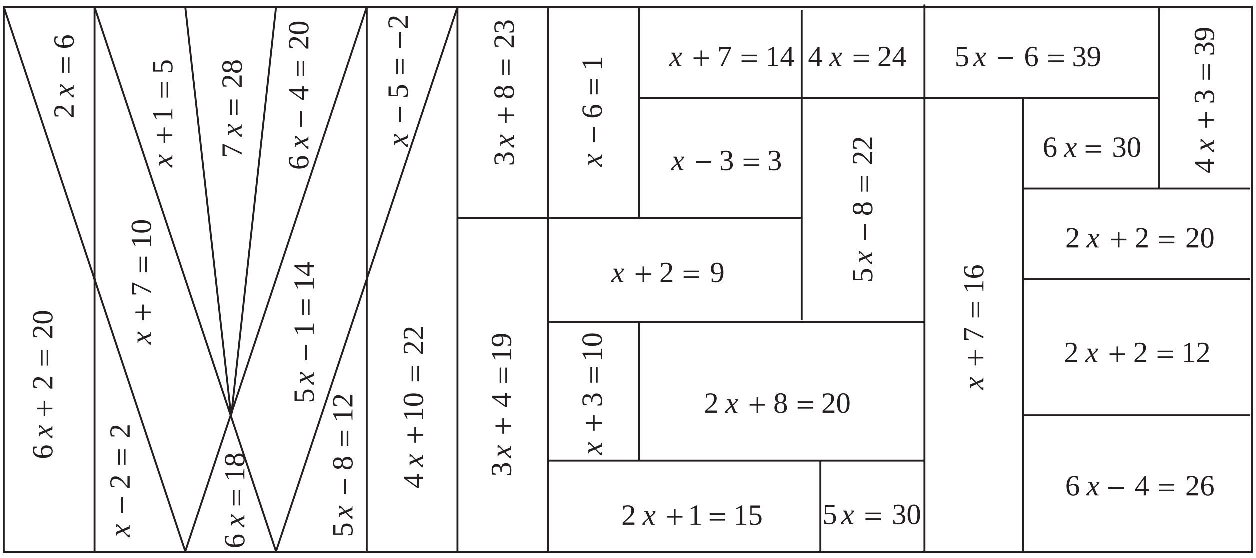
Game 3

$3x = 36$	$5x + 3 = 13$	$x + 8 = 22$	$x + 9 = 14$	$4x = 28$
$6x - 2 = 16$	$2x - 1 = 7$	$2x + 3 = 17$	$4x - 5 = 15$	$5x + 8 = 28$
$\frac{x}{4} = 7$	$x - 7 = 8$	$\frac{x}{3} = 5$	$\frac{x + 2}{5} = 2$	$4x + 1 = 17$
$\frac{x + 1}{3} = 2$	$x - 2 = 4$	$x - 7 = 2$	$x - 8 = 14$	$3x + 6 = 18$
$x + 6 = 11$	$4x + 1 = 29$	$x - 9 = 2$	$2x - 3 = 7$	$4x + 2 = 22$

ACTIVITY 12.4 *Colour Coding*

Solve the equations in the diagram and then colour each section using the following code:

$x = 3$	Red	$x = 4$	Blue
$x = 5$	Orange	$x = 6$	Purple
$x = 7$	Yellow	$x = 9$	Green



What letters do the shapes form?

Extension

Produce a puzzle like this for your initials and try it out on a friend.

ACTIVITY 12.5 *Hero's Method*

$\sqrt{2}$ is an example of an *irrational* number, that is, a number which *cannot* be expressed in the form of $\frac{p}{q}$ $((q \neq 0)$ where p and q are integers. Other examples of irrational numbers are $\sqrt{3}$, $\sqrt{5}$ and π.

We can use calculators or computers to find very accurate approximations to these numbers; for example

$$\sqrt{2} = 1.41421\ 35623\ 73095\ 04880\ 16887\ 24209\ 69807\ 85697\ \ldots$$

These values have to be worked out; here we show how you can use an algorithm, known as Hero's Method, to find the square root of $\sqrt{2}$ to any desired accuracy. (Hero' (or Heron) was a geometer and worker in mechanics who lived in Egypt in the first century.)

The algorithm is based on the fact that, if x is an approximation to $\sqrt{2}$, a better approximation is given by $\frac{1}{2}\left(x + \frac{2}{x}\right)$. We will see how it works:

Algorithm	*Example*		
STEPS			
1. Choose first approximation to $\sqrt{2}$; call it x.	$x = 1$	$x = 1.5$	$x = 1.41667$
2. Calculate next approximation $\frac{1}{2}\left(x + \frac{2}{x}\right)$	$\frac{1}{2}\left(1 + \frac{2}{x}\right)$ $= 1.5$	$\frac{1}{2}\left(1.5 + \frac{2}{1.5}\right)$ $= 1.41667$	$\frac{1}{2}\left(1.41667 + \frac{2}{1.41667}\right)$ $= 1.41422$
3. Return to step 1 with x replaced by the new approximation	$x = 1.5$	$x = 1.41667$	$x = 1.41422$

So after just *three* iterations (repeats of the process), checking the value on a calculator, we see that the approximation for $\sqrt{2}$ is correct to the 4th decimal place.

1. How many iterations are needed to produce 3 decimal place accuracy with starting value:

   (a) $x = 1.4$, (b) $x = 2$, (c) $x = 10$?

2. By changing Step 2 of the algorithm to $\frac{1}{2}\left(x + \frac{3}{x}\right)$, determine, correct to the 4th decimal place, the value of $\sqrt{3}$.

Extension

1. Explain why the method works.
2. Deduce a similar way of finding the square root of any positive number.

ACTIVITIES 12.1 - 12.3 *Notes for Solutions*

Notes and solutions given only where appropriate.

12.1 1. H E L L O H A N N A H

12.2 Possible solutions:

$3 = b(d-c)-a$

$4 = a(d+b-c)$

$5 = a+d+b-c$

$6 = (d-c)\times(a+b)$

$7 = \dfrac{c+d}{b}-a$

$8 = \dfrac{c+d}{ab}$

$9 = \dfrac{c+d}{ab}+a$

$10 = bd-a-c$

$11 = bd-ac$

$12 = bd+a-c$

$13 = d+c-a-b$

$14 = c+d-ab$

$15 = c+d-b+a$

$16 = (c+d)(b-a)$

$17 = b+c+d-a$

$18 = a(b+c+d)$

$19 = a+b+c+d$

$20 = d(a+b)-c$

$21 = \dfrac{cd}{a+b}$

$22 = bc-a+d$

$23 = a(bc+d)$

$24 = bd-a+c$

$25 = a(bd+c)$

$26 = bd+a+c$

$27 = b(a+d)+c$

12.3

Game 1

3	13	3	5	2
15	6	7	14	2
7	5	20	21	25
5	12	91	10	51
9	32	14	12	22

Game 2

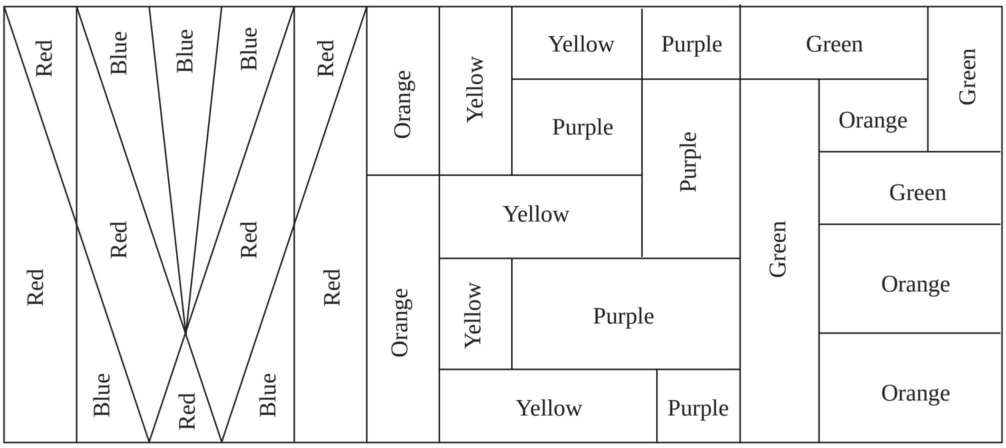
1	7	7	3	10
6	4	3	10	3
11	8	2	3	5
6	16	4	6	15
1	18	7	8	5

Game 3

12	2	14	5	7
3	4	7	5	4
28	15	15	8	4
5	6	9	22	4
5	7	11	5	5

ACTIVITIES 12.4 - 12.5 *Notes for Solutions*

12.4



M E P

12.5 The purpose behind this activity is twofold – firstly to introduce the idea of *irrational* numbers, and secondly to introduce the idea of an *algorithm.*

For some pupils, it might be better to use the algorithm in its given form.

For more able pupils, it would be better to write the algorithm as:

$$\boxed{x_{n+1} = \frac{1}{2}\left(x_n + \frac{2}{x_n}\right), \text{ given } x_0} \qquad n = 1, 2, \ldots$$

1. (a) 1 iteration (b) 2 iterations (c) 6 iterations

2. $x = 2$

$\frac{1}{2}\left(2 + \frac{3}{2}\right)$

$= 1.75$

$x = 1.75$

$x = 1.75$

$\frac{1}{2}\left(1.75 + \frac{3}{1.75}\right)$

$= 1.732143$

$x = 1.732143$

$x = 1.732143$

$\frac{1}{2}\left(1.732143 + \frac{3}{1.732143}\right)$

$= 1.732051$

$x = 1.732051$

This is correct to the 4th decimal place.

Extension

1. If the series converges to $\sqrt{2}$, then

$$x = \frac{1}{2}\left(x + \frac{2}{x}\right)$$

i.e.
$$x = \frac{1}{2}x + \frac{1}{x}$$

$$\frac{1}{2}x = \frac{1}{x}$$

$$x^2 = 2 \Rightarrow x = \sqrt{2}$$

2. Sequence to find $\sqrt{a}$ given by $x_{n+1} = \frac{1}{2}\left(x_n + \frac{a}{x_n}\right)$.