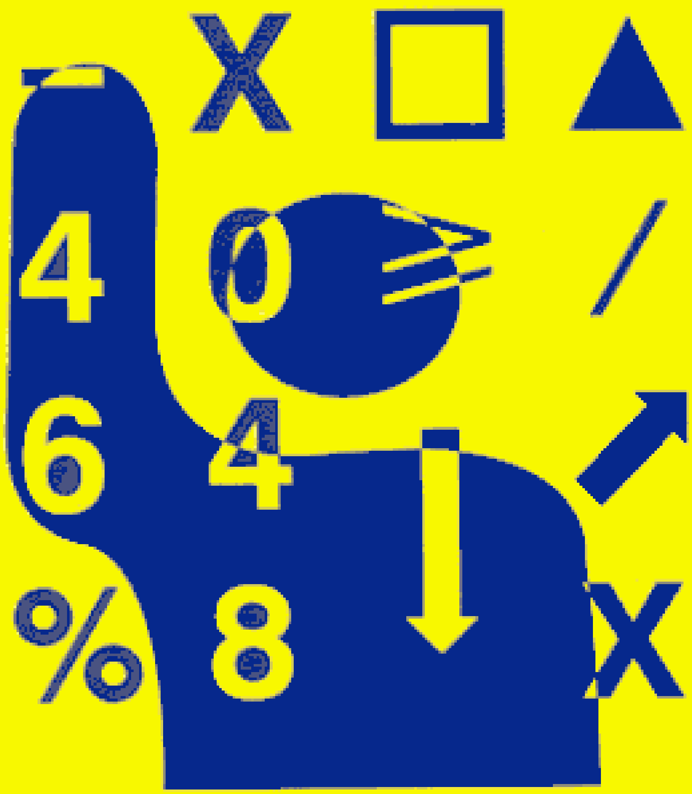
Mathematics Enhancement Programme

Primary Demonstration Project

10 A Probability

Help Booklet



Support for Primary Teachers in Mathematics

Primary Project
funded by
Pricewaterhouse
Coopers

in association with
British Steel
Garfield Weston Foundation

Sponsored by

ESSO

CIMT
School of Education
University of Exeter

Mathematics Enhancement Programme

Help Module 10

PROBABILITY

Part A

Contents of Part A

Contents of Part B

CIMT
University of Exeter

Sponsored by
ESSO

Gatsby Technical
Education Project

PREFACE

This is one of a series of *Help Modules* designed to help you gain confidence in mathematics. It has been developed particularly for primary teachers (or student teachers) but it might also be helpful for non-specialists who teach mathematics in the lower secondary years. It is based on material which is already being used in the *Mathematics Enhancement Programme: Secondary Demonstration Project*.

The complete module list comprises:

1.	ALGEBRA	6.	HANDLING DATA
2.	DECIMALS	7.	MENSURATION
3.	EQUATIONS	8.	NUMBERS IN CONTEXT
4.	FRACTIONS	9.	PERCENTAGES
5.	GEOMETRY	10.	PROBABILITY

Notes for overall guidance:

- Each of the 10 modules listed above is divided into 2 parts. This is simply to help in the downloading and handling of the material.
- Though referred to as 'modules' it may not be necessary to study (or print out) each one in its entirely. As with any self-study material you must be aware of your own needs and assess each section to see whether it is relevant to those needs.
- The difficulty of the material in **Part A** varies quite widely: if you have problems with a particular section do try the one following, and then the next, as the content is not necessarily arranged in order of difficulty. Learning is not a simple linear process, and later studies can often illuminate and make clear something which seemed impenetrable at an earlier attempt.
- In **Part B, Activities** are offered as backup, reinforcement and extension to the work covered in Part A. **Tests** are also provided, and you are strongly urged to take these (at the end of your studies) as a check on your understanding of the topic.
- The marking scheme for the revision test includes B, M and A marks. Note that:

  M *marks* are for method;

  A *marks* are for accuracy (awarded only following a correct M mark);

  B *marks* are independent, stand-alone marks.

We hope that you find this module helpful. Comments should be sent to:

Professor D. N. Burghes
CIMT, School of Education
University of Exeter
EXETER EX1 2LU

The full range of Help Modules can be found at

www.ex.ac.uk/cimt/help/menu.htm

10 *Probability* Introductory Notes

Historical Background

The origins of probability are not entirely clear, but we do know of discussions between Pascal and his friend, the Chevalier de Méré, in which, for example, they considered the problem,

"Are you more likely to obtain one six in 4 tosses of one fair die than to obtain at least one double six in 24 tosses of two fair dice?"

Some of these types of problems were published by Huygan in 1657 in his little tract *'On reasoning in Games of Dice'*. Other famous mathematicians had considered similar problems; for example, Galileo considered the problem

"Are you more likely to obtain a total of 9 when three fair dice are tossed than a total of 10?"

and Pepys asked Norton

"Which is more likely – one 6 when six dice are tossed or two 6s when 12 dice are tossed?"

These types of problems led to the Binomial distribution for probabilities, which includes the well known Pascal's triangle of coefficients, but this is beyond the scope of this module.

Key Issues

Introduction

Probability is a remarkably interesting area of mathematics, used by many people to solve real problems. You see it for example, in airline safety – figures such as "1 in 100 million" are given for the probability of an engine failure. Nearer to home, the probability of winning the jackpot on the National Lottery is about 1 in 14 million and many people in this country regularly (or occasionally) bet on large races such as the Grand National or invest money in Premium Bonds. The concepts of probability are used in these types of games to ensure that the organisers, on average, always win and you, the punter, normally lose!

Some experiments have been suggested in the activities, and we would encourage you to adopt a practical approach to teaching Probability when appropriate. What you think is obvious is not always so and experimentation could, and should be an important aspect of this topic.

Language / Notation

- There are some key words that are needed in this module; these include

  Outcomes – events that can occur after an experiment.

  Probability space – the complete set of outcomes for the experiment.

  Relative frequency – the frequency of an event divided by the total frequency, and is used as an estimate for the probabilities of that event.

  Independent event – when the result of one event happening does not affect the probability of the other.

  Mutually exclusive event – when two events cannot happen at the same time.

- You should also make sure that you are happy with the differences between terms such as

  impossible
  unlikely
  possibly
  likely
  certain

- The usual way of writing probabilities is either as a fraction or decimal; e.g. $\frac{1}{4}$ or 0.25. Other notations include, for example, '25%' or '1 in 4', but these are not encouraged for GCSE assessment.

- We talk about a 'fair' coin or a 'fair' dice meaning that all the outcomes are equally likely. For a 'fair' coin:

  $$p(\text{H}) = p(\text{T}) = \frac{1}{2} \text{ etc.}$$

  The alternative is that the coin or dice is 'biased'.

Key Points

- The probability of any event p, must satisfy $0 \le p \le 1$.

- The sum of the probabilities if all outcomes to an experiment must be 1.

- When using tree diagrams, you always multiply along the branches to determine probabilities of combined events.

- Probabilities can either be found by

  symmetry – when all outcomes are the equally likely,

  experiment – when probabilities can be estimated.

- For independent events, A and B,

  $$p(\text{A and B}) = p(\text{A}) \times p(\text{B})$$

- For mutually exclusive events, A and B,

$$p(\text{A or B}) = p(\text{A}) + p(\text{B})$$

Misconceptions

- The probability of an event must be ≤ 1. Any probability answer that is > 1 must be incorrect.

See Activity 10.1

- Adding rather than multiplying probabilities (often A level candidates have problems here!) – for example, the probability of getting 3 'sixes' in three throws of a dice is

$$\frac{1}{6} \times \frac{1}{6} \times \frac{1}{6} \text{ not } 3 \times \frac{1}{6} = \frac{1}{2}$$

- If you obtain 4 Heads in a row when tossing a fair coin, then the probability of Heads on the fifth throw is still $\frac{1}{2}$. (This is often feels in conflict with the result that over a period of many tosses of the coin, the number of Heads will approximately equate to the number of Tails).

Key Concepts

1. Experimental probability

   (Probability of event = $\dfrac{\text{frequency of event}}{\text{total frequency}}$)

2. Theoretical probability

   (If all outcomes are equally likely,

   Probability of particular outcome = $\dfrac{\text{no. of ways of obtaining outcome}}{\text{total no. of possible outcomes}}$)

3. Independent events

   (If two events, A and B, are independent,

   $$p(\text{A and B}) = p(\text{A}) \times p(\text{B}))$$

4. Mutually exclusive events

   (If two events, A and B, are mutually exclusive,

   $$p(\text{A or B}) = p(\text{A}) + p(\text{B}))$$

WORKED EXAMPLES and EXERCISES

10 Probability

10.1 Probabilities

Probabilities are used to describe how likely or unlikely it is that something will happen. Weather forecasters often talk about how likely it is to rain, or perhaps snow, in particular parts of the country.

Worked Example 1

(a) When you roll a dice, which number are you most likely to get?

(b) If you rolled a dice 600 times how my sixes would you expect to get?

(c) Would you expect to get the same number of ones?

Solution

(a) You are equally likely to get any of the six numbers.

(b) You would expect to get a six in about $\frac{1}{6}$ of the throws, so 100 sixes.

(c) Yes, in fact you would expect to get about 100 of each number.

Worked Example 2

Use one of the following to describe each one of the statements (a) to (d).

Certain

Very likely

Likely

Unlikely

Very unlikely

Impossible

(a) It will snow tomorrow.

(b) It will rain tomorrow.

(c) You win a car in a competition tomorrow.

(d) You are late for school tomorrow.

Solution

(a) *Very unlikely* for most places in Britain, especially in summer.

(b) *Likely* or *Very likely* in Britain in winter.

(c) *Very unlikely* if you have entered the competition. *Impossible* if you have not entered the competition.

(d) *Very unlikely*, unless the school bus breaks down.

10.1

Exercises

1. If you toss a coin 500 times, how many times would you expect to land:

   (a) on its side,

   (b) heads up,

   (c) tails up?

2. A tetrahedron is a shape with 4 faces. The faces are numbered 1, 2, 3 and 4. The tetrahedron is rolled 200 times. How many times would you expect it to land on a side numbered

   (a) 4 (b) 2 (c) 5?

3. Describe each of the following events as:

   Impossible,

   Unlikely,

   Likely,

   Certain.

   (a) You roll a normal dice and score 7.

   (b) You fall off your bike on the way home from school.

   (c) You complete all your maths homework correctly.

   (d) Your favourite football team wins their next match.

   (e) Your parents decide to double your pocket money next week.

   (f) You have chips with your next school dinner.

   (g) The school bus is on time tomorrow.

4. Describe two events that are:

   (a) Certain,

   (b) Impossible,

   (c) Likely to happen,

   (d) Unlikely to happen.

5. How many sixes would you expect to get if you rolled a dice:

   (a) 60 times,

   (b) 120 times,

   (c) 6000 times,

   (d) 3600 times?

10.1

6. Nisha tossed a coin a large number of times and got 450 heads. How many times do you think he tossed the coin?

7. Karen rolled a dice and got 250 twos.
   (a) How many times do you think she rolled the dice?
   (b) How many sixes do you think she got?

8. Stuart chooses a playing card from a full pack 100 times. How many times would you expect him to get:
   (a) a red card,
   (b) a black card,
   (c) a heart,
   (d) a diamond?

10.2 Simple Probability

Probabilities are given values between 0 and 1. A probability of 0 means that the event is *impossible*, while a probability of 1 means that it is *certain*. The closer the probability of an event is to 1, the more likely it is to happen. The closer the probability of an event is to 0, the less likely it is to happen.

Worked Example 1

When you toss a coin, what is the probability that it lands heads up?

Solution

When you toss a coin there are two possibilities, that it lands heads up or tails up. As one of these must be obtained,

$$p(\text{heads}) + p(\text{tails}) = 1$$

But both are equally likely so

$$p(\text{heads}) = p(\text{tails}) = \frac{1}{2}.$$

Worked Example 2

The probability that it rains tomorrow is $\frac{2}{3}$.

What is the probability that is does not rain tomorrow?

10.2

Solution

Tomorrow it must either rain or not rain, so,

$$p(\text{rain}) + p(\text{no rain}) = 1.$$

The probability it rains is $\frac{2}{3}$, so

$$\begin{aligned} \frac{2}{3} + p(\text{no rain}) &= 1 \\ p(\text{no rain}) &= 1 - \frac{2}{3} \\ &= \frac{1}{3}. \end{aligned}$$

So the probability that it does not rain is $\frac{1}{3}$.

Exercises

1. What is the probability that it does not rain tomorrow, if the probability that it does rain tomorrow is:

   (a) 0.9, (b) $\frac{3}{4}$ (c) $\frac{1}{2}$ (d) $\frac{1}{5}$?

2. Ben plays snooker with his friends. The probability that he beats Gareth is 0.8 and the probability that he beats Matthew is 0.6.

   (a) What is the probability that Gareth beats Ben?

   (b) What is the probability that Matthew beats Ben?

3. The probability that a train is late arriving at its destination is 0.02. What is the probability that it is not late?

4. Joshua has bought a trick coin in a joke shop. When he tosses it the probability of getting a head is $\frac{1}{5}$. What is the probability of getting a tail?

5. A weather forecaster states that the probability that it will snow tomorrow is $\frac{3}{7}$.

   (a) Find the probability that it does not snow tomorrow.

   (b) Is it more likely to snow or not to snow tomorrow?

6. The probability that it will snow during the winter in a certain city is 0.01. What is the probability that it does not snow?

7. A school basketball team play 20 matches each year. The probability that they win any match is $\frac{3}{5}$.

   (a) What is the probability that they lose a match?

   (b) How many matches can they expect to win each year?

10.2

8. When Claire plays battle chess on her home computer the probability that she wins depends on the level at which she plays the game.

Level	**Probability Claire wins**
Easy	0.9
Medium	0.4
Hard	0.1

What is the probability that the computer wins if the level is set to:

(a) Medium (b) Hard (c) Easy?

9. A child is selected at random from a school. The probability the child is a girl is $\frac{11}{20}$, the probability that the child is left handed is $\frac{1}{11}$ and the probability that the child wears glasses is $\frac{4}{13}$.

Find the probabilities that a child selected at random,

(a) is a boy (b) is right handed (c) does not wear glasses.

10. It has been estimated that the probability that a person has blue eyes is $\frac{4}{9}$.

Is it true that the probability that a person has brown eyes is $\frac{5}{9}$?

11. A machine makes compact discs. The probability that a perfect compact disc will be made by this machine is 0.85.

Work out the probability that a compact disc made by this machine will not be perfect. *(LON)*

12. Here are three possible events

A A coin when tossed will come down heads.

B It will snow in August in London

C There will be a baby born tomorrow.

Which of the three events is

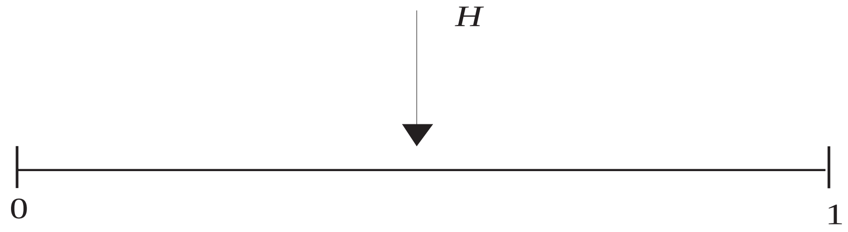
(a) most likely to happen?

(b) least likely to happen?

(LON)

10.2

13.



A probability line is shown above.

The arrow *H* on the line shows the probability that, when a coin is tossed, it will come down 'heads'.

(a)

Copy the probability line and put an arrow *S* on the line to show the probability that it will snow where you live tomorrow.

Explain why you put your arrow in that position.

(b) Put an arrow, *L* on the line to show the probability that the next lorry you see travelling on the road will have a male driver.

Explain why you put your arrow in that position.

(MEG)

10.3 Outcome of Two Events

When dealing with probabilities for two events, it is important to be able to identify all the possible outcomes. Here are examples to show the methods that can be used.

Method A : Systematic Listing

Worked Example 1

For a special meal customers at a pizza parlour can choose a pizza with *one* of the following toppings.

Ham
Mushroom
Salami
Peperoni
Tuna

and a drink from the following list

Cola
Diet Cola
Orange

How many possible combinations of toppings and drinks are there?

10.3

Solution

Using the first letter of each drink and topping, it is easy to see that Cola (c)) could be combined with any of the five toppings to give CH, CM, CS, CP, CT. Here 'CH' means 'Cola' drink and 'Ham' topping, etc.

Similarly, for Diet Cola (D), you have

DH, DM, DS, DP, DT

and for Orange (O)

OH, OM, OS, OP, OT

You can see that there are $3 \times 5 = 15$ possible outcomes.

This method of listing will always work but it might be slow, particularly if there are more than 2 choices to be made.

Method B : 2-way Tables

Worked Example 2

A six-sided die and a coin are tossed. List all the possible outcomes.

Solution

The coin can land leads (denoted by H) or tails (T), whilst the die can show 1, 2, 3, 4, 5 or 6. So for heads on the coin, the possible outcomes are

H1, H2, H3, H4, H5 and H6

whilst for tails, they are

T1, T2, T3, T4, T5 and T6.

The listing method used here can be conveniently summarised in a 2-way table.

		Die					
		1	2	3	4	5	6
Coin	H	H1	H2	H3	H4	H5	H6
	T	T1	T2	T3	T4	T5	T6

This method works well but cannot be used if there are more than 2 choices to be made.

10.3

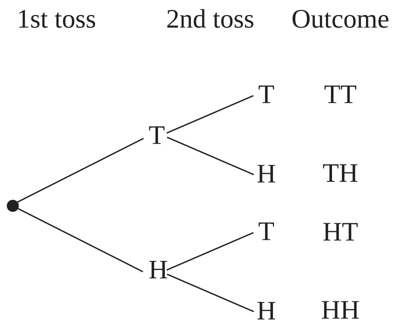
Method C : Tree Diagrams

Worked Example 3

A coin is tossed twice. List all the possible outcomes.

Solution

You can use a tree diagram to represent this solution.



Note that 'TH' is not the same as 'HT'.

This is an excellent method, but can lead to problems if you have too many branches.

Exercises

1. Two dice are rolled together. Complete the table below to show all the outcomes as total scores.

Second die

First die

	1	2	3	4	5	6
1						
2						
3						
4						
5						
6						

2. Three flavours of ice cream, vanilla (V), mint (M) and raspberry ripple (R), are available at a shop. Each is served with a topping of either chocolate (C) or strawberry (S).

   One possible order is for vanilla ice cream with chocolate topping (VC).

   Write a list of all the other possibilities.

10.3

3. A bag contains two balls which are the same size. One is green and one is red. You take a ball out of the bag, put it back, then take another.

   Make a list of all the possible outcomes for the colours of the two balls.

4. Three boys, Ben, John and Nigel, decide to hold a competition in the gym. They will do sit-ups and then press-ups.

   If Ben wins the sit-ups and John wins the press-ups, the outcome would be represented as BJ.

   (a) What does NB represent?

   (b) Make a list of all the 9 possible outcomes.

   (c) If only Ben and John take part in the competition there will be fewer possible outcomes. List the outcomes in this case.

   (d) If Timothy also takes part in the competition, list all the possible outcomes for the four competitors.

5. Packets of cornflakes contain a free model dinosaur. There are four different models, the Brontosaurus (B), the Stagosaurus (S), Tyrannosaurus-Rex (T) and Diplodocus (D). A mother buys two packets of cornflakes for her children. List all combinations of free gifts possible when the packets are opened.

6. At a school Christmas Fair three different sorts of prizes can be won in a lucky dip. One is a cassette tape (C), one is a diary (D) and the other a book (B).

   List all the possible outcomes for a girl who has two goes at the lucky dip.

7. For breakfast, Rachel will drink either fruit juice (F) or cold milk (M) and will eat cornflakes (C), honey-crunch loops (H) or toast (T).

   Complete a copy of the table below to show the possible outcomes for her choice of breakfast.

		Drinks	
		F	M
Food	H		
	T		
	C		

8. List the possible outcomes when 3 coins are tossed.

10.3

9. (a) A bag contains 2 red marbles, 1 blue marble and 1 yellow marble.

A second bag contains 1 red marble, 2 blue marbles and 1 yellow marble.
A marble is drawn from each bag.

Complete the table showing all the possible pairs of colours.

		Marble from second bag			
		R	B	B	Y
Marble from first bag	R	RR	RB	RB	RY
	R	RR			
	B	BR			
	Y	YR			

(b) 2 marbles are drawn from a third bag.

The probability that they are both of the same colour is $\frac{5}{9}$.

What is the probability that they are of different colours?

(MEG)

10.4 Finding Probabilities Using Relative Frequency

Sometimes it is possible to calculate values for the probability of an event by symmetry arguments, like tossing a coin and getting a head. For other events probabilities can be estimated by using results of experiments or records of past events.

Worked Example 1

In February 1995 it rained on 18 days. Use this information to estimate the probability that it rains on a day in February.

Solution

It rained on 18 out of the 28 days, so the relative frequency of rain is

$$\frac{18}{28} = \frac{9}{14}.$$

So the probability that it rains can be estimated as $\frac{9}{14}$.

10.4

Worked Example 2

Hitesh carries out an experiment with a piece of buttered toast. He drops it 50 times and finds that 35 times it lands butter side down. Use these results to estimate the probability that a piece of toast lands butter side down when dropped.

Solution

The toast landed butter side down 35 of the 50 times, so the relative frequency is $\frac{35}{50} = \frac{7}{10}$.

So the probability that the toast lands butter side down can be estimated as $\frac{7}{10}$.

Exercises

1. (a) Conduct an experiment with a drawing pin, by dropping it in the same way a large number of times. You could drop it 100 times and record whether it lands point up or point down.

   (b) Use your results to estimate the probability that a drawing pin lands point up.

2. (a) Obtain a short stick, such as a cocktail stick. On a sheet of A4 paper draw parallel lines that are 6 cm apart. Drop the stick onto the sheet of paper a large number of times and record whether or not it lands on a line.

   (b) Use your results to estimate the probability that the stick lands on a line.

3. When you toss a coin you would expect to get a head half of the time.

   (a) Toss a coin 20 times and record the results. How well do they compare with your expectation?

   (b) Toss the coin another 30 times, so that you have 50 results. How well do they compare with your expectation now?

4. Andrew observed that the school bus was late on 6 of the 24 school days in March. Estimate the probability that the bus was late on any one day.

5. A football team plays on average 40 matches each season and wins 32 of them.

   (a) Estimate the probability that this team wins a match.

   (b) Give a reason why this probability could change.

10.4

6. Six children play regularly in a chess club. The number of games that each child has won is recorded in the table below.

Player	**Games Won**	**Games Lost**
Timothy	4	10
Andrew	7	3
Daniella	3	9
Rachel	4	16
Charles	6	12
Maria	12	6

(a) Use this data to find the probability that each child wins a match.

(b) Which child is the best player?

(c) Which child is the worst player?

(d) If Charles played Timothy, who do you think would be most likely to win?

7. A garage records the number of cars that they sell each week over a 24 week period. The numbers for each week are given below.

3, 4, 8, 6, 5, 7, 4, 3, 6, 5, 2, 4,

5, 7, 6, 9, 2, 4, 5, 6, 7, 4, 3, 5.

Use this data to establish the probability that in any week;

(a) more than 5 cars are sold,

(b) fewer than 5 cars are sold,

(c) exactly 5 cars are sold.

8. A gardener plants 40 seeds and 32 of them produce healthy plants.

(a) Estimate the probability that a seed produces a healthy plant.

(b) If 120 seeds were planted, how many healthy plants can the gardener expect to obtain?

Investigation

A girl types 3 different letters and 3 different addresses on 3 envelopes. She puts the letters into the envelopes randomly and sends then to 3 of her friends, A, B and C. What is the probability that

(a) only one of her friends will receive the correct letter,

(b) only two of them will receive the correct letters?

10.5 Determining Probabilities

When the outcomes of an event are all equally likely, then probabilities can be found by considering all the possible outcomes.

For example, when you toss a coin there are two possible outcomes, either heads or tails.

So
$$p(\text{head}) = \frac{1}{2}$$
$$p(\text{tail}) = \frac{1}{2}$$

The probability of an outcome is given by

$$\frac{\text{number of ways of obtaining outcome}}{\text{number of possible outcomes}}$$

provided all the outcomes are equally likely.

Worked Example 1

A card is taken at random from a full pack of playing cards with no jokers. What is the probability that the card:

(a) is an ace, (b) is black,

(c) is a heart, (d) has an even number on it?

Solution

First note that each card is equally likely to be selected, and that there are 52 possible outcomes.

(a) There are 4 aces, so

$$p(\text{ace}) = \frac{4}{52} = \frac{1}{13}.$$

(b) There are 26 black cards, so

$$p(\text{black}) = \frac{26}{52} = \frac{1}{2}.$$

(c) There are 13 hearts in the pack, so;

$$p(\text{heart}) = \frac{13}{52} = \frac{1}{4}.$$

10.5

(d) There are 20 cards with even numbers on them, so;

$$p(\text{even number}) = \frac{20}{52} = \frac{5}{13}.$$

Worked Example 2

In a class of 30 children, 16 are girls, 4 wear glasses and 3 are left handed.

A child is chosen at random from the class. What is the probability that this child is:

(a) a girl, (b) right-handed, (c) wearing glasses.

Solution

All the children in the class are equally likely to be selected, when the choice is made at random.

(a) In the class there are 16 girls, so

$$p(\text{girl}) = \frac{16}{30} = \frac{8}{15}.$$

(b) There are 3 left handed children and so the other 27 must be right handed.

So,

$$p(\text{right handed}) = \frac{27}{30} = \frac{9}{10}.$$

(c) There are 4 children wearing glasses so,

$$p(\text{wears glasses}) = \frac{4}{30} = \frac{2}{15}.$$

Exercises

1. Richard takes a card at random from a full pack of playing cards. What is the probability that his card:

   (a) is a diamond, (b) is a spade, (c) is a seven,

   (d) is a king, (e) has a prime number on it?

2. Repeat question 1, for a pack of playing cards containing 2 jokers (a total of 54 cards).

3. When you roll an ordinary die, what is the probability of obtaining:

   (a) a six, (b) a five,

   (c) an even number, (d) a prime number?

10.5

4. A new game includes an octagonal roller with faces numbered from 1 to 8. When the roller is rolled, what is the probability of obtaining:

   (a) a number 8,
   (b) a number 1,
   (c) an odd number,
   (d) a number greater than 3,
   (e) a number less than 3?

5. In a class of 32 children, 20 have school lunches and the rest bring sandwiches. What is the probability that a child chosen at random from the class brings sandwiches?

6. In a lucky dip at a school fair, a tub contains 50 prizes at the start of the fair. There are 20 superballs, 10 pens, 10 toy cars and 10 packets of sweets.

   What is the probability that the first person to visit the lucky dip:

   (a) wins a superball,
   (b) does not win a pen,
   (c) wins a packet of sweets,
   (d) does not win a toy car.

   If the first person wins a pen, what is the probability that the second person:

   (e) wins a pen,
   (f) does not win a toy car,
   (g) wins a packet of sweets?

7. A coach sets off from Plymouth with 18 passengers. It stops at Exeter, where another 12 passengers join the coach. At Taunton it stops again and 20 more passengers get on board. When the coach arrives at its destination all the passengers get off and one is chosen at random to be interviewed about the journey.

   Find the probabilities that this passenger:

   (a) was on the coach for the whole journey,
   (b) got on the coach at Exeter,
   (c) got on the coach at Exeter or Plymouth,
   (d) got on the coach at Exeter or Taunton.

8. Liam has the following coins in his pocket:

   £1, 50p, 20p, 10p, 2p.

   He selects *one* coin at random to put in a charity collection box.

   What is the probability that he:

   (a) gives more than 20p,
   (b) has less than £1 left in his pocket,
   (c) has more than 70p left in his pocket,
   (d) gives away less than half the money in his pocket?

10.5

9. Five different types of model dinosaurs are being given away in cornflakes packets. A model dinosaur is put into each packet at random and five dinosaurs are needed for a complete set.

   (a) Ben already has 3 of the 5 models. What is the probability he gets a different one in the next packet he opens?

   (b) Adam only needs one more dinosaur to complete his set. What is the probability that he gets this dinosaur in the next packet he opens?

   (c) Ian has only one dinosaur in his collection. What is the probability that he gets the same one in his next packet?

10. A bag contains 5 red counters, 3 green counters and 2 blue counters. Counters are taken out of the bag at random, but are not put back into the bag.

    (a) What is the probability that the first counter taken out is green?

    (b) If the first counter is green, what is the probability that the second counter is green?

    (c) If the first two counters are green, what is the probability that the third counter is green?

    (d) If a red counter is followed by a blue counter, what is the probability that the third counter is green?

11. Graham has a bag of 30 marbles. There are 7 red marbles in the bag. He chooses a marble at random from the bag.

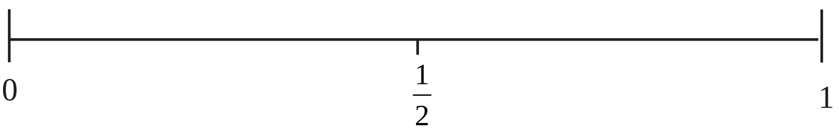
    What is the probability that

    (a) he gets a red marble? (b) he gets a marble which is not red?

(NEAB)

12. (a) Make a copy of the line below and mark with an *H* the probability of getting a head when one coin is tossed.

    (b) On the same line, mark with an *S* the probability of getting a 5 when a six - sided dice is thrown.



(LON)

13. In a raffle 200 tickets are sold.

    (a) Helen buys one ticket. What is the probability that she wins first prize?

    There are lots of prizes.

    (b) The probability that Helen wins a prize is $\frac{1}{10}$. How many prizes are there?

(SEG)

10.5

14. Shara is shown a selection of ski-holiday brochures. There are three for Italy, two for Austria and five for Switzerland.

    Shara takes one of these brochures at random.

    (a) What is the probability that it is for Italy?

    (b) What is the probability that it is **not** for Austria?

(SEG)

15. To play a game you spin the pointer.
    You win the prize on which the pointer stops.

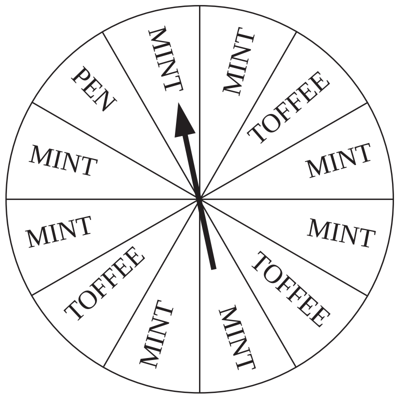
    Richard has one spin.

    (a) Which prize is Richard most likely to win?

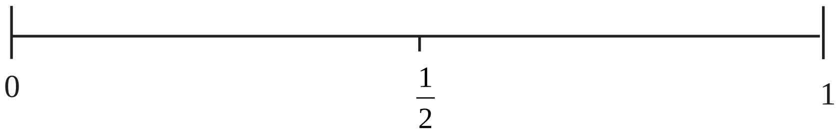
    (b) Explain your answer to part (a).

    Donna has one spin.

    (c) Make a copy of the line shown below and mark with a P the probability that Donna will win a pen.



    (d) On the line mark with a W the probability that Donna will win a watch.



(LON)

Just For Fun

Arron, Paul and Mary repeatedly take turns tossing a die. Arron begins; Paul always follows Arron; Mary always follows Paul; and Arron always follows Mary. What is the probability that Mary will be the first one to toss a six?

Investigations

1. *How many pupils must you gather together so as to ensure that at least two pupils have birthdays falling in the same month?*

2. *A secondary school has an enrolment of 1100. Is it possible that there will be four pupils in the school whose birthdays fall on the same day of the year? Explain your answers.*

10.6 Probability of Two Events

When two events take place, and every outcome is equally likely to happen, the probability of a particular combined outcome can be readily found from the formula

$$\text{probability} = \frac{\text{number of successful outcomes}}{\text{total number of outcomes}}$$

The next examples show how this formula is used.

Worked Example 1

Two dice are thrown together. Find the probability that the total score is 9.

Solution

The table shows all the possible outcomes and total scores.

Second die

First die	**1**	**2**	**3**	**4**	**5**	**6**
1	2	3	4	5	6	7
2	3	4	5	6	7	8
3	4	5	6	7	8	9
4	5	6	7	8	9	10
5	6	7	8	9	10	11
6	7	8	9	10	11	12

There are 36 possible outcomes, and each one is equally likely to occur.

The outcomes that give a total of 9 have been circled. There are 4 such outcomes.

Now the probability can be found.

$$P(9) = \frac{4}{36} = \frac{1}{9}$$

10.6

Worked Example 2

A spinner which forms part of a children's game can point to one of four regions, A, B, C or D, when spun. What is the probability that when two children spin the spinner, it points to the same letter?

Solution

The table shows all the possible outcomes.

Second child

First child	**A**	**B**	**C**	**D**
A	AA	AB	AC	AD
B	BA	BB	BC	BD
C	CA	CB	CC	CD
D	DA	DB	DC	DD

There are 16 possible outcomes. Each is equally likely to occur. The outcomes that are the same for both children have been circled. There are four outcomes of this type.

The probability that both have the same letter is

$$\frac{4}{16} = \frac{1}{4}$$

Note

It is expected that fractions are used for expressing probabilities, but using decimals is equally acceptable.

Exercises

1. When two coins are tossed together the possible outcomes are as shown in the table.

Second coin

First coin	**H**	**T**
H	HH	HT
T	TH	TT

(a) What is the probability that both coins show heads?

(b) What is the probability that only one coin shows a tail?

(c) What is the probability that both coins land the same way up?

10.6

2. A coin is tossed and a die is rolled. Copy and complete the table below to show the possible outcomes.

		Die					
		1	**2**	**3**	**4**	**5**	**6**
Coin	**H**	H1					
	T						

What is the probability of obtaining

(a) a head and a 6,

(b) a tail and an odd number,

(c) a head and an even number,

(d) a head and a number greater than 2,

(e) an even number?

3. (a) Use this table, which shows the outcomes when two dice are rolled, to find the probabilities of each event described below.

		Second die					
		1	**2**	**3**	**4**	**5**	**6**
First die	**1**	2	3	4	5	6	7
	2	3	4	5	6	7	8
	3	4	5	6	7	8	9
	4	5	6	7	8	9	10
	5	6	7	8	9	10	11
	6	7	8	9	10	11	12

(i) A score of 7.

(ii) A score of 5.

(iii) A score that is an even number.

(iv) A score of more than 8.

(v) A score of less than 6.

(b) What score are you most likely to get when you roll *two* dice?

10.6

4. A school cook decides at random what flavour squash to serve at lunch time. She chooses from blackcurrant, orange and lemon.

(a) Complete a copy of the table to show the possible outcomes from two consecutive days.

(b) What is the probability that she serves:

(i) blackcurrant on both days,

(ii) the same flavour on both days,

(iii) lemon or blackcurrant on both days?

Second day

		B	**L**	**O**
First day	**B**	BB		
	L			LO
	O			

(c) Clare is allergic to lemon squash. What is the probability that she is unable to drink the squash on two consecutive days?

5. A young couple decide that they will have two children. There is an equal chance that each child will be a boy or a girl.

(a) Find the probability that both children are boys.

(b) Find the probability that both children are of the same sex.

6. A game contains two tetrahedral dice which have 4 faces numbered 1 to 4.
The two dice are thrown, and the total score is noted.

(a) Find the probability that a score of 3 is obtained.

(b) Find the probability of getting a score of more than 4.

(c) Which score is most likely?

7. A bag contains one red ball, one blue ball and one green ball. One ball is taken out of the bag. A second ball is also taken out, without replacing the first ball. The table shows the possible outcomes.

(a) Explain why some entries in the table have been marked with an X. How many possible outcomes are there?

(b) What is the probability that the red ball is selected?

(c) What is the probability that the green ball is left in the bag?

Second ball

		R	**B**	**G**
First ball	**R**	X	RB	RG
	B	BR	X	BG
	G	GR	GB	X

8. Two darts are thrown at a dartboard so that they are equally likely to hit any number between 1 and 20. Ignore doubles, trebles and the bull's-eye.

(a) How many possible outcomes are there?

(b) What is the probability of a score of 2?

(c) What is the probability that both darts hit the same number?

(d) What is the probability of a score of 17?

10.6

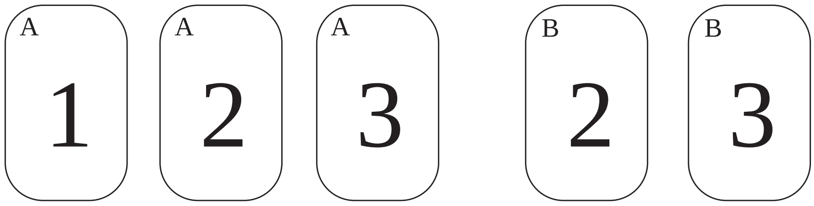
9. Three coins are tossed at the same time. Find the probabilities that

   (a) they all land the same way up,

   (b) they all land with heads showing,

   (c) at least one coin lands showing tails.

10. The diagram shows two sets of cards A and B.



   (a) One card is chosen at random from set A. One card is chosen at random from set B.

   (i) List all the possible outcomes.

   The two numbers are added together.

   (ii) What is the probability of getting a total of 5?

   (iii) What is the probability of getting a total that is **not** 5?

   A new card is added to the set B. It is

   One card is chosen at random from set A. One card is chosen at random from the new set B.

   (b) (i) How many possible outcomes are there now?

   (ii) Explain why adding the new card *does not* change the number of outcomes that have a total of 5.

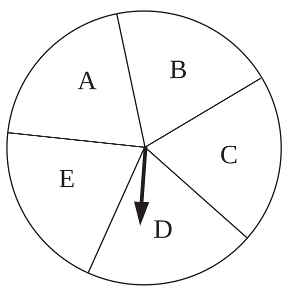
   (iii) Explain why adding the new card *does* change the probability of getting a total of 5.

(SEG)

11. This diagram shows an unbiased spinner used in a game.

   It is divided into five equal sections.

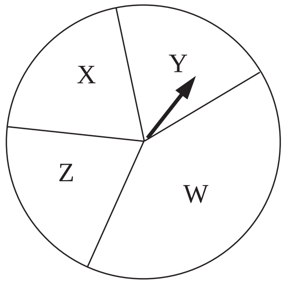
   The arrow is spun once.



   (a) What is the probability that the arrow will land on section *A*?

10.6

Another unbiased spinner is shown on the right.



Section W is twice the size of each of the other sections.

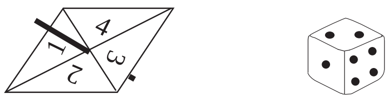
(b) What is the probability that the arrow on this spinner will land on section W?

(c) When both arrows are spun at once, one outcome could be AW.

List the other possible outcomes.

(NEAB)

12. A four sided spinner is spun and a dice is rolled.



The two scores are then multiplied to give a result.

(a) Complete the table to show all the possible results.

Dice

		1	2	3	4	5	6
Spinner	1						
	2						
	3						
	4						

(b) The spinner is spun once and the dice is rolled once.

What is the probability of getting a result of 12?

(SEG)

13. (a) A bag contains 2 red marbles, 1 blue marble and 1 yellow marble. A second bag contains 1 red marble, 2 blue marbles and 1 yellow marble.

A marble is drawn from each bag.

Complete the table showing all the possible pairs of colours.

Marble from second bag

		R	B	B	Y
Marble from first bag	R	RR	RB	RB	RY
	R	RR			
	B	BR			
	Y	YR			

(b) 2 marbles are drawn from a third bag. The probability that they are both of the same colour is $\frac{5}{9}$.

What is the probability that they are of different colours?

(MEG)

10.7 Use of Tree Diagrams

Tree diagrams can be used to find the probabilities for two events, when the outcomes are not necessarily equally likely.

Worked Example 1

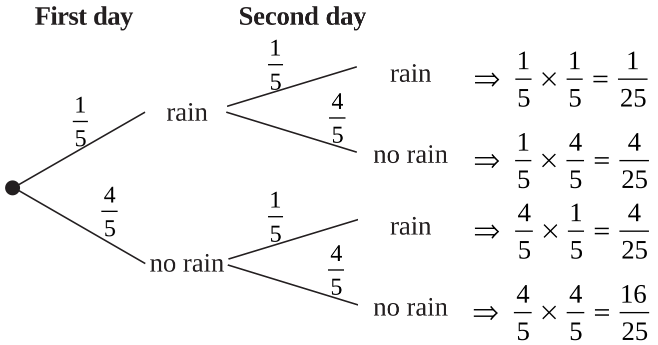
If the probability that it rains on any day is $\frac{1}{5}$, draw a tree diagram and find the probability

(a) that it rains on two consecutive days,

(b) that it rains on only one of two consecutive days.

Solution

The tree diagram shows all the possible outcomes. Then the probability of each event can be placed on the appropriate branch of the tree. The probability of no rain is $1-\frac{1}{5}=\frac{4}{5}$.



The probability of each outcome is obtained by multiplying together the probabilities on the branches leading to that outcome. For rain on the first day, but not on the second, the probability is

$$\frac{1}{5}\times\frac{4}{5}=\frac{4}{25}$$

(a) The probability that it rains on two consecutive days is given by the top set of branches, and is $\frac{1}{25}$.

(b) There are two outcomes where there is rain on only one of the two days. These are rain – no-rain, with a probability of $\frac{4}{25}$ and no-rain – rain with a probability of $\frac{4}{25}$.

10.7

The probability of rain on only one day is found by adding these two probabilities together:

$$\frac{4}{25}+\frac{4}{25}=\frac{8}{25}$$

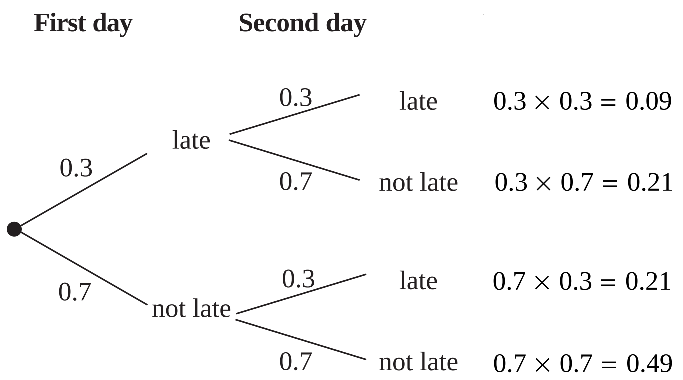
Worked Example 2

The probability that Jenny is late for school is 0.3. Find the probability that on two consecutive days she is:

(a) never late, (b) late only once.

Solution

The tree diagram shows the possible outcomes and their probabilities. Note that the probability of not being late is $1-0.3=0.7$.



The probabilities on each set of branches are multiplied together to give the probability of that outcome.

(a) The probability that Jenny is never late is given by the bottom set of branches and has probability 0.49.

(b) The probability that she is late once is given by the two middle sets of branches which both have a probability 0.21. So the probability that she is late once is given by

$$0.21+0.21=0.42$$

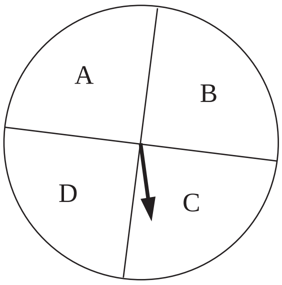
Note

The method shown here also works for problems when the outcomes are equally likely (as in the previous method) – it is sometimes rather cumbersome though to draw all the branches.

The next example is the same as Example 2 in the previous section, but this time the tree diagram method will be used.

10.7

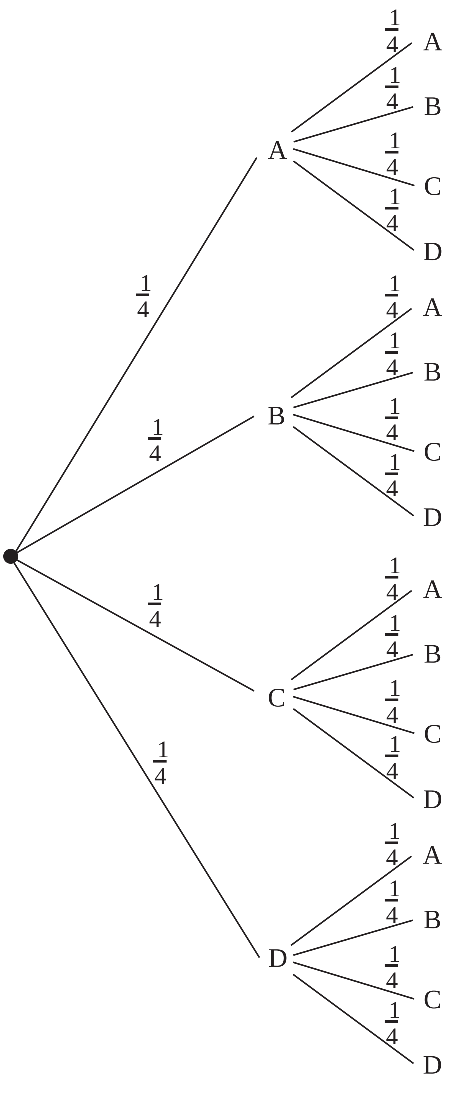
Worked Example 3



A spinner that forms part of a children's game can point to one of four regions, A, B, C or D, when spun. What is the probability that when two children spin the spinner, it points to the same letter?

Solution

This time, let us use the tree diagram approach.



$$\Rightarrow p(\text{AA}) = \frac{1}{4} \times \frac{1}{4} = \frac{1}{16}$$

$$\Rightarrow p(\text{BB}) = \frac{1}{4} \times \frac{1}{4} = \frac{1}{16}$$

$$\Rightarrow p(\text{CC}) = \frac{1}{4} \times \frac{1}{4} = \frac{1}{16}$$

$$\Rightarrow p(\text{DD}) = \frac{1}{4} \times \frac{1}{4} = \frac{1}{16}$$

So the probability of both children obtaining the same letter is

$$\frac{1}{16} + \frac{1}{16} + \frac{1}{16} + \frac{1}{16} = \frac{1}{4} \quad \text{(as obtained before)}$$

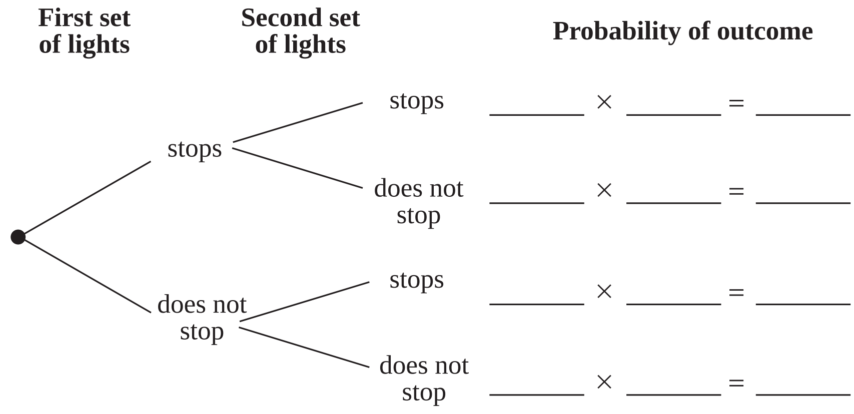
10.7

Exercises

1. On a route to school a bus must pass through two sets of traffic lights. The probability that the bus has to stop at a set of lights is 0.6.

   (a) What is the probability that the bus does not have to stop at a set of traffic lights?

   (b) Copy the tree diagram below and add the correct probabilities to each branch.



   (c) What is the probability that the bus gets to school without having to stop at a traffic light?

   (d) What is the probability that the bus stops at both sets of traffic lights?

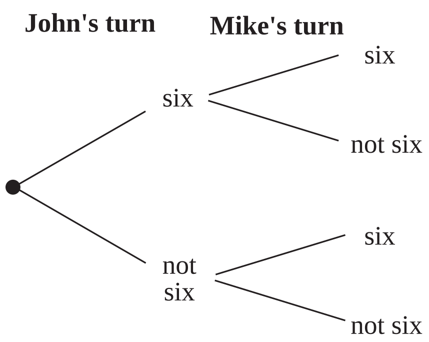
   (e) What is the probability that the bus stops at one set of traffic lights?

2. Two children are playing a game. They take it in turns to start. Before they start they must throw a six. John starts first.

   (a) What is the probability of throwing a six?

   (b) Copy the tree diagram and add the appropriate probabilities to each branch.

   Also calculate the probability of each outcome shown on the tree diagram.



   (c) Find the probability that:

   (i) both children start the game on their first throws,

   (ii) only one of them starts the game on their first throw,

   (iii) neither of them starts the game on their first throw.

10.7

3. Draw a tree diagram to show the possible outcomes when two coins are tossed. Include the probabilities on your tree diagram.

   Find the probability of obtaining:

   (a) two heads, (b) no heads, (c) only one head.

4. Paul travels to London on the early train. The probability that he arrives late is $\frac{1}{10}$.

   He catches the train on two consecutive days.

   What is the probability that he arrives:

   (a) on time on both days,

   (b) on time on at least one day,

   (c) late on both days.

5. When Jackie's phone rings the probability that the call is for her is $\frac{3}{4}$.

   (a) What is the probability that a call is not for Jackie?

   (b) Draw a tree diagram that includes probabilities to show the possible outcomes when the phone rings twice.

   (c) Find the probabilities that:

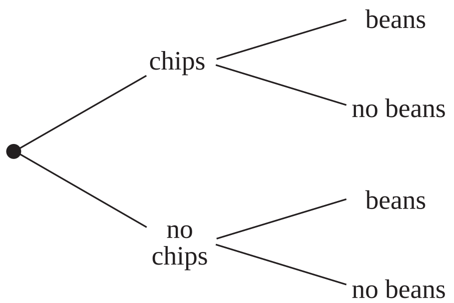
      (i) both calls are for Jackie,

      (ii) only one call is for Jackie,

      (iii) neither call is for Jackie.

6. In a school canteen the probability that a child has chips with their meal is 0.9 and the probability that they have baked beans is 0.6.

   (a) Copy and complete the tree diagram below.



   (b) What is the probability that a child has:

      (i) both chips and beans,

      (ii) chips but not beans,

      (iii) neither chips nor beans?

10.7

7. To be able to drive a car unsupervised you must pass both a theory test and a practical driving test. The probability of passing the theory test is 0.8 and the probability of passing the practical test is 0.6.

   (a) What is the probability of failing:

      (i) the theory test,

      (ii) the practical test?

   (b) What is the probability that someone:

      (i) passes both tests,

      (ii) fails both tests?

8. Matthew and Adam play squash together. The probability that Adam wins is 0.52.

   (a) Find the probabilities that, out of two games,

      (i) Adam wins two,

      (ii) Matthew wins two,

      (iii) they win one each.

   (b) Which of the outcomes is the most likely?

9. Veronica calls for her friends, Kathryn and Fionna. The probability that Kathryn is not ready to leave is 0.2 and the probability that Fionna is not ready is 0.3

   Use suitable tree diagrams to find the probability that:

   (a) both Fionna and Kathryn are ready to leave,

   (b) one of them is not ready to leave,

   (c) Kathryn is not ready to leave on two successive days,

   (d) Kathryn is ready to leave on two consecutive days.

10. A die has 6 faces of which 3 are green, 2 yellow and 1 red.

    Find the probabilities of the following outcomes if the die is rolled twice.

    (a) Both faces have the same colour.

    (b) Both faces are red.

    (c) Neither face is green.

11. (a) Draw a tree diagram to show the possible outcomes when a coin is tossed three times.

    (b) Find the probability of obtaining:

      (i) 3 heads and 3 tails,

      (ii) at least 2 heads,

      (iii) exactly one tail.

10.7

12. On average, Alice comes to tea on 2 days out every 5. If Alice comes to tea, the probability that we have jam tarts is 0.7.

    If Alice does not come to tea, the probability that we have jam tarts is 0.4.

    (a) Draw a tree diagram to illustrate this information.

    Write the appropriate probability on each branch.

    (b) What is the probability that we will have jam tarts for tea tomorrow?

(MEG)

13. John has 8 red socks and 6 white socks all mixed up in his sock drawer.
    He takes 2 socks at random from the drawer.

    (a) If the first sock that John takes is red, what is the probability that the second sock will also be red?

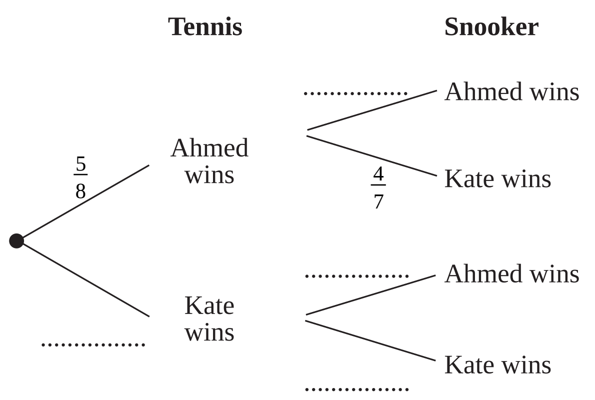
    (b) What is the probability that John will take 2 socks of the same colour?

(LON)

14. Ahmed and Kate play a game of tennis. The probability that Ahmed will win is $\frac{5}{8}$.

    Ahmed and Kate play a game of snooker. The probability that Kate will win is $\frac{4}{7}$.

    (a) Copy and complete the probability tree diagram below.



    (b) Calculate the probability that Kate will win both games.

(LON)

15. A dice is biased as follows:

    The probability of scoring a 6 is 0.4. The probability of scoring a 5 is 0.2.

    (a) Julia throws the dice once. Calculate the probability that the score will be 5 or 6.

    (b) Jeff throws the dice twice. Calculate the probability that both scores will be 6's.

(SEG)

10.7

16. A fair dice is thrown three times.

(a) What is the probability of throwing 3 sixes?

(b) What is the probability of throwing a six on the first throw, a six on the second throw but not a six on the third throw?

(c) What is the probability of throwing exactly two sixes in the three throws?

(d) What is the probability of throwing at least two sixes in the three throws?

(SEG)

10.8 Multiplication for Independent Events

Two events are *independent* if one event happening does not affect the probability of the other event. In this case the probability of two events A and B occurring is given by

$$p(\text{A and B}) = p(\text{A}) \times p(\text{B}).$$

Worked Example 1

A die is rolled twice. If event A is the first roll shows a six and event B is the second roll shows a six,

(a) are events A and B independent?

(b) Find p(A and B).

Solution

(a) The events are independent as the number obtained on the first roll does not affect the second roll.

(b) $p(\text{A}) = \frac{1}{6}$ and $p(\text{B}) = \frac{1}{6}$

so
$$\begin{aligned} p(\text{A and B}) &= p(\text{A}) \times p(\text{B}) \\ &= \frac{1}{6} \times \frac{1}{6} \\ &= \frac{1}{36} \end{aligned}$$

Worked Example 2

A spinner in game has 3 sections of equal size that are coloured red, blue and green. Let the events B, R and G be:

B: the spinner lands a blue

G: the spinner lands on green

R: the spinner lands on red.

(a) Are these events independent?

10.8

(b) Find the probability that when the spinner is spun twice the following outcomes are obtained.

(i) Red both times.

(ii) Red and green in any order.

(iii) Both are the same colour.

Solution

(a) The events are independent as the result of one spin does not affect the next.

(b) The probabilities of each event are

$$p(\text{B}) = \frac{1}{3} \quad p(\text{R}) = \frac{1}{3} \quad p(\text{G}) = \frac{1}{3}.$$

(i) The probability is given by

$$\begin{aligned} p(\text{R and R}) &= p(\text{R}) \times p(\text{R}) \\ &= \frac{1}{3} \times \frac{1}{3} \\ &= \frac{1}{9} \end{aligned}$$

(ii) For red and green in any order, two outcomes must be considered: Red then Green and Green then Red.

$$\begin{aligned} p(\text{G and R}) &= p(\text{G}) \times p(\text{R}) \\ &= \frac{1}{3} \times \frac{1}{3} \\ &= \frac{1}{9} \end{aligned} \qquad \begin{aligned} p(\text{R and G}) &= p(\text{R}) \times p(\text{G}) \\ &= \frac{1}{3} \times \frac{1}{3} \\ &= \frac{1}{9} \end{aligned}$$

Hence the probability of a red and green in any order is given by:

$$\begin{aligned} p(\text{R and G}) + p(\text{G and R}) &= \frac{1}{9} + \frac{1}{9} \\ &= \frac{2}{9}. \end{aligned}$$

(iii) For both to be the same colour the outcomes, R and R, G and G, B and B must be considered.

From (i) $\quad p(\text{R and R}) = \dfrac{1}{9}$

Similarly $\quad p(\text{B and B}) = \dfrac{1}{9}$

and $\quad p(\text{G and G}) = \dfrac{1}{9}.$

10.8

The probability that both spins are the same colour is given by:

$$p(\text{R and R}) + p(\text{B and B}) + p(\text{G and G}) = \frac{1}{9} + \frac{1}{9} + \frac{1}{9}$$
$$= \frac{3}{9}$$
$$= \frac{1}{3}.$$

Exercises

1. For each pair of events A and B listed below, decide whether or not it is likely that the events are independent.

   (a) A: It rains today.

   B: It rains tomorrow.

   (b) A: It rains on Monday this week.

   B: It rains on Monday next week.

   (c) A die is rolled twice.

   A: The first roll shows a 3.

   B: The second roll shows a 5.

   (d) A baby is born.

   A: Its left eye is blue.

   B: Its right eye is blue.

   (e) Joshua and James are brothers.

   A: Joshua catches measles.

   B: James catches measles.

   (f) Daniel cycles to school.

   A: Daniel's bicycle has a puncture.

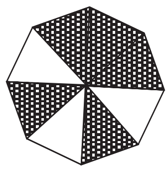
   B: Daniel is late for school.

2. The spinner shown in the diagram has eight sections of equal size; each one is coloured white or black.

   The events B and W are:

   B: the spinner lands on black,

   W: the spinner lands on white.



10.8

(a) Find the following probabilities:

(i) p(B) (ii) p(W) (iii) p(B and B)

(iv) p(W and W) (v) p(B and W) (vi) p(W and B).

(b) If the spinner is spun twice find the probabilities of the following outcomes.

(i) White is obtained both times.

(ii) A different colour is obtained on each spin.

(iii) The same colour is obtained on each spin.

3. A bag contains 7 red balls and 3 green balls. A ball is taken out and replaced. A second ball is then taken out.

R is the event that a Red ball is selected. G is the event that a Green ball is selected.

(a) Find the following probabilities

(i) p(R) (ii) p(G) (iii) p(G and G)

(iv) p(R and R) (v) p(G and R) (vi) p(R and G).

(b) Find the probability that if two balls are taken in turn;

(i) they are both red,

(ii) they are different colours,

(iii) they are the same colour.

4. A die is thrown twice. Find the probability that

(a) two odd numbers are obtained,

(b) the same two numbers are obtained.

5. The probability that Nigel is late for work is 0.2. The probability that Karen is late for work is 0.3. Assume that these events are independent.

(a) Find the probability that they are both late for work.

(b) Would your answer be the same if you knew that Karen was going to give Nigel a lift in her car?

6. In a school 20% of the children are colour blind and 10% are left handed. If a child is selected at random, what are the probabilities that they are:

(a) neither colour blind or left handed,

(b) colour blind and left handed,

(c) left handed but not colour blind.

10.8

7. Assume that 50% of the population of Britain are women, that 20% of the population are vegetarians and that men and women are equally likely to be vegetarians. Find the probability that a person selected at random is:

(a) a male vegetarian,

(b) a meat-eating female,

(c) a meat-eating male,

(d) a female vegetarian.

8. Ben's mum always gives him cheese or jam sandwiches in his packed lunch. The probability of her giving him cheese sandwiches is 0.3. She also gives him a chocolate biscuit or a wafer bar. The probability that she gives him a chocolate biscuit is 0.4. Find the probability that Ben's packed lunch contains:

(a) a wafer bar and jam sandwiches,

(b) a chocolate biscuit on two consecutive days.

9. When Wendy plays Tetrix on her computer, the probability that she scores more than 1000 points is $\frac{1}{7}$. One day Wendy plays two games of Tetrix.

(a) What is the probability that she scores over 1000 in both games?

(b) What is the probability that she scores less than 1000 in both games?

(c) What is the probability that she scores over 1000 in one of the two games?

(d) If Wendy plays three games instead of two, what is the probability that she scores over 1000 in all three games?

10. Once a week John checks his car. The probability that he needs to pump up a tyre is $\frac{1}{20}$. The probability he has to add oil is $\frac{1}{10}$ and the probability he has to add water is $\frac{1}{5}$.

(a) Is it reasonable to assume that the events described above are independent?

For questions (b) and (c), assume that the events are independent.

(b) What is the probability that John does not need to do anything to his car?

(c) What is the probability that John has to do one thing to his car, that is blow up a tyre, add oil or add water?

11. The probability that a woman in Mathsville is a least 165 cm tall is 0.15.

The probability that a woman in Mathsville is coloured-blind is 0.02.

These probabilities are independent of each other.

(a) What is the probability that a woman in Mathsville is both colour-blind and at least 165 cm tall?

(b) What is the probability that a woman in Mathsville is less than 165 cm tall and is not colour-blind?

(NEAB)

10.8

12. Tom cycles to school on Fridays. There is a probability of $\frac{1}{5}$ that he will be late and a probability of $\frac{4}{5}$ that he will not be late.

(a) Calculate the probability that Tom will be late on two consecutive Fridays.

(b) Using *L* for 'late' and *N* for 'not late', complete the table to show the possible outcomes of Tom's journeys for the two Fridays.

1st Friday	2nd Friday
L	*L*

Hence calculate the probability that Tom will be late on at least one of the two Fridays.

(MEG)

13. A student takes examinations in Mathematics, English and French.

The probability that she passes Mathematics is 0.7.

The probability that she passes English is 0.8.

The probability that she passes French is 0.6.

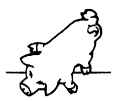
Given that her results in each subject are independent, find the probability that

(a) she fails Mathematics or French or both;

(b) she fails English or French but *not* both.

(SEG)

14. In the game of 'Pass the Pig', two identical toy pigs are thrown. Each pig can land in one of five positions. The five positions and the probabilities that the pig will land in each of these positions are shown in the table.

Position	Sider	Trotter	Razorback	Snouter	Leaning Jowler
					
Probability	0.57	0.2	0.2	0.02	0.01

Both pigs are thrown.

Work out the probability that they will both land in the 'Trotter' position.

(LON)

10.8

15. When I answer the telephone the call is never for me.

Half the calls are for my daughter Janette, one-third of them are for my son Glen and the rest are for my wife Barbara.

(a) I answer the telephone twice this evening.

Calculate the probability that

(i) the first call will be for Barbara,

(ii) both calls will be for Barbara.

(b) The probability that both these two telephone calls are for Janette is $\frac{1}{4}$.

The probability that they are both for Glen is $\frac{1}{9}$.

Calculate the probability that either they are both for Janette or both for Glen.

(NEAB)

10.9 Mutually Exclusive Events

If two events cannot happen or take place at the same time, then they are called *mutually exclusive* events. For example when tossing a single coin the events 'heads' and 'tails' are mutually exclusive because they cannot both be obtained at the same time.

If A and B are mutually exclusive events then the probability of obtaining A or B is given by:

$$P(\text{A or B}) = P(\text{A}) + P(\text{B}).$$

Worked Example 1

State whether or not the pairs of events describe below are mutually exclusive.

(a) A: A die is rolled and shows a 6.
B: A die is rolled and shows an odd number.

(b) A: Selecting a child with blue eyes from a class.
B: Selecting a left handed person from a class.

Solution

(a) It is not possible for a die to show a six and an odd number at the same time, so these events are mutually exclusive.

(b) It is possible to select a person who has blue eyes and is left handed, so these events are not mutually exclusive.

10.9

Worked Example 2

When Andrew buys a can of drink the probabilities of selecting particular brands are given in the table below:

Drink	Probability
Cola	$\frac{1}{2}$
Lemonade	$\frac{1}{9}$
Fizzo	$\frac{1}{4}$

Find the probabilities that he selects:

(a) Cola or Lemonade (b) Fizzo or Lemonade

(c) none of the drinks listed above.

Solution

(a)
$$\begin{aligned} p(\text{Cola or Lemonade}) &= p(\text{Cola}) + p(\text{Lemonade}) \\ &= \frac{1}{2} + \frac{1}{9} = \frac{9}{18} + \frac{2}{18} \\ &= \frac{11}{18} \end{aligned}$$

(b)
$$\begin{aligned} p(\text{Fizzo or Lemonade}) &= p(\text{Fizzo}) + p(\text{Lemonade}) \\ &= \frac{1}{4} + \frac{1}{9} \\ &= \frac{9}{36} + \frac{4}{36} \\ &= \frac{13}{36} \end{aligned}$$

(c) First find the probability that he chooses one of the drinks from the list.

$$\begin{aligned} p(\text{Cola or Lemonade or Fizzo}) &= p(\text{Cola}) + p(\text{Lemonade}) + p(\text{Fizzo}) \\ &= \frac{1}{2} + \frac{1}{9} + \frac{1}{4} \\ &= \frac{18}{36} + \frac{4}{36} + \frac{9}{36} \\ &= \frac{31}{36} \end{aligned}$$

then

$$\begin{aligned} p(\text{drink not from list}) &= 1 - p(\text{drink from list}) \\ &= 1 - \frac{31}{36} \\ &= \frac{5}{36}. \end{aligned}$$

10.9

Exercises

1. When a die is rolled, the following outcomes can be used to describe the result.

   A: An odd number is obtained.

   B: An even number is obtained.

   C: A prime number is obtained.

   Copy and complete the following statements.

   A and ___ are mutually exclusive.

   B and ___ are not mutually exclusive.

2. Decide if each pair of events A and B given below are mutually exclusive.

   (a) A: Winning a football match.
   B: Losing a football match.

   (b) A: Selecting a diamond from a pack of playing cards.
   B: Selecting a King from a pack of playing cards.

   (c) A: Arriving late on a train journey.
   B: Not arriving early on a train.

   (d) A: Selecting an ace from a pack of cards.
   B: Selecting a queen from a pack of cards.

   (e) A: It rains tomorrow.
   B: It is sunny tomorrow.

3. When Plymouth Argyle football team play in a league match the probability that they win is 0.4 and the probability that they draw is 0.3. What is the probability that they lose?

4. When Samantha, Annie and Katie play a game, the probability that Samantha wins is $\frac{1}{2}$ and the probability that Annie wins is $\frac{1}{3}$. What is the probability that Katie wins?

5. A bag contains a number of balls, which are yellow, blue or green. The probability of selecting a ball at random and getting a green is $\frac{1}{7}$ and the probability of getting a yellow is $\frac{3}{7}$.

   (a) What is the probability of getting a blue ball?

   (b) If the bag contains 4 green balls, how many yellow balls does it contain?

   (c) If the bag contains 6 blue balls, how many balls does the bag contain in total?

10.9

6. When a car arrives at a set of traffic lights the probability that a green light is showing is $\frac{7}{20}$ and the probability that a red light is showing is $\frac{3}{10}$.

(a) What is the probability that neither the green or red lights are showing?

(b) Explain why the probability that an amber light is showing, is greater than your answer to (a).

7. A bag contains 6 red counters, 5 blue counters and 4 pink counters. A counter is selected from the bag at random.

Find, the probability that the counter is:

(a) either red or pink, (b) not pink,

(c) not red, (d) blue or pink.

8. A bag contains a number of balls of different colours. The probability of obtaining a ball of a particular colour is given in the table below.

Colour	**Probability**
Red	$\frac{3}{8}$
Green	$\frac{1}{4}$
Blue	$\frac{1}{5}$

What is the probability that a ball taken from the bag is:

(a) red or green,

(b) not blue or green,

(c) not one of the colours listed above?

9. As part of a project, a group of students classify each day as *Dry*, *Damp*, *Wet* or *Very wet*. The probability that a day will be *Dry* or *Damp* is 0.6. The probability that a day is *Wet* is 0.3.

(a) What is the probability that a day is *Very wet*?

The probability that is *Damp* is twice the probability that it is *Dry.*

(b) What is the probability that it is *Damp* or *Wet*?

10.9

10. A pack contains cards that are coloured pink, yellow or black. When a card is chosen at random the probability of obtaining a black or pink card is $\frac{5}{7}$ and the probability of obtaining a black or yellow card is $\frac{3}{5}$.

    Find the probability of containing a card of each colour.

11. In a class of 25 students there are 8 students with size 6 feet, 4 students with size 7 feet and 3 students that are left handed.

    Find:

    (a) the probability that a student in this class has size 6 or size 7 feet,

    (b) the probability that a student in this class is right-handed.

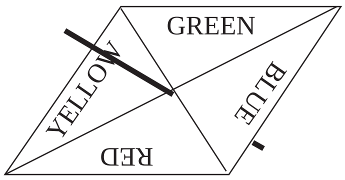
    Is it possible to find the probability that:

    (c) a student has size 6 feet and is left-handed.,

    (d) has feet that are bigger than size 7,

    (e) has feet that are not size 6 or size 7?

12. The spinner shown is biased.



    The probabilities of getting a particular colour are shown in the table below:

    (a) Complete the table to show the probability of getting *Green*.

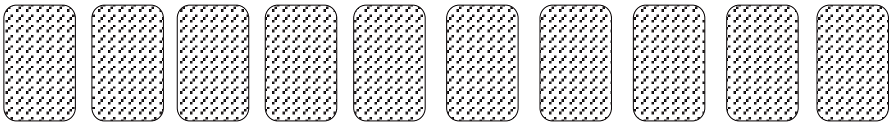
Colour	*Red*	*Yellow*	*Blue*	*Green*
Probability	0.4	0.1	0.3	

    (b) The spinner is spun once. What is the probability of getting either *Red* or *Blue*?

    (c) The spinner is spun 50 times. Approximately how many times would you expect to get *Red*?

10.9

13. A pack of 52 playing cards consists of equal numbers of clubs, diamonds, hearts and spades. Ten cards are removed from the pack and placed face down on a table.



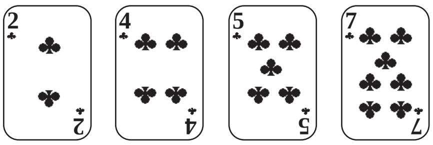
When one of these cards is taken at random the following probabilities apply:

Type of card	**Probability**
club	0.4
heart	0.2
spade	0.1
diamond	0.3

Number on card	**Probability**
2	0.2
3	0.2
4	0.1
5	0.3
7	0.2

Four of the ten cards are clubs.

They are numbered 2, 4, 5 and 7.



One of the ten cards is taken from the table at random.

(a) What is the probability that it is *not* a diamond?

(b) What is the probability that it is a club or a diamond?

(c) What is the probability that it is a club or numbered 3?

(d) Explain why the probability that it is a club or numbered 5 is *not* 0.4 + 0.3.

(SEG)

14. A bag contains a total of 20 beads. There are 6 red beads, 9 blue beads and 5 white beads.

(a) A bead is taken at random from the bag. The probability that it is red is 0.3.

(i) What is the probability that it is white?

(ii) What is the probability that it is **not** white.

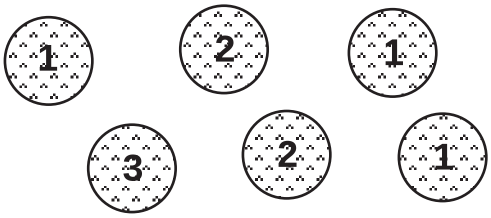
All the beads are taken from the bag, numbered 1, 2, 3 or 4 and then replaced.

10.9

When a bead is taken from the bag at random the probability of each number is as shown in the table.

Number on bead	**Probability**
1	0.3
2	0.4
3	0.2
4	0.1

The red beads are numbered,



(b) A bead is taken at random from the bag.

(i) What is the probability that it is red or numbered 4?

(ii) Explain why the probability of getting a red bead or a bead numbered 2 is *not* 0.3 + 0.4.

(SEG)

Answers to Exercises

10.1 Probabilities

1. (a) 0 (b) about 250 (c) about 250
2. (a) 50 (b) 50 (c) 0
3. (a) Impossible (b) Unlikely (c) Likely or Unlikely
   (d) Likely or Unlikely (e) Unlikely (f) Likely (g) Likely
5. (a) 10 (b) 20 (c) 1000 (d) 600
6. About 900
7. (a) about 1500 (b) about 250
8. (a) 50 (b) 50 (c) 25 (d) 25

10.2 Simple Probability

1. (a) 0.1 (b) $\frac{1}{4}$ (c) $\frac{1}{2}$ (d) $\frac{4}{5}$
2. (a) 0.2 (b) 0.4
3. 0.98
4. $\frac{4}{5}$
5. (a) $\frac{4}{7}$ (b) not to snow
6. 0.99
7. (a) $\frac{2}{5}$ (b) 12
8. (a) 0.6 (b) 0.9 (c) 0.1
9. (a) $\frac{9}{20}$ (b) $\frac{10}{11}$ (c) $\frac{9}{13}$
10. No
11. 0.15
12. (a) C (b) B
13. (a) near to 0 (b) near to 1

10.3 Outcome of Two Events

2. VC, VS, MC, MS, RC, RS
3. GG, RG, GR, RR
4. (a) Nigel wins sit ups and Ben wins press ups
   (b) BJ, BN, BB, JB, JN, JJ, NB, NJ, NN (c) BJ, BB, JB, JJ
   (d) BJ, BN, BT, BB, JB, JN, JT, JJ, NB, NJ, NT, NN, TB, TN, TJ, TT

Answers

10.3

5. BS, BT, BD, BB, ST, SD, SS, TD, TT, DD

6. CD, CB, CC, DB, DD, BB

7.

	F	M
H	HF	HM
T	TF	TM
C	CF	CM

8. HHH ; HHT, HTH, THH ; HTT, THT, TTH ; TTT

9. (b) $\frac{4}{9}$

10.4 Finding Probabilities Using Relative Frequency

4. $\frac{1}{4}$

5. (a) $\frac{4}{5}$

6. (a) $\frac{2}{7}, \frac{7}{10}, \frac{1}{4}, \frac{1}{5}, \frac{1}{3}, \frac{2}{3}$ (b) Andrew (c) Rachel (d) Charles

7. (a) $\frac{3}{8}$ (b) $\frac{5}{12}$ (c) $\frac{5}{24}$

8. (a) $\frac{4}{5}$ (b) 96

10.5 Determining Probabilities

1. (a) $\frac{1}{4}$ (b) $\frac{1}{4}$ (c) $\frac{1}{13}$ (d) $\frac{1}{13}$ (e) $\frac{4}{13}$

2. (a) $\frac{13}{54}$ (b) $\frac{13}{54}$ (c) $\frac{2}{27}$ (d) $\frac{2}{27}$ (e) $\frac{8}{27}$

3. (a) $\frac{1}{6}$ (b) $\frac{1}{6}$ (c) $\frac{1}{2}$ (d) $\frac{1}{2}$

4. (a) $\frac{1}{8}$ (b) $\frac{1}{8}$ (c) $\frac{1}{2}$ (d) $\frac{5}{8}$ (e) $\frac{1}{4}$

5. $\frac{3}{8}$

6. (a) $\frac{2}{5}$ (b) $\frac{4}{5}$ (c) $\frac{1}{5}$ (d) $\frac{4}{5}$ (e) $\frac{9}{49}$ (f) $\frac{39}{49}$ (g) $\frac{10}{49}$

7. (a) $\frac{9}{25}$ (b) $\frac{6}{25}$ (c) $\frac{3}{5}$ (d) $\frac{16}{25}$

Answers

10.5

8. (a) $\frac{2}{5}$ (b) $\frac{1}{5}$ (c) 1 (d) $\frac{4}{5}$

9. (a) $\frac{2}{5}$ (b) $\frac{1}{5}$ (c) $\frac{1}{5}$

10. (a) $\frac{3}{10}$ (b) $\frac{2}{9}$ (c) $\frac{1}{8}$ (d) $\frac{3}{8}$

11. (a) $\frac{7}{30}$ (b) $\frac{23}{30}$

12. (a) $\frac{1}{2}$ (b) $\frac{1}{6}$

13. (a) $\frac{1}{200}$ (b) 20

14. (a) $\frac{3}{10}$ (b) $\frac{4}{5}$

15. (a) Mint (b) $p(\text{mint}) = \frac{2}{3}$, $p(\text{toffee}) = \frac{1}{4}$, $p(\text{pen}) = \frac{1}{12}$ (c) $\frac{1}{12}$ (d) 0

10.6 Probability of Two Events

1. (a) $\frac{1}{4}$ (b) $\frac{1}{2}$ (c) $\frac{1}{2}$

2. (a) $\frac{1}{12}$ (b) $\frac{1}{4}$ (c) $\frac{1}{4}$ (d) $\frac{1}{3}$ (e) $\frac{1}{2}$

3. (a) (i) $\frac{1}{6}$ (ii) $\frac{1}{9}$ (iii) $\frac{1}{2}$ (iv) $\frac{5}{18}$ (v) $\frac{5}{18}$ (b) 7

4. (b) (i) $\frac{1}{9}$ (ii) $\frac{1}{3}$ (iii) $\frac{8}{9}$ (c) $\frac{1}{9}$

5. (a) $\frac{1}{4}$ (b) $\frac{1}{2}$

6. (a) $\frac{1}{8}$ (b) $\frac{5}{8}$ (c) 5

7. (a) 6 (b) $\frac{2}{3}$ (c) $\frac{1}{3}$

8. (a) 400 (b) $\frac{1}{400}$ (c) $\frac{1}{20}$ (d) $\frac{1}{25}$

Answers

10.6

9. (a) $\frac{1}{4}$ (b) $\frac{1}{8}$ (c) $\frac{7}{8}$

10. (a) (i) 1, 2 ; 1, 3 ; 2, 2 ; 2, 3 ; 3, 2 ; 3, 3 (ii) $\frac{1}{3}$ (iii) $\frac{2}{3}$ (b) (i) 9

11. (a) $\frac{1}{5}$ (b) $\frac{2}{5}$ (c) AX , AY , AZ , BW , BX , BY , BZ , CW , CX , CY, CZ , DW , DX , DY , DZ , EW , EX , EY , EZ

12. (b) $\frac{1}{8}$

13. (b) $\frac{4}{9}$

10.7 Use of Tree Diagrams

1. (b) $0.6 \times 0.6 = 0.36$; $0.6 \times 0.4 = 0.24$; $0.4 \times 0.6 = 0.24$; $0.4 \times 0.4 = 0.16$
   (c) 0.16 (d) 0.36 (e) 0.48

2. (a) $\frac{1}{6}$ (c) (i) $\frac{1}{36}$ (ii) $\frac{5}{18}$ (iii) $\frac{25}{36}$

3. (a) $\frac{1}{4}$ (b) $\frac{1}{4}$ (c) $\frac{1}{2}$

4. (a) $\frac{81}{100}$ (b) $\frac{99}{100}$ (c) $\frac{1}{100}$

5. (a) $\frac{1}{4}$ (c) (i) $\frac{9}{16}$ (ii) $\frac{3}{8}$ (iii) $\frac{1}{16}$

6. (b) (i) 0.54 (ii) 0.36 (iii) 0.04

7. (a) (i) 0.2 (ii) 0.4 (b) (i) 0.48 (ii) 0.08

8. (a) (i) 0.2704 (ii) 0.2304 (iii) 0.4992 (b) (iii)

9. (a) 0.56 (b) 0.38 (c) 0.04 (d) 0.64

10. (a) $\frac{7}{18}$ (b) $\frac{1}{36}$ (c) $\frac{3}{4}$

11. (b) (i) $\frac{1}{4}$ (ii) $\frac{1}{2}$ (iii) $\frac{3}{8}$

12. (b) 0.52

13. (a) $\frac{7}{13}$ (b) $\frac{43}{91}$

Answers

10.7

14. (b) $\frac{3}{14}$

15. (a) 0.6 (b) 0.16

16. (a) $\frac{1}{216}$ (b) $\frac{5}{216}$ (c) $\frac{5}{72}$ (d) $\frac{2}{27}$

10.8 Multiplication for Independent Events

1. NI - not independent I - independent (a) NI (b) I (c) I
   (d) NI (e) NI (f) NI

2. (a) (i) $\frac{5}{8}$ (ii) $\frac{3}{8}$ (iii) $\frac{25}{64}$ (iv) $\frac{9}{64}$ (v) $\frac{15}{64}$ (vi) $\frac{15}{64}$
   (b) (i) $\frac{9}{64}$ (ii) $\frac{15}{32}$ (iii) $\frac{17}{32}$

3. (a) (i) $\frac{7}{10}$ (ii) $\frac{3}{10}$ (iii) $\frac{9}{100}$ (iv) $\frac{49}{100}$ (v) $\frac{21}{100}$ (vi) $\frac{21}{100}$
   (b) (i) $\frac{49}{100}$ (ii) $\frac{21}{50}$ (iii) $\frac{29}{50}$

4. (a) $\frac{1}{4}$ (b) $\frac{1}{6}$

5. (a) 0.06 (b) No

6. (a) 0.72 (b) 0.02 (c) 0.08

7. (a) 0.1 (b) 0.4 (c) 0.4 (d) 0.1

8. (a) 0.42 (b) 0.16

9. (a) $\frac{1}{49}$ (b) $\frac{36}{49}$ (c) $\frac{12}{49}$ (d) $\frac{1}{343}$

10. (a) No (b) $\frac{171}{250}$ (c) $\frac{283}{1000}$

11. (a) 0.003 (b) 0.833

12. (a) $\frac{1}{25}$ (b) $\frac{8}{25}$

13. (a) 0.58 (b) 0.44

14. 0.04

15. (a) (i) $\frac{1}{6}$ (ii) $\frac{1}{36}$ (b) $\frac{13}{36}$

Answers

10.9 Mutually Exclusive Events

1. B ; C
2. (a) Yes (b) No (c) No (d) Yes (e) No
3. 0.3
4. $\frac{1}{6}$
5. (a) $\frac{3}{7}$ (b) 12 (c) 14
6. (a) $\frac{7}{20}$ (b) Not mutually exclusive
7. (a) $\frac{2}{3}$ (b) $\frac{11}{15}$ (c) $\frac{3}{5}$ (d) $\frac{3}{5}$
8. (a) $\frac{5}{8}$ (b) $\frac{11}{20}$ (c) $\frac{7}{40}$
9. (a) 0.1 (b) 0.7
10. Pink : $\frac{2}{3}$ Yellow : $\frac{2}{7}$ Black : $\frac{11}{35}$
11. (a) $\frac{13}{25}$ (b) $\frac{22}{25}$ (c) No (d) No (e) Yes : $\frac{12}{25}$
12. (a) 0.2 (b) 0.7 (c) 20
13. (a) 0.7 (b) 0.7 (c) 0.6 (d) not mutually exclusive
14. (a) (i) 0.25 (ii) 0.75 (b) (i) 0.4 (ii) not mutually exclusive