

1

The **base set** (B) of numbers for this question is:

$$B = \{-3, -2, -1, 0, 1, 2, 3, 4, 5\}$$

Which of these numbers can be used instead of the letters to make the statements true?

- a) $5.4 + \boxed{-2} = 3.4$ b) $-3 + x = 4$ $x = 7$ c) y is divisible by 3
 d) $3 + z < 9$ **All positive** e) $-2.6 \times t \geq 2$ $y = -3, 0, 3$
 $t = -3, -2, -1$

2

Which numbers in the given **base set** (B) can be used instead of the letters to make the equations true?

- a) $x - 9 = 3$ $x = 12$ $B = \{\text{whole numbers}\}$
 b) $y + 8 = 7$ $y = -1$ $B = \{0, 1, 2, 3, 4\}$ **None in base set (B)**
 c) $z - 6 = 6 - z$ $z = 6$ $B = \left\{5\frac{1}{3}, 5\frac{2}{3}, 6, 6\frac{1}{3}, 6\frac{2}{3}\right\}$
 d) $3 + t \times 3 = (3 + t) \times 3$ $B = \{-2, -1, 0, 1, 2, 3, 4, 5\}$ **Not possible**
 e) $3 \times t + 3 = (t + 1) \times 3$ **All** $B = \left\{-7, -3\frac{1}{5}, -0.21, 0, 0.375, 6\frac{1}{7}\right\}$ **Possible**
 f) $4 \times u - 2 = u + 10$ $u = 4$ $B = \{-1, 4, 9, 14\}$
 g) $|v + 3| = v + 3$ $v \geq -3$ $B = \{-5, -4, -3, -2, -1, 0, 1, 2, 3\}$
 h) $(w + 1) \times (w - 2) = 0$ $B = \{-3, -2, -1, 0, 1, 2, 3\}$
 $w = 2$ or $w = -1$

3

Which numbers can be written instead of the letters to make the statements true?
Solve the equations and inequalities in your exercise book.

- a) $a + b = b + a$ $2 + 3 = 3 + 2$ b) $x - (-4) = 2.1$ $x = -1.9$
 c) $y \times (-5) = 30$ $y = -6$ d) $2 \times (x + 1) = 2 \times x + 5$ **Not possible**
 e) $(u - 2) \times 3 = -6 + 3 \times u$ $u = 2$ f) $44 \div z < 11$ $z = 5$
 g) $u \times v = v \times u$ $2 \times 3 = 3 \times 2$ h) $t - 3 \geq 2 \times t$ $t = -3$

4

Write an equation about the relationship between the given data. Solve the equation, then check your result in context.

- a) I think of a number. If I subtract 8 from 3 times my number, the result is 19.
What is my number? $3x - 8 = 19$ $3x = 27$ $x = 9$
- b) I think of a number. If I divide my number by 5, then subtract 11 from the quotient, the result is 8.
What is my number? $\frac{x}{5} - 11 = 8$ $\frac{x}{5} = 19$ $x = 95$
- c) I think of a number. I add 28 to 4 times my number, then divide the sum by 4. I subtract my number from the quotient and the difference is 7.
What is my number? $((4x + 28) \div 4 - x) = 7$
 $x + 7 - x = 7$

1

a) Complete this table.

x	-3	-2	-1	0	1	2	3	4	5	6	
$2 \times x - 1$	-7	-5	-3	-1	1	3	5	7	9	11	Inequality
$2x - 1$											$2x < 6$

b) Solve the inequality $2 \times x - 1 < 5$, if the **base set** is: $2x - 1 < 5$ $x < 3$

- i) the set of **integers** (Z) ii) the set of **natural** numbers (N)
 (-3, -2, -1, 0, 1, 2) 3, 4, 5, 6 (1, 2) 3, 4, 5, 6
 iii) the set of all the numbers that you know. (Q: **rational** numbers)
(-3, -2, -1, 0, 1, 2) 3 4 5 6

The number in () hold for the inequality

2

Write a formula about the relationship between the data for:

- a) the area of a rectangle with sides a and b ab
- b) the perimeter of a rectangle with sides e and f $2(e + f)$
- c) the area of a square with side c c^2
- d) the perimeter of a square with side t $4t$
- e) the area of a square with diagonal e $(\frac{e}{2})^2 = \frac{e^2}{4}$
- f) the surface area of a cube with edge c $6c^2$
- g) the volume of a cube with edge a a^3
- h) the volume of a cuboid with edges a, b, c abc

3

The difference between two numbers is 19.

- a) What is the other number if: i) the smaller number is x $y - x = 19$
 ii) the greater number is y ?
- b) Write the sum of the two numbers using only one letter, x . $2x + 19$
- c) What are the two numbers if their sum is 40?
 $x = 10.5$ $y = 29.5$

4

A natural number is 3 times another natural number.

- a) If the smaller number is y , what is the greater number? $3y$
- b) Write the sum of the two numbers. $3y + y = 4y$
- c) Calculate the smaller number if the sum of the two numbers is 324. $y = 81$

5

In a box there are b apples. In a second box there are 7 apples more than b . In a third box there are 5 apples less than b .

- a) How many apples are in each box? $b + (7 + b) + (b - 5) = 3b + 2$
- b) How many apples are in the 3 boxes altogether? $3b + 2$
- c) How many apples are in the first box if there are 77 apples in all 3 boxes?

$$\begin{aligned} 3b + 2 &= 77 \\ 3b &= 75 \\ b &= 25 \end{aligned}$$

1

Solve the equations and check your results.

- a) $x + 2.7 = 11$ **$x = 8.3$** b) $-6.2 + y = 3$ **$y = 9.2$** c) $z - (-3) = -2$ **-5**
 d) $\frac{x}{4} \times 3 = \frac{9}{4}$ **$x = 3$** e) $u \div (-3) = 6$ **$u = -18$** f) $(-42) \div v = 6$ **$v = -7$**

2

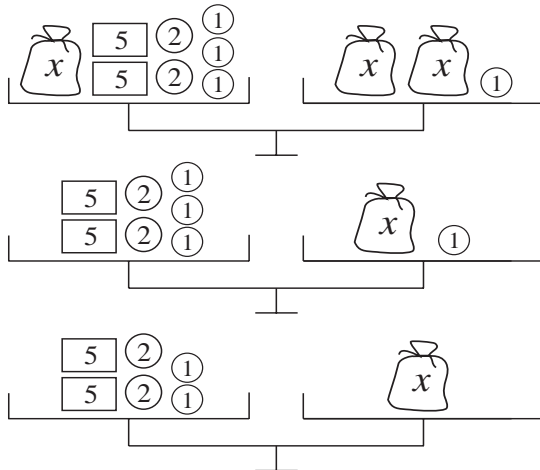
Solve the inequalities.

- a) $a + (-5) < -13$ **$a < -8$** b) $b - \left(-\frac{7}{6}\right) \geq 1\frac{5}{9}$ **$b \geq \frac{7}{18}$**
 c) $c - (+63) \leq -17$ **$c \leq 46$** d) $d + \frac{2}{3} > -2\frac{1}{4}$ **$d > -\frac{38}{12}$**

3

Write an equation about the diagram. Solve the equation by changing the sides equally.

Follow the steps.



$$x + 17 = 2x + 1$$

$$x = 16$$

$$17 = x + 1$$

$$x = 16$$

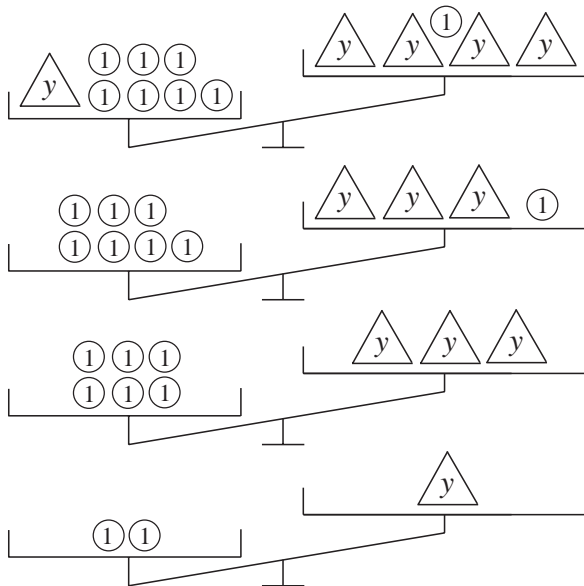
$$16 = x$$

$$x = 16$$

4

Write an inequality about the diagram. Solve the inequality by changing the sides equally.

Follow the steps.



$$y + 7 > 4y + 1$$

$$y < 2$$

$$7 > 3y + 1$$

$$y < 2$$

$$6 > 3y$$

$$y < 2$$

$$2 > y$$

$$y < 2$$

5

Draw diagrams to help you solve this equation.

$$3 \times x - 2 = x + 6$$

$$3x - 2 = x + 6$$

$$2x - 2 = 8$$

$$2x = 10$$

$$x = 5$$

1

Write each operation using abbreviations (e.g. 'a' instead of 'apricots') then do the operation.

- a) 140 apricots + 27 apricots – 52 apricots $140a + 27a - 52a = 115a$
- b) 150 apples ÷ 5 $150a \div 5 = 30a$
- c) 83 boxes × 3 $83b \times 3 = 249b$
- d) 63 stamps ÷ 9 stamps $63s \div 9 = 7$
- e) $\frac{4}{7}$ of 84 potatoes $\frac{4}{7} \times 84p = 48p$
- f) 4 apples + 10 apples + 5 bananas – 2 apples – 4 bananas $4a + 10a + 5b - 2a - 4b$

2

Study the diagram to help you understand the equation. Solve the equation and check the result. (The first step is given.)

etc. $72 - 3x = 6x$

3

Write an inequality about the diagram. Solve the inequality and check your result by filling in the table.

$2y + 3 \geq 3y - 7$

$3 \geq y - 7$
 $y - 7 \leq 3$
 $y \leq 10$

LHS RHS

y	-1	0	3	7	9	10	11	16
Left side (L)	1	3	9	17	21	23	25	35
Right side (R)	-10	-8	2	14	20	23	26	41
L ≥ R	>	>	>	>	>	=	<	<

4

Solve the equations and check the results in your exercise book.

- a) $5 \times x = 2 \times x + 12$ $5x = 2x + 12$ $x = 4$
- b) $5 \times y + 3 = 2 \times y + 9$ $3y = 6$ $y = 2$
- c) $4 \times z - 5 = 10 + (-z)$ $4z - 5 = 10 - z$ $z = 3$
- d) $45 - d = 25 - d$ (Not possible)
- e) $\frac{x}{4} + 5 = 8$ $\frac{x}{4} = 3$ $x = 12$
- $5y + 3 = 2y + 9$

5

We weighed out equal packs in kg. What can you write about the mass, *m*, of one pack?

Show the possible values for each pack on an appropriate segment of the number line.

- a) $7 \times m + 1 \leq 22$ $7m \leq 21$ $m \leq 3$
- b) $4 \times m + 32 > 12 \times m$ $4m + 32 > 12m$ $32 > 8m$ $8m < 32$ $m < 4$
- c) $29.5 < 5 \times m + 2 < 32$ $6 \leq m \leq 5.5$
- $29.5 < 5m + 2 < 32$
 $5m + 2 < 32$
 $5m < 30$
 $m < 6$

$29.5 < 5m + 2$
 $27.5 < 5m$
 $m > 5.5$

1

Write an equation about the relationship of the given data. Solve the equation, then check that your result is correct.

- a) I think of a number. If I add 4 to 5 times my number, the result is 39.
 What is my number? $5x + 4 = 39$ $5x = 35$ $x = 7$
- b) I think of a number. If I divide my number by 2, then add 7 to the quotient, the result is twice my number.
 What is my number? $\frac{x}{2} + 7 = 2x$ $1\frac{1}{2}x = 7$ $x = 4\frac{2}{3}$
- c) I think of two numbers. Their sum is 27 and their difference is 5.
 What are my two numbers? $x + y = 27$ $x - y = 5$
 $(y = 11, x = 16)$ $x = 5 + y$ $5 + y + y = 27$

2

Write a formula about the relationship of the given data.

- a) The **area** of a triangle with base b and height h : $(b \times h) \div 2 = \frac{bh}{2}$
- b) The **perimeter** of a regular octagon with side a : $8a$
- c) The **surface area** of a cuboid with edges a , b and c : $2(ab + ac + cb)$
- d) The **volume** of a cuboid with edges a , $2a$ and $3a$: $2a \times 3a \times a = 6a^3$
- e) The **surface area** of a cuboid with edges a , a and $2a$: $2(2a^2 + 2a^2 + a^2) = 10a^2$

3

Solve the inequalities. Check that your solutions are correct.

- a) $a - (-4) > -2$ $a + 4 > -2$ $a > -6$
- b) $\frac{4}{5} + (-b) \leq \frac{3}{10}$ $b \geq \frac{3}{10}$
- c) $c + (+4) \geq +4$ $c + 4 \geq 4$ $c \geq 0$ $c \geq 0$

4

Solve the equations and check your results.

- a) $x + 6.2 = 9.3$ $x = 3.1$
- b) $-3.7 + y = 5$ $y = 8.7$
- c) $z \times 2 = \frac{1}{4}$ $z = \frac{1}{8}$
- d) $3 \times a = a + 5$ $3a = a + 5$ $2a = 5$ $a = 2.5$
- e) $5 \times b + 2 = 3 \times b - 8$ $5b + 2 = 3b - 8$ $2b = -10$ $b = -5$
- f) $\frac{c}{3} - 2 = 8$ $\frac{c}{3} = 10$ $c = 30$

5

The relationship between adult shoe sizes in Europe (E) and in the UK (U) is:

$$E = \frac{5}{4} \times U + \frac{127}{4}$$



- a) Fill in the table.

U	3	4	5	6	7	8	9	10	11	12
E	35.5	36.75	38	39.25	40.5	41.75	43	44.25	45.4	46.75

- b) What is the nearest shoe size in the UK to each of these European shoe sizes:

- i) 40 7.5 ii) 38 5 iii) 45 10.5 if there are also half sizes in the UK?

1

Solve the equations and check your results.

a) $2 \times a + a = \frac{21}{40}$ $2a + a = \frac{21}{40}$ $3a = \frac{21}{40}$ $a = \frac{7}{40}$

c) $35 \div c = 14$ $c = \frac{35}{14}$

e) $(4 - e) \times 5 = -5 \times e + 20$
 $20 - 5e = -5e + 20$
 $e = 0$

$\frac{4b}{9} = b - 1$
 $4b = 9b - 9$
 $b = \frac{9}{5} = 1\frac{4}{5}$

d) $7 \times (d - 2) = d - 2$ $7d - 14 = d - 2$
 $d = 2$

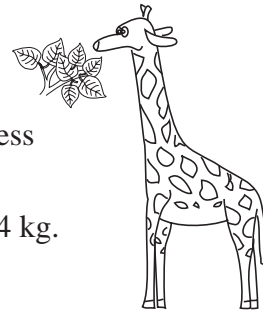
f) $6 \times f - 3 \times f = \frac{3}{8} + f \times 3$
 $6f - 3f = \frac{3}{8} + 3f$ Not possible

2

Gerry Giraffe eats the same amount of leaves every day.

His keeper told the other keepers in the zoo that:

- three of Gerry's daily portions plus five kilograms are less than 29 kg, but $3x + 5 < 29$ $x < 8$
- five of Gerry's daily portions plus 4 kg are more than 34 kg. $5x + 4 > 34$ $x > 6$



What could be the mass of Gerry's daily portion of leaves?

- Write two inequalities about the portion of leaves.
- Solve the inequalities and note the values which make **both** inequalities true.
- Write your answer in a sentence. $6 < x < 8$

Portion size: Between 6 and 8kg

3

Show the relationship among the data by writing an equation. Solve the equation and check the result in the given context.

- a) One fifth of a barrel is 20 litres less than the capacity of the whole barrel.

What is the capacity of the barrel? $\frac{1}{5}x = x - 20$ $x = 25$

- b) On Monday, a shop sold x kg of honey. On Tuesday it sold 11 kg more than on Monday, and on Wednesday it sold 5 kg more than on Monday.



How much honey did the shop sell on each of these days if the total amount of honey sold was 220 kg? $(x) + (11 + x) + (5 + x) = 3x + 16$
 $3x = 220 - 16$ $68, 79, 73\text{kg}$
 $3x = 204$ $M \quad T \quad W$

- c) In one container there is twice as much water as there is in a second container.

If we took 30 litres of water out of the first container and 12 litres of water out of the second container, both containers would hold the same amount of water.

How much water is in each container? $2x - 30 = x - 12$
 $x = 8$
36 and 18 litres

- d) Let ① mean £1, [-1] mean -£1 and  mean an £x banknote in an envelope.

Betty has: ① [-1] [-1] [-1]   $2x - £2$

Larry has: ① ① ① ① ① ① ① ① ① [-1]  $£8 + x$
 $£9 - £1 + x$

If Betty has the same amount of money as Larry, what value of banknote is in each envelope?

$2x - 2 = 8 + x$
 $x = 10$
 $x = £10$

1

The **base set** is the set of natural numbers. (N)

Write an equation or an inequality, solve it and check your result.

Which number am I thinking of?

- a) I add 5 to 3 times my number and the result is 53. $3x + 5 = 53$ $3x = 48$ $x = 16$
- b) I subtract 18 from 7 times my number and the result is 269. $7x - 18 = 269$ $x = 41$
- c) I subtract 4 times my number from 7 times my number and the result is 156. $7x - 4x = 156$ $x = -37.25$
- d) I add 6 to 5 times my number and the result is less than 26. $5x + 6 < 26$
 $x < 4$
- e) The difference between 7 times and 5 times my number is greater than 50.
 $7x - 5x = 2x$ $2x > 50$ $x > 25$

2

Solve each problem in two ways, with and without an equation.

2:1
14:7

- a) *Daffy Duck* is twice as old as *Donald Duck*. If the sum of their ages is 21 months, how old is *Daffy* and how old is *Donald*?
 $2x = 14$ $x = 7$ $3x = 21$ $x = 7$
- b) A 120 cm long stick is cut into two pieces so that one of the pieces is 30 cm longer than the other piece. How long is each piece?
 $x + x + 30 = 120$ $2x = 90$ $x = 45$
- c) Liz has twice as many marbles as Julia. If Liz gave Julia 10 marbles they would both have the same amount. How many marbles do Liz and Julia each have?
 $2x - 10 = x + 10$ $x = 20$ $2x = 40$

3

Solve the problems by writing equations.

$37 + x = 25$
 $x = -12$

- a) Joe's bank balance is £37. Joe's and Charlie's bank balances add up to £25.
i) Explain how this is possible. x £-12 in debt
ii) How much money is in Charlie's account? £-12
- b) Claire's bank balance is £2.50. Claire's and Mike's bank balances add up to £31.
How much money does Mike have in his account?
 $\text{£2.50} + x = \text{£31}$
 £28.50
- c) Colin's balance is £4.50 less than Pete's balance.
How much do they each have if the sum of their balances is £2.20?
 $x + (x - \text{£4.50}) = \text{£2.20}$
 $2x = \text{£6.70}$
 $x = \text{£3.35}$

4

Solve the problems by writing equations.

- a) Alex has £100 in cash, is £300 in debt and has two savings bonds of equal value. Ben is £100 in debt, has £400 in cash and has 3 such savings bonds.
If Ben has 3 times as much money as Alex, how much is a savings bond worth? £300
- b) Adam has £100 in cash, is £300 in debt and has a savings bond. $100 - 300 + x$
Matthew has £500 in cash, is £100 in debt, and also has a savings bond. $500 - 100 + x$
Norah has £100 in cash, is £200 in debt and has two savings bonds. $100 - 200 + 2x$
If the sum of the two boys' money is greater than twice Norah's money, how much can one of their savings bonds be worth?
 $x < 200$
Saving bond $< \text{£200}$

$100 - 300 + 2x$
 $-100 + 400 + 3x$

1

Solve the equations and inequalities. Check your results.

- a) $x - \frac{2}{5} = \frac{7}{10}$ $x = \frac{11}{10}$
- b) $y - \frac{3}{4} > \frac{3}{4} - y$ $2y > \frac{6}{4}$ $y > \frac{3}{4}$
- c) $\frac{4}{5} + u = u + \frac{12}{15}$ $u = u$
- d) $\frac{2}{3} \times t = \frac{6}{30}$ $\frac{6}{30} \times \frac{30}{20} = \frac{6}{20} = \frac{3}{10}$
- e) $0.2 \times v + 0.85 \leq 1.7 \times v - 0.8$ $w + 0.3 = 0$ $w = -0.3$
 $v \geq \frac{11}{20}$

2

Write an equation for each question. Solve the equation and check your result.

- a) Brian said, "If I add my number to a quarter of my number I get $12\frac{1}{2}$."
 What is Brian's number? $x + \frac{x}{4} = 12.5$ $\frac{5}{4}x = 12.5$ $x = 10$
- b) Tom said, "If I add a quarter of my number to half of my number I get the same result as if I had taken 2 away from 4 fifths of my number."
 What is Tom's number? $\frac{x}{4} + \frac{x}{2} = \frac{4}{5}x - 2$ $\frac{3}{4}x = \frac{4}{5}x - 2$ $x = 40$
- c) Two identical bottles contain 2.2 litres of squash altogether. One bottle is $\frac{2}{3}$ full
 $\frac{2}{5}x + \frac{4}{5}x = 2.2$
 and the other bottle is $\frac{4}{5}$ full. How much squash is there in a full bottle?
 $x = 1.5$ litres

3

Write an equation for each question. Solve the equation and check your result.

- a) 130% of a number is the same as adding 10.8 to 35% of the number.
 What is the number? $130x = 10.8 + 35x$ $x = \frac{216}{19}$
- b) Lilly has £150 in her purse. This amount is £60 less than 1 sixth of all her money.
 How much money does Lilly have? $150 = \frac{x}{6} - 60$ $x = \text{£}1\ 260$
- c) I am thinking of a number x . When I subtract 5 thirds of my number from the number itself, then add $-\frac{1}{2}$ to the difference, the result is $-\frac{7}{6}$.
 What is my number? $x - \frac{5}{3}x - \frac{1}{2} = -\frac{7}{6}$
 $x = 1$

4Solve each problem with and without an equation. $\frac{3}{8}x + 2 = x - 17$

- a) Tim has covered $\frac{3}{8}$ of his planned route plus an additional 2 km.
 He still has 17 km to go. How long is Tim's route? $x = 30.4\text{km}$
- b) Belinda spent half of her money plus another £40. Then she spent half of what was left plus £40. Her money has just run out.
 How much money did Belinda have at first? $\text{£}240$

1

Solve the equations and inequalities in your exercise book.

- a) i) $3.7 \times x - 2.4 < 4.9 \times x + 1.2$ $x < -3$ ii) $3.7 \times x - 2.4 = 4.9 \times x + 1.2$ $x = -3$
 iii) $3.7 \times x - 2.4 > 4.9 \times x + 1.2$ $x > -3$
- b) i) $\frac{3}{14} + x < -2$ $x < -\frac{31}{14}$ ii) $\frac{3}{14} + x = -2$ $x = -\frac{31}{14}$ iii) $\frac{3}{14} + x > -2$ $x > -\frac{31}{14}$
 $x < \frac{10}{3}$
- c) i) $3 \times x - (-10) < 20$ ii) $3 \times x - (-10) = 20$ $3x + 10 = 20$
 iii) $3 \times x - (-10) > 20$ $3x + 10 > 20$ $x > \frac{10}{3}$ $3x = 10$ $x = \frac{10}{3}$

2

Solve the problem with or without an equation.

Two cities, A and B, are 105 km apart.

A cyclist starts from A and cycles to B at a steady speed of 15 km per hour.

At exactly the same time, another cyclist starts from B and cycles to A at a steady speed of 20 km per hour.

- a) When will the cyclists meet? **3 hours** b) Where will the cyclists meet? **45 km from A**
- b) When will they arrive at their destinations? **A 7 hours**
B $5\frac{1}{2}$ hours

3

Town A is 288 km from town B.

Cindy leaves A at 08:00 and drives at a steady speed of 48 km per hour to B.

Dan leaves B at 10:00 and drives at a steady speed of 80 km per hour to A.

a) **$2\frac{1}{4}$ hours**

- a) When will they meet each other? b) Where will they meet each other? **108 km from A**
- c) When will they reach their destinations? **A 2:00 pm**
B 1:36 pm

4

Solve the problems by writing equations.

- a) *Piggy* runs off at a speed of 5 metres per second. Two seconds later, *Doggy* chases *Piggy* at a speed of 7 metres per second.

When and where will *Doggy* catch up with *Piggy*? **After 7 seconds, 35m**

- b) The area of a rectangular garden is 150 m².
If its length is 7.5 m, what is its width? **$150 \div 7.5 = 20m$**
- c) The perimeter of a rectangular garden is 55 m and its length is 20 m.
What is its width? **$55 - 20 - 20 = 15$ (7.5m)**
- d) The area of a square garden is 196 m². How long is each side? **14m**
- e) The perimeter of a square garden is $60\frac{4}{5}$ m. How long is each side? **15.2m**
- f) The surface area of a cuboid is 58 cm². Its base edges are 4 cm and 2 cm.
What is the height of the cuboid? **Height = 3.5m**
- g) The volume of a cuboid is 28 cm³. Its base edges are 4 cm and 2 cm.
What is its height? **Height 3.5m**

1

Write an equation for each question. Solve the equation and check your result.

- a) Russell said, "If I add half of my number to a quarter of my number, I get $10\frac{1}{2}$."
 What is Russell's number? $\frac{x}{2} + \frac{x}{4} = \frac{3x}{4} = 10\frac{1}{2}$ $x = 14$
- b) Margaret said, "If I add two thirds of my number to 5, I get the same result as subtracting 10 from my number."
 What is Margaret's number? $\frac{2}{3}x + 5 = x - 10$ $x = 45$
- c) Liz said, "If I divide my number by 10 and then add 10, I get the same result as dividing my number by 5 and adding 5."
 What is Liz's number? $\frac{x}{10} + 10 = \frac{x}{5} + 5$ $x = 50$

2

City A is 350 km from city B. A train leaves A at 09:00 and moves at a steady speed of 125 km per hour towards B. Another train leaves B at 10:00 and moves at a steady speed of 100 km per hour towards A.

- a) When will the trains pass each other? **1 hour 33 mins** b) Where will they pass each other? **Approx 190km (194.4km)**
- c) At what time will each train reach its destination? **After 90 mins (approx)**
Train from A: 11.48am
Train from B: 1.30pm

3

Solve the problems by writing equations.

$$240 = x + (x + 20) + (x - 20)$$

$$x = 80$$

- a) A 240 cm length of wire was cut into three pieces. The shortest and longest pieces were each 20 cm longer or shorter than the third piece.
 What was the length of the third piece of wire? **80cm**
- b) Sara has 3 times as many sweets as Louis. If Sara gives 8 sweets to Louis, they will both have the same amount. How many sweets do they each have? **Sara = 24**
Louis = 8
- c) George spent half of his money on Monday, then he spent half of the money he had left on Tuesday. After spending half of the money he had left on Wednesday, he now has only £2. $x - (\frac{x}{2} + (\frac{x}{2} \div 2) + \frac{x}{8}) = 2$ **£16**
 How much money did George have at first?
 $x - (\frac{x}{2} + \frac{x}{4} + \frac{x}{8}) = 2$ $x - \frac{7}{8}x = 2$ $\frac{1}{8}x = 2$ $x = 16$

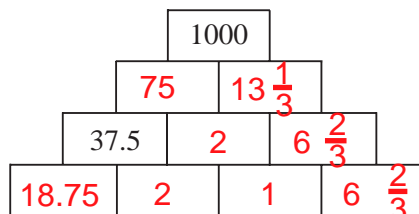
4

Solve these equations and check your results.

- a) $3a + 2a = 12$ $a = \frac{12}{5}$ b) $42 \div b = 3$ $b = 126$ c) $2c + 4 = 3$ $c = -\frac{1}{2}$
- d) $2d + 5d = 3d + \frac{1}{2}$ $4d = \frac{1}{2}$ $d = \frac{1}{8}$ e) $\frac{e}{9} - 2 \div 9 = e \div 15$ $e = 5$ f) $f^2 = f \times 111$ $f = 111$

5

Find a rule and fill in the missing numbers.



Rule: Multiply bricks adjacent to each other to give the product above.

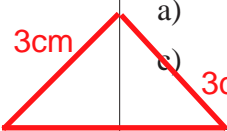
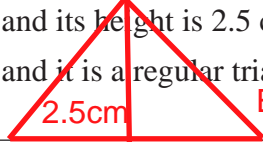
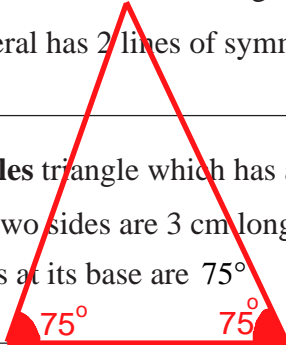
1

If the statement is true, write 'T' in the box and if it is false, write 'F'.

- a) A **cuboid** has 8 vertices, 6 faces and 10 edges. **12 edges** F
- b) Every **cube** has 6 faces, 8 vertices and 12 edges. T
- c) A **circle** is a 2-dimensional shape. T
- d) A **line segment** is a 2-D shape. F
- e) Every cuboid is a **prism**. T
- f) Any prism is a cuboid. F
- g) If the diagonals of a quadrilateral are equal and **bisect** each other, the quadrilateral is a rectangle. **(True for a square)** F
- h) If a quadrilateral has 2 lines of symmetry, it is a **rhombus**. F

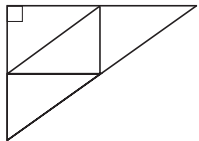
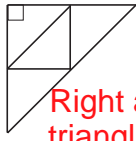
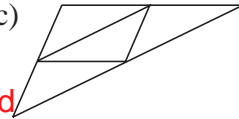
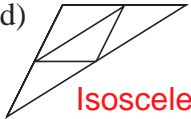
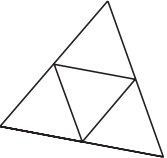
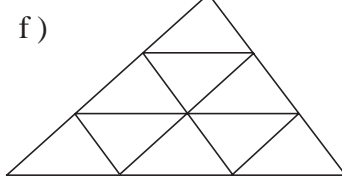
2

Construct an **isosceles** triangle which has a base side of 5 cm:

- a) and its other two sides are 3 cm long 
- b) and its height is 2.5 cm 
- c) and the angles at its base are 75° 
- d) and it is a regular triangle. **Equilateral Triangle**

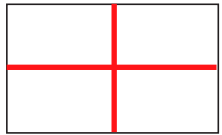
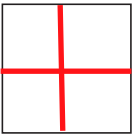
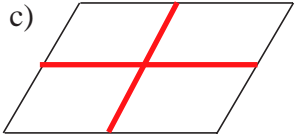
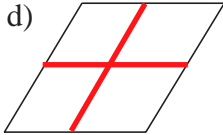
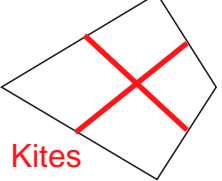
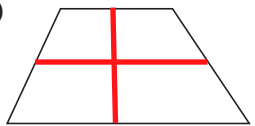
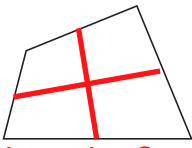
3

These triangles are made up of **congruent** triangles. The triangles in b), d) and e) are **isosceles** triangles.

- a) 
 - b)  **Right angled triangle**
 - c) 
 - d)  **Isosceles triangles each with an obtuse angle**
 - e)  **4 Equilateral Triangle every angle is 60**
 - f) 
- Find relationships for each shape and write mathematical statements about them.

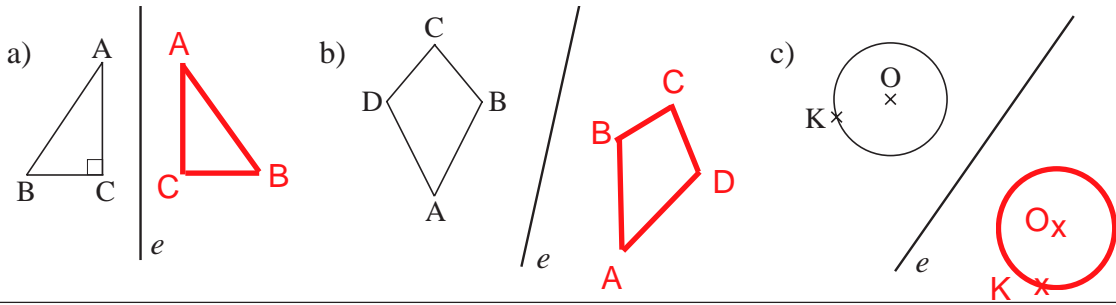
4

Mark the midpoints of the sides of each quadrilateral and join them up in order. Write the names of the polygons you have made.

- a)  **Rectangles**
- b)  **Squares**
- c)  **Parallelograms**
- d)  **Rhombus**
- e)  **Kites**
- f)  **Trapezium**
- g)  **Irregular Quadrilaterals**

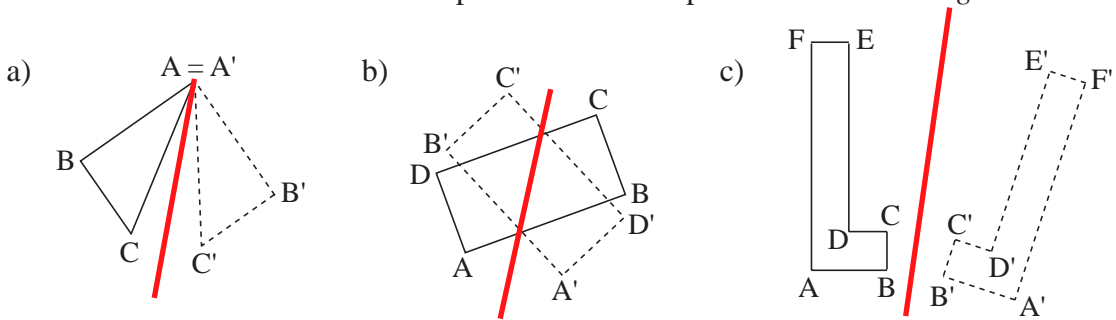
1

Reflect each shape in the given *mirror line*. Use a ruler and a pair of compasses.



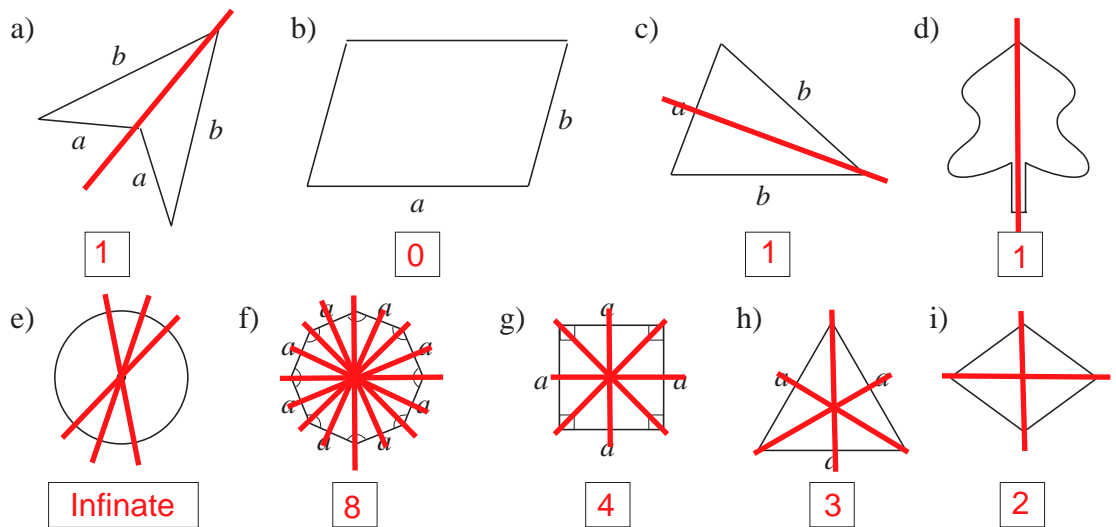
2

Draw the *mirror line* in the correct place for each shape and its *mirror image*.

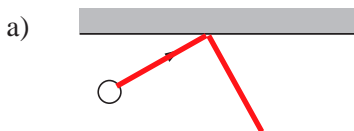


3

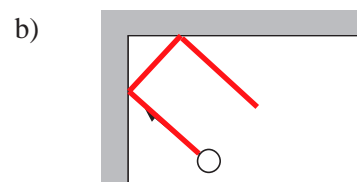
Which shapes are symmetrical? Draw the lines of symmetry where appropriate. Write the number of lines of symmetry below each shape.



4



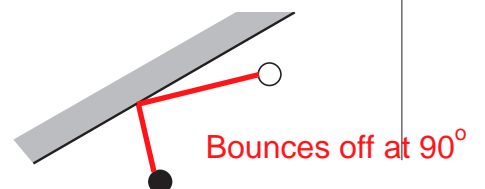
Draw the path of the billiard ball after it has rebounded off the edge of the billiard table.



5

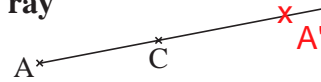
We want the *black* billiard ball to hit the *white* ball after rebounding off the edge of the billiard table.

Draw the path it should take. Explain why you drew it.



1

a) Measure the length of segment AC and mark A' on the **ray** so that it is the reflection of A in C.



b) Complete the statements.

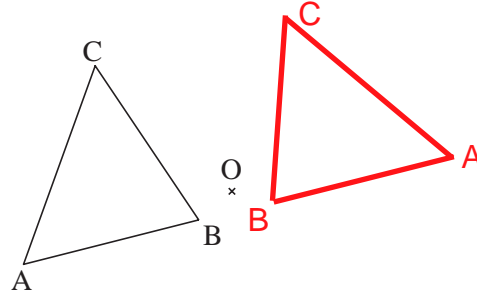
i) $AC = CA'$

ii) C is the midpoint of AA' .

2

Reflect triangle ABC in point O.

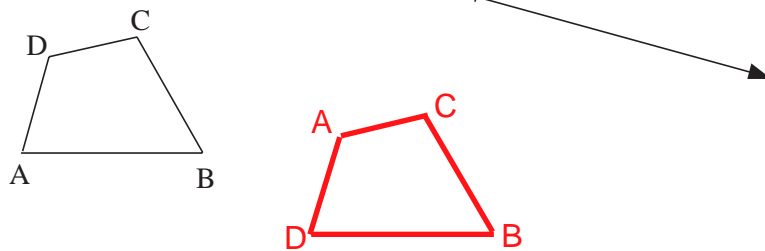
Use a ruler and a pair of compasses.



3

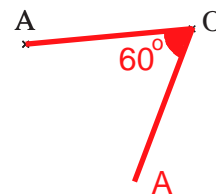
Translate quadrilateral ABCD in the direction, and by the distance, shown by the arrow.

Use a ruler and a pair of compasses.



4

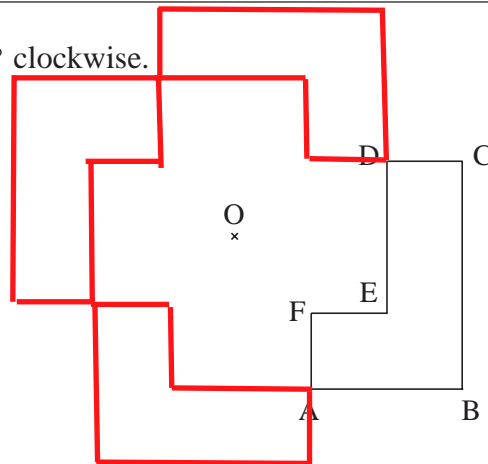
Rotate point A around centre O by 60° anticlockwise.



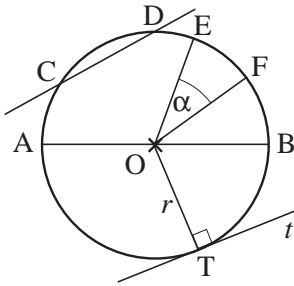
5

Rotate the shape around centre O by 90° clockwise.

Use a ruler and a pair of compasses (and a protractor if you wish).



1



Complete the statements about the diagram.

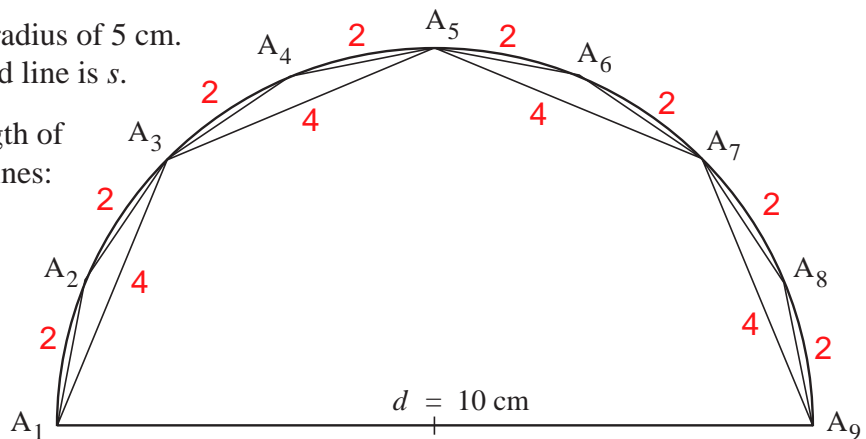
- a) OT is a **Radius** of the circle.
- b) O is the **Centre** of the circle.
- c) AB is a **Diameter** of the circle.
- d) **Line segment CD** is a **Segment** of the circle.
- e) The smaller shape EOF is a **Sector** of the circle.
- f) The curve EF is an **Arc** on the **circumference** of the circle.
- g) $\angle EOF (= \alpha)$ is the **Centre Angle** of the smaller sector EOF.
- h) **Line CD** is an **Secant** of the circle. i) t is a **Tangent** to the circle.

2

This **semicircle** has a radius of 5 cm. The length of its curved line is s .

- a) Measure the length of the two broken lines:
 $A_1 A_3 A_5 A_7 A_9$
(4cm) 16cm

 $A_1 A_2 A_3 \dots A_9$
(2cm) 16cm



- b) Write the lengths of the curved line, s , and the two broken lines in increasing order. **15.7, 16, 16**
- c) If the ratio of the **circumference** to the **diameter** of any circle is about 3.14, what is the length of the curved line of the semi-circle? **15.17cm**
- d) Compare the lengths of the 3 lines and write their ratio. **15.17, 16, 16**

3

If the circumference of a circle with diameter 1 unit is about 3.14 units, calculate the circumference of a circle which has:

- a) a 1 cm diameter **3.14cm** b) a 7 cm diameter **21.98m** c) a 1 m diameter **3.14m**
- d) a 5 m diameter **15.7m** e) a 1 cm radius **6.28cm** f) a 3 cm radius **18.84cm**
- g) a 1 m radius **6.28m** h) a 2 m radius. **12.56m**

4

If $\pi \approx 3.14$, calculate the circumference of a circle which has a:

- a) 10 cm diameter **31.4** ii) 8 m radius **50.24m** c) 4 m radius **25.12m** d) radius r . **$2\pi r = 6.28\pi$**

5

Write the length of the circumference of a circle using π if its:

- a) radius is 11 cm **22π** b) diameter is 2.5 m **2.5π** c) diameter is d **πd** d) radius is r . **$2\pi r$**

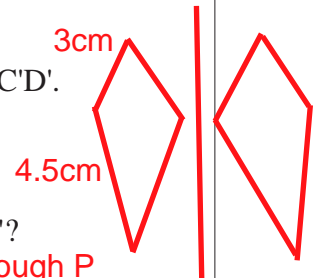
1

If the statement is true, write 'T' in the box and if it is false, write 'F'.

- a) Every **isosceles triangle** has angles of 60° . F
- b) No **rectangle** has adjacent equal sides. F
- c) The **diameter** of a circle is twice the length of its radius. T
- d) The **circumference** of a circle is its radius multiplied by π . F
- e) There is a **prism** which has congruent faces. T
- f) A square-based **pyramid** has 5 vertices, 5 faces and 8 edges. T
- g) If the diagonals of a quadrilateral **bisect** each other at right angles, the quadrilateral is a **rhombus**. T
- h) A **tangent** to a circle can touch the circle at more than 1 point. F

2

- a) Construct a **deltoid** which has adjacent sides 3 cm and 4.5 cm long and the angle between the two 3 cm sides is 45° . Label its vertices ABCD.
- b) What length are its diagonals? **2cm, 7cm**
- c) What other information could have been given in a) to enable you to construct the deltoid? **Angle ABC, lengths of diagonals**
- d) Draw an axis, m , and **reflect** the deltoid in m . Label its vertices A'B'C'D'.
- e) Mark a point, P. Rotate the *mirror image* by 90° around P. Label the vertices of the rotated deltoid A''B''C''D''.
- f) In what other way could ABCD have been transformed to A''B''C''D''?
Reflection in a diagonal mirror line, through P



3

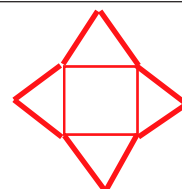
- If the **circumference** of a circle with diameter 1 unit is about 3.14 units ($\approx \pi$),
- a) calculate the approximate length of the circumference of a circle which has:
 - i) a 1.5 cm diameter **4.71cm**
 - ii) a 3.5 cm diameter **10.99**
 - iii) radius 3π units. **59.1576 units**
 - b) write the length of the circumference of a circle using π if its:
 - i) diameter is 8.9 cm **8.9π**
 - ii) radius is 1.62 cm **3.24π**
 - c) radius is 0.5 cm. **π**

4

- A farmer has 200 m of fencing to construct a rectangular pen for his sheep.
- a) If he wants to enclose as much grass as possible, what dimensions should he use? **50m by 50m = 2 500m²**
 - b) The farmer thought of using a long straight stone wall for one side of the pen. What dimensions could he now use to enclose the greatest area of grass? **50 by 100 = 5 000m²**
 - c) What is the ratio of the two areas? **Give your answer as a percentage. = 2:1 200%**

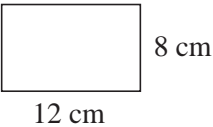
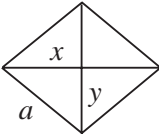
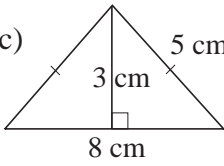
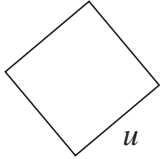
5

Draw 3 different nets for a **square-based pyramid**. eg.



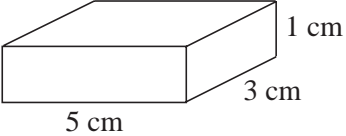
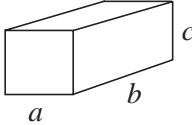
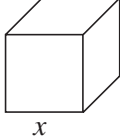
1

Write below each **polygon** its perimeter and area.

<p>a) </p> <p>$P = 40\text{cm}$ $A = 96\text{cm}^2$</p>	<p>b) </p> <p>$P = 4a$ $A = 2xy$</p>	<p>c) </p> <p>$P = 18\text{cm}$ $A = 12\text{cm}^2$</p>	<p>d) </p> <p>$P = 4u$ $A = u^2$</p>
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2

Write below each **polyhedron** its surface area and volume.

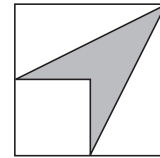
<p>a) </p> <p>$A = 46\text{cm}$ $V = 15\text{cm}^3$</p>	<p>b) </p> <p>$A = 2(ac + bc + ab)$ $V = abc$</p>	<p>c) </p> <p>$A = 6x$ $V = (x)(x)(x) = x^3$</p>
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3

HMC
1982
Age 11

In the diagram, the points on the two sides are midpoints.
What part of the area of the square has been shaded?

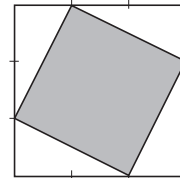
$\frac{1}{4}$ of the square is shaded



4

HMC
1995
Age 11

In the diagram, the sides of the large square are 3 units long.
The sides of the large square have been divided into 3 equal parts and some of the dividing points have been joined up.
What is the area of the shaded square? $9 - 4 = 5\text{cm}^2$



5

HMC
1993
Age 12

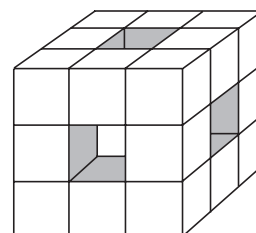
A wooden cube was cut by some planes. The cuts were parallel to two opposite faces.
The sum of the surface areas of the pieces formed is 3 times the surface area of the cube.
How many planes cut the cube? **6 plane cutes**

6

HMC
1996
Age 11

Imagine a cube built from 27 small 1 cm cubes.
The middle cube in each face is removed and so is the small cube at the centre of the large cube.
What is:

- a) the surface area of the remaining solid **72cm^2**
- b) the volume of the remaining solid? **20cm**

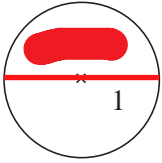


1

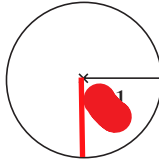
The circumference of a circle with radius 1 unit is $2 \times \pi \approx 6.28$ units.

Colour an arc of the given length on the circumference of the circle.

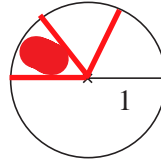
a) π



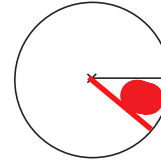
b) $\frac{\pi}{2}$



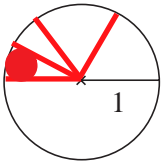
c) $\frac{\pi}{3}$



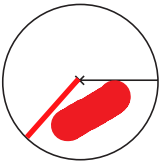
d) $\frac{\pi}{4}$



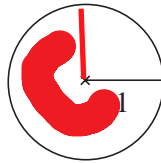
e) $\frac{\pi}{6}$



f) $\frac{3}{4}\pi$



g) $\frac{3}{2}\pi$



2

HMC
1992
Age 11

- a) Is there a square in which the numerical value of its perimeter length (in cm) is equal to the numerical value of its area (in cm²)? **4 by 4**
- b) Is there a cube in which the numerical value of its surface area (in cm²) is equal to the numerical value of its volume (in cm³)? **6 by 6 by 6**

3

HMC
1993
Age 11

You have 60 **congruent** small cubes and you want to build different kinds of cuboids .

You must use all the unit cubes to build each cuboid. **(1,1,60)(1,2,30)(2,3,10)(3,4,5)(1,5,12)(1,4,15)**

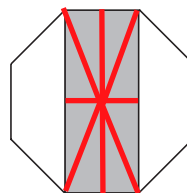
How many different sized cuboids could be built? **6 cuboids**

4

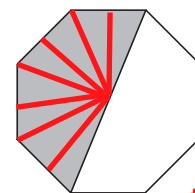
HMC
1983
Age 12

The two regular octagons are **congruent**.

Show that the two shaded areas are equal.



8 right angled triangles



8 right angled triangles

5

HMC
1991
Age 12

The shorter side of a rectangle is 2 units and each of its diagonals is 4 units.

- a) What size are the angles formed by the diagonals? **60° and 120°**
- b) What size are the angles formed by the diagonals and the sides? **60° and 30°**

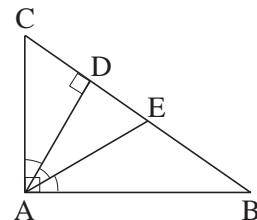
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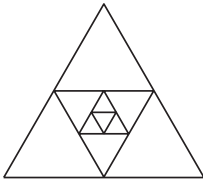
HMC
1994
Age 12

In this right-angled triangle, the lines DA and EA divide the right angle CAB into 3 equal parts.

DA is perpendicular to the **hypotenuse** BC.
E is the midpoint of BC.

What size are the acute angles of triangle ABC? **30° and 60°**


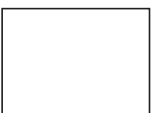
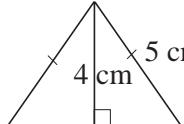
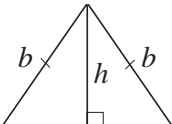


<p>1</p> <p>HMC 1980 Age 11</p>	<p style="text-align: center;">1 2 3 □</p> <p>This number is in a number system which is based on a number less than 10.</p> <p>We know that:</p> <ul style="list-style-type: none"> • when the number is divided by 2, there is a remainder of 1, • when the number is divided by 3, there is a remainder of 0, • when the number is divided by 4, there is a remainder of 3. <p>What is the base number? 1239</p>
<p>2</p> <p>~</p> <p>HMC 1986 Age 11</p>	<p>a) Number the days of 2004 in order. This is their ordinal value. <i>Example:</i> 1st of January is 1; 5th of February is $31 + 5 = \mathbf{36}$, etc.</p> <p>b) Multiply each date in every month by the ordinal value of the month. This is their product value. <i>Example:</i> 11 April: $11 \times 4 = \mathbf{44}$; 31 October: $31 \times 10 = \mathbf{310}$, etc. (c) Answer in lesson plan 32</p> <p>c) How many days were there in the year 2004 when the ordinal value and the product value were equal? No 31, 37, 41, 43 etc (46 Prime numbers) $366 - 46 = 320$ days</p>
<p>3</p> <p>HMC 1988 Age 11</p>	<p>I cut a rectangle into two parts by drawing a straight line. Then I cut one of the two parts into two polygons by drawing another straight line. Then I cut one of the two polygons by drawing another straight line, and so on.</p> <p>After I had drawn 100 dividing lines, I counted the vertices of all the polygons I had formed. I counted 300 vertices.</p> <p>Is this possible? Give a reason for your answer.</p> <p style="text-align: right;">Verticies follow rule $2n + 4$ where $n =$ number of cuts</p> <p style="text-align: center;">yes $300 = 2n + 4 \Rightarrow n = 148$</p>
<p>4</p> <p>HMC 1993 Age 11</p>	<p>Prove that if all the natural numbers from 1 up to and including a number which has units digit 5 (in a base 10 number system), the sum will be divisible by 5.</p> <p style="text-align: center;">sum of 1 to 15 is 120 (see lesson plan for more detailed answer) 120 is divisible by 5</p>
<p>5</p> <p>HMC 1984 Age 12</p>	<p>Liz started to write the whole numbers from 1 and now she is writing the 2893rd digit. Which whole number is she now writing?</p> <p style="text-align: center;">1 000</p>
<p>6</p> <p>HMC 1987 Age 12</p>	<p>Four equilateral triangles have been drawn, one inside the other. The area of the innermost, smallest triangle is 1 square unit.</p> <p>What is the sum of the areas of the 4 triangles?</p> <p style="text-align: center;">$4 \times 16 = 64\text{cm}^2$</p> <div style="text-align: right;">  <p style="text-align: center;">1, 4, 16</p> </div>

<p>1</p>	<p>The first 10 positive integers are multiplied together. How many zeros are at the right-hand side of the product? Two zero's 3628800</p>
<p>2</p> <p><i>HMC</i> <i>1986</i> <i>Age 12</i></p>	<p>The first 100 positive whole numbers are multiplied together. In the product, which digit is in the place value 24th from the right? 'zero' product = 9.3×10^{157}</p>
<p>3</p>	<p>Imagine that this fraction is simplified as far as possible.</p> $\frac{1 \times 2 \times 3 \times 4 \times 5 \times 6 \times 7 \times 8 \times 9 \times 10}{2^{10}} \quad \frac{\underline{3 \times 5 \times 3 \times 7 \times 9 \times 5}}{4}$ <p>[2^{10} means the product of 10 factors and each factor is 2.]</p> <p>Which number will be the denominator of the simplified fraction? 4</p>
<p>4</p> <p><i>HMC</i> <i>1987</i> <i>Age 12</i></p>	<p>Imagine that this fraction is simplified as far as possible.</p> $\frac{1 \times 2 \times 3 \times 4 \times 5 \times \dots \times 98 \times 99 \times 100}{2^{100}} \quad \text{see lesson plan}$ <p>[2^{100} means the product of 100 factors and each factor is 2.]</p> <p>Which number will be the denominator of the simplified fraction? 8</p>
<p>5</p> <p><i>HMC</i> <i>1990</i> <i>Age 12</i></p>	<p>Some consecutive whole numbers, from 1 to a positive, whole number which is greater than 1, are added together. 1, 2, 3, 4, 5.... add</p> <p>Which digit can be in the units place value in the sum? (0, 1, 3, 5, 6, 8)</p> <p><i>Give a reason for your answer.</i> 1, 3, 6, 10, 15, 21, 28, 36, 45, 55, 66, 78, 91, 105, 120, 136, etc</p>
<p>6</p> <p><i>HMC</i> <i>1991</i> <i>Age 12</i></p>	<p>A new volume in a series of books is published every 7 years. When the 7th volume was published, the sum of all the year numbers in which a book was published was 13 727.</p> <p>In which year was the first volume in the series published? 1 940</p>
<p>7</p> <p><i>HMC</i> <i>1999</i> <i>Age 12</i></p>	<p>The whole numbers from 1 to 1999 are added together.</p> <p>Is the sum the square of a natural number? sum = 1 999 000</p> <p><i>Give a reason for your answer.</i> 1 999 000 \neq square numbers</p>

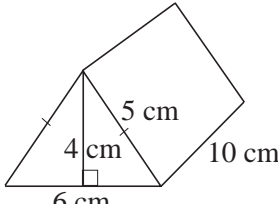
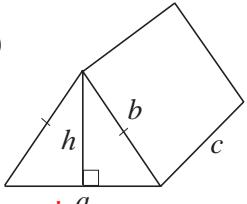
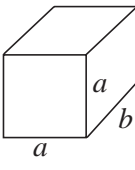
1

Write the perimeter and area below each **polygon**.

<p>a)  3 cm</p> <p>4 cm</p> <p>$P = 14\text{cm}$</p> <p>$A = 12\text{cm}^2$</p>	<p>b)  b</p> <p>a</p> <p>$P = 2a + 2b$</p> <p>$A = ab$</p>	<p>c)  5 cm</p> <p>4 cm</p> <p>6 cm</p> <p>$P = 16\text{cm}$</p> <p>$A = 12\text{cm}^2$</p>	<p>d)  b</p> <p>b</p> <p>h</p> <p>a</p> <p>$P = a + 2b$</p> <p>$A = \frac{a \times h}{2}$</p>
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2

Write the surface area and volume below each **prism**.

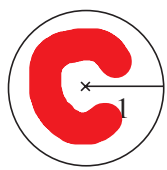
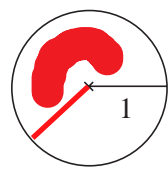
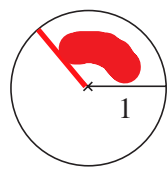
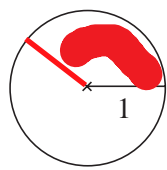
<p>a)  5 cm</p> <p>4 cm</p> <p>6 cm</p> <p>10 cm</p> <p>$A = 120\text{cm}^2$</p> <p>$V = 120\text{cm}^3$</p>	<p>b)  b</p> <p>h</p> <p>a</p> <p>c</p> <p>$A = \frac{ah}{2} \times 2 + 2bc + ac$</p> <p>$V = \frac{ah}{2} \times c$</p>	<p>c)  a</p> <p>a</p> <p>b</p> <p>$A = 2a^2 + 4ab$</p> <p>$V = a^2b$</p>
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3

How many different cuboids could be built from 40 small unit cubes?
 (All the unit cubes should be used for each cuboid.) **4**
 (1, 2, 20)(1, 4, 10)(1, 5, 8)(1, 1, 40)

4

The **circumference** of a circle with **radius** 1 unit is $2 \times \pi \approx 6.28$ units.
 Colour an **arc** of the required length on the circumference of each circle.

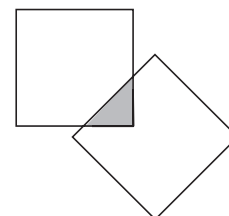
<p>a) 2π</p> 	<p>b) $\frac{5\pi}{4}$</p> 	<p>c) $\frac{2\pi}{3}$</p> 	<p>d) $\frac{5\pi}{6}$</p> 
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5

These two squares **intersect** to form a triangle (shaded).

Can 2 squares intersect to form polygons with:

- a) 4 sides **Y** b) 5 sides **Y** c) 6 sides **Y**
 d) 7 sides **Y** e) 8 sides **Y** f) 10 sides? **N**



6

In this *magic square*, the sum of the numbers in each row, column and diagonal is 42.
 Fill in the missing numbers.

10	26	6
10	14	18
22	2	18

1

HMP

Decide whether each statement is true or false and write T or F in the box.

- a) The **product** of two numbers can be less than each of the two numbers.
- b) The **arithmetic mean** of two negative numbers can be positive.
- c) There is an **isosceles** triangle which has two right angles.
- d) There is a positive fraction less than 1 which is equal to its **reciprocal**.
- e) If a **product** is zero, at least one of its factors is zero.
- f) If the areas of two triangles are equal, the triangles are **congruent**.
- g) There is a quadrilateral which is both a **deltoid** and a **parallelogram** but is not a square.

Rhombus**2**

HMP

Frank, Charlie, Johnny and George went to visit their friend. The surnames of the 4 boys, in no particular order, are Little, Grant, Tailor and Miller.

Miller arrived first, then Johnny, then Little. George arrived last.

Each boy gave his friend a present. Miller gave a magic cube, Frank gave a pen, George gave a bar of chocolate and Tailor gave a book.

What is the full name of each boy? **No information linking first and second names**

3HMC
1997
Age 12

In a Canadian city, 80% of the population speaks English and 70% speaks French. Every inhabitant can speak either French or English.

What percentage of the population can speak both languages? **50%**

4HMC
2000
Age 11

Ten pupils took part in a mathematics competition in which 5 problems were set. Thirty-five answers were handed in.

We know that there was a pupil who handed in only 1 answer, a pupil who handed in 2 answers and a pupil who handed in 3 answers.

Show that there must be a pupil who answered all five problems.

$$\textcircled{5}, 1, 4, 4, 4, 4, 4, 4, 2, 3 = 35$$

5HMC
1985
Age 12

A Year 6 class of 42 pupils took part in a special Physical Education lesson. The pupils could choose from basketball, swimming and gymnastics.

We know that 20 of them did swimming, 19 did gymnastics and 18 played basketball.

We also know that 7 pupils swam and played basketball, 8 pupils swam and did gymnastics and 6 pupils did gymnastics and played basketball.

How many pupils took part in all 3 sports? **6 pupils**

6HMC
2000
Age 11

A shooting practice target is shaped like an equilateral triangle and each of its sides is 1 metre long.

If 10 shots hit the target, show that two of the shots must be less than 34 cm apart.

$$\text{Area} = 4300\text{cm}^2$$

**Shots that hit the target
in the corner regions < 34cm apart**

1Complete the **arithmetic laws**. Try them with numbers if necessary.

a) $a + (-b) - (+c) - (-d) =$

b) $(a - b) \times c = ac - bc$

c) $x \times y + x \times z = xy + xz = x(y + z)$

d) $(a - b) \div c = \frac{a - b}{c}$

e) $u \div w + v \div w = \frac{u}{w} + \frac{v}{w} = \frac{1}{w}(u + v)$

f) $2 \times f + 3 \times f - 4 \times f = 2f + 3f - 4f = f$

g) $6t - 4t - 9t = -7t$

h) $\frac{a \times c}{b \times c} = \frac{ac}{bc}$

i) $\frac{a + b}{c} = \frac{a}{c} + \frac{b}{c} = \frac{1}{c}(a + b)$

j) $\frac{a}{b} - \frac{c}{d} = \frac{ad - cb}{bd}$

k) $\frac{a \times n}{n} = \frac{an}{n} = a$

l) $\frac{a}{b} \times b = a$

m) $\frac{a}{b} \div c = \frac{a}{bc}$

n) $\frac{a}{b} \times \frac{c}{d} = \frac{ac}{bd}$

o) $\frac{a}{b} \div \frac{c}{d} = \frac{a}{b} \times \frac{d}{c} = \frac{ad}{bc}$

2HMC
1985
Age 11How is it possible to share 7 equal sized loaves of bread among 12 hungry people **without** cutting each loaf into 12 pieces?

$\frac{1}{4} + \frac{1}{3}$ for each person

Try to solve it using as few cuts as possible.

$12 \times \frac{1}{4} = 3 \quad 12 \times \frac{1}{3} = 4 \quad 3 + 4 = 7$

3HMC
1991
Age 11

What is the smallest positive whole number which has units digit 6 and if we move this digit from the right-hand side to the left-hand side but leave the other digits unchanged, we get 4 times the original number?

153 846 (for worked solution see lesson plan)

4HMC
1980
Age 12

We multiply the digits of a 3-digit whole number, then multiply the digits of the product.

We can represent the number and the two products in this way:

$$\begin{array}{ccc} 1 & 4 & 4 \\ \triangle & \circ & \circ \end{array}; \quad \begin{array}{ccc} 1 & 6 & \\ \triangle & \square & \square \end{array}; \quad \begin{array}{c} 6 \\ \square \end{array}$$

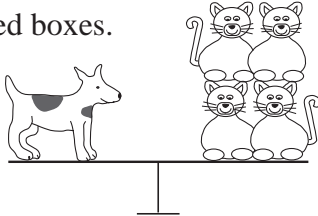
The same shape means the same digit.

What was the original number? *Explain your reasoning.* (see lesson plan)**5**HMC
1992
Age 12

We put £255 into 8 envelopes, seal the envelopes and write on each how much money it contains. There is a different amount in each envelope.

Without opening any of the envelopes we can pay **any** whole amount from £1 to £255.

How much money is in each envelope? £1, £2, £4, £8, £16, £32, £64, £128

<p>1</p>	<p>Solve the equations and inequalities. Check your results. (The base set is in brackets.)</p> <p>a) $x - 5 > -5$ (Z) $x > 0$</p> <p>b) $(-3) \times (4 \times y) + 8 \leq 2 \times (5 \times y + 6)$ (Q) $-12y + 8 \leq 10y + 12$ $22y \geq -4$ $-4 \leq 22y$ $y \geq -\frac{2}{11}$</p> <p>c) $\frac{3 \times t}{8} - 2 + \frac{2 \times t}{3} = -\frac{5}{6} + \frac{5 \times t}{12}$ (Q) $t = \frac{28}{15}$</p> <p>d) $\frac{3 \times v + 5}{-2} = -4$ (N) $3v + 5 = 8$ $3v = 3$ $v = 1$</p>
<p>2</p> <p>HMC 1985 Age 11</p>	<p>I have 18 coins (2 p and 5 p pieces) in my pocket.</p> <p>If I had as many 5 p coins as I have 2 p coins and as many 2 p coins as I have 5 p coins, I would have twice as much money as I have now.</p> <p>How much money do I have? $16 \times 2p + 2 \times 5p = 42p$ altogether</p>
<p>3</p> <p>HMC 1981 Age 11</p>	<p>Steve and a dog can be balanced on a seesaw by 5 equal-sized boxes.</p> <p>The dog and 2 cats can be balanced on the seesaw by 3 of the boxes and the dog can be balanced by 4 cats.</p> <p>How many cats are needed to balance Steve?</p> <p>six cats will balance Steve</p> 
<p>4</p> <p>HMC 1991 Age 11</p>	<p>The average age of the 11 members of a football team is 22 years. Total = 11 x 22 = 242</p> <p>When one member of the team was sent off because of a bad tackle, the average age of the rest of the team was 21 years. $10 \times 21 = 210$</p> <p>How old is the player who was sent off? $242 - 210 = 32$ years old</p>
<p>5</p> <p>HMC 1992 Age 11</p>	<p>If I had four times as much money as I have now, my money would be as much over £1000 as the amount I have now is less than £1000.</p> <p>How much money do I have? £400</p>
<p>6</p> <p>HMC 1995 Age 11</p>	<p>A lorry and a car started from two cities at the same time and travelled towards each other at steady speeds. The lorry took 6 hours to cover the distance between the two cities and the car took 4 hours.</p> <p>(see lesson plan)</p> <p>After what amount of time did they pass each other?</p> <p>They passed each other after 2 hours 24 mins</p>
<p>7</p> <p>HMC 1984 Age 12</p>	<p>A matchbox contains some matches. If we double the number of matches then take away 8, then double the number of matches left and take away 8 again, then do the same for a third time, the box will be empty.</p> <p>(See lesson plan)</p> <p>How many matches are in the matchbox? 7 matches in the box</p>

1

Find a relationship between the corresponding values and complete the table.

Show the data in a graph in your exercise book.

a)	x	0	-1	$\frac{1}{2}$	2	$\frac{2}{3}$	$-\frac{1}{6}$	2.5	-4	$\frac{1}{6}$	0.2	$-\frac{1}{2}$
	y	0	-3	$\frac{3}{2}$	6	2	$-\frac{3}{6}$	7.5	-12	$\frac{1}{2}$	0.6	$-\frac{3}{2}$

$y = 3x$ $x = \frac{y}{3}$ $\frac{y}{x} = 3$ ($x \neq 0$)

b)	u	1	$-\frac{1}{2}$	2	$\frac{1}{3}$	-4	$\frac{1}{2}$	$\frac{8}{3}$	$\frac{4}{6}$	-3	-12		-4
	v	4	-8	2	12	-1	8	$\frac{3}{2}$	6	$-\frac{4}{3}$	$-\frac{1}{3}$	0	-1

$v = \frac{4}{u}$ $u = \frac{4}{v}$ $u \times v = 4$ $v \neq 0$

2

Solve the problems. Think about the ratio between the quantities.

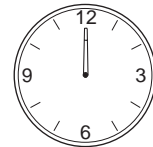
- a) If $\frac{4}{5}$ kg of apples cost £2.40, what is the price of $\frac{2}{3}$ kg of apples? **£2**
- b) $\frac{1}{5}$ kg of strawberries costs £1.50, including the cost of the punnet. If all punnets cost 10 p, what would you pay for 400 g of strawberries in a punnet?
£2.80 + 10p = £2.90
- c) When our car used $5\frac{1}{3}$ litres of petrol every 100 km, a full tank lasted for 864 km.
46.08 tank size
If our car had used $7\frac{1}{5}$ litres of petrol every 100 km, how far could we have driven with a full tank?
46.08 ÷ 7.2 = 6.4
6.4 × 100 = 640km

3HMC
1990
Age 11

The big hand and the little hand of a clock coincide at 12 o'clock.

When will the two hands of the clock next be in a straight line?

12h 32 $\frac{8}{11}$ mins

**4**HMC
2000
Age 11

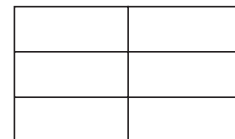
Dad could dig the garden in 2 hours.

His elder son, Benny, could dig the garden in 3 hours.

His younger son, Charlie, could dig the garden in 6 hours.

If they all worked together, how long would it take the three of them to dig the garden?

$$\frac{t}{2} + \frac{t}{3} + \frac{t}{6} = 1 \qquad t = 1 \text{ hour}$$

**5**

The ratio of the lengths of the sides of a right-angled triangle is 3 : 4 : 5.

If the area of the triangle is 24 cm², what is the length of each of its sides?

6, 8 and 10

1

Decide whether each statement is true or false and write T or F in the box.

- a) If the areas of two rectangles are equal, the rectangles are **congruent**. F
- b) All equilateral triangles are **similar**. T
- c) The **arithmetic mean** of two numbers is always positive. F
- d) There is an **isosceles** triangle which has three equal angles. T
- e) The diagonals of a **parallelogram** intersect at right angles. (Rhombus) F
T
- f) If the areas of two squares are equal, the squares are **congruent**. T

2Complete the **arithmetic laws**. Try them with numbers first if necessary.

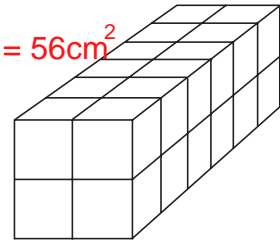
- a) $(a + b) \times c = ac + bc$
- b) $(a + b) \div c = \frac{a+b}{c}$
- c) $\frac{a}{b} + \frac{c}{d} = \frac{ad + cb}{bd}$
- d) $\frac{a}{b} \times \frac{b}{a} = \frac{ab}{ba} = 1$
- e) $\frac{a}{b} \div a = \frac{a}{b} \times \frac{1}{a} = \frac{1}{b}$
- f) $\frac{a - b}{c} = \frac{a}{c} - \frac{b}{c}$

3

- a) Show that when $x = 8$ and $y = 24$, $\frac{1}{x} + \frac{1}{y} = \frac{1}{6}$. $\frac{1}{8} + \frac{1}{24} = \frac{4}{24} = \frac{1}{6}$
- b) Find other positive integers, x and y , which also make the equation true.
 $x = 7 \quad y = 42 \quad x = 9 \quad y = 18 \quad x = 10 \quad y = 15$

4

- a) How many unit cubes were needed to make this cuboid? 24cm^3
- b) What is the surface area of the cuboid? $4 + 4 + 4(2 \times 6) = 56\text{cm}^2$
- c) What are the dimensions of a cuboid made from 24 unit cubes which has:
- the **smallest** possible surface area $3 \times 4 \times 2$
 - the **greatest** possible surface area? $1 \times 1 \times 24$

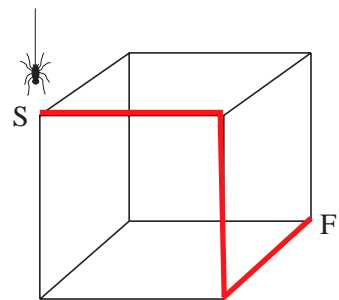
**5**

The sketch shows a wire frame in the shape of a cube.
The length of each edge is one unit.

A spider starts at S and has to reach F.

It can only go **down** the vertical edges and cannot go along any wire more than once.

- a) What is the length of its **shortest** path from S to F? **3**
- b) What is the length of its **longest** path from S to F? **7**
- c) How many different paths are possible? $2 + 4 + 4 + 4 = 14$ different paths



1

- a) Write five numbers using **non-zero** digits so that their ratio is $1 : 2 : 3 : 4 : 5$.
Use each digit only once.

..... **9** **18** **27** **36** **45**

- b) Use **all** possible digits once each to make five numbers in the ratio $1 : 2 : 3 : 4 : 5$.

..... **18** **36** **54** **72** **90**

2

The perimeter of an **isosceles** triangle is 36.8 cm. The length of the base side is 2 thirds of the length of the adjacent side.

- a) What length are the sides of the triangle? **13.8, 13.8, 9.2**

- b) What is the ratio of the 3 sides? **$1 : 1 : \frac{2}{5}$ - $3 : 3 : 2$**

3

The ratio of two natural numbers is $3 : 2$ and they are both multiples of 6. If we divide them by 6, the first quotient is 4 greater than the second quotient.

What are the two numbers? **72 : 48**

4

HMC
1987
Age 11

In a mathematics competition, 9 pupils got through to the final round. In the final round, 6 tenths of the girls solved at least two problems correctly.

How many boys and how many girls reached the final of the competition?

4 girls and 5 boys reached the final

5

HMC
1990
Age 11

I have £2 in my two pockets altogether.

If I transfer a quarter of the money that I have in one pocket plus an additional 20 p from the same pocket to the other pocket, I would have an equal amount of money in each pocket.

How much money do I have in each pocket?

£1.60 in one pocket and 40p in another

6

HMC
1997
Age 11

Sally owns a hotel. She has seen some material which matches the colour scheme in her public rooms exactly. She needs 51 m^2 of material to make cushions and drapes.

However, Sally has been told that when the material is washed, it shrinks by $\frac{1}{16}$ of its

length and by $\frac{1}{18}$ of its width, so she intends to wash the material before she uses it.

How many square metres of unshrunk material should she buy?

Sally should buy 57.6 m^2 of material

7

HMC
1983
Age 11

Which is more:

$0.75 < 0.7500000625$
 $\frac{3}{4}$ or $\frac{3000001}{4000001}$?

1

Freddie Fox decided that in future he would tell lies on Mondays, Wednesdays and Fridays but he would always tell the truth on the other days of the week.

One day, Freddie said, "Tomorrow I will tell the truth."

On what day of the week could he have said it?

Saturday (see lesson plan)

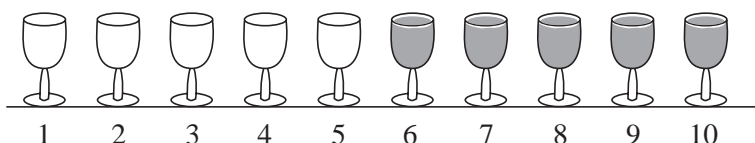
**2**

Is it possible for four whole numbers to have an odd number as their sum and an odd number as their product? If so, write the numbers. If not, say why not.

Impossible (see the lesson plan for detailed explanation)

3

Five empty glasses and five glasses full of grape juice are standing in a row.



How can you make the empty and full glasses alternate by touching only 2 glasses?

Pour the contents of 7 and 9 into 2 and 4!

4

On an old, smudged sheet of paper we can see this writing:

72 barrels: £ 5 37.8 4

but the digits marked ● are illegible. What could the price of a barrel have been?

$£537.84 \div 72 = £7.47$

5

Ben had to make a 4-digit number, choosing from the digits 1, 2, 3, 4, 5 and 6. He was allowed to use a digit more than once.

Ben wrote his number on a piece of paper and put it in his pocket. The rest of the class had to guess Ben's number.

The first suggestion was 4215. Ben said that two digits were correct but only one of them was in the correct place-value column.

The second suggestion was 2365. Ben said that again two digits were correct but only one of them was in the correct place-value column.

The third suggestion was 5525. This time Ben said that no digits were correct.

What do you think Ben's number could be?

Ben's number could be 4 163 or 6 314

6

We know this information about a certain square and a certain rectangle.

- Their areas are equal.
- The perimeter of the square is 4 fifths of the perimeter of the rectangle.
- The long side of the rectangle is 4 times the length of its short side.
- The perimeters, areas and sides of the 2 shapes are whole numbers less than 100.

What could be the lengths of the sides of the square and the rectangle?

Square could be 2, 4, 6 or 8

Rectangle could be
1 by 4, 2 by 8, 3 by 12
4 by 16

1

Harvey put £500 into a savings account which had a yearly interest rate of 7.5%.

- a) How much interest would be added to the account at the end of the 1st year? **£37.50**
- b) If Harvey did not touch his account, how much would the account be worth at the end of the 2nd year? **£577.81**
- c) Harvey wants his account to reach £700 before he spends any of it. How many years will Harvey have to wait before this happens? **5 years**
£717.81

2

A cuboid is made from 1 cm cubes. Its edge lengths are 2 cm, 4 cm and 5 cm.


If the outside of the cuboid is painted *blue*, how many of the 1 cm cubes will have:

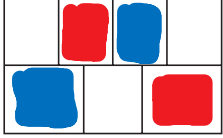
- a) 3 *blue* faces **8**
- b) 2 *blue* faces **20**
- c) 1 *blue* face **12**
- d) no *blue* faces? **0**


3

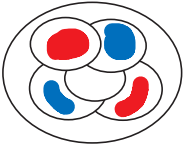
Colour these maps using the smallest number of different colours possible.

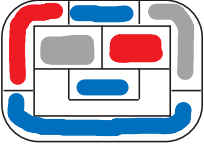
Adjacent countries must not have the same colour but countries with the same colour can meet at a point.

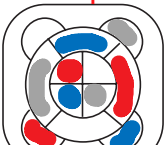
a)  **2**

b)  **3**

c)  **4**

d)  **3**

e)  **4**

f)  **4**

4

If x and y are positive integers, find as many values as possible for x and y which make this equation true.

$$\frac{x+y}{xy} = \frac{1}{8} \qquad \frac{1}{x} + \frac{1}{y} = \frac{1}{8} \qquad \frac{1}{24} + \frac{1}{12} = \frac{3}{24} = \frac{1}{8}$$

5

Most bar codes on grocery products have 8 or 13 digits. The last digit is a **check digit**, designed to check that the bar code reader has read the number correctly.

For example, the last digit in 5070 9274

is designed so that $3 \times (5 + 7 + 9 + 7) + 1 \times (0 + 0 + 2 + 4) = 90$ is divisible by 10.

- a) What is the check digit, c , for these numbers?
 - i) 5008 473c **9**
 - ii) 5120 173c **9**
 - iii) 8300 720c **0**
- b) The bar code reader reads a code as 5070 4827. If **one** number has been read incorrectly, what could the real number have been?
50 704 828 or 50 704 927

1

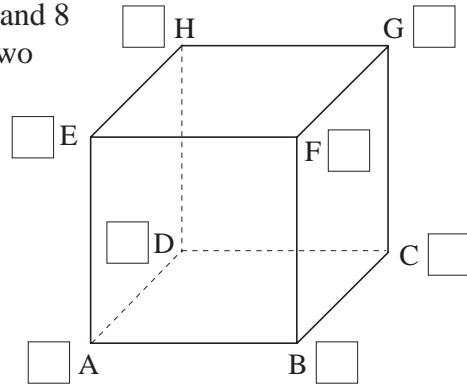
HMC
1986
Age 11

We want to assign the numbers 1, 2, 3, 4, 5, 6, 7 and 8 to the vertices of a cube so that the sums of the two numbers on each edge are all different.

Is this possible?

Give a reason for your answer.

see lesson plan for detailed explanation of why it is impossible



2

HMC
1987
Age 11

The numbers 1, 2, 3, . . . , 10 and 11 were each written on a small piece of paper. The pieces of paper were mixed up and put into two boxes.

Adam added the numbers in one box and Becky added the numbers in the other box.

Becky said, "Isn't it interesting? The sum of my numbers is exactly six times the sum of Adam's numbers."

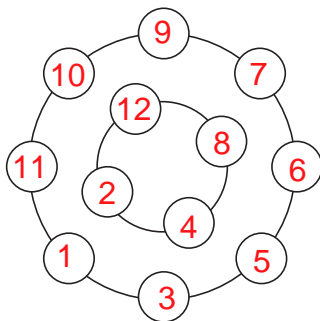
Adam said, "I think there must be a mistake in our calculations."

Is Adam correct? Give a reason for your answer.

Yes (see lesson plan for explanation)

3

HMC
1989
Age 11



Write the numbers from 1 to 12 in the two **concentric** circles so that:

- each *inner* number is even,
- the sum of the *outer* numbers is twice the sum of the *inner* numbers.

see lesson plan for the other solution

4

HMC
1997
Age 11

The members of a club rented a room for their meeting.

Ten members attended the meeting and they each paid the same amount towards the hire of the room.

If another five members of the club had attended the meeting, everyone would each have paid £10 less.

How much did it cost to hire the meeting room? £300 (see lesson plan)

5

HMC
1998
Age 11

Ben picked some apples from his apple tree and put them in a box in his garage.

That day, Ben made an apple pie with one third of the apples in the box.

The next day he ate one third of the remaining apples and on the following day he gave one third of what was left to his neighbour.

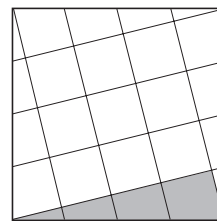
If 8 apples were left in the box, how many apples did Ben pick from the tree?

Ben picked 27 apples from the tree

1HMC
1980
Age 12

The sides of a square are each divided into 4 equal parts.
Some of the points are joined up as shown in the diagram.

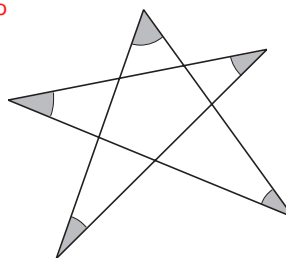
What part of the area of the whole square is the area of the shaded part? **The shaded part is $\frac{2}{17}$ th of the original shape**

**2**HMC
1984
Age 12

What is the sum of the shaded angles? **180°**

Explain how you worked out the solution.

See detailed explanation in lesson plan

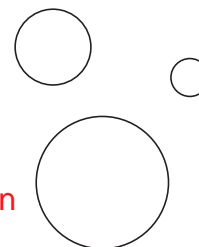
**3**HMC
1985
Age 12

We say that two circles touch each other if they have exactly **one** common point.

How many circles which touch each of the 3 circles in the diagram can you imagine in the plane?

There are 8 circles

See lesson plan for solution

**4**HMC
1986
Age 12

We marked 7 points on a plane and joined them up so that any two different points are on a straight line.

When we had finished, we had drawn 14 different straight lines.

Show how the 7 points could have been drawn. **$4 \times 3 + 1 + 1 = 14$**

5HMC
1990
Age 12

David asked his friend to guess how much money he had. He gave him this clue.

*My money could be made up in 20 different ways using just £5 notes and £2 coins but it could **not** be made up with only £2 coins.*

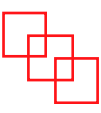

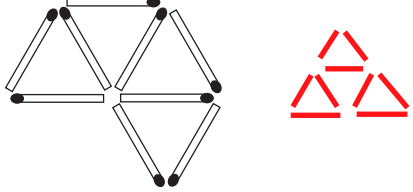
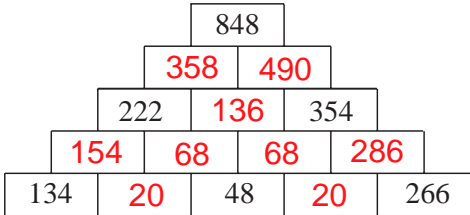
How much money did David have? **£195 (see solution in lesson plan)**

6HMC
1995
Age 12

What is the smallest, positive, whole number which gives:

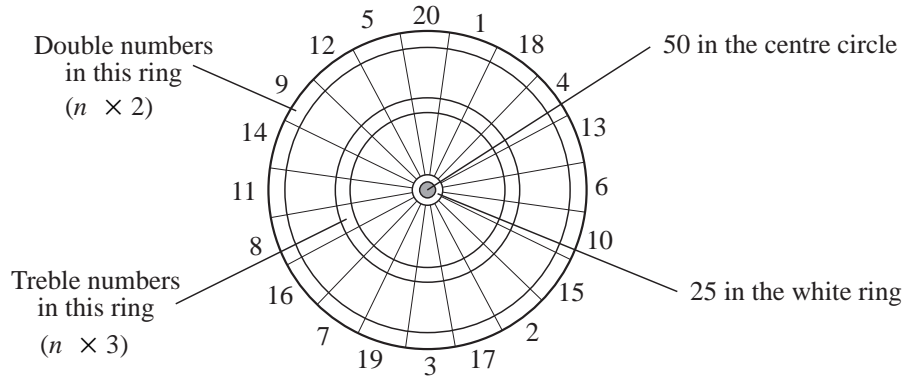
- a remainder of 1 when when it is divided by 3
- a remainder of 2 when it is divided by 4
- a remainder of 3 when it is divided by 5
- a remainder of 4 when it is divided by 6?

58 is the smallest positive whole number

<p>1</p>	<p>A small group of soldiers need to cross a river but the bridge has been destroyed. The river is very deep and its current is so swift that it is too dangerous for the soldiers to swim across.</p> <p>Two children are playing in a boat on the river bank. This boat is so small that only the two children or a single soldier can fit inside it.</p> <p>Is it possible for the group of soldiers to cross the river using the boat? <i>Give a reason for your answer.</i> Yes, see solution in lesson plan</p>
<p>2</p>	<p>a) Using 24 matchsticks of equal length, form 4 squares with 1 unit sides and 3 squares with 2 unit sides. </p> <p>b) Form 6 equilateral triangles from 12 matchsticks of equal length. </p>
<p>3</p>	<p>Change the position of only 2 matchsticks so that there are 5 triangles.</p> 
<p>4</p>	<p>Complete the diagram so that the sum of every two adjacent numbers is the number directly above them.</p> 
<p>5</p>	<p>A farm goose saw a flock of wild geese land on his pond. The farm goose said, "There must be a hundred geese in your flock!"</p> <p>One of the wild geese overheard him and said, "There aren't one hundred of us but if there were twice as many of us, then another half of us, then another quarter of us and if you joined our flock, then there would be a hundred geese in our flock."</p> <p>How many wild geese landed on the pond? 36 geese landed in the pond (see lesson pan for explanation)</p>
<p>6</p>	<p>We have 30 silver coins. Although they all look the same, we know that one of the coins is fake and is lighter than the others.</p> <p>If we tried to find out which coin is fake using a 2-pan balance, what is the least number of weighings we would need to do? The least is 3 weighings and the most is 4 weighings</p>
<p>7</p>	<p>The product of the length of a ship in metres, the age of its captain and the number of children he has is 32 118. Each of the three numbers is a whole number.</p> <p>How old is the captain of the ship? 53 years old, ship length = 101m Number of children = 6</p>

1

This is a diagram of a *dart board*.

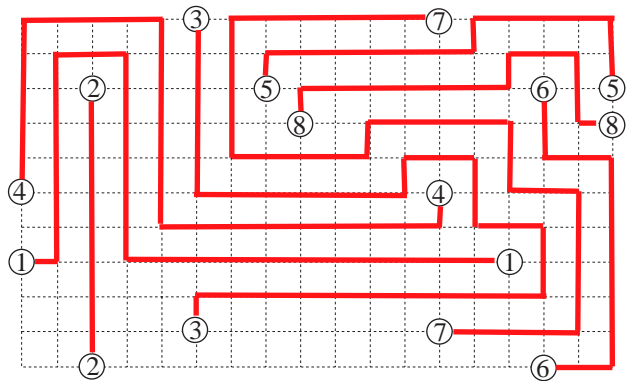


- a) What different scores between 50 and 60 can you get with one dart?
 51 (treble 17) 54 (treble 18) 57 (treble 19)
- b) If you throw 3 darts one after the other and all of them score, what is the:
- i) **highest** score possible 180 ii) **lowest** score possible? $3 \times 1 = 3$
 $3 \times \text{treble } 20$
- c) Using 1, 2 or 3 darts, what is the lowest score that is **impossible** to get?
 163

2

Join up the corresponding pairs of numbers by drawing lines **only** along the grid lines.

The lines should **not** cross or touch one another.



3

A domino is a 2×1 rectangle.



- a) How many different sizes of rectangles can you make with:
- i) 3 2 ii) 6 3 iii) 11 2 iv) n 2n

- b) Many of the rectangles can be split along a **fault line**, as shown with 6 dominoes.

What are the dimensions of the smallest rectangle, excluding a single domino, which does **not** have a fault line? 5×6



4

Braille is a code system based on a pattern of 6 dots. Each dot can be raised or flat.

How many different codes are possible using:

- a) 2 dots

 4

- b) 3 dots

 8

- c) 4 dots

 16

- d) 6 dots?

 64