Mathematics Enhancement Programme

TEACHING SUPPORT: Year 6

FACTS TO KNOW AND REMEMBER

Multiplication tables Up to 10 × 10

S I Units

10 mm	=	1 cm
1000 mm	=	1 m
100 cm	=	1 m
1000 m	=	1 km

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10 ml	=	1 cl
1000 ml	=	1 litre
100 cl	=	1 litre

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1000 g	=	1 kg
1000 kg	=	1 tonne

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60 seconds	=	1 minute
60 minutes	=	1 hour
24 hours	=	1 day
7 days	=	1 week
52 weeks	=	1 year
12 months	=	1 year

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Imperial Units

12 inches	=	1 foot
3 feet	=	1 yard
1760 yards	=	1 mile

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16 oz	=	1 lb
14 lb	=	1 stone

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8 pints	=	1 gallon

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Conversion between units

Length	1 inch	$\approx$	25.4 mm = 2.54 cm	1 cm	$\approx$	0.3937 inch
	1 foot	$\approx$	0.3048 m	1 m	$\approx$	3.281 feet
	1 mile	=	1.609 km	1 km	$\approx$	0.6214 mile

(so $1 \text{ km} \approx \frac{5}{8}$ mile and $1 \text{ mile} \approx \frac{8}{5}$ km)

Mass	1 oz	$\approx$	28.35 g	1 g	$\approx$	0.0353 oz
	1 kg	$\approx$	2.205 lb	1 lb	$\approx$	0.4536 kg

Capacity	1 litre	$\approx$	1.76 pints	1 pint	$\approx$	0.568 litres
	1 litre	$\approx$	0.22 gallons	1 gallon	$\approx$	4.545 litres

Temperature

$$x\,°\text{C} = \left(\frac{9x}{5} + 32\right)°\text{F}$$

$$y\,°\text{F} = \left(y - 32\right) \times \frac{5}{9}\,°\text{C}$$

($°\text{C} \equiv$ degrees Celsius, $°\text{F} \equiv$ degrees Fahrenheit)

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Numbers

$$1 \text{ th} = \frac{1}{1000}$$

$$1 \text{ h} = \frac{1}{100}$$

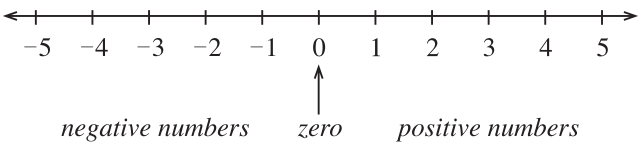
$$1 \text{ t} = \frac{1}{10}$$

$$1 \text{ T} = 10$$

$$1 \text{ H} = 10 \text{ T} = 100$$

$$1 \text{ Th} = 10 \text{ H} = 100 \text{ T} = 1000$$

Negative Numbers



Roman Numerals

1	I
5	V
10	X
50	L
100	C
500	D
1000	M

Even / Odd

Whole numbers ending in 0, 2, 4, 6, 8 are EVEN (and divisible by 2 with no remainder).

Whole numbers ending in 1, 3, 5, 7, 9 are ODD (and have remainder 1 when divided by 2).

Divisor or Factor and Multiple

Any whole number that divides exactly into a whole number with no remainder is called a *divisor* or *factor* of the number.

For example, 1, 2, 3, 4, 6 and 12 are all divisors (or factors) of 12.

Any whole number that can be divided by a whole number with no remainder is called a *multiple* of the number.

For example, 5, 10, 15, 20, . . . are all multiples of 5.

Any whole number is *divisible by 3* if the sum of its digits is divisible by 3.

For example, 123 $(1 + 2 + 3 = 6$, so 123 is divisible by 3. In fact, $123 \div 3 = 41)$

7212 $(7 + 2 + 1 + 2 = 12$, and 12 is divisible by 3, so 7212 is divisible by 3
In fact, $7212 \div 3 = 2404)$

Any whole number is *divisible by 9* if the sum of its digits is divisible by 9.

For example, 873 $(8 + 7 + 3 = 18$ and 18 is divisible by 9. In fact, $873 \div 9 = 97)$

Equivalent Fractions

$$\frac{1}{2} = \frac{2}{4} = \frac{4}{8} = \ldots$$

$$\frac{1}{10} = \frac{5}{50} = \frac{10}{100} = \ldots$$

Adding/Subtracting Fractions

$$\frac{a}{b} + \frac{c}{b} = \frac{a+c}{b}$$

(a, b and c are natural numbers, that is, numbers used for counting)

$$\frac{a}{b} - \frac{c}{b} = \frac{a-c}{b}$$

Decimals

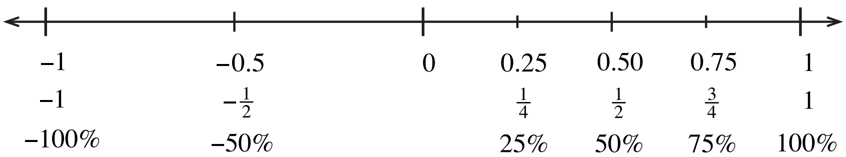
$$0.a = \frac{a}{10} \qquad (a = 0, 1, \ldots, 9) \qquad (\text{e.g. } 0.8 = \frac{8}{10})$$

$$0.ab = \frac{a}{10} + \frac{b}{100} \qquad (a, b = 0, 1, 2, \ldots, 9) \qquad (\text{e.g. } 0.82 = \frac{8}{10} + \frac{2}{100})$$

$$0.abc = \frac{a}{10} + \frac{b}{100} + \frac{c}{1000} \qquad (a, b, c = 0, 1, 2, \ldots, 9) \qquad (\text{e.g. } 0.823 = \frac{8}{10} + \frac{2}{100} + \frac{3}{1000})$$

Fraction, Decimal, Percentage Equivalents

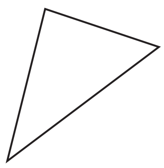
For example,



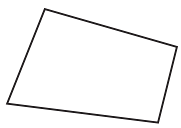
Similarly,

$$\frac{1}{10} = 0.1 \equiv 10\%, \qquad \frac{1}{20} = 0.05 \equiv 5\%, \qquad \text{etc.}$$

Shapes : 2D

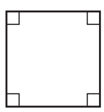


Triangle (3 straight sides)



Quadrilateral (4 straight sides)

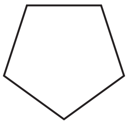
Rectangle (opposite sides equal and parallel, and four right angles)



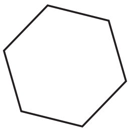
Square (all sides equal and four right angles)

(Note that all squares are rectangles and all rectangles are quadrilaterals.)

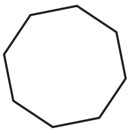
Polygon (any closed 2D shape with sides (edges) all straight lines)



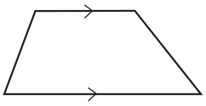
Pentagon (any 5-sided polygon: a regular pentagon has all internal angles of $108°$ and all sides the same length)



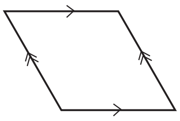
Hexagon (any 6-sided polygon; a regular hexagon has internal angles of $120°$ and all sides of equal length)



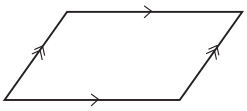
Octagon (any 8-sided polygon; a regular octagon has internal angles of $135°$)



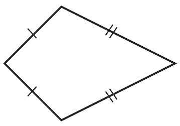
Trapezium (a quadrilateral with at least one pair of sides parallel)



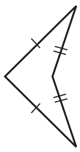
Rhombus (a quadrilateral with 4 equal sides; opposite sides are parallel)



Parallelogram (2 pairs of equal and parallel sides)

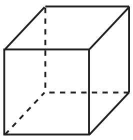


or

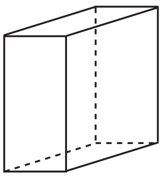


Deltoid (a quadrilateral with two pairs of adjacent sides equal)

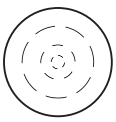
Shapes : 3D



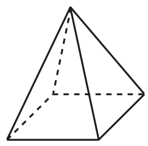
Cube (all sides equal so each face is a square)



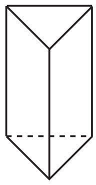
Cuboid (all opposite sides equal so each face is a rectangle)



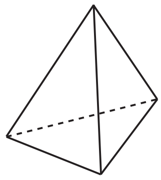
Sphere



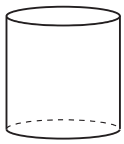
Square-based pyramid



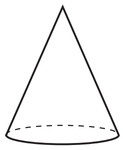
Triangle-based prism



Triangle-based pyramid



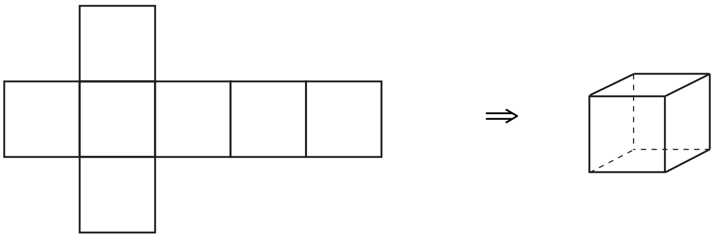
Cylinder

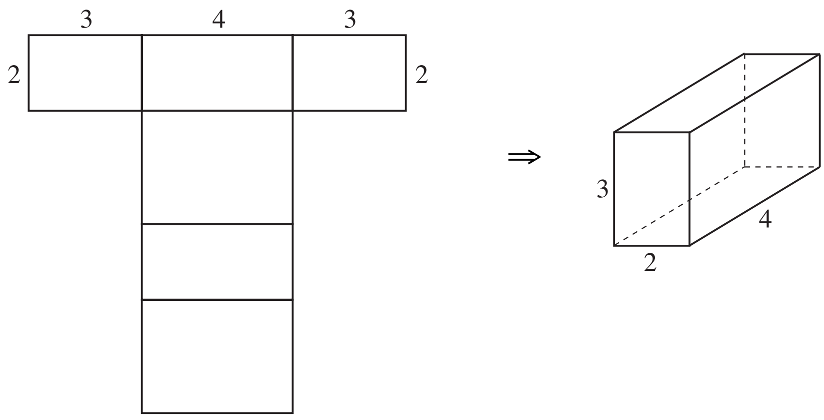


Cone

Nets

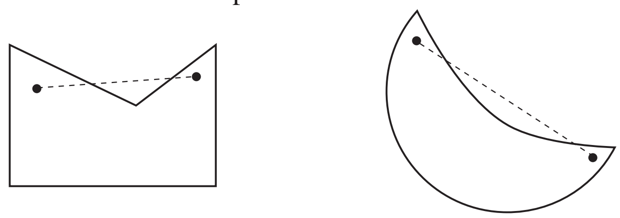
A *net* is a 2-D figure which can be folded to make a 3-D shape.



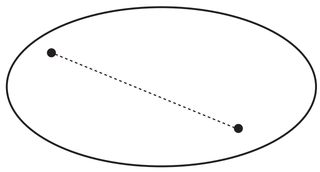


Convex and Concave Shapes

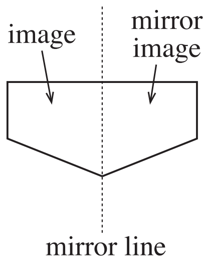
Concave: a straight line cannot always be drawn between any two points on the shape that is always inside the shape. In each of the examples below, the two points are *inside* the shape but the straight line drawn between them passes *outside* the shape.



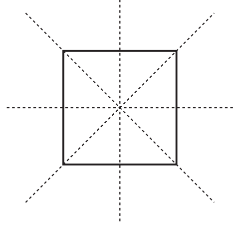
Convex: a straight line drawn between any two points on the shape will always lie *inside* the shape, as can be seen from the example below.



Symmetry



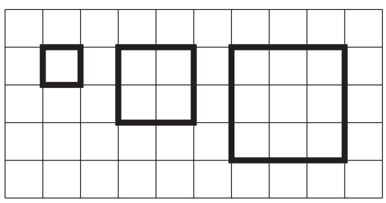
The whole shape has one line of symmetry.



Four lines of symmetry are shown here.

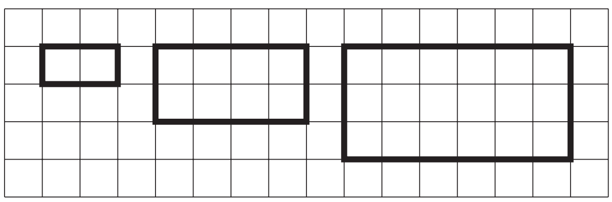
Similarity

(a)



These shapes are similar.

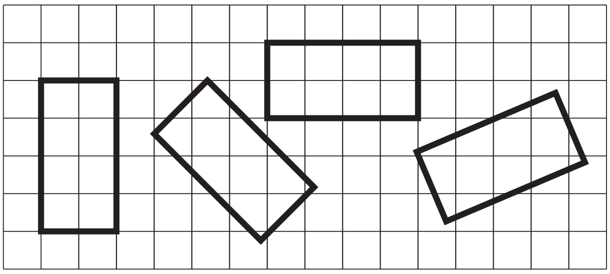
(b)



These shapes are similar.

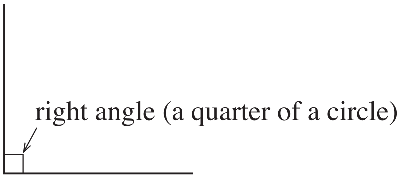
(The sides are in the same ratio, that is, 1 : 1 in (a) and 1 : 2 (i.e, 2 : 4 and 3 : 6) in (b).

Congruence

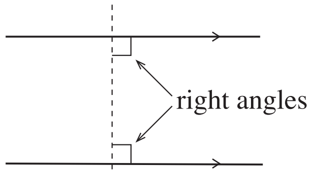


Congruent shapes are identical in shape and size but can be rotated or reflected; the 4 shapes shown are congruent.

Parallel and Perpendicular Lines



Lines are perpendicular

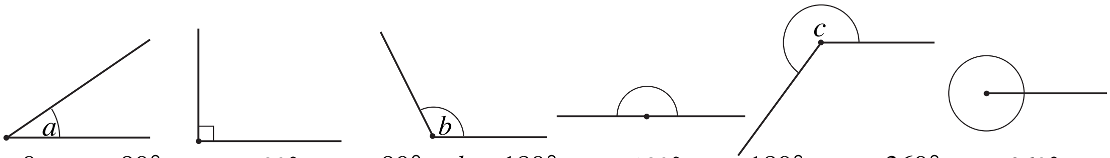


Lines are parallel

Angles

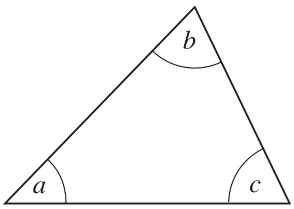
Angles about a point

acute *right* *obtuse* *straight* *reflex* *whole*



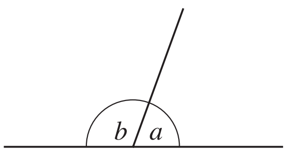
$0 < a < 90°$ | $90°$ | $90° < b < 180°$ | $180°$ | $180° < c < 360°$ | $360°$

Angles in a triangle



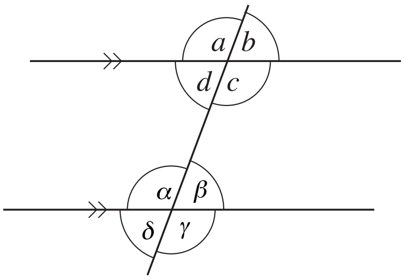
$a + b + c = 180°$

Angles on a straight line



$a + b = 180°$

Angles in parallel and intersecting lines

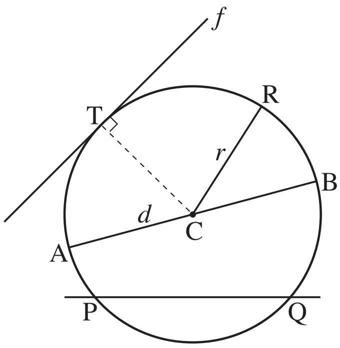


Angle a = angle c Angle b = angle d

Angle a = angle α Angle b = angle β

Angle c = angle γ Angle d = angle δ

Circles



C is the centre of the circle

AB is the diameter, d, of the circle

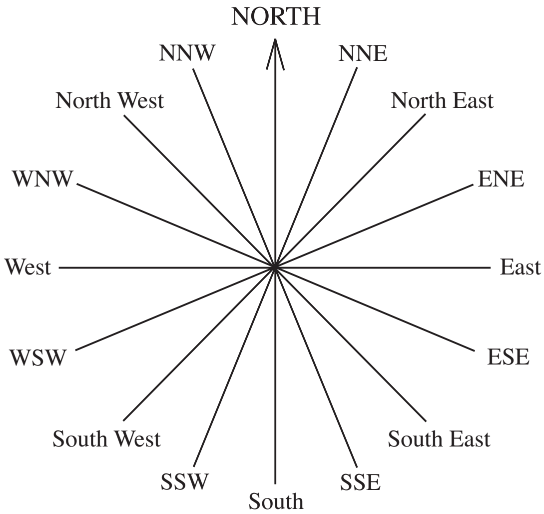
CR is a radius, r, of the circle ($d = 2r$)

RB is an arc of the circle

PQ is a chord

f is a tangent to the circle

Compass Points



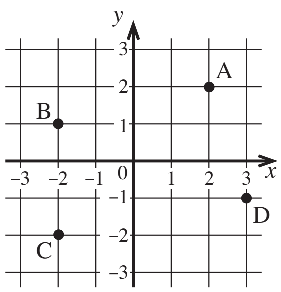
Coordinates

In the diagram opposite,

A is at the point with coordinates (2, 2),

B is at (−2, 1), C is at (−2, −2)

and D is at (3, −1).

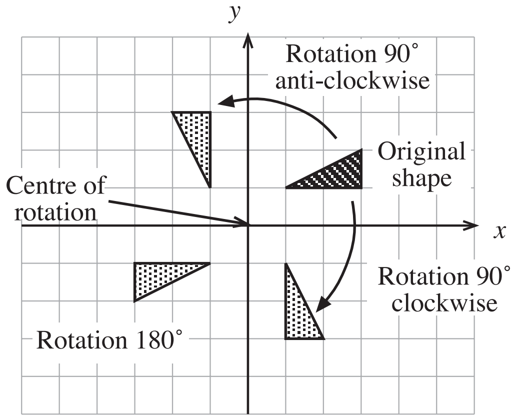


Transformations and Enlargements

Transformations are ways of moving a shape; for example, reflection, rotation and translation.

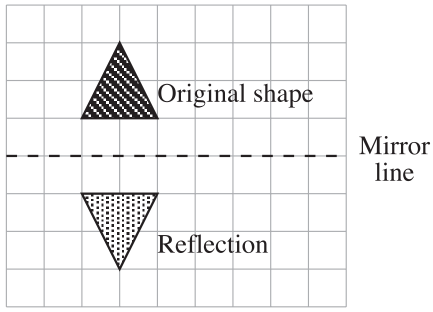
A *reflection* is obtained by drawing the image of a shape in a mirror line.

An example is shown opposite.



A *rotation* is obtained when a shape is rotated about a point, the *centre of rotation,* through a specified angle.

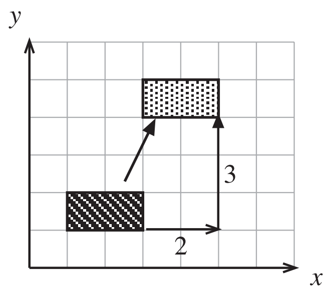
An example is shown opposite.



A *translation* moves a shape so that it is in a different position but retains the same size, area, angles and line lengths.

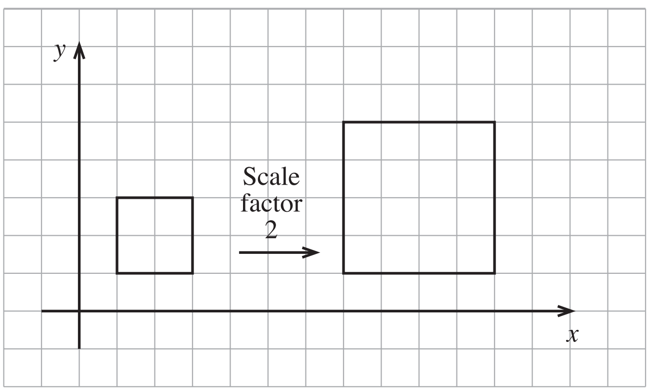
The diagram opposite shows the translation

$$\begin{Bmatrix} 2 \text{ in } x\text{-direction} \\ 3 \text{ in } y\text{-direction} \end{Bmatrix}$$



Enlargements are similar to transformations but they alter (enlarge or reduce) the size of the shape.

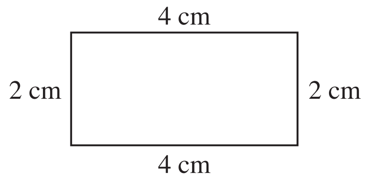
For example, the shape on the left below has been enlarged by a scale factor of 2 to give the image on the right.



Perimeter, Area and Volume

The *perimeter* is the total distance around the outside of a 2D shape.

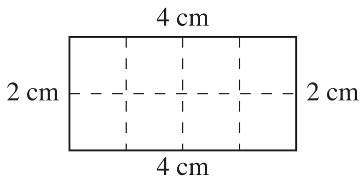
For example,



perimeter = 4 + 2 + 4 + 2 = 12 cm

The *area* is the quantity inside a 2D shape.

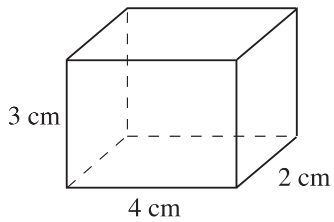
For example,



area = 8 square cm

The *volume* is the number of cubic units that will exactly fill a 3D shape.

For example,



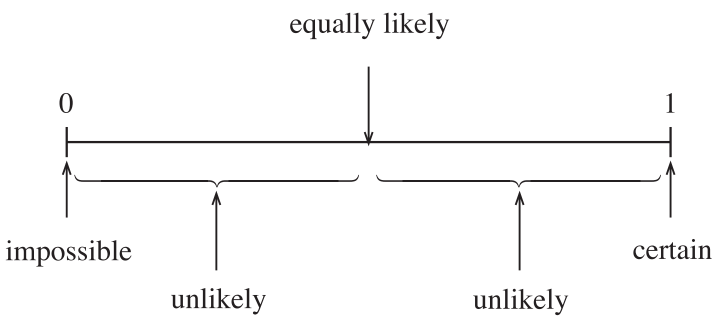
volume = 3 × 2 × 4 = 24 cubic cm

Probability

The probability of any outcome, *p*, must satisfy $0 \le p \le 1$.

The sum of the probabilities of all outcomes must equal 1.

Probabilities can be illustrated on a *probability line*, as shown below:



For finding probabilities by experiment:

$$\textit{probability of event} = \frac{\textit{frequency of event}}{\textit{total frequency}}$$

For equally likely outcomes:

$$\textit{probability of particular event} = \frac{\textit{number of ways of obtaining event}}{\textit{total no. of possible outcomes}}$$

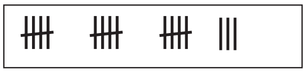
For example, when throwing a fair die,

$$p(6) = \frac{1}{6}, \quad p(\text{prime number}) = p(2, 3 \text{ or } 5) = \frac{3}{6} = \frac{1}{2}$$

Illustrating Data

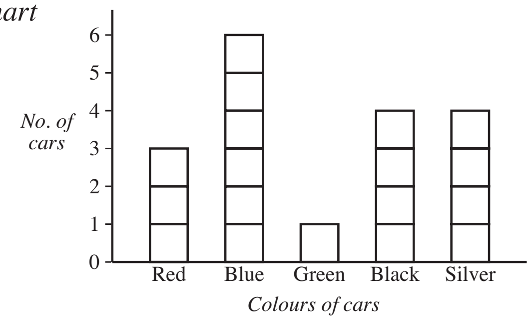
You can illustrate data with a:

Tally Chart



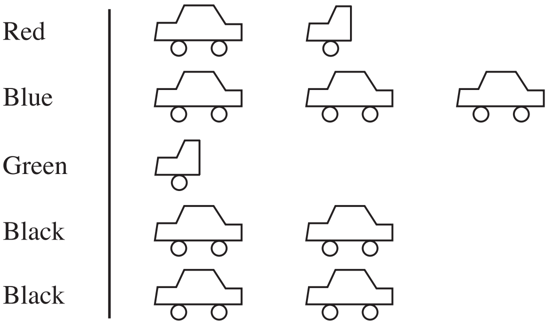
The tally chart represents 18 items of data

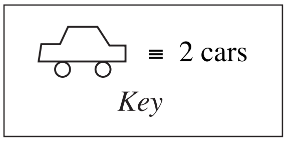
Bar Chart



The bar chart represents 18 items of data (3 Red, 6 Blue, 1 Green, 4 Black and 4 Silver cars)

Pictogram

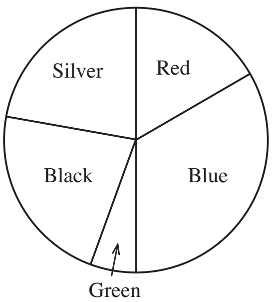




The pictogram represents the 18 cars above.

A pictogram must always have a key.

Pie Chart



The pie chart represents the 18 cars.

As there are 18 items of data, the angle representing each item is $\frac{360°}{18} = 20°$.

So the angle for Red $= 3 \times 20° = 60°$, Blue $= 120°$, etc.

Median of a set of numbers is the *middle value* when they are arranged in order.

For example,

$$2,\ 5,\ 3,\ 1,\ 4,\ 9,\ 8 \ \Rightarrow\ 1,\ 2,\ 3,\ 4,\ 5,\ 8,\ 9$$

Mean of a set of numbers is the *average value* calculated by adding all the numbers in the set and then dividing by the total number of numbers in the set.

For example,

$$2,\ 5,\ 3,\ 1,\ 4,\ 9,\ 8 \ \Rightarrow\ \text{mean} = \frac{2+5+3+1+4+5+8}{7}$$

$$= \frac{28}{7}$$

$$= 4$$

Mode of a set of numbers (or objects) is the number (or object) that has the highest frequency, that is, occurs most often.

For example, for the set of numbers

$$4,\ 7,\ 3,\ 2,\ 7,\ 1,\ 3,\ 5,\ 4,\ 7$$

we have

	Frequency		
1	I	1	
2	I	1	
3	II	2	
4	II	2	
5	I	1	
6		0	
7	III	3	← highest frequency

So the mode is 7 as it occurs most frequently.

Range of a set of numbers is the difference between the highest and the lowest values in the data set.

For example, for the data set $\{ 5,\ 7,\ 2,\ 5,\ 1,\ 2,\ 3,\ 5,\ 6 \}$,

the range is $7 - 1 = 6$.