

Unit 3 *Linear Programming*

Introduction

Learning objectives

After studying this unit you should

- be able to formulate linear programming problems from contextual problems
- be able to identify feasible regions for linear programming problems
- be able to find solutions to linear programming problems using graphical means
- be able to apply the simplex method using slack variables.

Notes

The methods of linear programming were originally developed between 1945 and 1955 by American mathematicians to solve problems arising in industry and economic planning. Many such problems involve constraints on the size of the workforce, the quantities of raw materials available, the number of machines available and so on. The problems that will be solved usually have two variables in them and can be solved graphically, but problems occurring in industry have many more variables and have to be solved by computer. For example, in oil refineries, problems arise with hundreds of variables and tens of thousands of constraints.

Another application is in determining the best diet for farm animals such as pigs. In order to maximise the profit a pig farmer needs to ensure that the pigs are fed appropriate food and sufficient quantities of it to produce lean meat. The pigs require a daily allocation of carbohydrate, protein, amino acids, minerals and vitamins. Each involves various components. For example, the mineral content includes calcium, phosphorus, salt, potassium, iron, magnesium, zinc, copper, manganese, iodine, and selenium. All these dietary constituents should be present, in correct amounts.

Undoubtedly linear programming is one of the most widespread methods used to solve management and economic problems, and has been applied in a wide variety of situations and contexts.

Key points

- All linear programming problems take the form of finding the *optimum* value (maximum or minimum) of a *linear* objective function subject to a number of *linear* constraints.

- In 2 dimensions, the objective function will be of the form

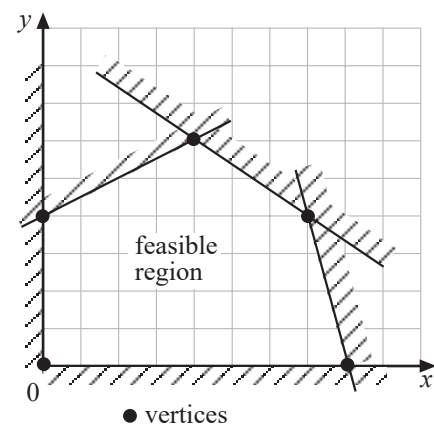
$$P = ax + by \quad (a, b \text{ positive constants})$$

subject to a number of constraints of the form

$$cx + dy \leq e \quad (c, d, e \text{ constants})$$

together with $x \geq 0, y \geq 0$.

- The constraints define the feasible region in which all constraints are satisfied.
- The objective function is optimised at one of the vertices of the feasible region.



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Glossary

- *Objective function*: the function (e.g. cost, profit, time) that needs to be optimised (maximum or minimum). For example, $P = 5x + 7y$.
- *Constraints*: the conditions that need to be satisfied when optimising the objective function. For example, $x + 2y \leq 10$, $x + y \geq 5$, $x \geq 0$, $y \geq 2$, etc.
- *Feasible region*: the region that is defined when all the constraints are satisfied.