11 Algebraic Manipulation

11.1 Equations, Formulae and Identities

In this section we discuss the difference between equations, formulae and identities, and then go on to make use of them.

> An *equation* contains unknown quantities; for example,
>
> $$3x + 2 = 11$$
>
> This equation can be solved to determine x.
>
> A *formula* links one quantity to one or more other quantities; for example,
>
> $$A = \pi r^2$$
>
> This formula can be used to determine A for any given value of r.
>
> An *identity* is something that is always true for any values of the variables that are involved; for example,
>
> $$2(x + y) \equiv 2x + 2y$$
>
> and
>
> $$(x + y)^2 \equiv x^2 + 2xy + y^2$$
>
> If any pair of values of x and y are substituted, then the left hand side of an identity will generate the same value as the right hand side of that identity.

Example 1

The formula $C = \frac{5}{9}(F - 32)$ is used to convert temperatures in degrees Fahrenheit to degrees Celsius.

(a) If $F = 41$, calculate C.

(b) If $F = 131$, calculate C.

Solution

(a)
$$C = \frac{5}{9} \times (41 - 32)$$
$$C = \frac{5}{9} \times 9$$
$$C = 5$$

(b) $C = \frac{5}{9} \times (131 - 32)$

$C = \frac{5}{9} \times 99$

$C = 55$

Example 2

A formula states that $v = u + at$.

Calculate v if $u = 10$, $a = 6.2$ and $t = 20$.

Solution

When substituting into equations, you need to be aware that the BODMAS rule applies automatically.

$$
\begin{aligned}
v &= u + at \\
v &= 10 + 6.2 \times 20 \\
v &= 10 + 124 \\
v &= 134
\end{aligned}
$$

Example 3

Solve the following equations:

(a) $7x = 21$

(b) $x - 5 = 12$

(c) $2x + 1 = 6$

(d) $5x - 8 = 22$

Solution

(a)
$$
\begin{aligned}
7x &= 21 \\
x &= \frac{21}{7} && \textit{Dividing both sides by 7} \\
x &= 3
\end{aligned}
$$

(b)
$$
\begin{aligned}
x - 5 &= 12 \\
x &= 12 + 5 && \textit{Adding 5 to both sides} \\
x &= 17
\end{aligned}
$$

(c)
$$
\begin{aligned}
2x + 1 &= 6 \\
2x &= 6 - 1 && \textit{Subtracting 1 from both sides} \\
2x &= 5 \\
x &= \frac{5}{2} && \textit{Dividing both sides by 2} \\
x &= 2\frac{1}{2}
\end{aligned}
$$

(d) $5x - 8 = 22$

$$5x = 22 + 8 \qquad \textit{Adding 8 to both sides}$$

$$5x = 30$$

$$x = \frac{30}{5} \qquad \textit{Dividing both sides by 5}$$

$$x = 6$$

Example 4

One of the following statements is *not* an identity. Which one?

A $\qquad \dfrac{x+y}{2} \equiv \dfrac{x}{2} + \dfrac{y}{2}$

B $\qquad x - y \equiv y - x$

C $\qquad x^2 + y^2 \equiv (x+y)^2 - 2xy$

Solution

An identity will be true for any pair of values x and y. We could test each statement with $x = 5$ and $y = 10$.

Left-hand-side of A $= \dfrac{x+y}{2} = \dfrac{5+10}{2} = \dfrac{15}{2} = 7.5$

Right-hand-side of A $= \dfrac{x}{2} + \dfrac{y}{2} = \dfrac{5}{2} + \dfrac{10}{2} = 2.5 + 5 = 7.5$

Therefore LHS of A $=$ RHS of A if $x = 5$ and $y = 10$.

LHS of B $= x - y = 5 - 10 = -5$

RHS of B $= y - x = 10 - 5 = 5$

Therefore LHS of A $\neq$ RHS of A if $x = 5$ and $y = 10$.

LHS of C $= x^2 + y^2 = 5^2 + 10^2 = 25 + 100 = 125$

RHS of C $= (x+y)^2 - 2xy = (5+10)^2 - 2 \times 5 \times 10 = 15^2 - 100$

$= 225 - 100 = 125$

Therefore LHS of C $=$ RHS of C if $x = 5$ and $y = 10$.

So statement B is *not* an identity. We have *not proved* that A and C are identities, but we know that they are true for certain values of x and y.

Exercises

1. Solve the following equations:

(a) $4x = 12$ (b) $x - 5 = 8$ (c) $x + 3 = 9$

(d) $15 + x = 20$ (e) $x - 3 = 9$ (f) $\frac{x}{2} = 7$

(g) $x + 7 = 22$ (h) $5x = 30$ (i) $\frac{x}{4} = 9$

2. Solve the following equations:

(a) $2x + 1 = 11$ (b) $4x - 3 = 21$ (c) $5x - 6 = 4$

(d) $5x - 9 = 26$ (e) $9x + 21 = 102$ (f) $10x - 5 = 35$

(g) $\frac{x}{2} + 3 = 4$ (h) $\frac{x}{7} - 9 = 2$ (i) $\frac{x}{2} + 3 = 11$

(j) $x + 6 = 5$ (k) $4x + 20 = 10$ (l) $\frac{x}{4} + 9 = 4$

3. In this question, use the formula,

$$C = \frac{5}{9}(F - 32)$$

Calculate C if:

(a) $F = 77$ (b) $F = 68$ (c) $F = 158$

4. In this question, use the formula,

$$v = u + at$$

Calculate v if:

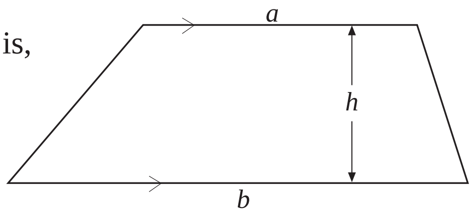
(a) $u = 2,\ a = 3$ and $t = 7$

(b) $u = 3.2,\ a = 0.8$ and $t = 5$

(c) $u = 30,\ a = 4$ and $t = 22$

(d) $u = 3.6,\ a = -0.2$ and $t = 40$

5. The formula for the area of a trapezium is,



$$A = \frac{1}{2}(a + b)h$$

Calculate A if:

(a) $a = 4,\ b = 10$ and $h = 6$ (b) $a = 2,\ b = 10$ and $h = 13$

(c) $a = 3.2,\ b = 2.8$ and $h = 3.2$ (d) $a = 4,\ b = 2.5$ and $h = 7.2$

6. If $a = 6$, $b = 7.5$ and $c = -2$, calculate:

(a) $a + b + c$ (b) $ab + c$ (c) $2a + 3b$

(d) $a + 2b + 3c$ (e) ac (f) $a^2 + b^2$

(g) $a^2 + c^2$ (h) $ab + bc$ (i) $a(b - c)$

7. A formula states:

$$y = 4x - 5$$

(a) Calculate y if $x = 3$.

(b) Determine x if $y = 23$.

(c) Determine x if $y = 8$.

8. The mean of three numbers is calculated using the formula,

$$m = \frac{x + y + z}{3}$$

(a) Calculate m if $x = 8$, $y = 17$ and $z = 2$.

(b) Determine x if $m = 5$, $y = 6$ and $z = 7$.

(c) Determine z if $m = 18$, $x = 19$ and $y = 20$.

9. Use the formula $C = \frac{5}{9}(F - 32)$ to determine F when:

(a) $C = 100$ (b) $C = 60$ (c) $C = 0$

10. Which of the following statements are *not* identities?

A $\frac{x}{y} \equiv \frac{y}{x}$

B $x \times y \equiv y \times x$

C $(x - y)^2 \equiv (y - x)^2$

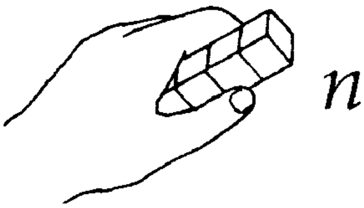
D $(a + b)^2 \equiv (a - b)^2$

E $2xy \equiv (x + y)^2 - x^2 - y^2$

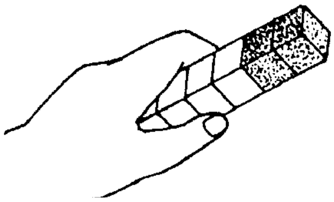
11. Jenny is holding a row of cubes.

You cannot see exactly how many cubes she is holding.

Call the number of cubes she is holding n.



(a) She joins on t*wo more* cubes.

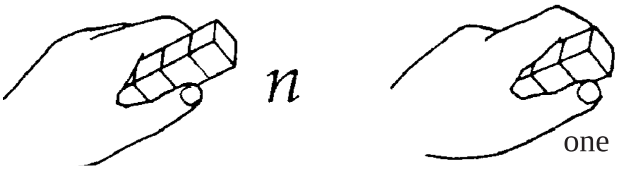


Write an expression for the total number of cubes she is holding now.

(b) Jenny starts again with n cubes.

One cube is *removed*.

Write an expression for the number of cubes she is holding now.

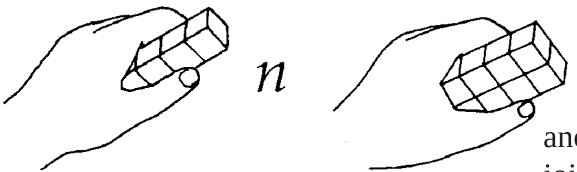


one cube
removed

(c) Jenny starts again with n cubes.

Another row of the same length is *joined on.*

Write an expression for the total number of cubes she is holding now.



another row
joined on

(d) Jacob also has some cubes in his hands.

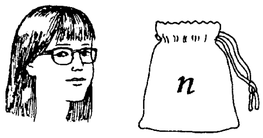
In one hand there are $2n - 1$.

In the other hand there are $2(n - 1)$ cubes.

Is Jacob holding the same number of cubes in each hand?
Explain your answer.

(KS3/97/Ma/Tier 5-7/P1)

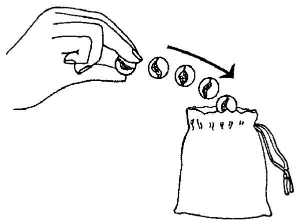
12. (a) Elin has a bag of marbles.



You cannot see how many marbles are inside the bag.

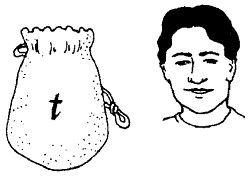
Call the number of marbles which Elin starts with in her bag n.

Elin puts 5 more marbles *into* her bag.



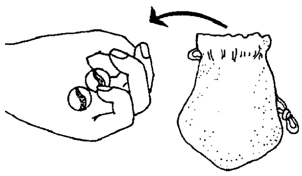
Write an expression to show the total number of marbles in Elin's bag now.

(b) Ravi has another bag of marbles.



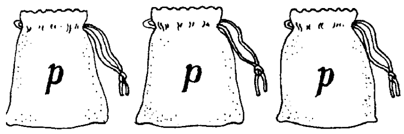
Call the number of marbles which Ravi starts with in his bag t.

Ravi takes 2 marbles *out* of his bag.



Write an expression to show the total number of marbles in Ravi's bag now.

(c) Jill has 3 bags of marbles.



Each bag has p marbles inside.

Jill takes some marbles out.

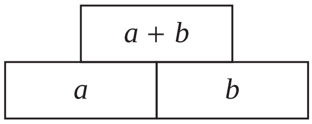
Now the total number of marbles in Jill's 3 bags is $3p - 6$.

Some of the statements below *could* be *true*.
Write down the letter of each statement which *could* be *true*.

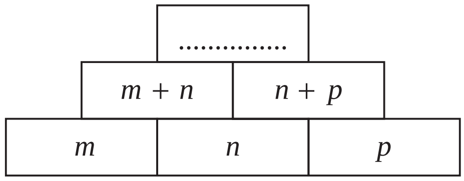
A	Jill took 2 marbles out of *one* of the bags, and *none* out of the other bags.
B	Jill took 2 marbles out of *each* of the bags.
C	Jill took 3 marbles out of *one* of the bags, and *none* out of the other bags.
D	Jill took 3 marbles out of each of *two* of the bags, and *none* out of the other bag.
E	Jill took 6 marbles out of *one* of the bags and *none* out of the other bags.
F	Jill took 6 marbles out of each of *two* of the bags, and *none* out of the other bag.

(KS3/98/Ma/Tier 5-7/P1)

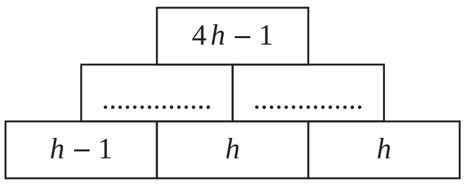
13. In these walls each brick is made by *adding* the *two* bricks underneath it.

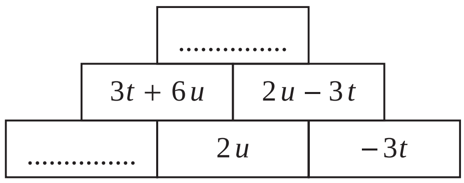


(a) Write an expression for the top brick in this wall.
Write your expression as simply as possible.



(b) Make a copy of the walls shown below and fill in the missing expressions.
Write your expressions as simply as possible.

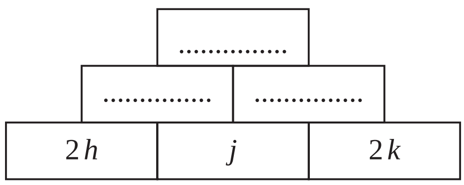




(c) In the wall below, h, j and k can be any whole numbers.

Explain why the top brick of the wall must *always* be an *even* number.

You can copy the wall and fill in the missing expressions if you want to.



(KS3/97/Ma/Tier 5-7/P2)

11.2 Simplifying Expressions

In this section we look at how to simplify expressions, in particular, how to remove brackets from both formulae and equations.

Collecting like terms

Examples

$$a + a + a = 3a$$

$$a + b + a = 2a + b$$

$$2y + 8y = 10y$$

$$x + x^2 + x^2 = x + 2x^2$$

Only like terms can be collected

Example 1

Simplify the following expressions,

(a) $4a + 2b + 3a + 6b$

(b) $3x - 4y + 2x - y$

(c) $x^2 + 4x + 2x^2 - x$

(d) $4a^2 + a + 2a^2 - 3a$

Solution

(a) $4a + 2b + 3a + 6b = 7a + 8b$

(b) $3x - 4y + 2x - y = 5x - 5y$

(c) $x^2 + 4x + 2x^2 - x = 3x^2 + 3x$

(d) $4a^2 + a + a^2 - 3a = 6a^2 - 2a$

Expanding Brackets

Every term in each bracket must be multiplied by every other item.

$$\begin{aligned} x(4x + 2) &= x \times 4x + x \times 2 \\ &= 4x^2 + 2x \end{aligned}$$

$$\begin{aligned} (x + 1)(x + 4) &= x \times x + x \times 4 + 1 \times x + 1 \times 4 \\ &= x^2 + 4x + x + 4 \\ &= x^2 + 5x + 4 \end{aligned}$$

Alternatively, you can expand brackets using the 'box' method, as shown opposite.

$$(x + 1)(x + 4) = x^2 + 1x + 4x + 4 = x^2 + 5x + 4$$

$\times$	x	$+1$
x	x^2	$+1x$
$+4$	$+4x$	$+4$

Example 2

Expand each of the following:

(a) $2(x+3)$ (b) $4(2x-6)$

(c) $x(x+2)$ (d) $2x(3x-2)$

Solution

(a) $2(x+3) = 2\times x + 2\times 3$

$= 2x+6$

(b) $4(2x-6) = 4\times 2x - 4\times 6$

$= 8x-24$

(c) $x(x+2) = x\times x + x\times 2$

$= x^2+2x$

(d) $2x(3x-2) = 2x\times 3x - 2x\times 2$

$= 6x^2-4x$

Example 3

Expand,

(a) $(x+6)(x+3)$ (b) $(x+4)(2x-5)$

Solution

(a) $(x+6)(x+3) = x\times x + x\times 3 + 6\times x + 6\times 3$

$= x^2+3x+6x+18$

$= x^2+9x+18$

or alternatively, using the box method,

$\times$	x	$+6$
x	x^2	$+6x$
$+3$	$+3x$	$+18$

$(x+6)(x+3) = x^2+6x+3x+18 = x^2+9x+18$

(b) $(x+4)(2x-5) = x\times 2x - x\times 5 + 4\times 2x - 4\times 5$

$= 2x^2-5x+8x-20$

$= 2x^2+3x-20$

Again, using the box method,

$\times$	x	$+4$
$2x$	$2x^2$	$+8x$
-5	$-5x$	-20

$$(x+4)(2x-5) = 2x^2 + 8x - 5x - 20 = 2x^2 + 3x - 20$$

Exercises

1. Simplify each of the following by collecting like terms:

(a) $4a + b + 2a$ (b) $4b + 2c + 6b + 3c$

(c) $4a + 5b - a + 2b$ (d) $14p + 11q - 8p + 3q$

(e) $6x - 4y + 8x + 9y$ (f) $11x + 8y + 3z - 2y + 4z$

(g) $16x - 8y - 3x - 4y$ (h) $11y + 12z - 10y + 4z + 2y$

2. Simplify each of the following:

(a) $3x + 3x^2 + 4x - x^2$ (b) $4y^2 + 4y - 2y^2 + 3y$

(c) $a^2 + a + 3a^2 - 2a$ (d) $6x^2 + 12x - 9x^2 + 3x$

3. Expand each of the following expressions by multiplying out the brackets:

(a) $3(x+6)$ (b) $4(x+2)$ (c) $3(x-1)$

(d) $4(2x+5)$ (e) $6(3x-5)$ (f) $7(2x-5)$

(g) $6(4-2x)$ (h) $8(3-5x)$ (i) $9(5x+10)$

4. Simplify each of the following expressions:

(a) $2(x+3) + 4(x+4)$ (b) $5(x-6) + 2(x+3)$

(c) $4(6-x) + 7(2x+1)$ (d) $11(x-2) + 4(7x+3)$

(e) $8(x-6) + 4(7-x)$ (f) $3(4-5x) + 6(3x-2)$

5. Expand each of the following expressions by multiplying out the brackets:

(a) $x(x+3)$ (b) $x(6x+1)$ (c) $x(3x-2)$

(d) $2x(4-x)$ (e) $6x(2x+4)$ (f) $5x(3x-7)$

(g) $11x(x-3)$ (h) $14x(2+3x)$ (i) $6x(4-2x)$

6. Expand each of the following expressions by multiplying out the brackets:

(a) $(x+4)(x+3)$ (b) $(x+2)(x+4)$ (c) $(x+1)(x+5)$

(d) $(x+6)(x-1)$ (e) $(x-4)(x+2)$ (f) $(x-3)(x+2)$

(g) $(x-4)(x-5)$ (h) $(x-3)(x-2)$ (i) $(x-7)(x-9)$

7. Simplify each of the following expressions:

(a) $(x+2)(x+4)+(x+1)(x+2)$

(b) $(x+3)(x+7)+(x-1)(x+5)$

(c) $(x+6)(x+2)-(x-2)(x+3)$

(d) $(x-4)(x-8)-(x-1)(x-9)$

8. Expand each expression:

(a) $(2x+1)(3x+2)$ (b) $(4x-7)(2x+1)$

(c) $(3x+5)(2x-8)$ (d) $(4x+5)(3x-8)$

(e) $(8x+2)(3x-3)$ (f) $(6x-5)(3x-7)$

9. Simplify:

(a) $(3x+2)(5x+9)+(4x-2)(3x-5)$

(b) $(4x+6)(5x+1)-(2x+3)(3x+1)$

(c) $(6x-5)(x+1)-(2x+7)(3x-5)$

10. Expand:

(a) $(x+1)^2$ (b) $(x-2)^2$ (c) $(x+3)^2$

(d) $(x+5)^2$ (e) $(x-7)^2$ (f) $(x-8)^2$

(g) $(x+10)^2$ (h) $(x-12)^2$ (i) $(x+4)^2$

(j) $(2x+3)^2$ (k) $(4x-7)$ (l) $(3x+2)^2$

(m) $(4x+1)^2$ (n) $(5x-2)^2$ (o) $(6x-4)^2$

11. Expand:

(a) $(x+1)(x-1)$ (b) $(x+3)(x-3)$

(c) $(x+7)(x-7)$ (d) $(x+9)(x-9)$

(e) $(x+12)(x-12)$ (f) $(2x+1)(2x-1)$

(g) $(3x+2)(3x-2)$ (h) $(4x+7)(4x-7)$

12. Expand:

(a) $(x+1)^3$ (b) $(2x+1)^3$ (c) $(x-5)^3$

13. Here are some algebra cards:

$\boxed{n \div 2}$ $\boxed{2+n}$

$\boxed{n+2}$ $\boxed{n^2}$

$\boxed{n}$ $\boxed{2n}$

$\boxed{n-2}$ $\boxed{n+n}$

$\boxed{n^3}$ $\boxed{2n-n}$

(a) One of the cards will always give the same answer as $\boxed{\frac{n}{2}}$.

Which card is it?

(b) One of the cards will always give the same answer as $\boxed{n \times n}$.

Which card is it?

(c) *Two* of the cards will always give the same answer as $\boxed{2 \times n}$.

Which cards are they?

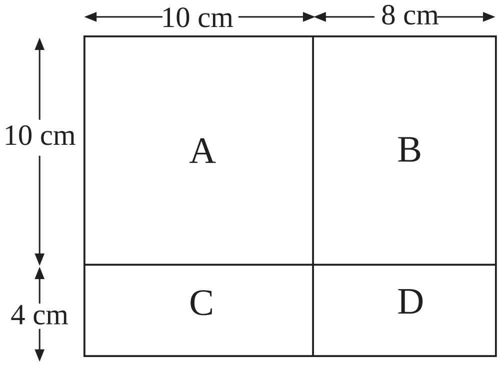
(d) Write a *new* card which will always give the same answer as

$\boxed{3n+2n}$.

(KS3/97/Ma/Tier 5-7/P1)

14. (a) (i) The diagram shows a rectangle 18 cm long and 14 cm wide. It has been split into *four smaller rectangles*, A, B, C and D. Write down the *area* of each of the small rectangles.

One has been done for you.



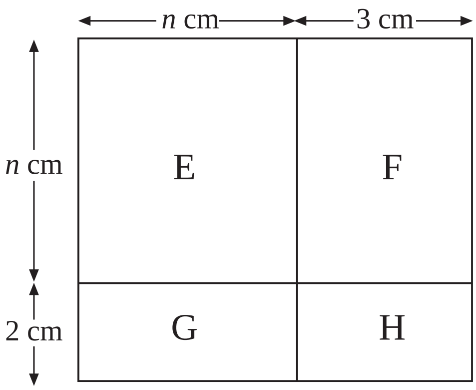
Area of Rectangle C $= 40 \text{ cm}^2$.

(ii) What is the area of the *whole* rectangle?

(iii) What is 18×14 ?

(b) (i) The diagram shows a rectangle $(n+3)$ cm long and $(n+2)$ cm wide. It has been split into *four smaller rectangles.* Write down a *number* or an *expression* for the *area* of *each small rectangle.*

One has been done for you.



Area of Rectangle F $= 3n \text{ cm}^2$.

(ii) What is $(n+3)(n+2)$ multiplied out?

(KS3/99/Ma/Tier 5-7/P1)

15. Multiply out and simplify these expressions:

(a) $3(x-2)-2(4-3x)$

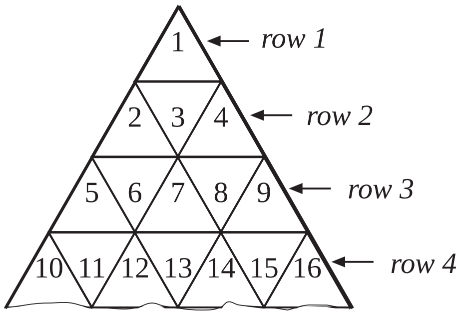
(b) $(x+2)(x+3)$

(c) $(x+4)(x-1)$

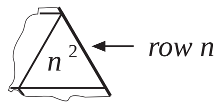
(d) $(x-2)^2$

(KS3/98/Ma/Tier 6-8/P1)

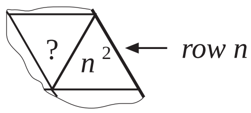
16. A number grid is inside a large triangle.
The small triangles are numbered consecutively.
The diagram shows the first 4 rows.



(a) An expression for the *last* number in row n is n^2.



Write an expression for the *last but one* number in row *n*.



(b) An expression for the *first* number in row n is n^2-2n+2.
Calculate the value of the first number in *row 10.*

(c) Make a copy of the table and complete it by writing an expression:

first number in row n	n^2-2n+2
second number in row n	

(d) Make a copy of the table and complete it by writing an expression:

centre number in *row* n	n^2-n+1
centre number in *row*	$(n+1)^2-(n+1)+1$

(e) Multiply out and simplify the expression $(n+1)^2-(n+1)+1$.
Show your working.

(KS3/99/Ma/Tier 6-8/P1)

11.3 Factorising

In this section we consider examples of the process of factorising, whereby the process of removing brackets is reversed and brackets are introduced into expressions.

Example 1

Factorise:

(a) $8x + 12$ (b) $35x + 28$

Solution

(a) Note that both terms are multiples of 4, so we can write,

$$8x + 12 = 4(2x + 3)$$

(b) Here both terms are multiples of 7, so

$$35x + 28 = 7(5x + 4)$$

Results like this can be checked by multiplying out the bracket to get back to the original expression.

Example 2

Factorise,

(a) $x^2 + 2x$ (b) $3x^2 - 9x$ (c) $x^3 - x^2$

Solution

(a) Here, as both terms are multiples of x, we can write,

$$x^2 + 2x = x(x + 2)$$

(b) In this case, both terms are multiples of x and 3, giving,

$$3x^2 - 9x = 3x(x - 3)$$

(c) In this example, both terms are multiples of x^2,

$$x^3 - x^2 = x^2(x - 1)$$

Sometimes it is possible to factorise in stages. For example, in part (b), you could have worked like this:

$$\begin{aligned} 3x^2 - 9x &= 3(x^2 - 3x) \\ &= 3x(x - 3) \end{aligned}$$

Example 3

Factorise:

(a) $x^2 + 9x + 18$ (b) $x^2 + 2x - 15$ (c) $x^2 - 7x + 12$

Solution

(a) This expression will need to be factorised into two brackets:

$$x^2 + 9x + 18 = (x \quad)(x \quad)$$

As the expression begins x^2, both brackets must begin with x. The two numbers to go in the brackets must multiply together to give 18 and add to give 9. So they must be 3 and 6, giving,

$$x^2 + 9x + 18 = (x + 3)(x + 6)$$

You can check this result by multiplying out the brackets.

(b) We note first that two brackets are needed and that both must contain an x, as shown:

$$x^2 + 2x - 15 = (x \quad)(x \quad)$$

Two other numbers are needed which, when multiplied give – 15 and when added give 2. In this case, these are – 3 and 5. So the factorisation is,

$$x^2 + 2x - 15 = (x - 3)(x + 5)$$

Check this result by multiplying out the brackets.

(c) Again, we begin by noting that,

$$x^2 - 7x + 12 = (x \quad)(x \quad)$$

We require two numbers which, when multiplied give 12 and when added give – 7. In this case, these numbers are – 3 and – 4.

$$x^2 - 7x + 12 = (x - 3)(x - 4)$$

Exercises

1. Factorise:

(a) $4x - 2$ (b) $6x - 12$ (c) $5x - 20$

(d) $4x + 32$ (e) $6x - 8$ (f) $8 - 12x$

(g) $21x - 14$ (h) $15x + 20$ (i) $30 - 10x$

2. Factorise:

(a) $x^2 + 4x$ (b) $x^2 - 3x$ (c) $4x - x^2$

(d) $6x^2 + 8x$ (e) $9x^2 + 15x$ (f) $7x^2 - 21x$

(g) $28x - 35x^2$ (h) $6x^2 - 14x$ (i) $5x^2 - 3x$

3. Factorise:

(a)	$x^3 + x^2$	(b)	$2x^2 - x^3$	(c)	$4x^3 - 2x^2$
(d)	$8x^3 + 4x^2$	(e)	$16x^2 - 36x^3$	(f)	$4x^3 + 22x^2$
(g)	$16x^2 - 6x^3$	(h)	$14x^3 + 21x^2$	(i)	$28x^3 - 49x^2$

4. (a) Expand $(x+5)(x-5)$.

(b) Factorise $x^2 - 25$.

(c) Factorise each of the following:

(i)	$x^2 - 49$	(ii)	$x^2 - 64$	(iii)	$x^2 - 100$
(iv)	$x^2 - a^2$	(v)	$x^2 - 4b^2$		

5. Factorise:

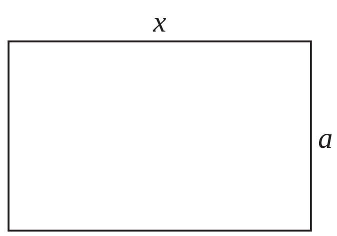
(a)	$x^2 + 7x + 12$	(b)	$x^2 + 8x + 7$	(c)	$x^2 + 11x + 18$
(d)	$x^2 + 12x + 27$	(e)	$x^2 + 17x + 70$	(f)	$x^2 + 6x + 8$
(g)	$x^2 + 16x + 28$	(h)	$x^2 + 18x + 77$	(i)	$x^2 + 16x + 63$

6. Factorise:

(a)	$x^2 + x - 2$	(b)	$x^2 + x - 20$	(c)	$x^2 - x - 12$
(d)	$x^2 - 13x + 36$	(e)	$x^2 - 10x + 16$	(f)	$x^2 + x - 42$
(g)	$x^2 + 13x - 30$	(h)	$x^2 - 17x + 72$	(i)	$x^2 - 2x - 99$

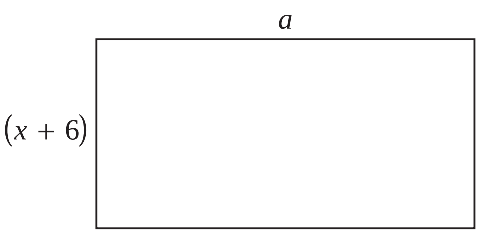
7. The area of the rectangle shown is $x^2 - 5x$.

Express a in terms of x.



8. The area of the rectangle shown is $x^2 + 11x + 30$.

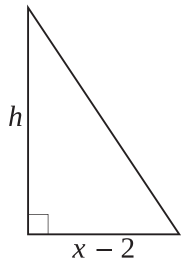
Express a in terms of x.



9. The area of the triangle shown is

$$\frac{1}{2}x^2 + \frac{3}{2}x - 5.$$

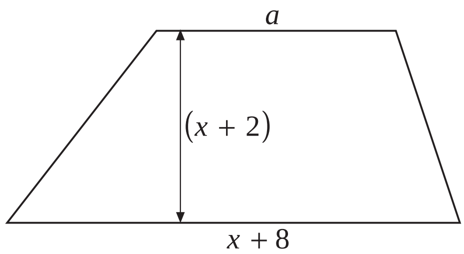
Express h in terms of x.



10. The area of the trapezium shown is

$$\frac{1}{2}x^2 + 10x + 18.$$

Determine a.



11.4 Using Formulae

In this section we make use of formulae and develop simple formulae ourselves. First we begin with some revision of working with *negative numbers.*

Example 1

If $a = 6$, $b = -5$, $c = -7$ and $d = 3$, calculate:

(a) $a + c$ (b) $a - b$ (c) bc (d) $b^2 + cd$

Solution

(a)
$$\begin{aligned} a + c &= 6 + (-7) \\ &= 6 - 7 \\ &= -1 \end{aligned}$$

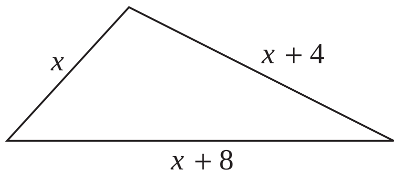
(b)
$$\begin{aligned} a - b &= 6 - (-5) \\ &= 6 + 5 \\ &= 11 \end{aligned}$$

(c)
$$\begin{aligned} bc &= (-5) \times (-7) \\ &= 35 \end{aligned}$$

(d)
$$\begin{aligned} b^2 + cd &= (-5)^2 + (-7) \times 3 \\ &= 25 + (-21) \\ &= 25 - 21 \\ &= 4 \end{aligned}$$

Example 2

A triangle has sides of length x, $x+4$ and $x+8$, as shown in the diagram.



(a) Write down a formula for the perimeter, p, of the triangle.

(b) Calculate the perimeter when $x = 10$.

(c) Calculate x when the perimeter is 45.

Solution

(a) $p = x + (x+4) + (x+8)$

$p = 3x + 12$

(b) $p = 3 \times 10 + 12$

$p = 30 + 12$

$p = 42$

(c) $45 = 3x + 12$

$33 = 3x$ *Subtracting 12 from both sides*

$x = \frac{33}{3}$ *Dividing both sides by 3*

$x = 11$

Example 3

A removal firm charges £80 plus £2 for every mile that their removal van travels.

(a) Write down a formula for the cost, £C, of a move of n miles.

(b) Calculate the cost of moving 262 miles.

(c) A move costs £500. How far did the removal van travel?

Solution

(a) $C = 80 + 2n$

(b) $C = 80 + 262 \times 2$

$C = 80 + 524$

$C = £604$

(c) $500 = 80 + 2n$

$420 = 2n$ *Subtracting 80 from both sides*

$n = \frac{420}{2}$ *Dividing both sides by 2*

$n = 210$ miles, so the van travelled 210 miles.

Exercises

1. If $p = 6$, $q = -2$, $r = 12$ and $s = -5$, calculate:

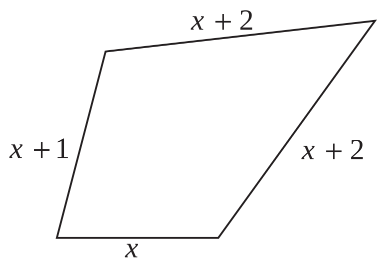
   (a) $p + q$ (b) $p - s$ (c) $r - q$

   (d) $q^2 + s^2$ (e) $p - r$ (f) $r + s^2$

   (g) $pq + r$ (h) $r - qs$ (i) $2r + 5q$

   (j) $6q - 2p$ (k) $4s - 2r$ (l) $3q + 7s$

2. The diagram shows a quadrilateral.

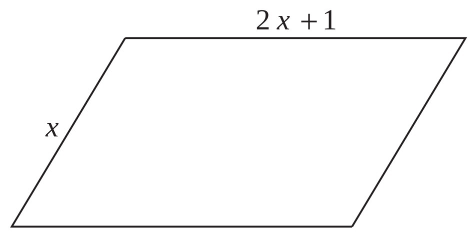


   (a) Write down a formula for the perimeter, p, of the quadrilateral.

   (b) Calculate the perimeter if $x = 14$.

   (c) Determine x if the perimeter is 73.

3. The diagram shows a parallelogram.

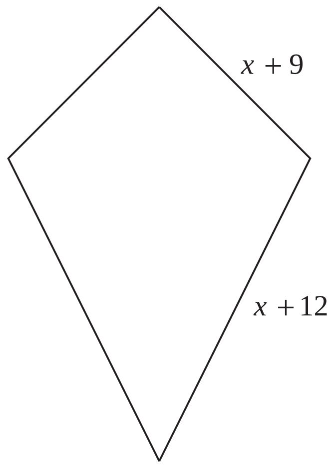


   (a) Write down a formula for the perimeter, p, of the parallelogram.

   (b) Calculate the perimeter when $x = 7$.

   (c) Determine x if the perimeter is 182.

4. The diagram shows a kite.

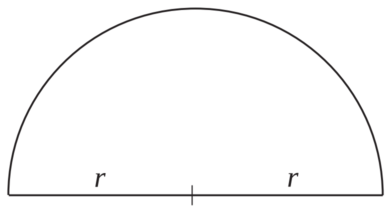


   (a) Write down a formula for the perimeter, p, of the kite.

   (b) Calculate the perimeter if $x = 9$.

   (c) Determine x if the perimeter is 118.

5. The semi-circle shown has radius r.



   (a) Write down formulae for the area and perimeter of the semi-circle, giving your answers correct to 2 decimal places.

   (b) Calculate the area and perimeter if $r = 5$ cm.

   (c) Determine r if the perimeter is 40 cm.

   (d) Determine r if the area is 18 cm^2.

6. A taxi firm charges £1.80 plus 50p per mile travelled.

   (a) Write down a formula for the cost, C pence, of travelling m miles.

   (b) Calculate the cost of a 3-mile journey.

   (c) The charge for a journey is £6. What is the distance travelled?

7. Ahmed runs a baked potato stall at the market. He makes a profit of 40p on each potato he sells but he has to pay £50 each day for the stall.

   (a) Write down a formula for the amount of money, in pounds, Ahmed makes on one day if he sells x potatoes.

   (b) Describe what happens if $x = 200$.

   (c) Describe what happens if $x = 100$.

   (d) How many potatoes must he sell to make a profit of £100 in one day?

8. Records for the weather suggest that on average, it is $22\ °\text{C}$ warmer in Miami than in Washington.

   The average temperature in Miami in $°\text{C}$ is M.

   The average temperature in Washington in $°\text{C}$ is W.

   (a) Write down a formula for M in terms of W.

   (b) Write down a formula for W in terms of M.

   (c) Determine M if W is -7.

   (d) Determine W if M is -3.

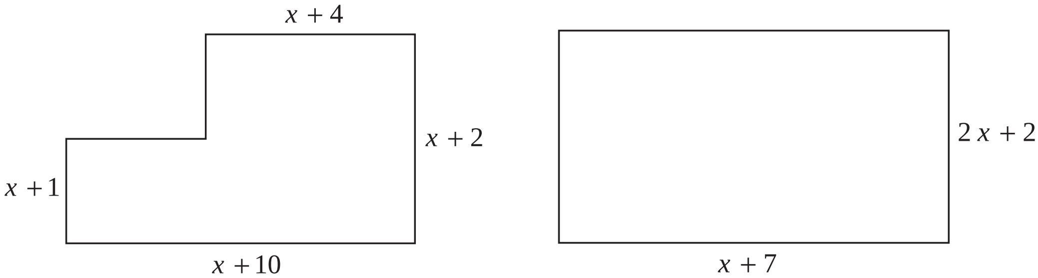
9. An engineer charges £20 plus £p per hour to repair central heating boilers. At one house a repair takes 3 hours and costs £71.

   (a) Determine the value of p.

   (b) Write down a formula for the cost, £C, of a repair that takes x hours.

   (c) A repair costs £96.50. How long does it take?

10. Alan claims that the two shapes shown have the same area.



   (a) Determine a formula for the area of each shape.

   (b) Is Alan correct?

11. Ice creams are sold as cones or tubs at the Beach Kiosk.
A cone costs 60 pence.
A tub costs 40 pence.

The income (F) in pence of the Beach Kiosk can be calculated from the equation $F = 60x + 40y$ where x is the number of cones sold and y is the number of tubs sold.

(a) On June 1st 1994, $x = 65$ and $y = 80$.

Work out the income.
Show your working.

(b) On June 2nd 1994, $F = 4800$ and $x = 50$.

Work out how many tubs were sold.
Show your working.

(c) During the first week of last summer 950 ice creams were sold.
437 of them were tubs.

What percentage of the ice creams sold were tubs?

(d) *Estimate* the total income in pounds for the summer of 1995 using the information in the box.

Last summer 14 723 ice creams were sold. Roughly the same number of ice creams is likely to be sold in the summer of 1995. The ratio of cones to tubs sold is likely to be about 1 : 1. The cost of a cone is to stay at 60 pence. The cost of a tub is to stay at 40 pence.

(i) Write down the number you will use instead of 14 723.

(ii) Write down the value you will use for the cost of an ice cream.

(iii) Write down your estimate of the total income for the summer of 1995.

(KS3/95/Ma/Levels 5-7/P1)

12. A robot accelerates at a constant rate. It can move backwards or forwards.

When the robot moves, three equations connect the following:

u its initial speed in m/s
v its final speed in m/s
a its acceleration in m/s^2
s the distance travelled in m
t the time taken in seconds

The equations are:

$$v = u + at \qquad v^2 = u^2 + 2as \qquad s = ut + \frac{1}{2}at^2$$

For a journey made by the robot:

u = 0.25 m/s   t = 3.5 seconds   a = – 0.05 m/s

Use the appropriate equation to find:

(a) The distance travelled.
Show your working.

(b) The robot's final speed.
Show your working.

(KS3/95/Ma/Levels 6-8/P2)

13. Magic squares were used to tell fortunes long ago in China.
They contain whole numbers starting from 1.
The numbers in each row add up to the magic number.

Eleri's magic square has 3 rows.
The magic number is 15.
The size is 3×3.

8	1	6
3	5	7
4	9	2

magic number 15

Tony made a magic square with more rows.
The magic number is 2925.
We do not know the size of his square.

?

magic number 2925

(a) When the size of a magic square is $n \times n$,

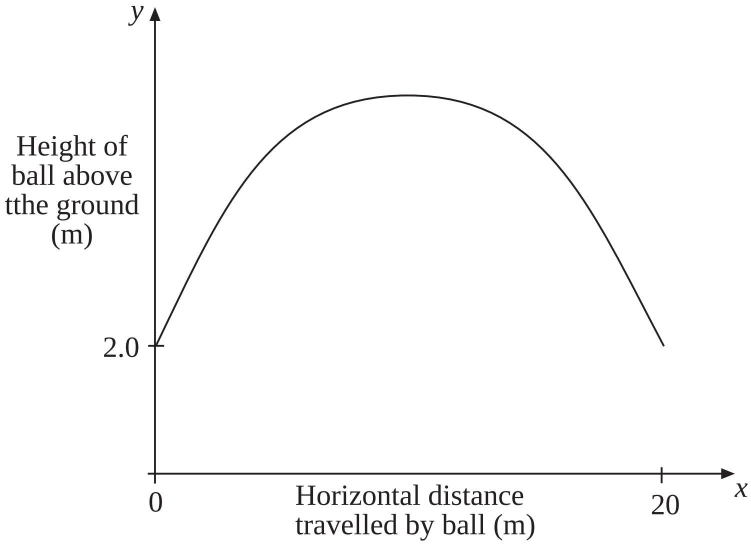
the magic number is $\dfrac{n\left(n^2+1\right)}{2}$.

Use $n = 3$ to check that this works for Eleri's magic square.

(b) Find the size of Tony's magic square.
You may use trial and improvement.

(KS3/94/Ma/5-7/P2)

14. Alan throws a ball to Katie who is standing 20 m away.
The ball is thrown and caught at a height of 2.0 m above the ground.



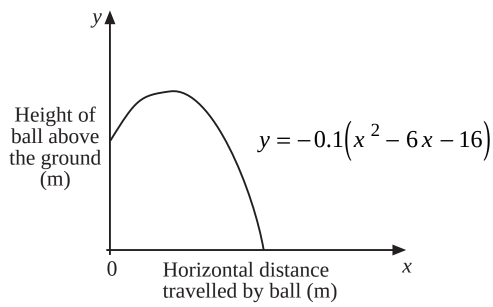
The ball follows the curve with equation

$$y = 6 + c(10 - x)^2 \quad \text{where } c \text{ is a constant.}$$

(a) Calculate the value of c by substituting $x = 0, y = 2$ into the equation.
Show your working.

Alan throws the ball to Katie again, but this time the ball hits the ground before it reaches her.

The ball follows the curve with equation $y = -0.1(x^2 - 6x - 16)$



(b) Calculate the height above the ground at which the ball left Alan's hand.
Show your working.

(KS3/97/Ma/Tier 6-8/P1)