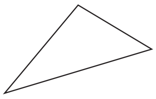
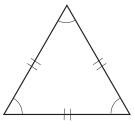
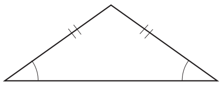
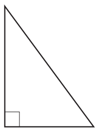
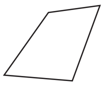
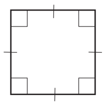
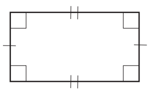
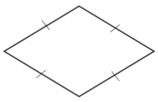
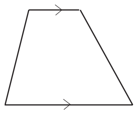
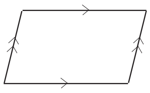
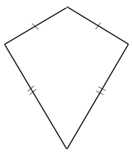
7 Transformations

7.1 Shapes

You should be familiar with the common 2-D shapes, but to recap, we give the names and definitions below.

NAME	*ILLUSTRATION*	*NOTES*
Triangle		3 straight sides
Equilateral Triangle		3 equal sides and 3 equal angles ($= 60°$)
Isosceles Triangle		2 equal sides and 2 equal angles
Right-angled Triangle		One angle $= 90°$
Quadrilateral		4 straight sides
Square		4 equal sides and 4 right angles
Rectangle		Opposite sides equal and 4 right angles
Rhombus		4 equal sides; opposite sides parallel
Trapezium		One pair of opposite sides parallel
Parallelogram		Both pairs of opposite sides equal and parallel
Kite		Two pairs of adjacent sides equal

Example 1

What could each one of the following shapes be if it has 4 sides and:

(a) opposite sides equal and parallel, (b) all sides equal,

(c) two adjacent angles are right angles?

Solution

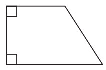
(a) It could be a

parallelogram, rhombus, rectangle or *square.*

(b) It could be a

rhombus or *square.*

(c) It could be a

trapezium, rectangle or *square.* (Trapezium )

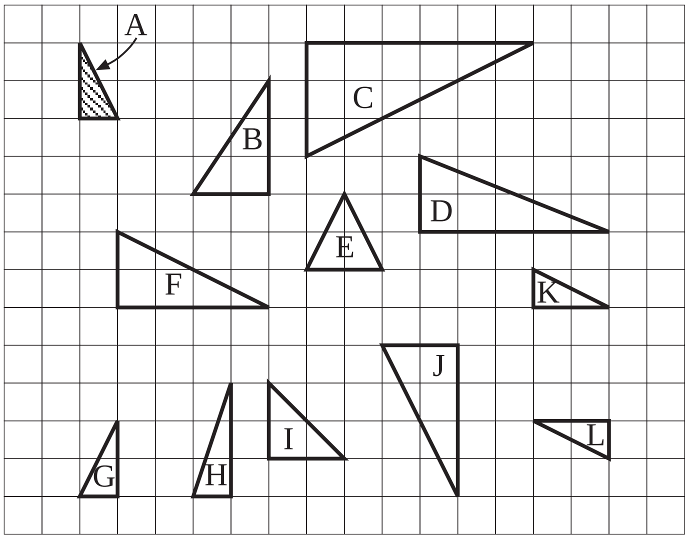
Example 2

For the grid opposite, name all shapes that are:

(a) *congruent,*

(b) *similar*

to shape A.



Solution

(a) *Congruent* to A means the *same size and shape* as A. The shapes congruent to A are G, L and K.

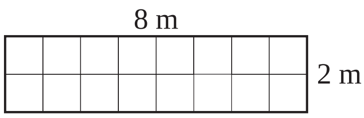
(b) *Similar* to A means the *same shape* as A but *not necessarily the same size* as A. The shapes similar to A are C, F, G, J, K and L.

Example 3

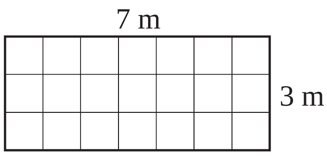
Using 20 m of fencing, design *four* different rectangular enclosures. For each one, find its area. Which shape gives the maximum area?

Solution

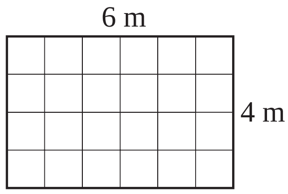
Possible shapes could be:



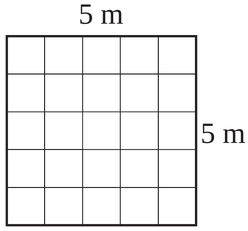
area $= 2 \times 8 = 16 \text{ m}^2$



area $= 3 \times 7 = 21 \text{ m}^2$



area = $4 \times 6 = 24 \text{ m}^2$



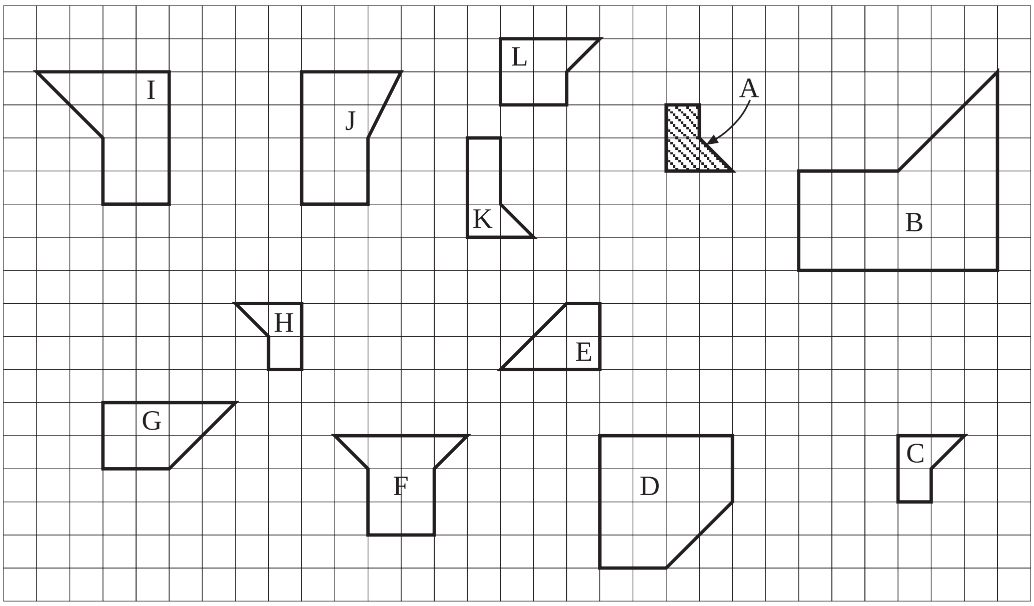
area = $5 \times 5 = 25 \text{ m}^2$

The square $(5 \text{ m} \times 5 \text{ m})$ gives the maximum area.

Exercises

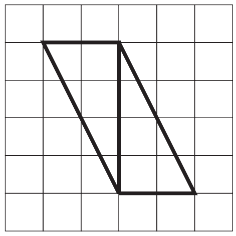
1. What could each one of the following shapes be if it has 4 sides and:
   (a) all angles right angles,
   (b) exactly one pair of opposite sides parallel, but not equal,
   (c) diagonals intersecting at right angles?

2. Which of the shapes in the diagram below are:
   (a) *congruent,* (b) *similar*
   to shape A ?

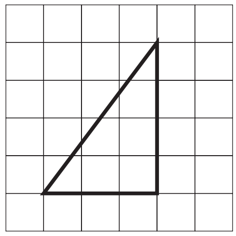


3. Using 40 cm of wire, design different rectangles. For each one, find its area. What shape gives the maximum area?

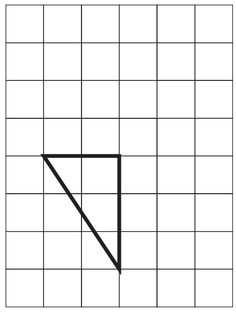
4. These two congruent triangles make a *parallelogram.*



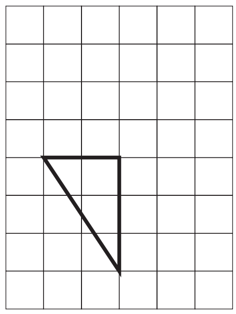
(a) On a copy of the grid opposite, draw another congruent triangle to make a *rectangle.*



(b) On a copy of the grid opposite, draw another congruent triangle to make a *bigger triangle.*

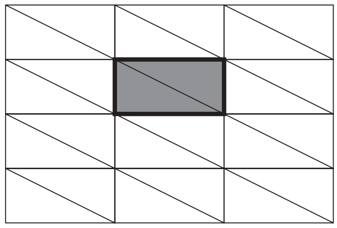


(c) On a copy of the grid opposite, draw another congruent triangle to make a *different bigger triangle.*

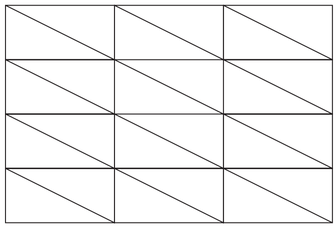


(KS3/98/Ma/Tier 3-5/P2)

5. Mike has a triangle grid. He shades in *2 triangles* to make a shape with 4 sides.



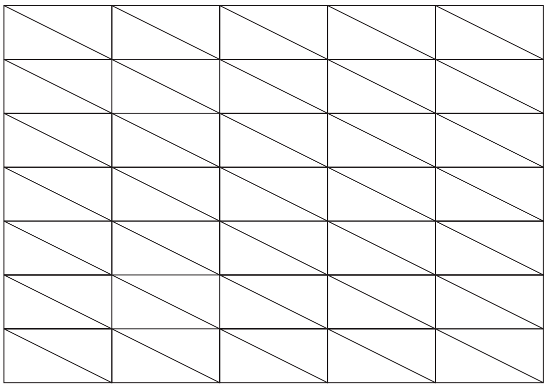
(a) Shade in *2 triangles* on a copy of the grid opposite to make a *different* shape with 4 sides.



(b) On another copy of the grid, shade in *2 triangles* to make another *different* shape with 4 sides.

(c) On another copy of the grid, shade in *4* small triangles to make a *bigger triangle.*

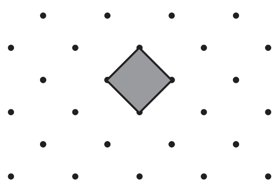
(d) On a copy of the grid below, shade in *more than 4* small triangles to make a *bigger triangle.*



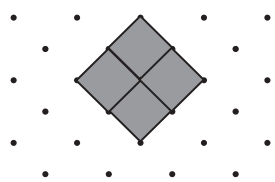
(KS3/97/Ma/Tier 3-5/P2)

6.

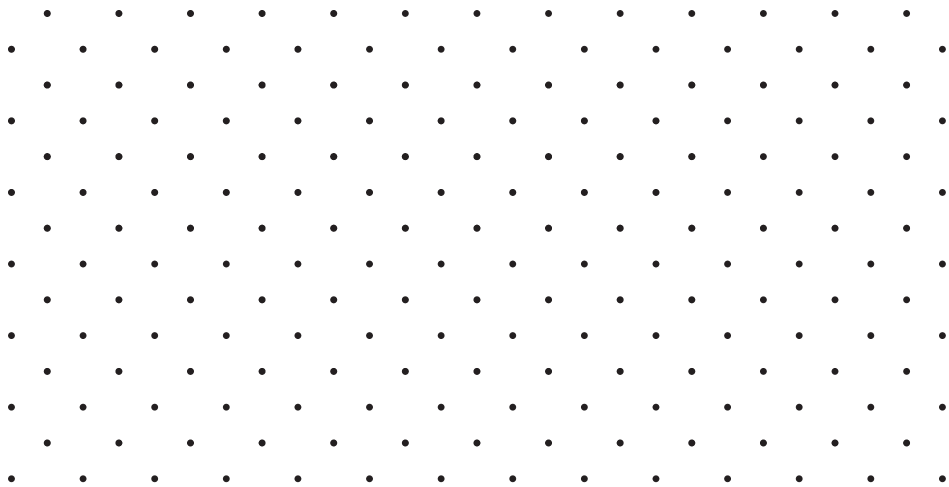
Kath puts 1 small square tile on a square dotty grid, like this:



Den makes a *bigger square* with 4 square tiles, like this:



(a) Scott has 9 small square tiles. On a copy of the following grid, show how Scott can make a *square* in the same way with 9 small square tiles.

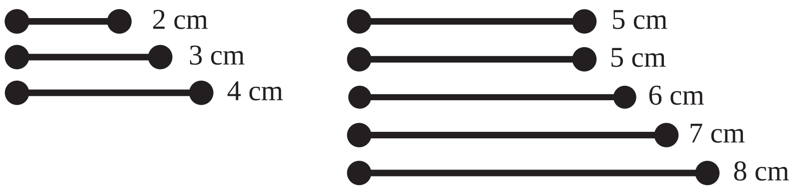


(b) On another copy of the grid, show how to make a *square* with *more than 9* of these small square tiles.
How many tiles are there in your square?

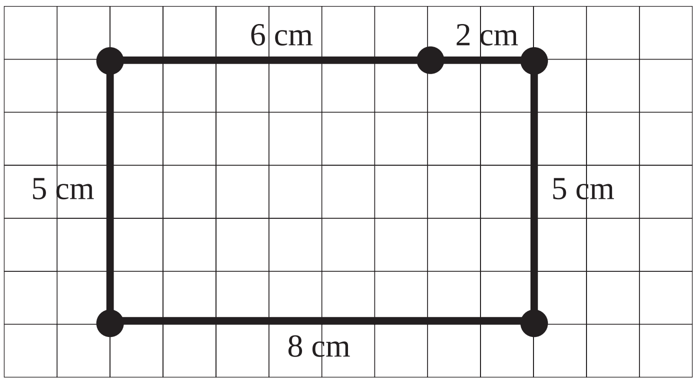
(c) Huw wants to make some more squares with the tiles. Write down 3 *other* numbers of tiles that he can use to make squares.

(KS3/96/Ma/Tier 3-5/P2)

7. Helen has *these eight rods.*



She can use 5 of her rods to make a rectangle.



(a) On a copy of the grid above, show how to make a *different rectangle* with a *different shape* with 5 of Helen's rods.

(b) On a larger grid, 13 squares by 10 squares, show how to make a rectangle with 6 of Helen's rods.

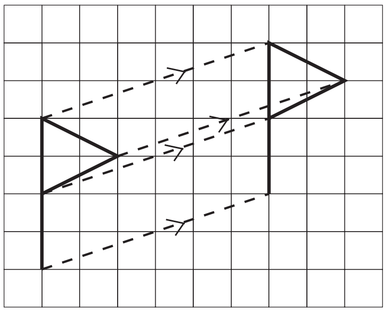
(c) On another large grid, show how to make a *square* with all 8 of Helen's rods.

(KS3/99/Ma/Tier 5-7/P1)

7.2 Translations

Under a *translation*, every point is moved by the *same amount* in the *same direction*. If each point moves distance a in the x-direction and distance b in the y-direction, we use the 'vector' notation $\begin{pmatrix} a \\ b \end{pmatrix}$ to describe this translation.

For example, the translation described by the column vector $\begin{pmatrix} 6 \\ 2 \end{pmatrix}$ is illustrated opposite; the translation moves the shape 6 units to the right and 2 units upwards.



Note that the actual shape *does not change its orientation*, only its position. It is *not reflected or rotated.*

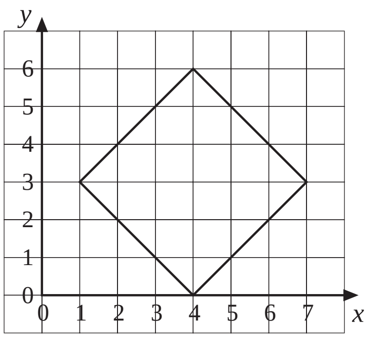
Example 1

(a) Draw the square with corners at the points with coordinates (4, 0), (1, 3), (4, 6) and (7, 3).

(b) The square is translated along the vector $\begin{pmatrix} 5 \\ 2 \end{pmatrix}$. Draw the new square obtained by the translation.

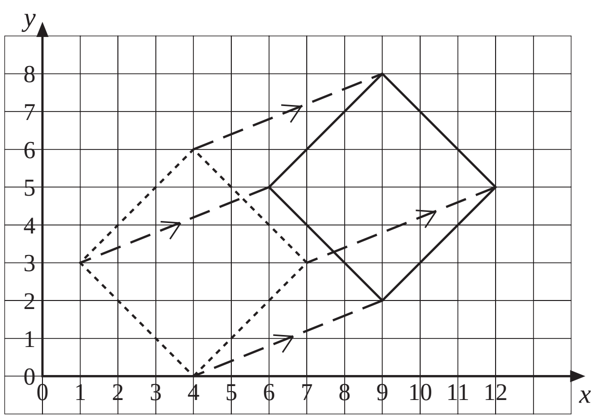
Solution

(a)



The diagram opposite shows the square.

(b)



For this translation each point should be moved 5 units to the right and 2 units up.

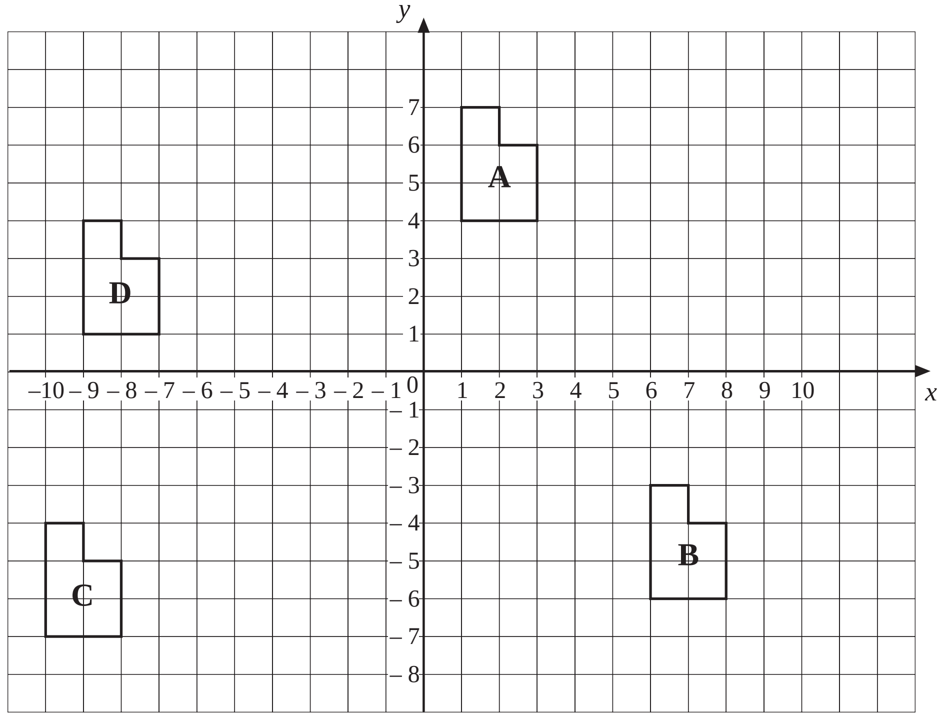
This diagram shows both squares and the vector that has been used to translate each corner.

Example 2

The diagram below shows the shapes A, B, C and D. Along what vector would you translate:

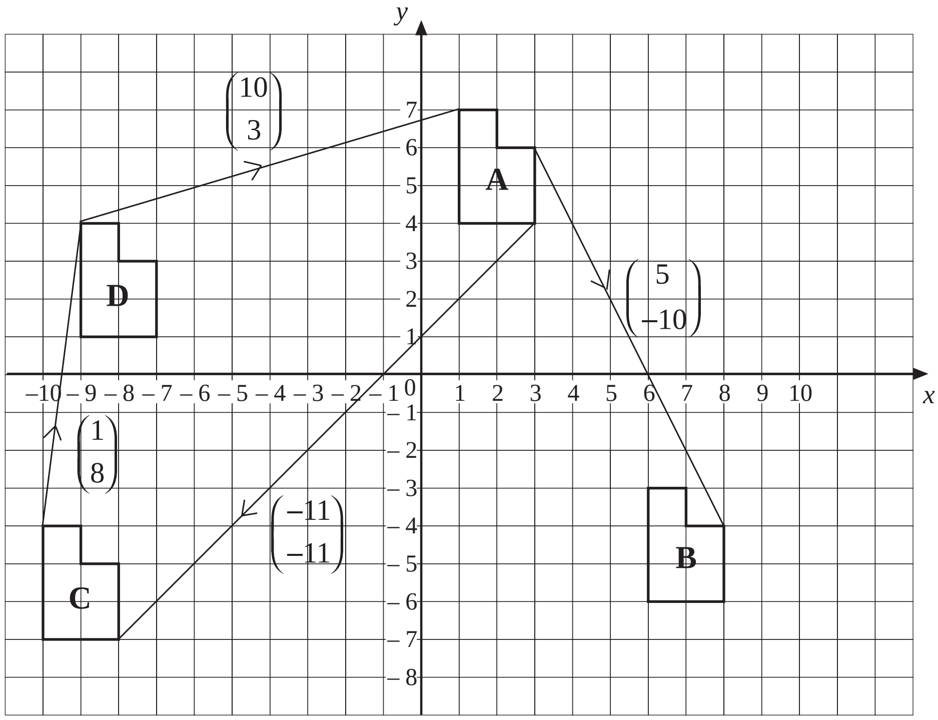
(a) D to A, (b) C to D,

(c) A to B, (d) A to C ?



Solution

The vector that describes each translation is shown on the following diagram:



(a) D to A $\begin{pmatrix} 10 \\ 3 \end{pmatrix}$, 10 to the right and 3 up.

(b) C to D $\begin{pmatrix} 1 \\ 8 \end{pmatrix}$, 1 to the right and 8 up.

(c) A to B $\begin{pmatrix} 5 \\ -10 \end{pmatrix}$, 5 to the right and 10 down.

(d) A to C $\begin{pmatrix} -11 \\ -11 \end{pmatrix}$, 11 to the left and 11 down.

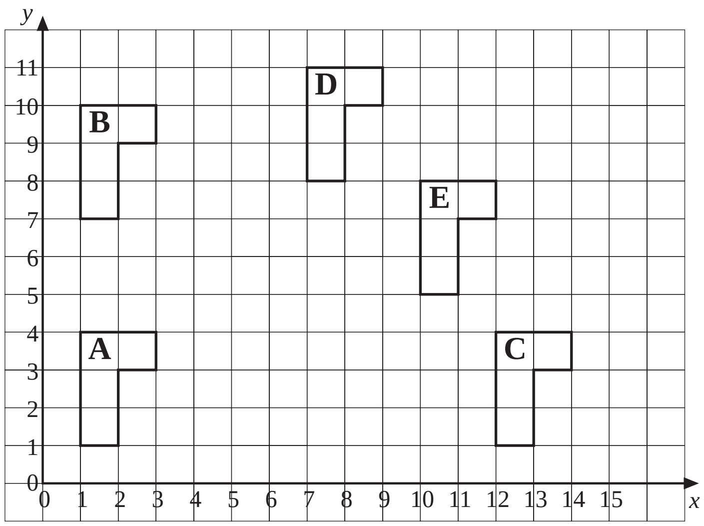
Exercises

1. (a) Draw the triangle which has corners at the points with coordinates (4, 1), (3, 5) and (1, 2).

   (b) Translate the triangle along the vector $\begin{pmatrix} 3 \\ 4 \end{pmatrix}$.

   (c) Write down the coordinates of the corners of the translated triangle.

2. The following diagram shows the shape A which is translated to give the shapes B, C, D and E:



Write down the vector that describes the translation from:

(a) A to B, (b) A to C, (c) A to D,

(d) A to E, (e) B to D.

3. (a) Join the points with coordinates (1, 1), (2, 3) and (5, 4) to form a triangle. Label this triangle A.

(b) Translate the triangle A along the vector:

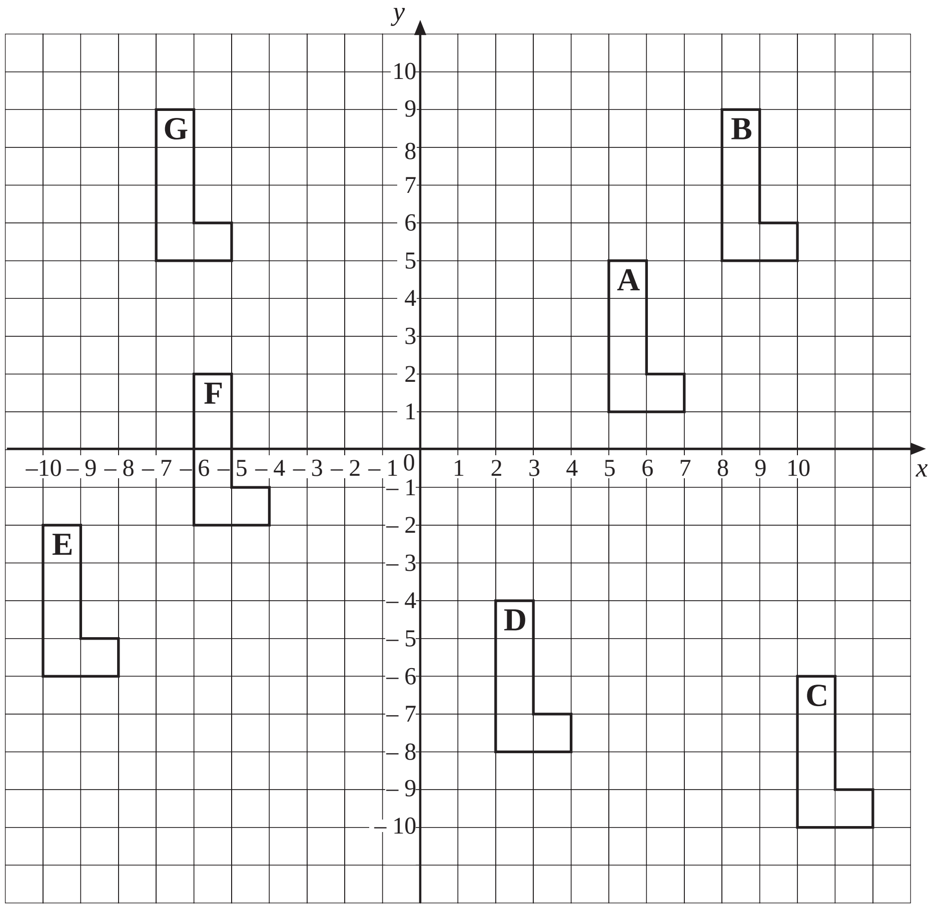
(i) $\begin{pmatrix} 2 \\ 1 \end{pmatrix}$, to obtain B,

(ii) $\begin{pmatrix} 1 \\ 3 \end{pmatrix}$, to obtain C,

(iii) $\begin{pmatrix} 2 \\ -5 \end{pmatrix}$, to obtain D,

(iv) $\begin{pmatrix} -1 \\ -2 \end{pmatrix}$, to obtain E.

4. Write down the vector needed to translate the shape A to each of the other shapes shown on the following diagram:

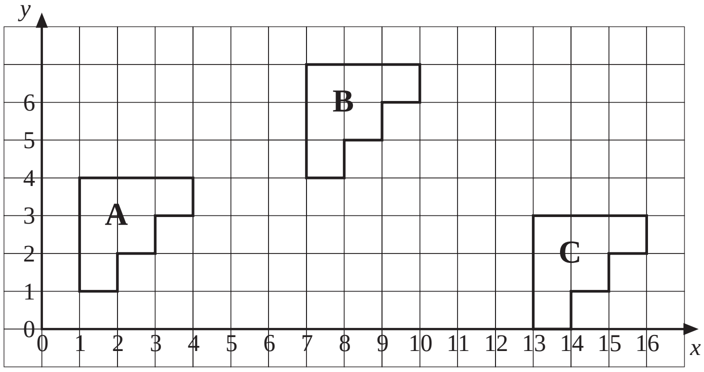


5. The point with coordinates (2, 3) is moved to the point with coordinates (7, 6) by a translation.

   (a) Describe the translation using a column vector.

   (b) Where would the point with coordinates (6, 1) move to under the same translation?

6. The diagram shows three shapes, A, B and C:



   Write down the vector for the translation that moves:

   (a) A to B,

   (b) B to C,

   (c) A to C.

   Describe any relationship between these vectors.

7. The shape A has corners at the points with coordinates

   (4, 2) (4, – 1) (6, – 3) and (6, 0)

   (a) What is this shape?

   (b) The shape is translated along the vector $\begin{pmatrix} 4 \\ -2 \end{pmatrix}$ to give shape B and then shape B is translated along the vector $\begin{pmatrix} -5 \\ 2 \end{pmatrix}$ to give C.

   Draw A, B and C.

   (c) What translation would take A straight to C ?

8. (a) Draw the triangle, A, that has corners at the points with coordinates (– 7, – 2), (– 5, – 5) and (– 4, – 2).

   (b) Translate this shape along the vector $\begin{pmatrix} 4 \\ 8 \end{pmatrix}$ to obtain B.

   (c) Describe the translation that would take B to A.

9. (a) Draw three lines by joining the points with coordinates

   (4, 2) and (2, 4);

   (6, 4) and (6, 6);

   (2, 6) and (4, 8).

   (b) Describe how to translate each line to form a hexagon made up of the original and translated lines.

10. A parallelogram has corners at the points A, B, C and D. The points A, B and C have coordinates (1, 2), (2, 5) and (5, 3) respectively.
    (a) Draw the parallelogram.
    (b) State the coordinates of the fourth corner, D.
    (c) Describe the translation that moves A B onto D C.
    (d) Describe the translation that moves A D onto B C.

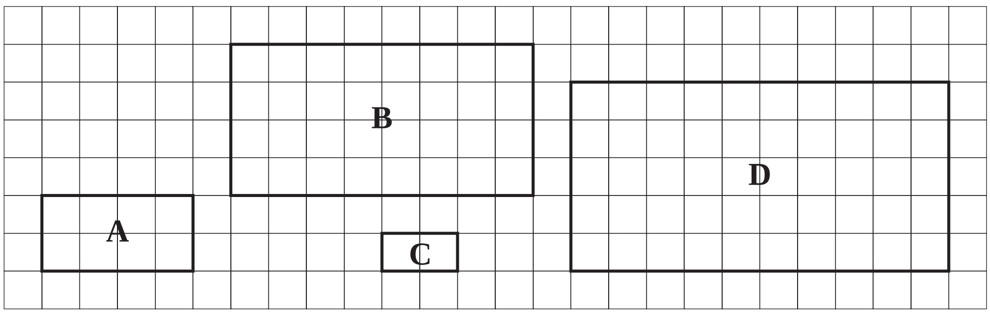
7.3 Enlargements

In this section we consider enlargements. We look at the use of the terms 'scale factor' and 'centre of enlargement'.

Example 1

The rectangle A, shown below, has been enlarged to give the shapes B, C and D.

Write down the scale factor for each enlargement.



Solution

A to B is scale factor 2 because the lengths are doubled.

A to C is scale factor $\frac{1}{2}$ because the lengths are halved.

A to D is scale factor 2.5 because the lengths are 2.5 times longer.

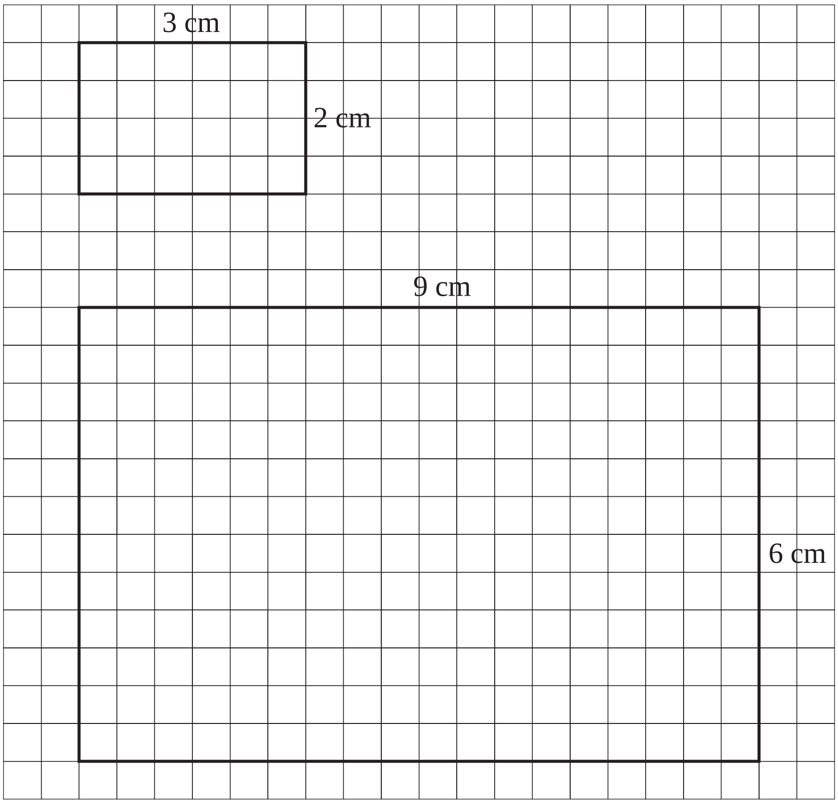
Example 2

A rectangle has sides of lengths 2 cm and 3 cm. It is enlarged with scale factor 3. Draw the original rectangle and the enlarged rectangle.

Solution

The lengths of the sides of the enlarged rectangle will be:

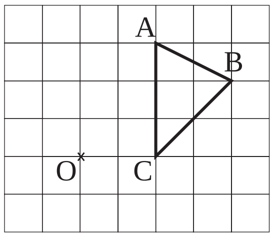
$$3 \times 2 \text{ cm} = 6 \text{ cm}$$
$$3 \times 3 \text{ cm} = 9 \text{ cm}$$



Examples 3 and 4 show how to use a centre of enlargement when enlarging a shape.

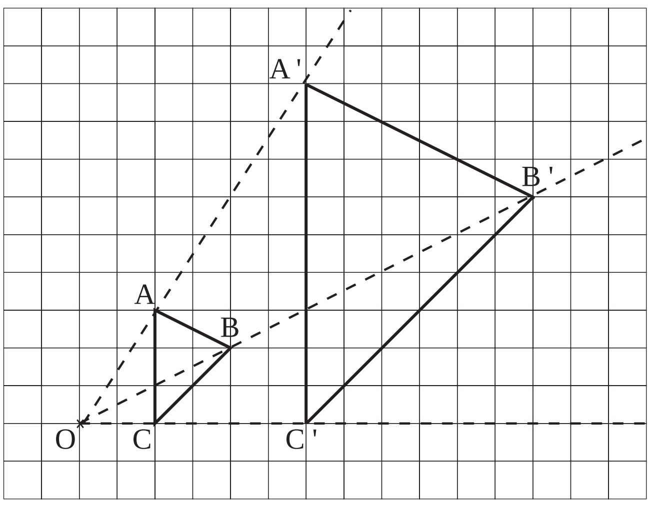
Example 3

The diagram shows the triangle A B C and the point O. Enlarge the triangle with scale factor 3, using O as the centre of enlargement.



Solution

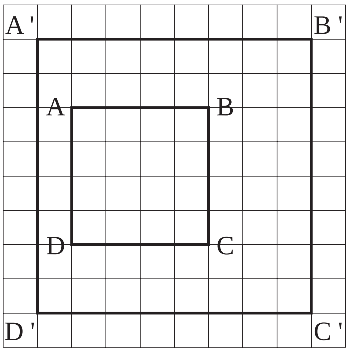
The diagram shows the 2 triangles; the explanation follows.

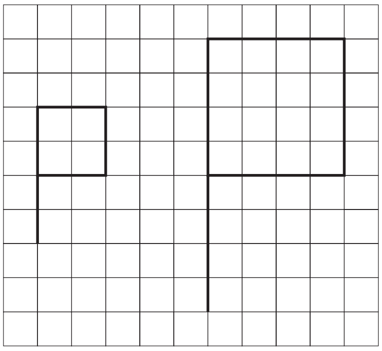


First draw lines from point O through A, B and C, as shown in the diagram. Measure the length O A and multiply it by 3 to get the distance from O of the image point A', i.e. $OA' = 3 \times OA$. Mark the point A' on the diagram. The images B' and C' can then be marked in a similar way and the enlarged triangle A' B' C' can then be drawn.

Example 4

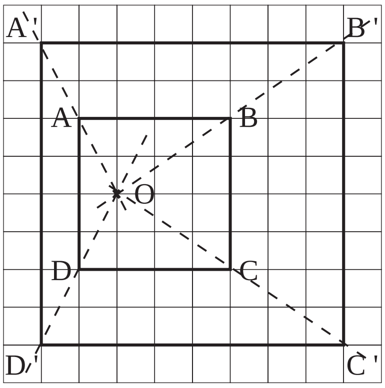
The following diagrams show two shapes that have been enlarged. Determine the centre of enlargement in each case.

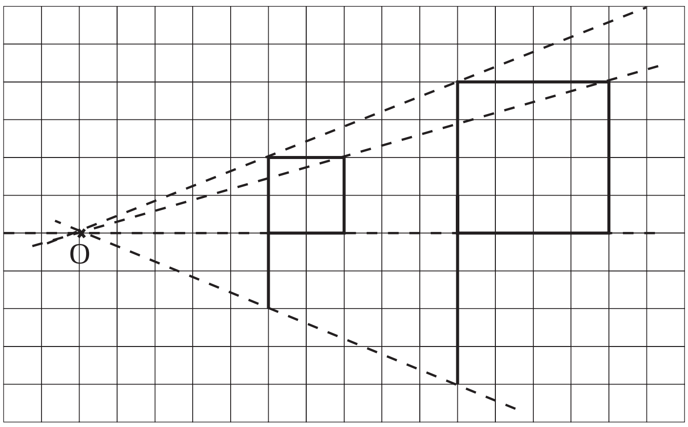




Solution

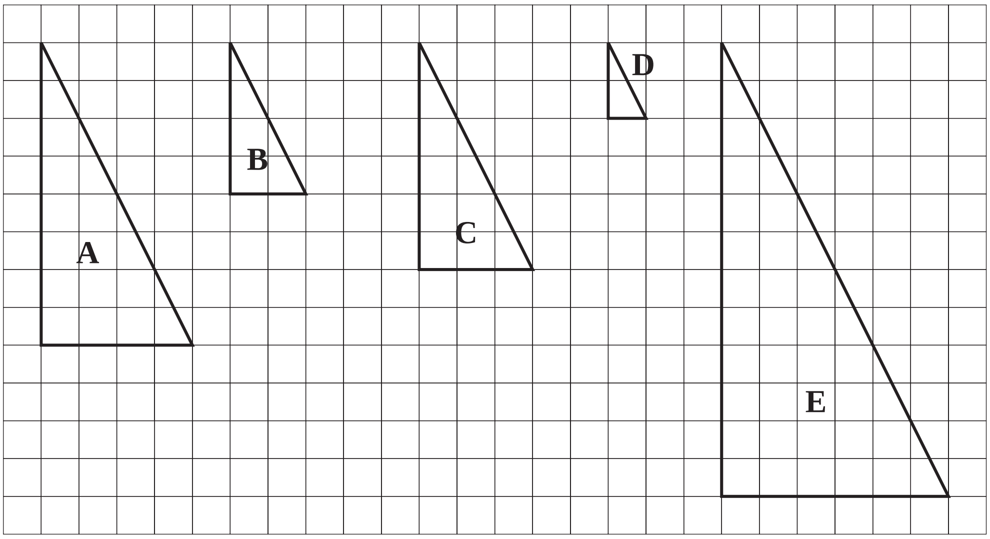
To find the centres of enlargement, draw lines through the corresponding corners of each shape. These lines will cross at the centre of enlargement, as shown below. The centres have been marked with the letter O in both diagrams.





Exercises

1. The following diagram shows 5 triangles, A, B, C, D and E:



What scale factor is used for each of the enlargements described below:

(a) B enlarged to A,
(b) C enlarged to E,
(c) D enlarged to E,
(d) D enlarged to A,
(e) B enlarged to C,
(f) B enlarged to D ?

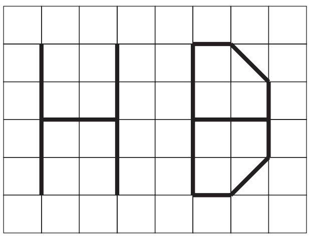
2. (a) Draw a rectangle that has sides of lengths 2 cm and 4 cm.

(b) Draw enlargements of this rectangle using scale factors 2, 3, and $\frac{1}{2}$.

3. (a) Construct a triangle that has sides of lengths 3 cm, 4 cm and 5.5 cm.

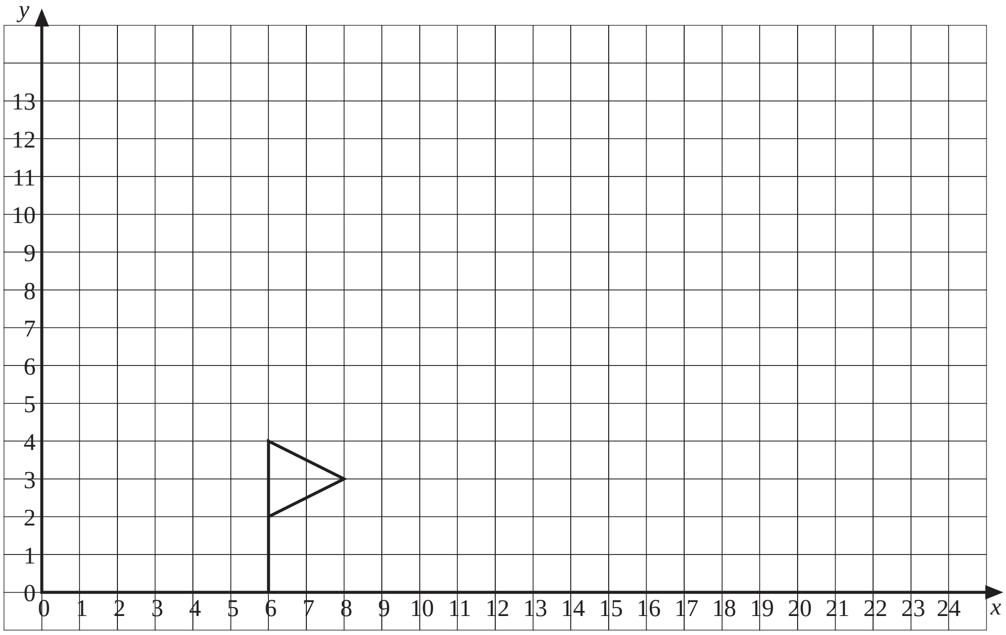
(b) Draw enlargements of this triangle using scale factors 2 and 3.

4. Hannah writes her initials as shown:



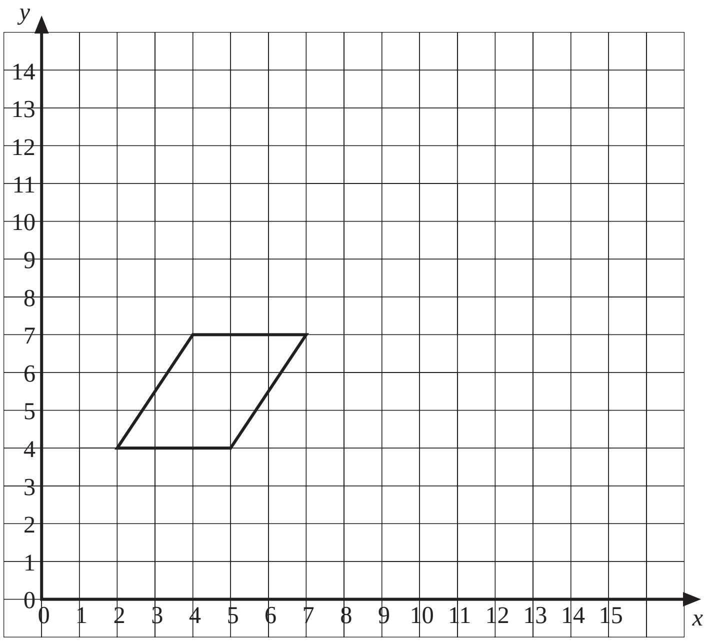
Enlarge her initials with scale factors 2 and $\frac{1}{2}$.

5. (a) Copy the following diagram:



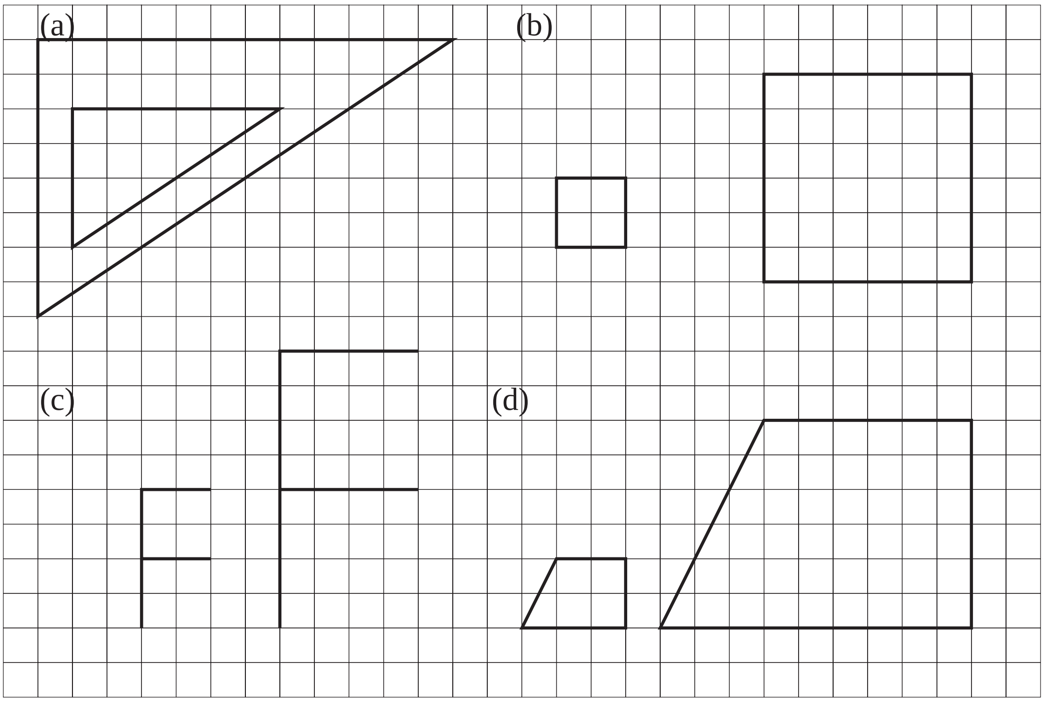
(b) Using (0, 0) as the centre of enlargement, enlarge the shape with scale factor 2 and scale factor 3.

6. (a) Copy the following diagram:



(b) Enlarge the shape with scale factor 2, using first (0, 0) as the centre of enlargement and then (1, 8) as the centre of enlargement.

7. For each of the following enlargements, copy the diagram and determine the centre of enlargement.



8. A triangle has corners at the points with coordinates (1, 2), (3, 3) and (0, 3). It is enlarged to give a triangle with corners at the points (5, 4), (11, 7) and (2, 7). Determine the scale factor of the enlargement and the coordinates of the centre of enlargement.

9. A trapezium has corners at the points with coordinates (1, 0), (3, 2), (3, 4) and (1, 5). It is enlarged with scale factor 3, using the point (0, 3) as the centre of enlargement.

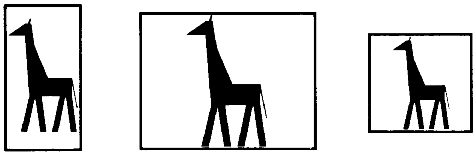
   Determine the coordinates of the corners of the enlarged trapezium.

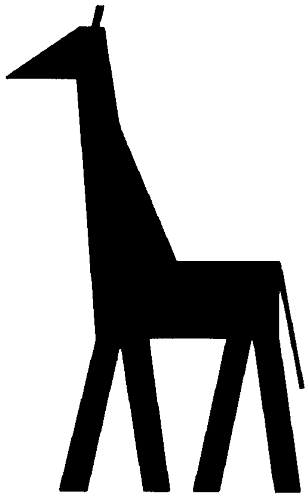
10. A parallelogram has corners at the points with coordinates (5, 1), (9, 3), (11, 9) and (7, 7). Enlarge this shape with scale factor $\frac{1}{2}$, using the point with coordinates (1, 3) as the centre of enlargement.

11. Jill has drawn an original picture of a giraffe for an animal charity.

    It measures 6.5 cm high by 4 cm wide.

    Different-sized copies of the original picture can be made to just fit into various shapes.

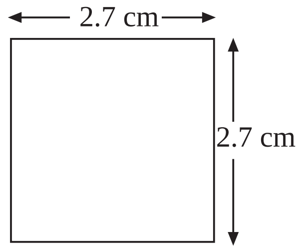




(a) Jill wants to enlarge the original picture so that it *just* fits inside a rectangle on a carrier bag. The rectangle measures 24 cm high by 12 cm wide.

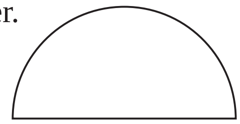
By what scale factor should she multiply the original picture? Show your working.

(b) Jill wants to multiply the *original picture* by a scale factor so that it *just* fits inside the square shown below for a badge.



By what scale factor should she multiply the original picture?

(c) The *original picture* is to be used on a poster.
It must fit inside a shape like this.

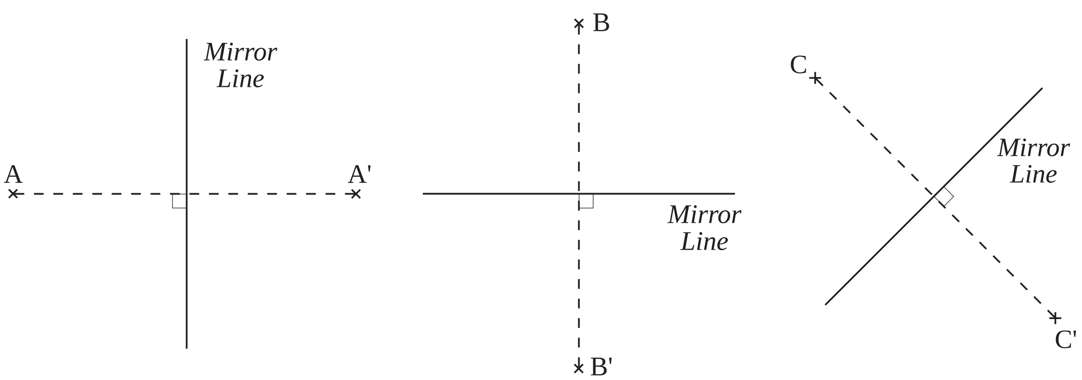


The shape is to be a semi-circle of radius 6.6 cm.
What would be the perimeter of the shape? Show your working.

(KS3/96/Ma/Tier 5-7/P2)

7.4 Reflections

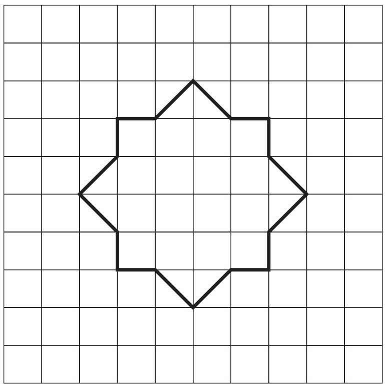
In this section we look at line symmetry and reflections of simple shapes, in horizontal, vertical and sloping lines. In a reflection, a point will move to a new position that will be the same distance from the mirror line, but on the other side. These distances will always be measured at right angles to the mirror line.



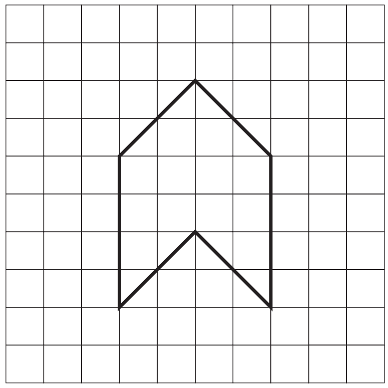
Example 1

Draw in the lines of symmetry for each of the following shapes:

(a)

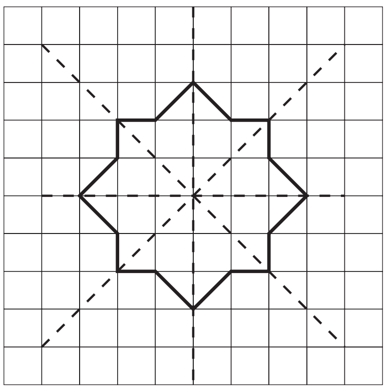


(b)



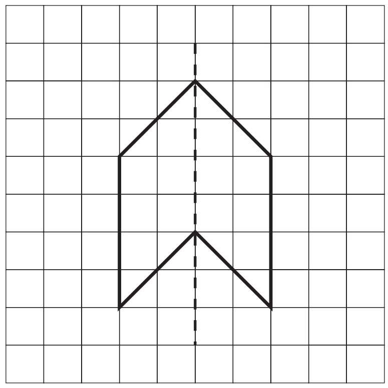
Solution

(a)



4 lines of symmetry

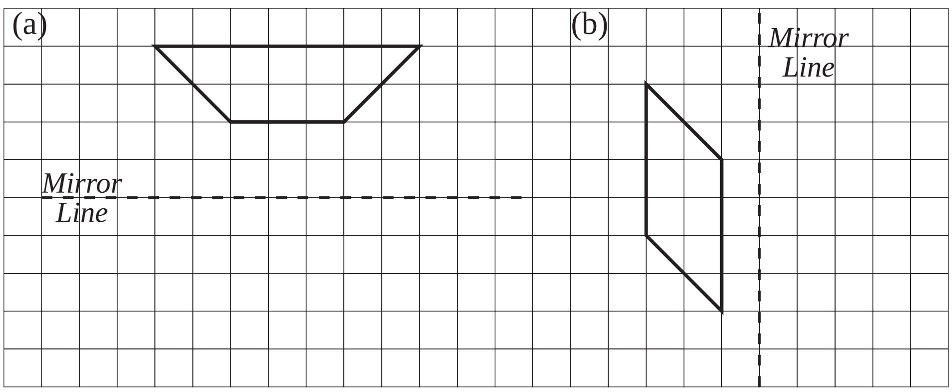
(b)



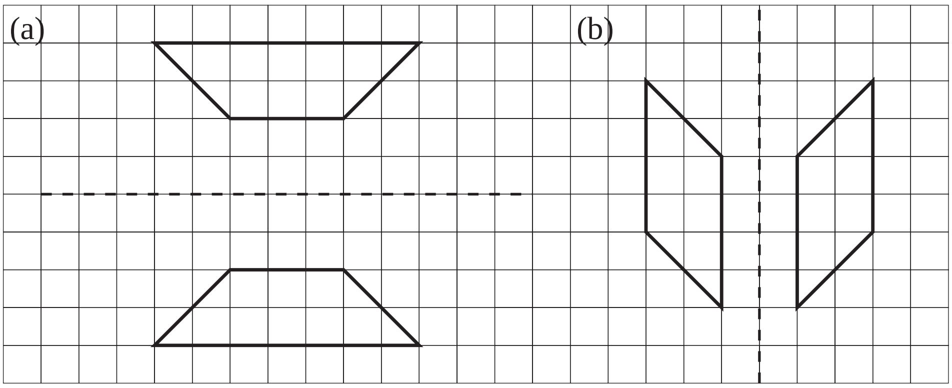
1 line of symmetry

Example 2

Draw the reflection of each of the following shapes in the given mirror line.



Solution



Example 3

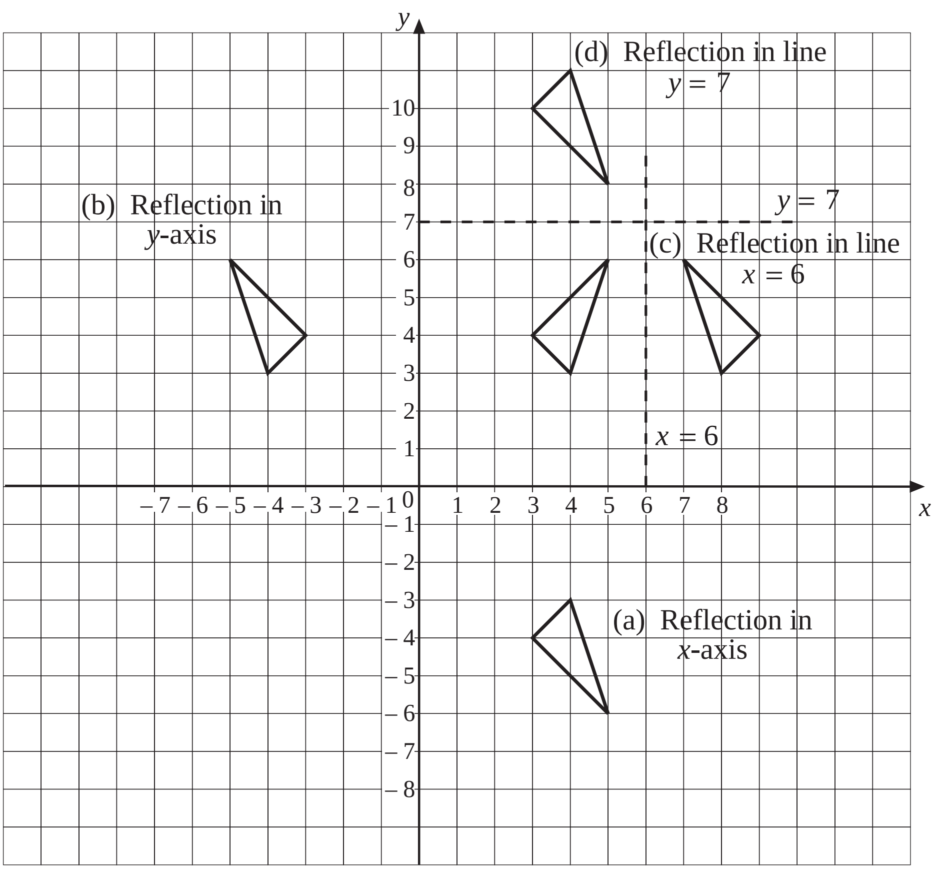
A triangle has corners at the points with coordinates (4, 3), (5, 6) and (3, 4).

Draw the reflection of the triangle in the:

(a) x-axis (b) y-axis,

(c) line $x = 6$ (d) line $y = 7$

Solution



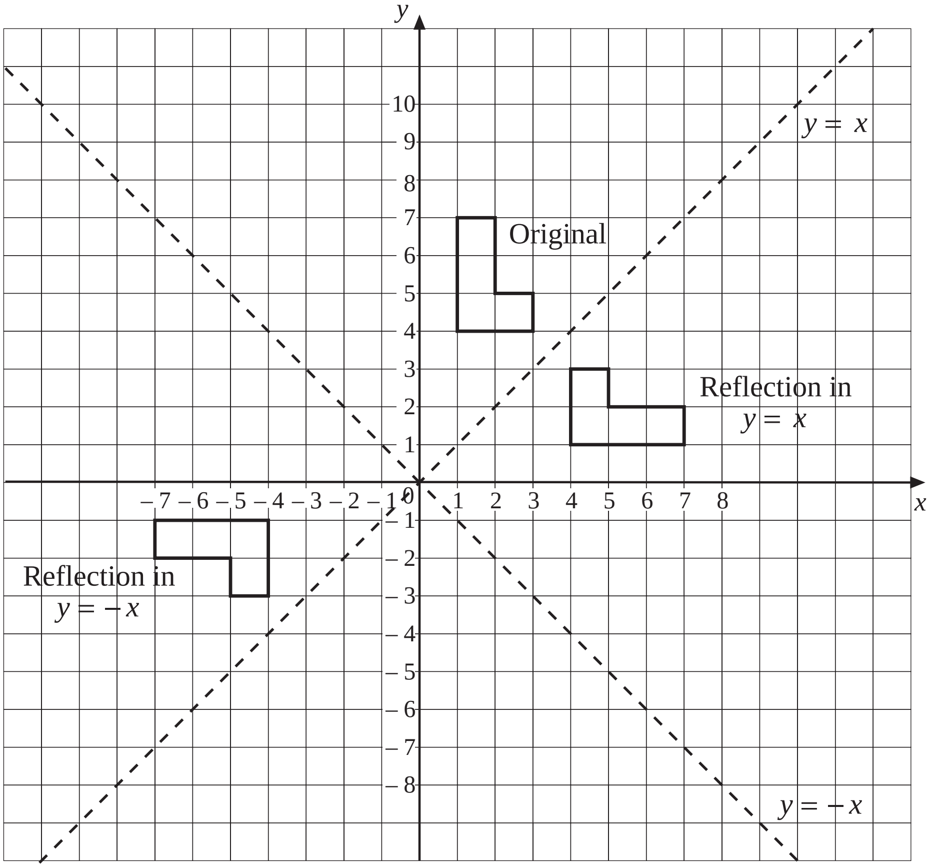
Example 4

An 'L' shape has corners at the points with coordinates (1, 4), (1, 7), (2, 7), (2, 5), (3, 5) and (3, 4).

Draw the reflection of the shape in the lines:

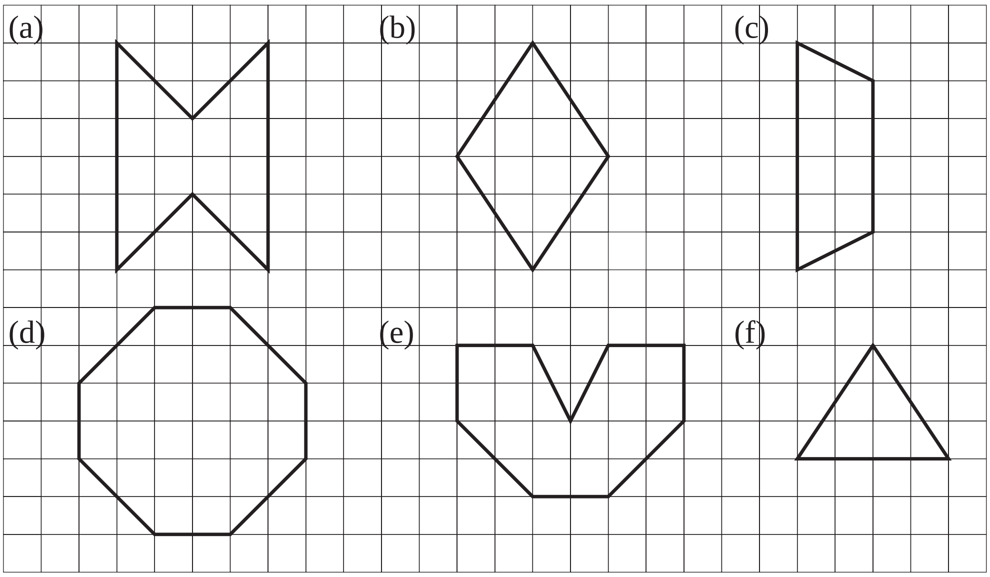
(a) $y = x$ (b) $y = -x$

Solution

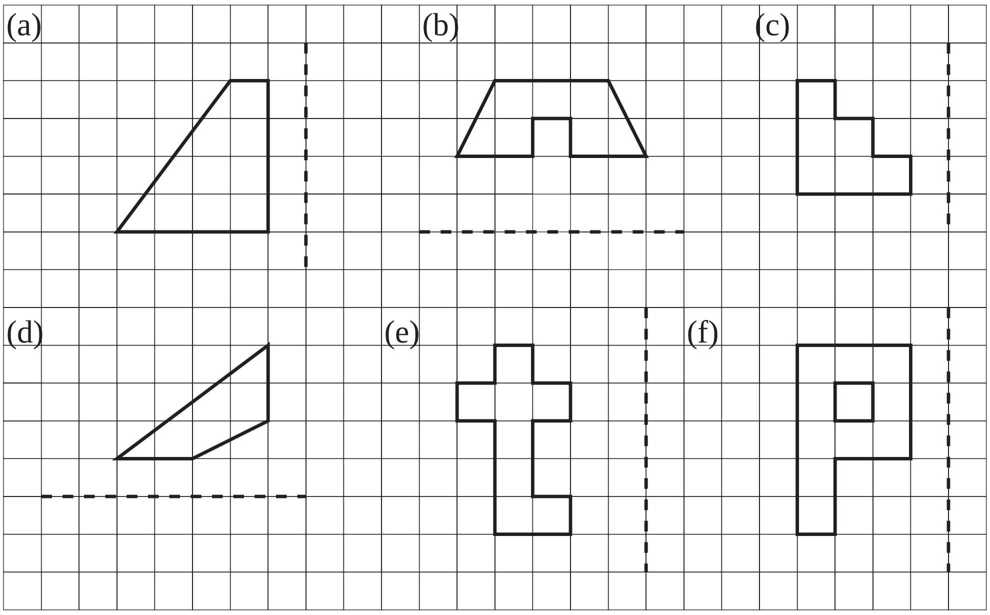


Exercises

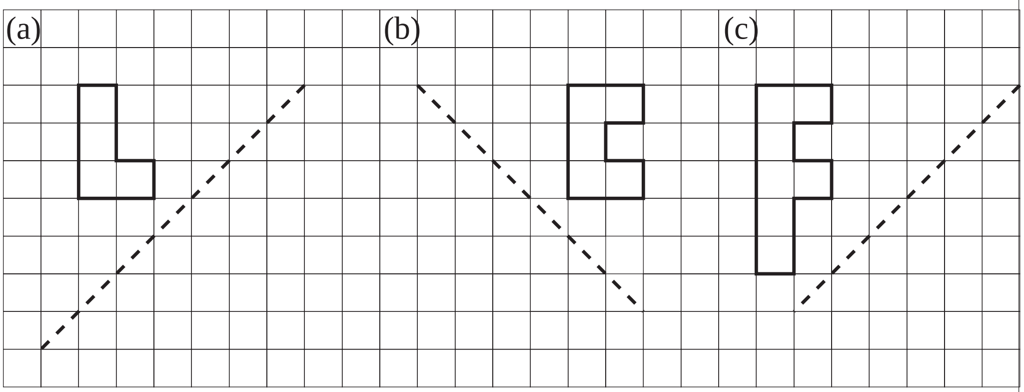
1. Copy the following shapes and draw in all their lines of symmetry.



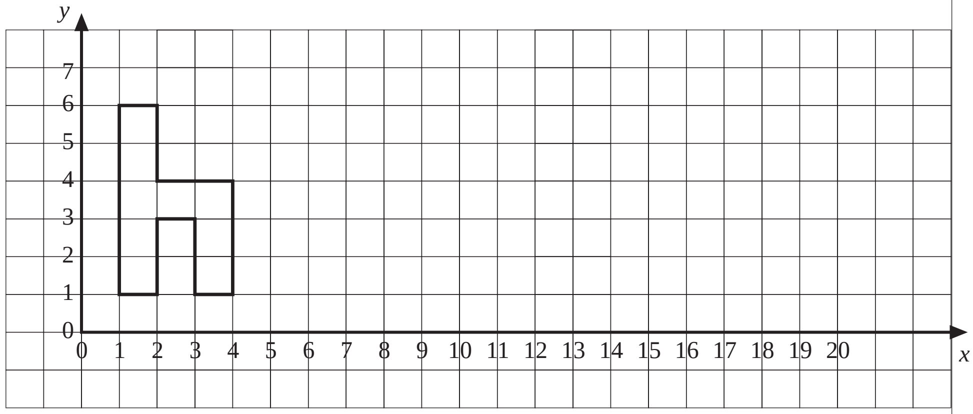
2. Draw the reflection of each of the following shapes in the line given:



3. Copy each of the following shapes and draw its reflection in the line shown:

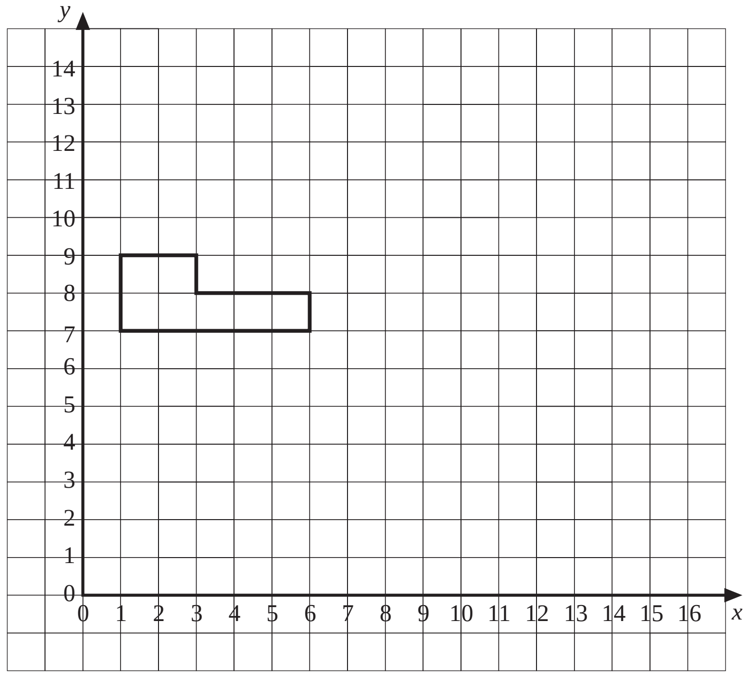


4. (a) Copy the following diagram:



(b) Reflect the shape in the lines $x = 8$ and $x = 11$.

5. (a) Copy the diagram shown.



(b) Reflect the shape in the lines $y = 10$, $y = 5$ and $x = 7$.

6. (a) Draw the triangle that has corners at the points with coordinates (1, 1), (4, 7) and (2, 5).

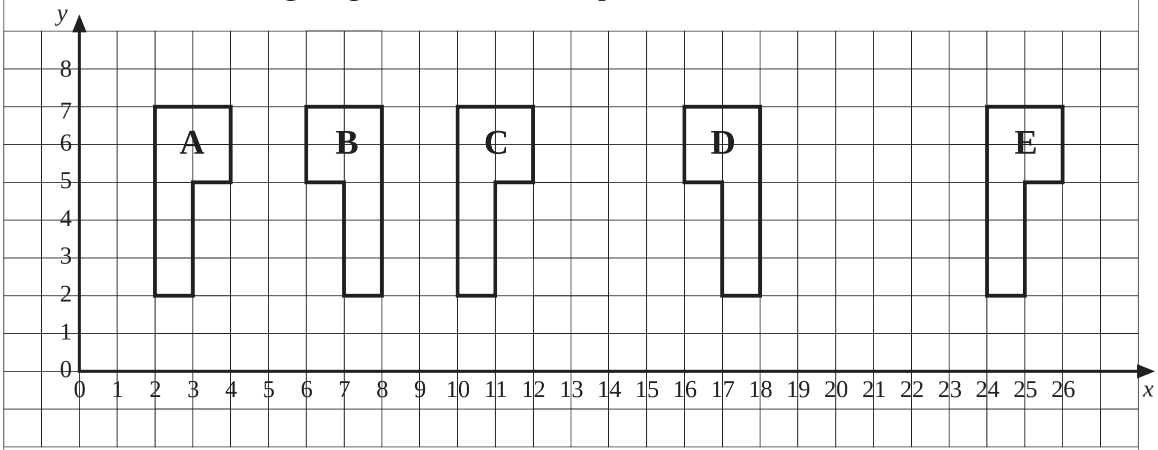
(b) Reflect the triangle in the lines:

(i) $x = 8$,

(ii) $x = -1$,

(iii) $y = -2$

7. The following diagram shows the shapes A, B, C, D and E.



Write down the equation of the mirror line for each of the following reflections:

(a) A to B (b) B to C (c) A to D

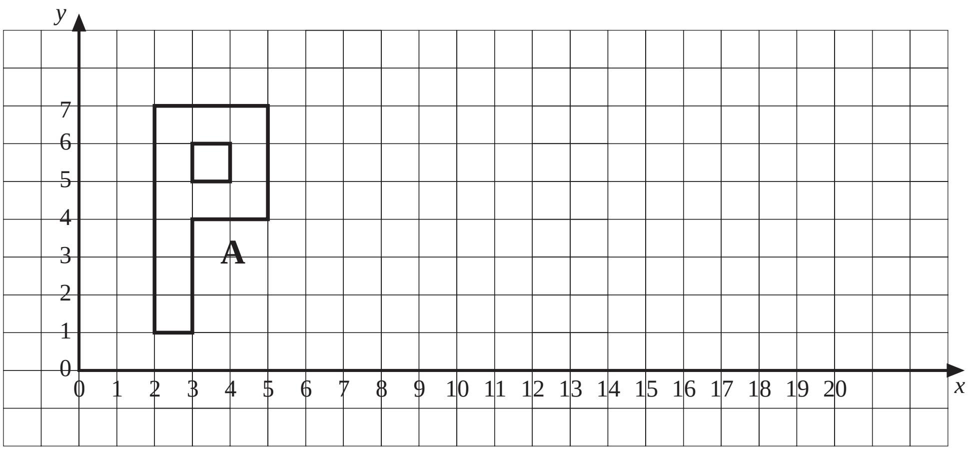
(d) B to E (e) D to E (f) C to D

8. (a) Draw the triangle which has corners at the points with coordinates (1, 4), (1, 7) and (3, 5).

(b) Reflect this shape in the line $y = x$ and state the coordinates of the corners of the reflected shape.

(c) Reflect the original triangle in the line $y = -x$ and state the coordinates of the corners of the reflected shape.

9. (a) Draw the shape A shown in the following diagram.



(b) Reflect the shape A in the line $x = 6$ to obtain shape B.

(c) Reflect the shape B in the line $x = 14$ to obtain shape C.

(d) Describe the translation that would take shape A straight to shape C.

10. Draw the triangle with corners at the points with coordinates (1, 3), (1, 8) and (6, 8). Reflect this triangle in the following lines:

(a) $x = 0$ (b) $y = 0$

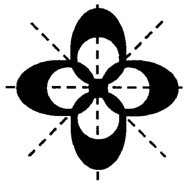
(c) $y = x$ (d) $y = -x$

11. These patterns have one or more lines of symmetry.

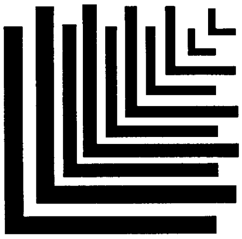
Draw *all* the lines of symmetry in each pattern.

You may use a mirror or tracing paper to help you.

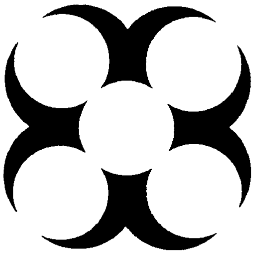
EXAMPLE



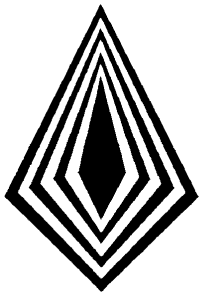
(a)



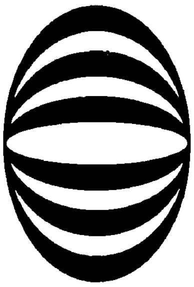
(b)



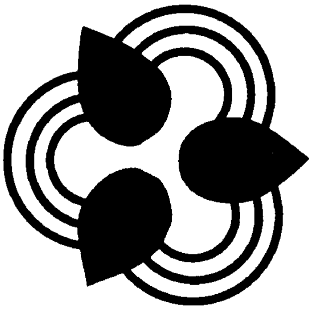
(c)



(d)

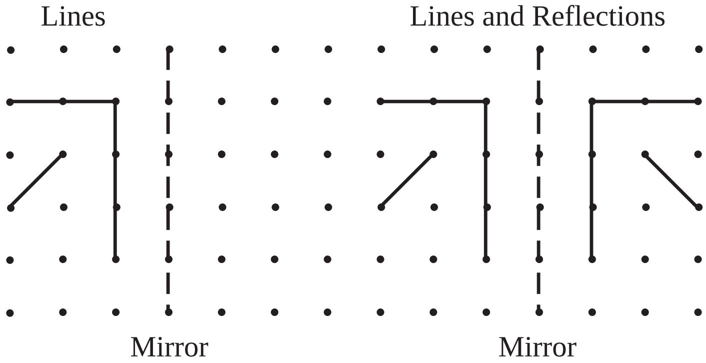


(e)



(KS3/94/Ma/3-5/P1)

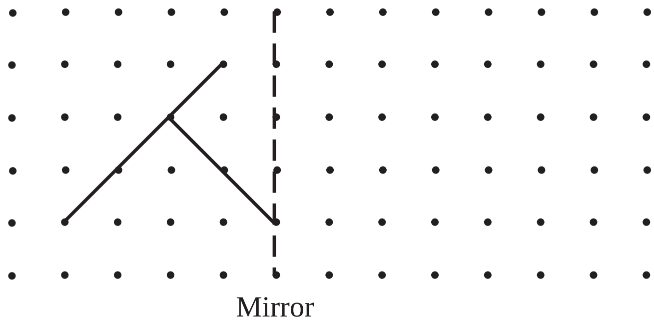
12. Nina is making Rangoli patterns. To make a pattern she draws some lines on a grid. Then she reflects them in a mirror line.



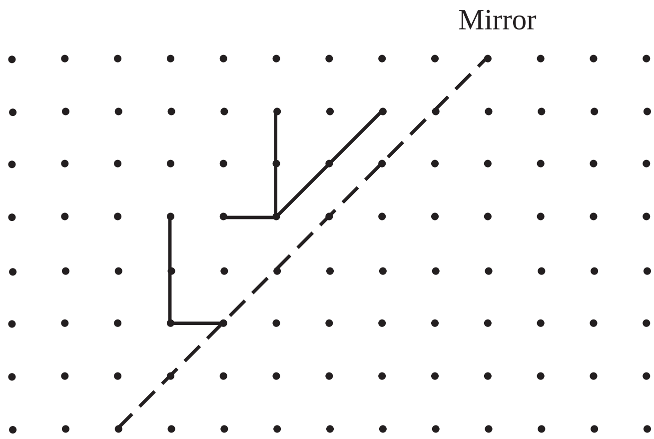
Make a copy each of the following grids and lines.

Reflect each group of lines in its mirror line to make a pattern. You may use a real mirror or tracing paper to help you.

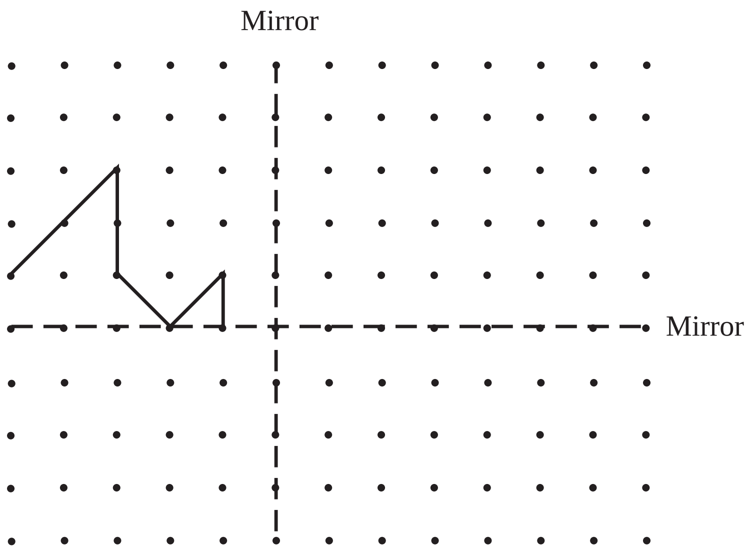
(a)



(b)



(c) Now use two mirror lines to make a pattern.
First reflect the group of lines in one mirror line to make a pattern.
Then reflect the whole pattern in the other mirror line.

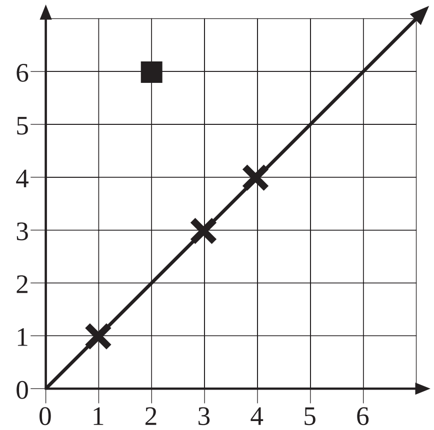


(KS3/95/Ma/Levels 3-5/P1)

13. (a) Three points on this line are marked with ✖.

Their coordinates are:
(1, 1), (3, 3) and (4, 4).

Look at the *numbers* in the coordinates of each point.
What do you notice?



(b) The point (? , $14\frac{1}{2}$) is *on* the line.
Write down its missing coordinate.

(c) The point ■ is *above* the line.

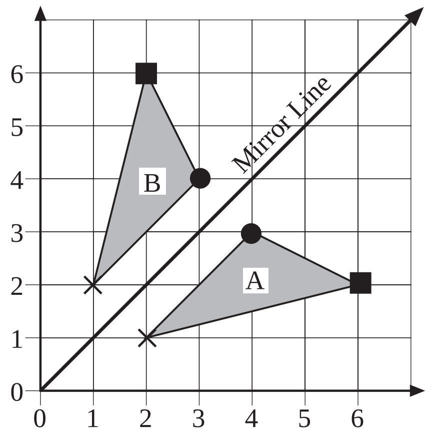
Four points are at (10, 10), (10, 12), (12, 10) and (12, 12).

Which one of these points is *above* the line? Explain why.

(d) The point (? , 15) is *above* the line. Write down a possible coordinate for the point.

(e) Look at triangles A and B.

	Triangle A	*Triangle B*
Coordinates of ●	(4, 3)	(3, 4)
Coordinates of ×	(2, 1)	(1, 2)
Coordinates of ■	(6, 2)	(2, 6)

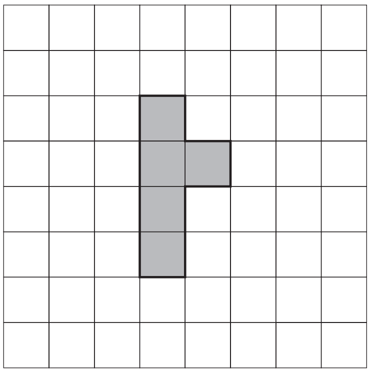


Triangle A was reflected onto triangle B.
What happened to the *numbers* in the coordinates of each corner?

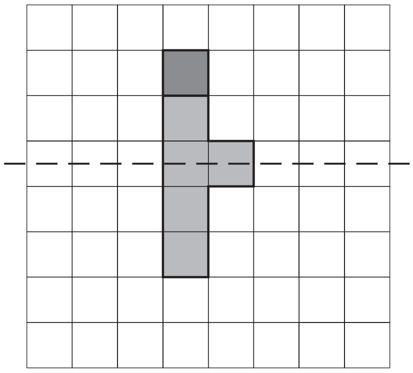
(f) Elen wants to reflect the point (20, 13) in the mirror line. What point will (20,13) go to?

(KS3/94/Ma/3-5/P2)

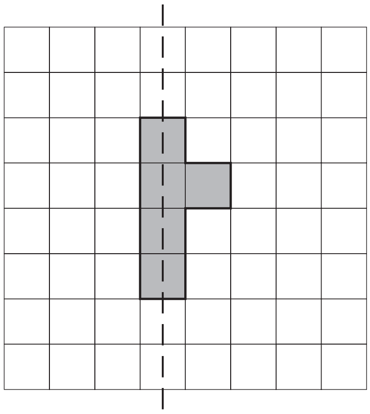
14. Catrin shades in a shape made of five squares on a grid:



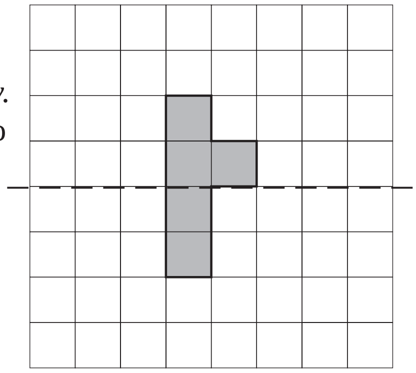
She shades in *1 more square* to make a shape which has the dashed line as a *line of symmetry*:



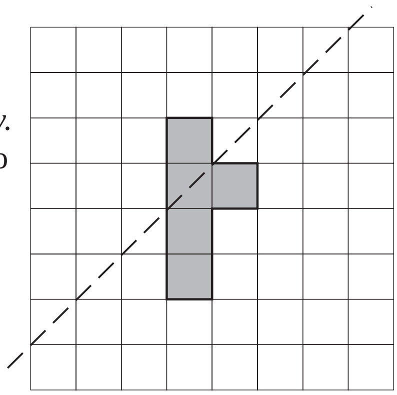
(a) On a copy of the grid opposite, shade in *1 more square* to make a shape which has the dashed line as a *line of symmetry.*



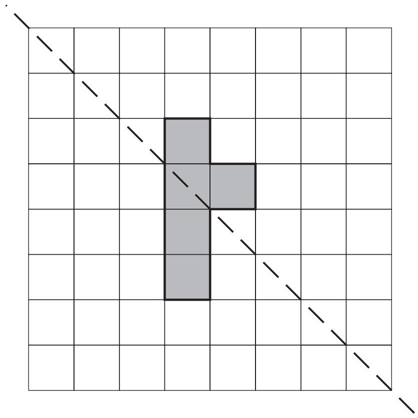
(b) On a copy of the grid opposite, shade in *1 more square* to make a shape which has the dashed line as a *line of symmetry.* You may use a mirror or tracing paper to help you.



(c) On a copy of the grid opposite, shade in *2 more squares* to make a shape which has the dashed line as a *line of symmetry.* You may use a mirror or tracing paper to help you.



(d) On a copy of the grid opposite, shade in 2 more *squares* to make a shape which has the dashed line as a *line of symmetry*. You may use a mirror or tracing paper to help you.



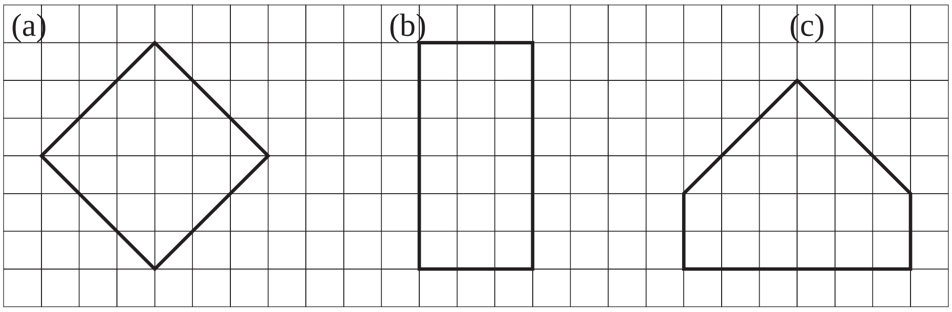
(KS3/96/Ma/Tier 3-5/P2)

7.5 Rotations

In this section we review rotational symmetry and draw rotations of shapes.

Example 1

State the order of rotational symmetry of each of the following shapes:



Solution

(a) Order 4. This means that the shape can be rotated 4 times about its centre before returning to its starting position. Each rotation will be through an angle of $90°$, and, after each one, the rotated shape will occupy the same position as the original square.

(b) Order 2

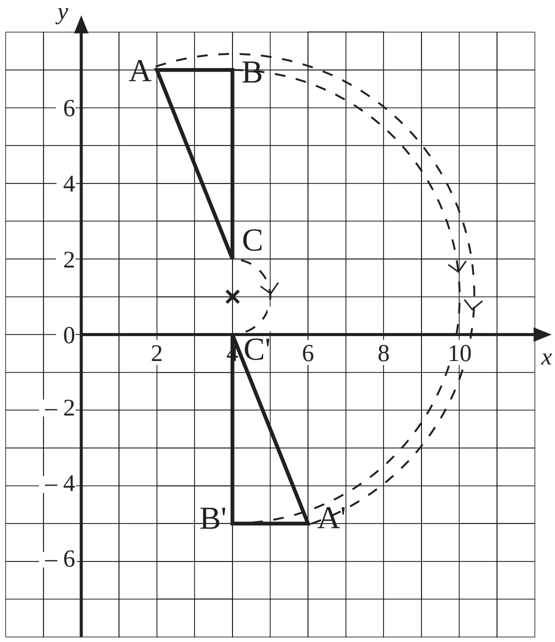
(c) Order 1. This means that the shape does *not* have rotational symmetry.

Example 2

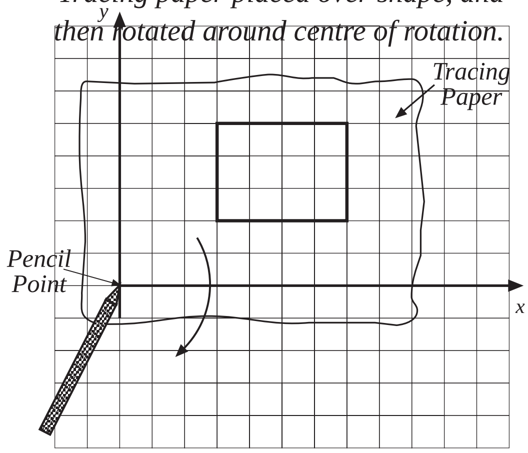
The corners of a rectangle have coordinates (3, 2), (7, 2), (7, 5) and (3, 5). The rectangle is to be rotated through $90°$ clockwise about the origin.
Draw the original rectangle and its position after being rotated.

Solution

The following diagram shows the original rectangle A B C D and the rotated rectangle A' B' C' D'. The curves show how each corner moves as it is rotated.
The easiest way to rotate a shape is to place a piece of tracing paper over the shape, trace the shape, and then rotate the tracing paper about the centre of rotation, as shown.



Tracing paper placed over shape, and then rotated around centre of rotation.



Example 3

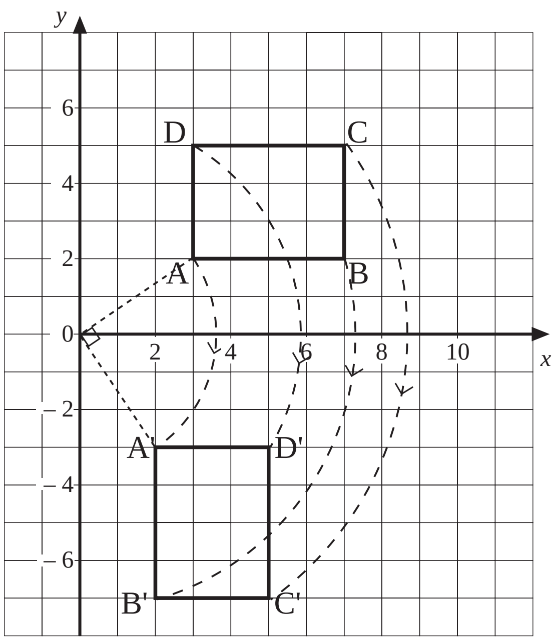
A triangle has corners at the points with coordinates (4, 7), (2, 7) and (4, 2).

(a) Draw the triangle.

(b) Rotate the triangle through 180 ° about the point (4, 1).

Solution

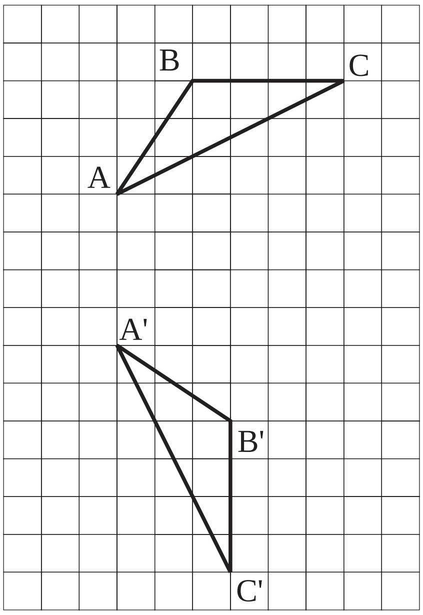
The diagram shows how the original triangle A B C is rotated about the point (4, 1) to give the triangle A' B' C'.



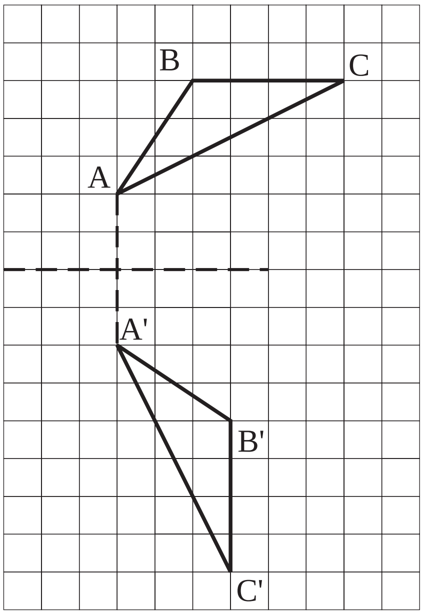
Example 4

The diagram shows the triangle A B C which is rotated through $90°$ to give A' B' C'.

Determine the position of the centre of rotation.



Solution

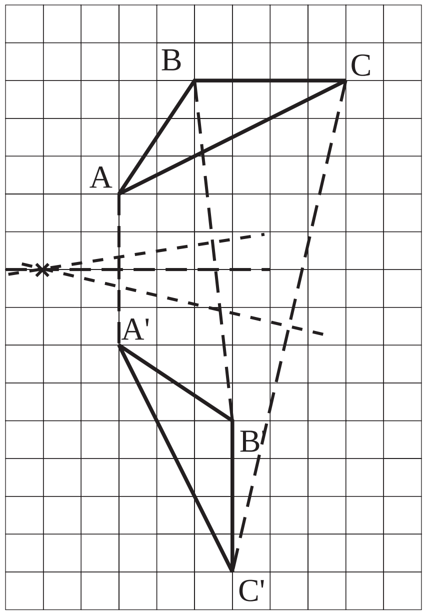


The first step is to join the points A and A' and draw the perpendicular bisector of this line.

The centre of rotation must be on this line.

Repeat the process, drawing the perpendicular bisectors of B B' and C C' as shown opposite.

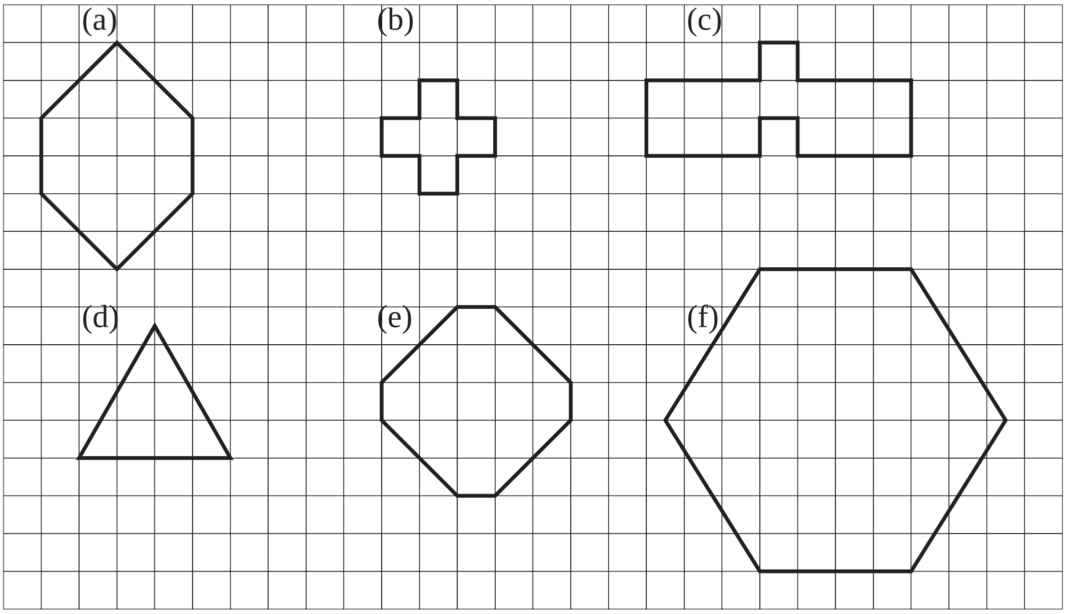
The point where the lines cross is the centre of rotation.



Note: For simple rotations you may be able to spot the centre of rotation without having to use the method shown above. Alternatively, you may be able to find the centre of rotation by experimenting with tracing paper.

Exercises

1. State the order of rotational symmetry of each of the following shapes:



2. Which of the capital letters have rotational symmetry?

3. A rectangle has corners at the points A (2, 4), B (6, 4), C (6, 6) and D (2, 6).
   (a) Draw this rectangle.
   (b) Rotate the rectangle through $90°$ clockwise about the point (0, 0).
   (c) Rotate the rectangle A B C D through $180°$ about the point (0, 0).

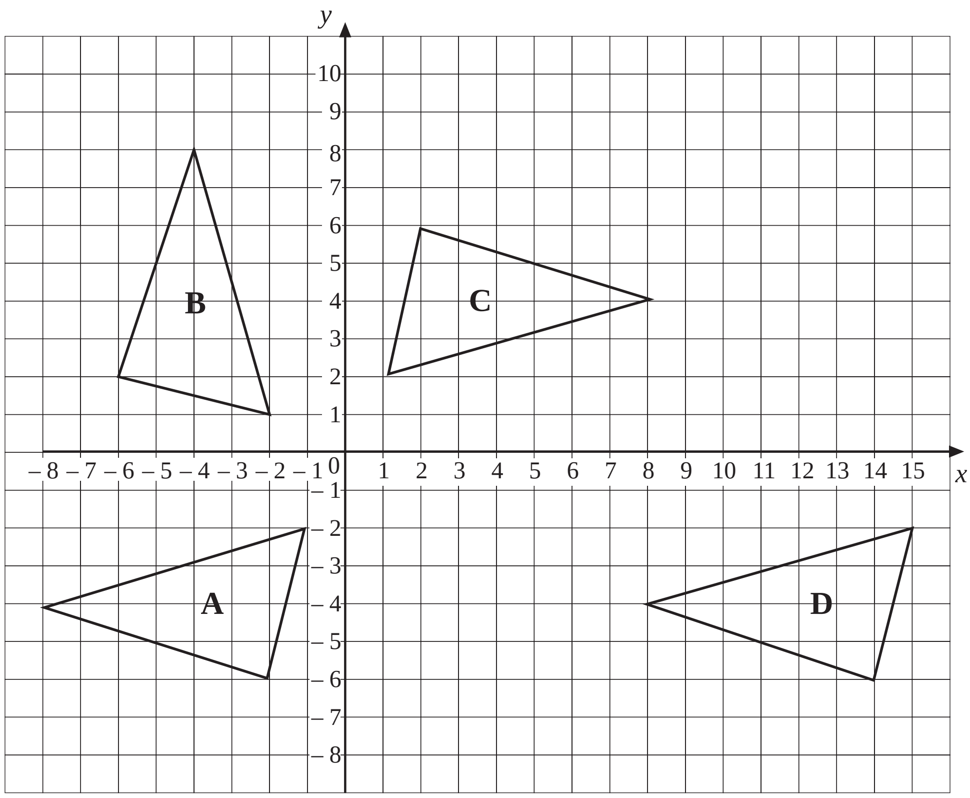
4. Rotate the rectangle formed by joining the points (1, 1), (3, 1), (3, 2) and (1, 2) through $90°$ clockwise about the origin.

5. A triangle has corners at the points with coordinates (4, 7), (3, 2) and (5, 1).

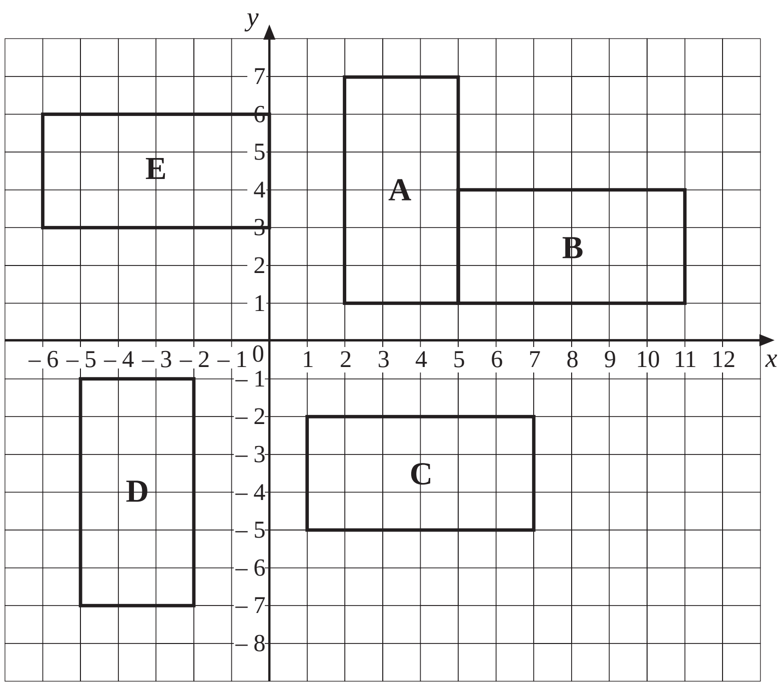
   Determine the coordinates of the triangles that are obtained by rotating the original triangle:
   (a) through $90°$ anticlockwise about (0, 3),
   (b) through $180°$ about (4, 0),
   (c) through $90°$ clockwise about (6, 2).

6. The following diagram shows the triangles A, B C and D.

   Describe the rotation that takes:
   (a) A to B,
   (b) A to C,
   (c) C to B,
   (d) C to D.



7. The following diagram shows the rectangles A, B, C and D.



Describe the rotation that takes:

(a) A to B,

(b) A to C,

(c) A to D,

(d) A to E.

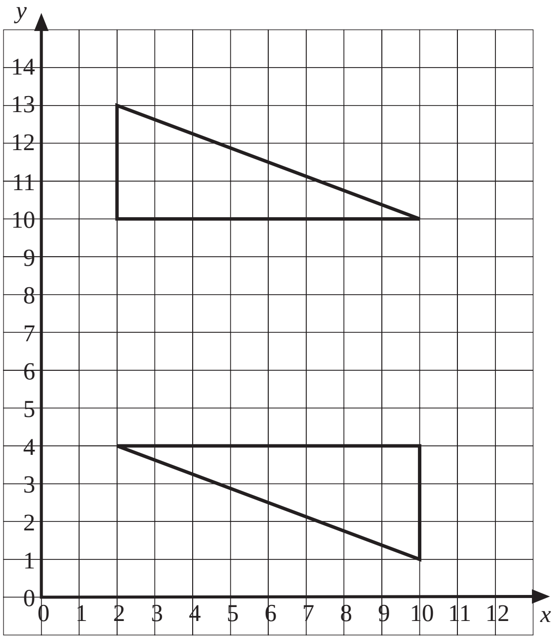
8. The triangle A has corners at the points with coordinates (1, 7), (3, 6) and (2, 4).

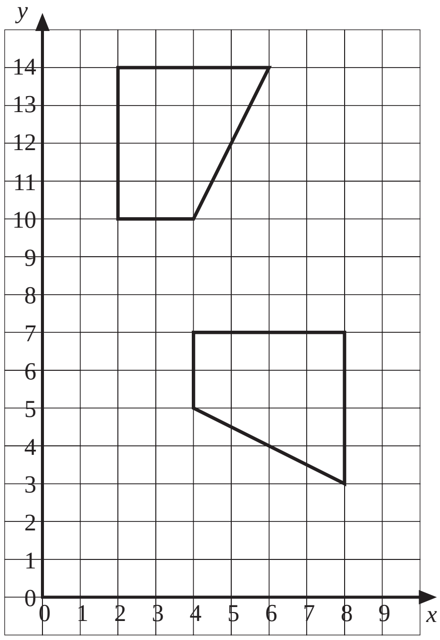
   (a) Rotate triangle A through 180 ° about the origin to get triangle B.

   (b) Rotate triangle B clockwise through 90 ° about the point (0, – 4) to get triangle C.

   (c) Write down the coordinates of the corners of triangle C.

9. The following diagrams show two rotations. Determine the coordinates of the centre of rotation in each case.

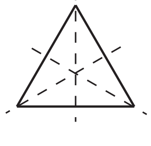




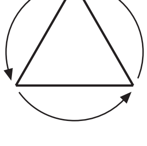
10. A triangle has corners at the points A (4, 2), B (6, 3) and C (5, 7). The triangle is rotated to give the triangle with corners at the points A' (3, – 1), B' (4, – 3) and C' (8, – 2).

    Describe fully this rotation.

11. An equilateral triangle has *3 lines of symmetry.*

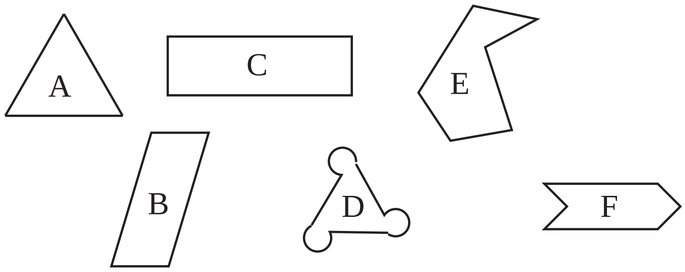


It has *rotational symmetry* of *order 3.*



Write the letter of each of the following shapes in the correct space in a copy of the table. You may use a mirror or tracing paper to help you.

The letters for the first two shapes have been written for you.

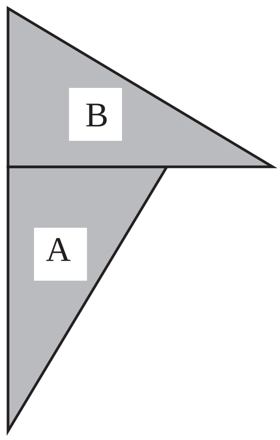


Number of Lines of Symmetry

Order of Rotational Symmetry	0	1	2	3	
1					
2	B				
3				A	

(KS3/99/Ma/Tier 5-7/P1)

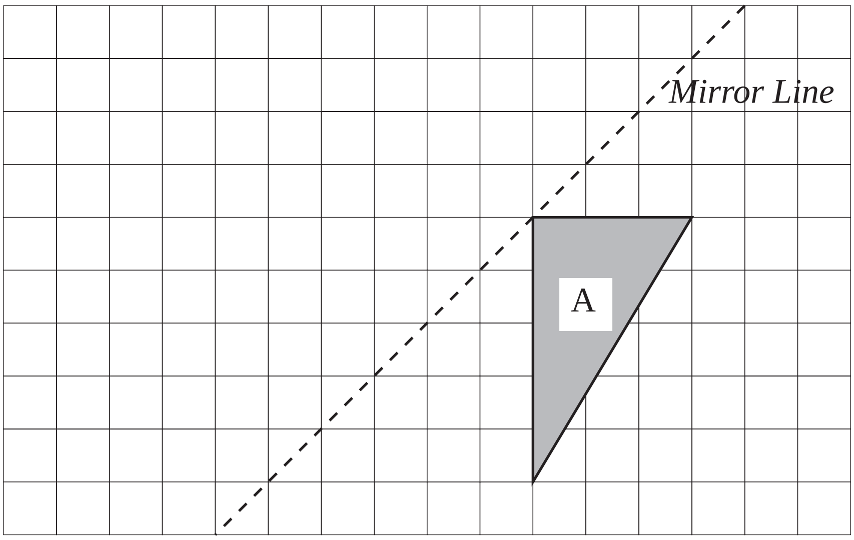
12.



(a) You can rotate triangle A onto triangle B.
Make a copy of the diagram and put a cross on the *centre of rotation.*
You may use tracing paper to help you.

(b) You can *rotate* triangle A onto triangle B.
The rotation is *anti-clockwise.*
What is the *angle* of rotation?

(c) On a copy of the diagram below, *reflect* triangle A in the mirror line. You may use a mirror or tracing paper to help you.



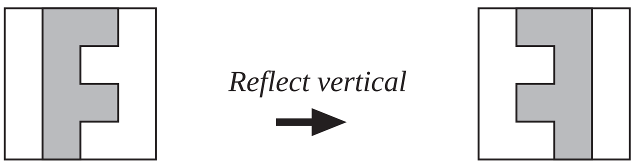
(KS3/99/Ma/Tier 4-6/P2)

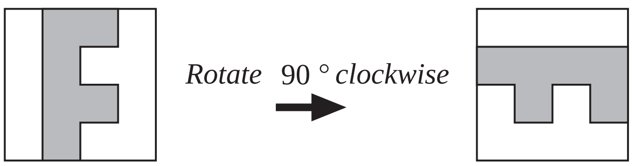
13. Julie has written a computer program to transform pictures of tiles. There are *only two instructions* in her program,

reflect vertical

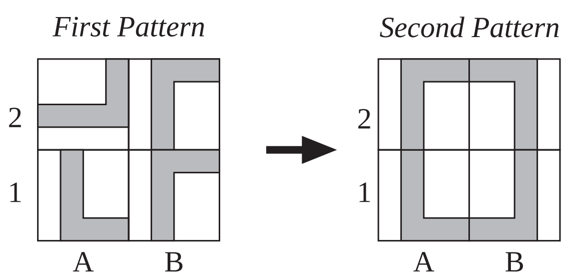
or

rotate 90 ° *clockwise.*





(a) Julie wants to transform the first pattern to the second pattern.



Copy and complete the following instructions to transform the tiles B1 and B2. You must use only *reflect vertical* or *rotate* 90 ° *clockwise.*

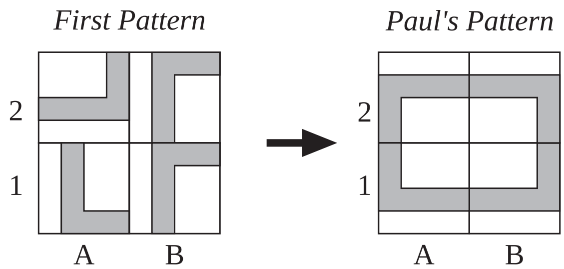
A1 *Tile is in the correct position.*

A2 *Reflect vertical, and then rotate* 90 °*clockwise.*

B1 *Rotate* 90 ° *clockwise and then* ....

B2 ....

(b) Paul starts with the first pattern that was on the screen.



Copy and complete the instructions for the transformations of A2, B1 and B2 to make Paul's pattern. You must use only *reflect vertical* or *rotate* 90 °*clockwise.*

A1 *Reflect vertical, and then rotate* 90 ° *clockwise.*

A2 *Rotate* 90 °*clockwise, and then* ....

B1 ....

B2 ....

(KS3/96/Ma/Tier 4-6/P1)

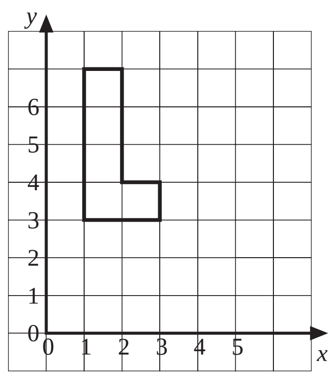
7.6 Combining Transformations

In this section we combine transformations. We see that sometimes 2 transformations are equivalent to a single transformation.

Here we use transformations from the following types:

Translations *Enlargements* *Reflections* *Rotations*

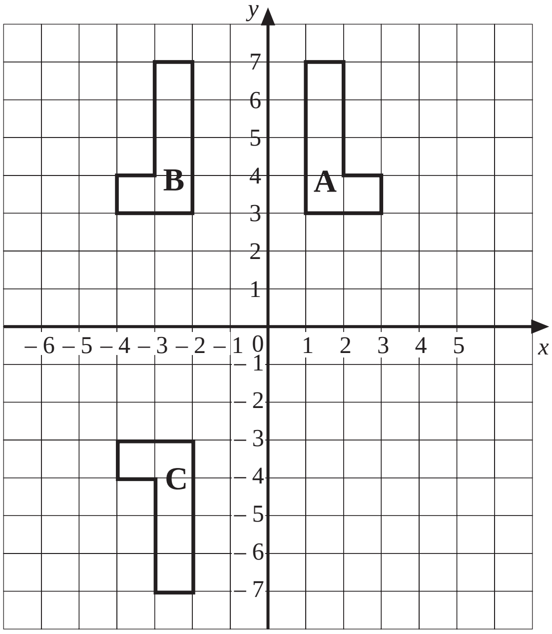
Example 1



The shape shown in the diagram is reflected first in the y-axis and its image is then reflected in the x-axis.

What *single* transformation would have the same result as these *two* transformations?

Solution



The diagram shows how the original shape A is first reflected to B, and B is then reflected to C.

A rotation of 180 ° about the origin would take A straight to C.

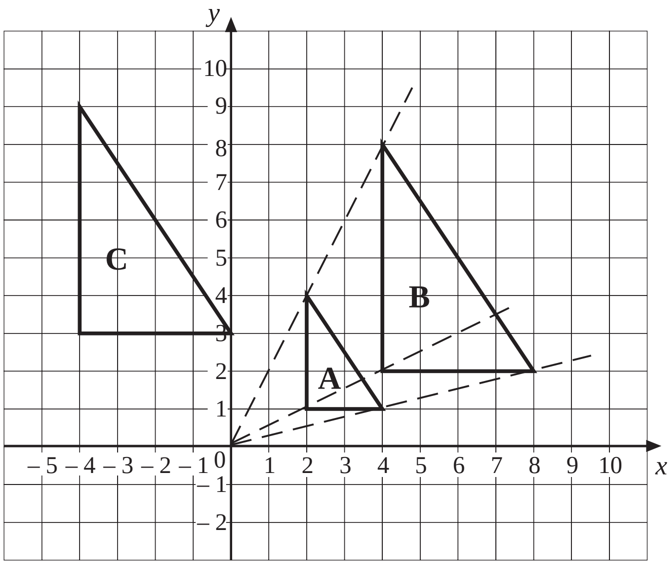
Example 2

A triangle is to be enlarged with scale factor 2, using the origin as the centre of enlargement. Its image is then to be translated along the vector $\begin{pmatrix} -8 \\ 1 \end{pmatrix}$.

The coordinates of the corners of the triangle are (2, 1), (2, 4) and (4, 1).

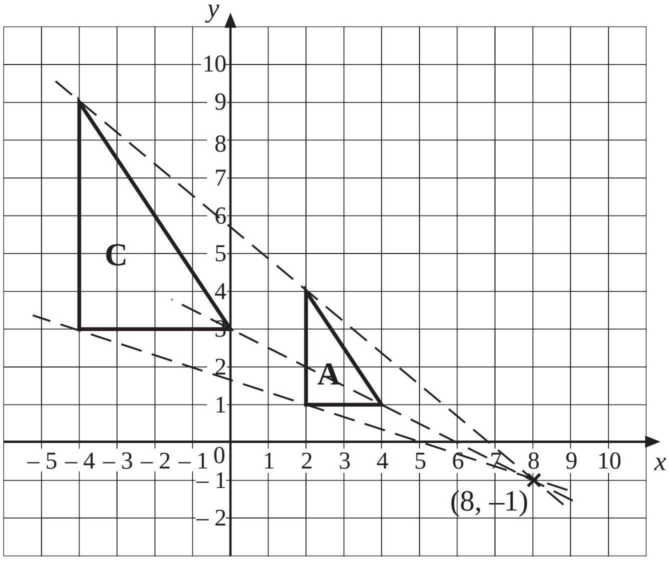
What *single* transformation would have the same result?

Solution



The diagram shows the original triangle, A; the enlargement takes it to B, which is then translated to C.

The triangle A could be enlarged with scale factor 2 to give C.



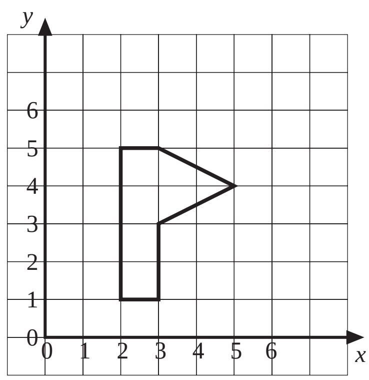
This diagram shows that the centre of enlargement would be the point (8, – 1).

The single transformation that will move triangle A to triangle C is an enlargement, scale factor 2, centre (8, – 1).

Exercises

1. (a) Reflect the shape shown in the *x*-axis and then reflect its image in the *y*-axis.

   (b) What single transformation would have the same result as these two transformations?



2. A rectangle has corners at the points with coordinates (1, 2), (3, 1), (5, 5) and (3, 6). It is first reflected in the *x*-axis and then its image is rotated through 180 ° about the origin.

   Describe how to move the rectangle from its original position to its final position, using only one transformation.

3. A shape is rotated through 180 ° about the origin and then its image is reflected in the *x*-axis.

   (a) Choose a shape and carry out the transformations described above.

   (b) What single transformation would have the same result as the two transformations described above?

4. A triangle has corners at the points with coordinates (2, 2), (3, 6) and (8, 6).

   (a) Draw the triangle and enlarge it with scale factor 2, using the origin as the centre of enlargement.

(b) Translate the enlarged shape along the vector $\begin{pmatrix} -3 \\ -6 \end{pmatrix}$.

(c) Describe fully the enlargement that would produce the final triangle from the original triangle.

5. (a) Draw the triangle with corners at the points (2, 1), (4, 1) and (4, 2).

(b) Reflect this shape in the line $y = x$.

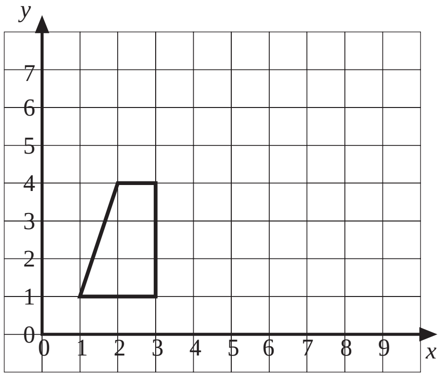
(c) Reflect the new triangle in the *y*-axis.

(d) What single transformation would have the same result as the two transformations described above?

6. (a) Reflect a shape of your choice in the line $y = x$ and then reflect the image in the line $y = -x$.

(b) Describe a single transformation that would have the same result.

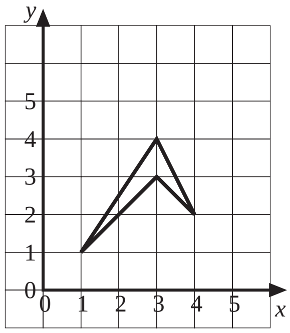
7. The shape shown in the following diagram is to be reflected in the line $x = 4$ and then its image is to be reflected in the line $y = 5$.



(a) Draw a diagram to show how the shape moves.

(b) What single transformation would have the same result?

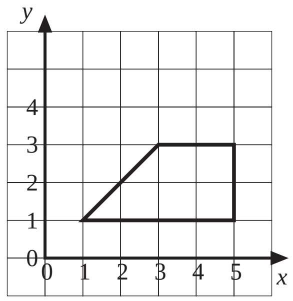
8. The shape shown in the diagram is translated along the vector $\begin{pmatrix} 10 \\ 0 \end{pmatrix}$.



(a) Draw the final position of the shape.

(b) Describe how the shape could be moved to this position using 2 reflections.

9. The shape shown in the diagram is to be enlarged with scale factor 3 using the point (0, 4) as the centre of enlargement.



(a) Draw the enlarged shape.

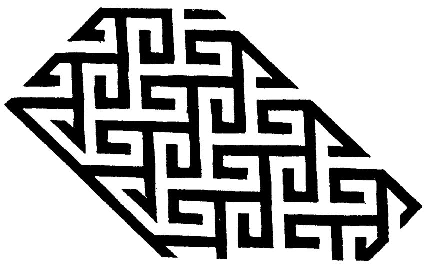
(b) The enlarged shape is translated along the vector $\begin{pmatrix} 0 \\ 8 \end{pmatrix}$. Draw the new position of the shape.

(c) Describe the single enlargement that would have the same result as the two transformations used above.

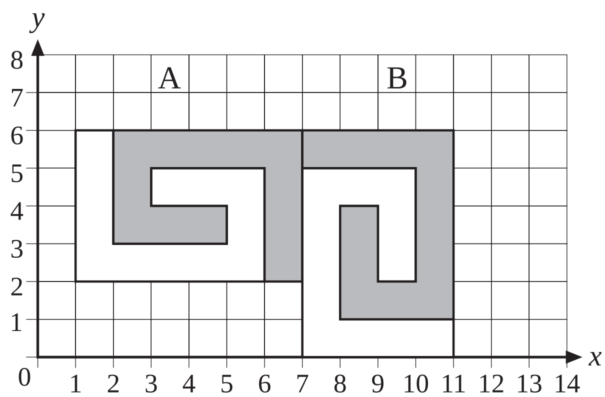
10. A shape is reflected in the line $y = x$, then in the line $y = -x$, and finally in the x-axis.

What single transformation would have the same result?

11. The following design is based on a Celtic pattern.



Part of the pattern is shown below:



The pattern is made of two rectangular blocks, A and B.

Use *two* transformations to map block A onto block B. Your transformations must be either rotations or reflections.

Mark any mirror lines or centres of rotation on a copy of the previous diagram.

Write down instructions for the first and second transformations.

Give coordinates of any centres of rotation, the amount of turn and direction of turn. Give the equations of any lines of reflection.

(KS3/95/Ma/Levels 9-10)