UNIT 3 *Pythagoras' Theorem* Teaching Notes

Historical Background and Introduction

Pythagoras' Theorem is arguably the most important elementary theorem in mathematics, since its consequences and generalisations have such wide-ranging implications. Although named after the 6th-century BC Greek philosopher and mathematician, it is in fact one of the earliest theorems known to ancient civilisation, and there is strong evidence that it was known 1000 years before Pythagoras!

There is, for example, evidence of Pythagorean triples (see Activity 2.4) in the Babylonian tablet of about 1700 BC. Part of this is reproduced below:

a	*b*	*c*
120	119	169
3456	3367	4825
4800	4601	6649
13 500	12 709	18 541
72	65	97
360	319	481
2700	2291	3541
960	799	1249
600	481	769
6480	4961	8161
60	45	75
2400	1679	2929
240	161	289
2700	1771	3229
90	56	106

Apparently, this is a random list of triples such that $a^2 + b^2 = c^2$, but note that a pattern emerges if you consider the ratio $\left(\frac{a}{b}\right)^2$.

How and why these triples were derived, or even if they were related to geometry at all, is not clear.

The Chinese discovered the general formula for Pythagorean triples in about 200 BC, in the form

$$x = ab, \quad y = \frac{2^2 - b^2}{2}, \quad z = \frac{a^2 + b^2}{2}$$

when a, b are odd integers, and $a > b$. By this stage though, its geometrical significance was clear.

The geometrical significance of this result was certainly clear to Pythagoras (572 - 497 BC) and his followers, but it also caused his group great problems! Their philosophy was that 'number is the substance of all things' and by 'number' they meant 'positive integers'. So all known objects had a number, or could be numbered and counted. Since this included lengths, they would assume that the side and diagonal of a square could be counted, i.e. there should exist a length such that the side and diagonal are integral multiples of it – but this, as we now know, is not true! Given the sides of the

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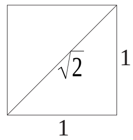
square to be of length 1, then the diagonal length is $\sqrt{2}$, and it is impossible to write



$$\sqrt{2} = k \times 1$$

for some integer *k*. When irrational numbers were discovered, in about 430 BC, the followers of Pythagoras gave up their basic philosophy that all things were made up of whole numbers, and the way opened for Greek mathematicians to develop many new theories.

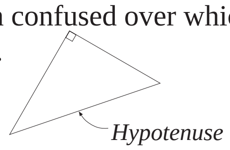
Routes		**Standard**	**Academic**	**Express**
3.1	Pythagoras' Theorem	✓	✓	✓
3.2	Calculating the Length of the Hypotenuse	✓	✓	✓
3.3	Calculating the Lengths of Other Sides	✗	✓	✓
3.4	Problems in Context	(✓)	✓	✓
3.5	Constructions and Angles	✓	✓	✓

Language	**Standard**	**Academic**	**Express**
Pythagoras' Theorem	✓	✓	✓
Right-angled triangle	✓	✓	✓
Pythagorean triples	(✓)	✓	✓
Hypotenuse	✓	✓	✓
Acute angles	✓	✓	✓
Obtuse angles	✓	✓	✓

(✓) *denotes extension work for these pupils*

Misconceptions

- pupils are often confused over which side is the hypotenuse, particularly when the triangle is orientated, e.g.



- pupils must understand that Pythagoras' Theorem applies *only* to right-angled triangles.

Challenging Questions

The following questions are more challenging than others in the same section:

	Section	*Question No.*	*Page*
Practice Book Y8A	3.2	10	52
" "	3.3	7, 8	54, 55
" "	3.5	9, 10	62

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