



Mathematics Enhancement Programme

Primary Demonstration Project

5B Geometry

Help Booklet



Support for Primary Teachers
in Mathematics

Primary Project
funded by
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Coopers

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ESSO

CIMT
School of Education
University of Exeter

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Mathematics Enhancement Programme

Help Module 5

GEOMETRY

Part B

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Preface
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PREFACE

This is one of a series of *Help Modules* designed to help you gain confidence in mathematics. It has been developed particularly for primary teachers (or student teachers) but it might also be helpful for non-specialists who teach mathematics in the lower secondary years. It is based on material which is already being used in the *Mathematics Enhancement Programme: Secondary Demonstration Project*.

The complete module list comprises:

- | | |
|--------------|-----------------------|
| 1. ALGEBRA | 6. HANDLING DATA |
| 2. DECIMALS | 7. MENSURATION |
| 3. EQUATIONS | 8. NUMBERS IN CONTEXT |
| 4. FRACTIONS | 9. PERCENTAGES |
| 5. GEOMETRY | 10. PROBABILITY |

Notes for overall guidance:

- Each of the 10 modules listed above is divided into 2 parts. This is simply to help in the downloading and handling of the material.
- Though referred to as 'modules' it may not be necessary to study (or print out) each one in its entirety. As with any self-study material you must be aware of your own needs and assess each section to see whether it is relevant to those needs.
- The difficulty of the material in **Part A** varies quite widely: if you have problems with a particular section do try the one following, and then the next, as the content is not necessarily arranged in order of difficulty. Learning is not a simple linear process, and later studies can often illuminate and make clear something which seemed impenetrable at an earlier attempt.
- In **Part B**, **Activities** are offered as backup, reinforcement and extension to the work covered in Part A. **Tests** are also provided, and you are strongly urged to take these (at the end of your studies) as a check on your understanding of the topic.
- The marking scheme for the revision test includes B, M and A marks.

Note that:

- | | |
|----------------|---|
| M marks | are for method; |
| A marks | are for accuracy (awarded only following a correct M mark); |
| B marks | are independent, stand-alone marks. |

We hope that you find this module helpful. Comments should be sent to:

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EXETER EX1 2LU

The full range of Help Modules can be found at

www.ex.ac.uk/cimt/help/menu.htm

ACTIVITIES

- Activity 5.1 Rotational and Line Symmetry
 - Activity 5.2 Symmetry of Regular Polygons
 - Activity 5.3 Special Quadrilaterals
 - Activity 5.4 Sam Loyd's Dissection
 - Activity 5.5 Overlapping Squares
 - Activity 5.6 Interior Angles in Polygons
 - Activity 5.7 Lines of Symmetry
 - Activity 5.8 Pythagoras' Theorem
 - Activity 5.9 Posting Parcels
- Notes for Solutions

ACTIVITY 5.1

Rotational and Line Symmetry

1. For each polygon below:

- (a) use dotted lines to show the lines of symmetry, if any;
- (b) check whether it has rotational symmetry and if so, state its order;
- (c) mark the centre of rotational symmetry with a cross (x).

(i)	(ii)	(iii)	(iv)
Order _____	Order _____	Order _____	Order _____
(v)	(vi)	(vii)	(viii)
Order _____	Order _____	Order _____	Order _____

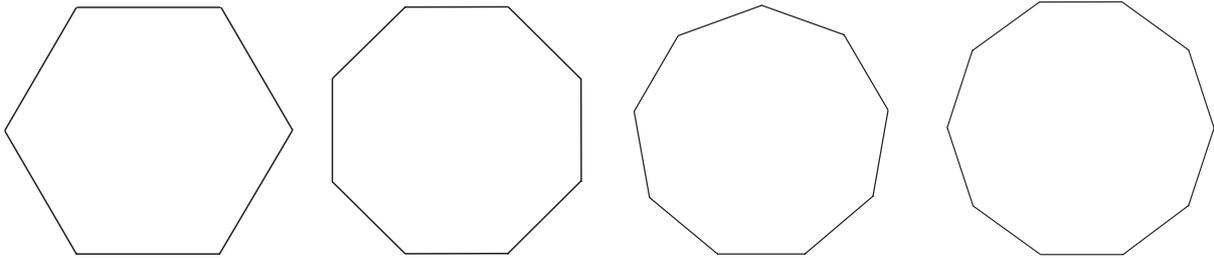
2. Use the results from Question 1 to complete the following table.

	Name of Polygon	Number of Lines of Symmetry	Order of Rotational Symmetry
(i)	<i>Isosceles triangle</i>		
(ii)	<i>Equilateral triangle</i>		
(iii)	<i>Rectangle</i>		
(iv)	<i>Square</i>		
(v)	<i>Parallelogram</i>		
(vi)	<i>Rhombus</i>		
(vii)	<i>Kite</i>		
(viii)	<i>Trapezium</i>		

ACTIVITY 5.2

Symmetry of Regular Polygons

1. For each of the following regular polygons, draw in the lines of symmetry and locate the centre of rotational symmetry.



2. Use your answers to Question 1 to complete the following table.

Name of Polygon	Number of sides	Number of Lines of Symmetry	Order of Rotational Symmetry
<i>Hexagon</i>			
<i>Octagon</i>			
<i>Nonagon</i>			
<i>Decagon</i>			

3. Use the completed table in Question 2 to find:

(a) the number of lines of symmetry,

(b) the order of rotational symmetry,

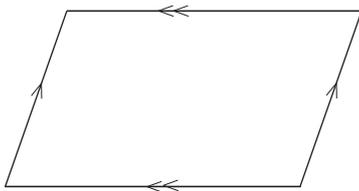
for

- (i) a regular 10-gon (ii) a regular 20-gon (iii) a regular n -gon.

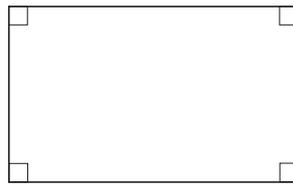
ACTIVITY 5.3

Special Quadrilaterals

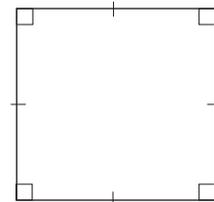
Fill in the table below to identify the properties of these special quadrilaterals.



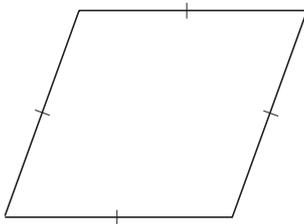
Parallelogram



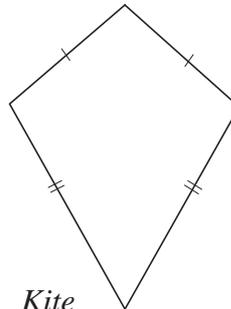
Rectangle



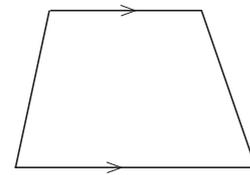
Square



Rhombus



Kite



Trapezium

Property						
All sides equal						
Opposite sides equal						
Opposite sides parallel						
Opposite angles equal						
Diagonals equal						
Diagonals bisect each other						
Diagonals intersect at right angles						
Longer diagonal bisects shorter diagonal						
Two pairs of adjacent sides equal but not all sides equal						
Only one pair of opposite sides parallel						
Only one pair of opposite angles equal						

ACTIVITY 5.4

Sam Loyd's Dissection

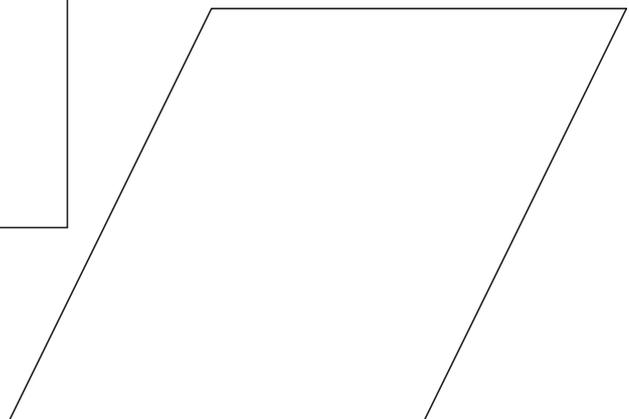
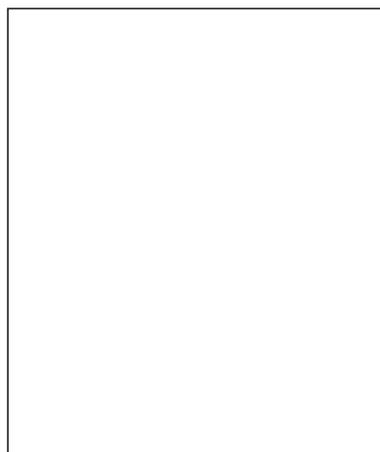
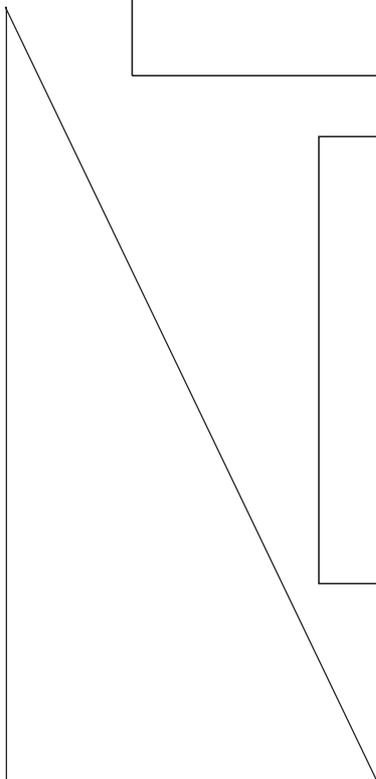
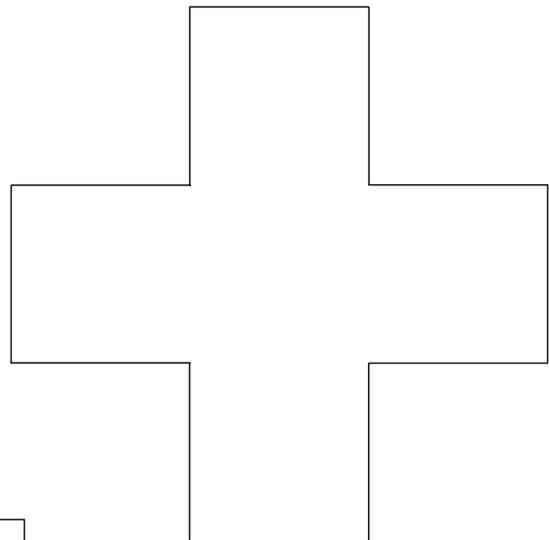
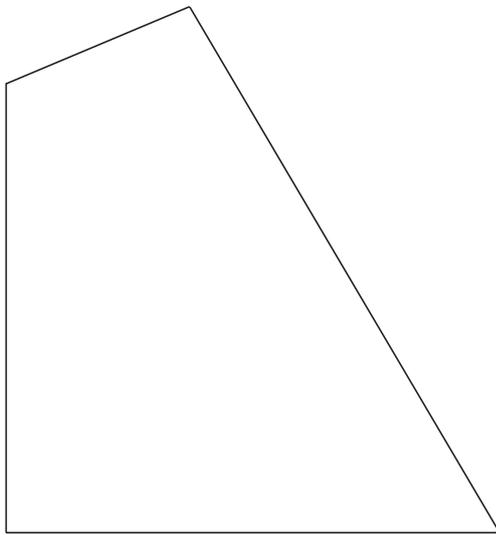
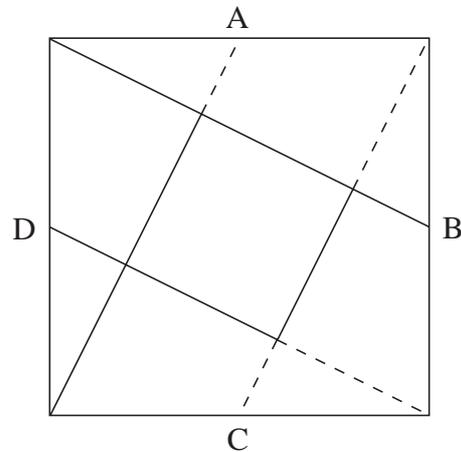
This famous dissection problem was designed by *Sam Loyd* in the 1920s.

Draw a 5 cm square as on the right. Find the mid-points (A, B, C and D in diagram) on each side and join them up.

Using the diagram as a guide, cut your square into 5 pieces along the **bold** lines.

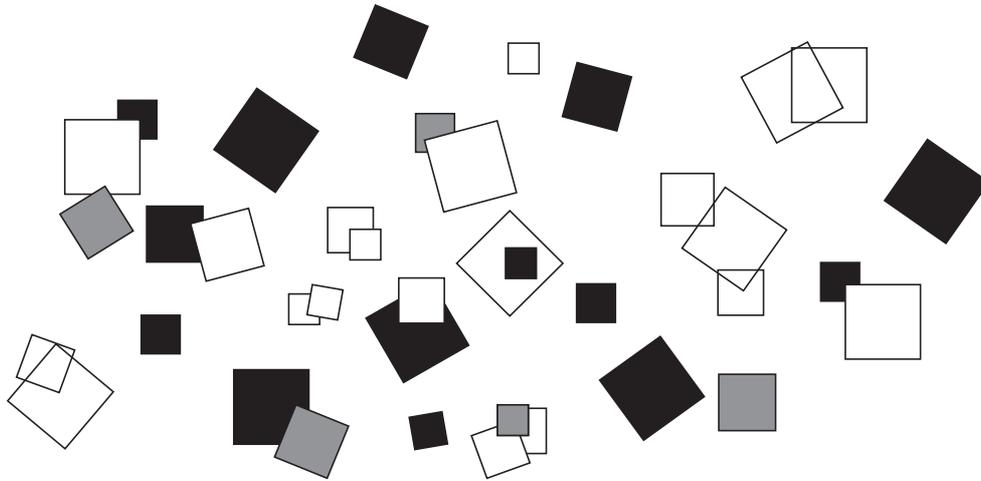
Do not cut along the dotted lines.

With the 5 pieces, try to make all the shapes below.



ACTIVITY 5.5

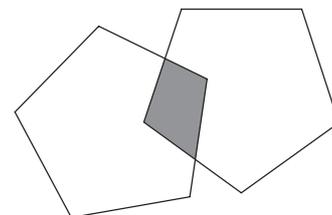
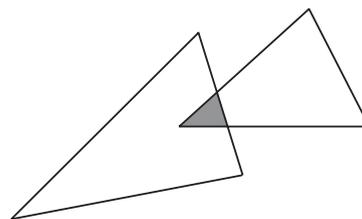
Overlapping Squares



Take two squares and put them down on a surface so that they overlap. The squares can be of any size, not necessarily the same.

1. Which of the following shapes can be formed by the overlap:
 - (a) rectangle (b) square (c) kite (d) rhombus?
2. Can two squares intersect so that a triangle is formed by the overlap?
3. Can two squares intersect so that the overlap forms a polygon of n sides for values of n equal to
 - (a) 5 (b) 6 (c) 7 (d) 8 (e) 9 (f) 10?

4. What happens when two triangles overlap?



Extensions

1. What happens when two pentagons overlap?
2. What happens when two *different* shapes, e.g. square and triangle, overlap?

ACTIVITY 5.6

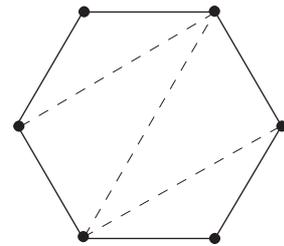
Interior Angles in Polygons

You can find the sum of the interior angles in any polygon by dividing it up into triangles with lines connecting the vertices.

For example, the hexagon shown opposite has been divided into 4 internal triangles.

The sum of all the interior angles of the hexagon is equal to the sum of all the angles in each triangle; so

$$\text{sum of interior angles} = 4 \times 180^\circ = 720^\circ.$$



1. Repeat the same analysis for the following shapes:

- (a) quadrilateral (b) pentagon (c) heptagon
 (d) octagon (e) nonagon (f) dodecagon.

2. Copy and complete the table.

Name of Polygon	Number of sides	Number of Triangles	Sum of Interior Angles
<i>Triangle</i>	3	1	180°
<i>Quadrilateral</i>			
<i>Pentagon</i>			
<i>Hexagon</i>	6	4	720°
<i>Heptagon</i>			
<i>Octagon</i>			
<i>Nonagon</i>			
<i>Dodecagon</i>			

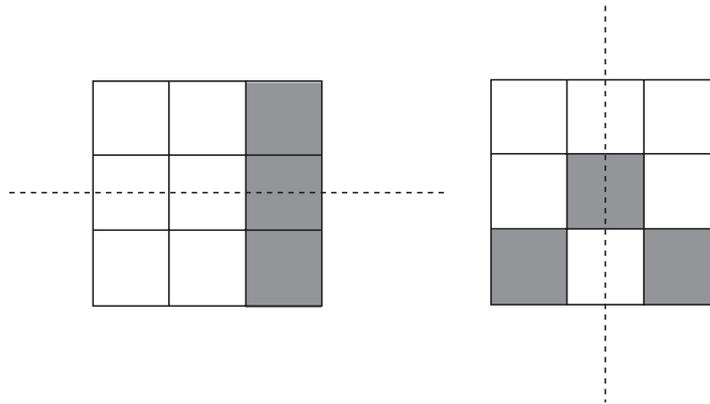
Extension

What is the formula for the sum of the interior angles of an n -gon?

ACTIVITY 5.7

Lines of Symmetry

Each of the 3×3 squares below has **3** shaded squares and **one** line of symmetry.



1. How many more ways can you find to shade 3 squares in a 3×3 square so that there is only one line of symmetry? Record your patterns.

2. (a) In a 3×3 square find a pattern of 3 shaded squares which has 2 lines of symmetry.
 (b) Is it the only one? If not, try to find all such patterns.

3. Using a 3×3 square, find all the possible patterns of **4** shaded squares which have
 - (a) one line of symmetry
 - (b) two lines of symmetry
 - (c) three lines of symmetry
 - (d) four lines of symmetry.

Extension

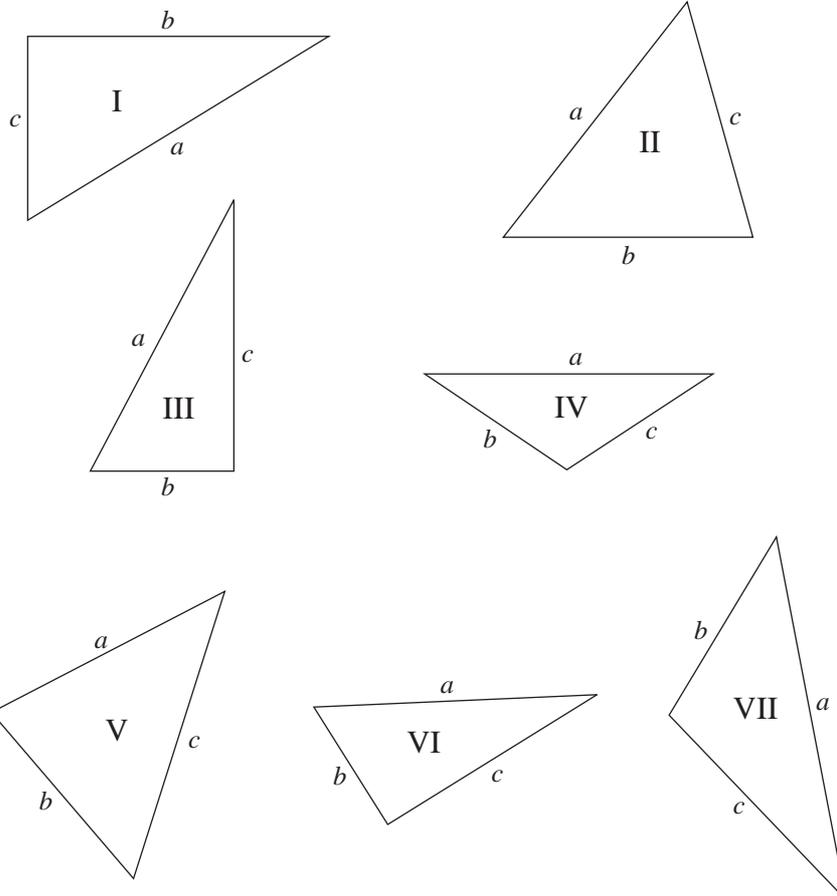
Do a similar study for a 4×4 square with different patterns of

- (a) 3 shaded squares
- (b) 4 shaded squares
- (c) 5 shaded squares
- (d) 6 shaded squares.

ACTIVITY 5.8

Pythagoras' Theorem

For each of the triangles below, measure the angles and the sides of each triangle. Record your answers in the table.



Triangle	Right Angled?	a	b	c	a^2	b^2	c^2	$a^2 - b^2 - c^2$
I								
II								
III								
IV								
V								
VI								
VII								

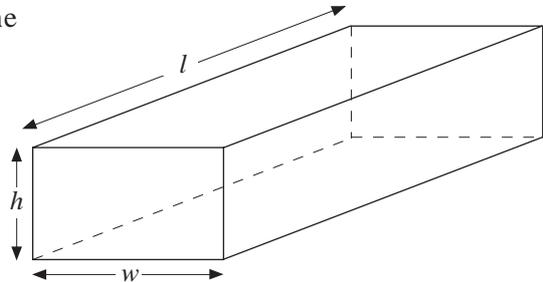
What can you say from the last column of results?

ACTIVITY 5.9

Posting Parcels

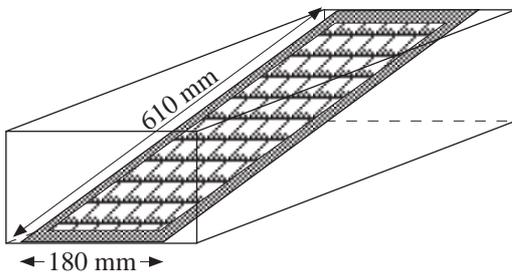
The *Royal Mail Data Post International* parcel service accepts parcels up to a maximum size as given in the rules below.

- Rule 1 *Length + height + width* must not exceed 900 mm.
- Rule 2 None of the *length, height, width* must exceed 600 mm.



1. Which of the following parcels would be accepted for this service?
 - (a) $l = 620$ mm, $h = 120$ mm, $w = 150$ mm
 - (b) $l = 500$ mm, $h = 350$ mm, $w = 150$ mm
 - (c) $l = 550$ mm, $h = 100$ mm, $w = 150$ mm

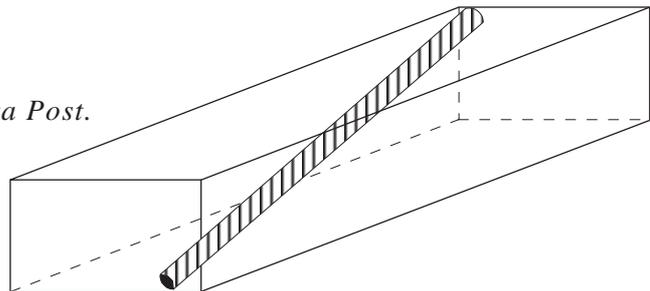
2.



A picture with frame, 610 mm by 180 mm, is to be placed diagonally in a rectangular box as shown.

Find suitable dimensions for the box so that it would be accepted for the *Data Post* service.

3. A long, thin tube is to be sent by *Data Post*.
What is the largest possible length that can be sent?



Extension

What are the dimensions of the rectangular box of *maximum volume* that can be sent through the *Data Post* service?

ACTIVITIES 5.1 – 5.9

Notes for Solutions

Notes and solutions are given only where appropriate.

5.1	2.	Polygon	No. of lines of symmetry	Order of rotational symmetry
		Isosceles	1	1
		Equilateral	3	3
		Rectangle	2	2
		Square	4	4
		Parallelogram	0	2
		Rhombus	2	2
		Kite	1	1
		Trapezium	0	1

5.2	2.	Polygon	No. of sides	No. of lines of symmetry	Order of rotational symmetry
		Hexagon	6	6	6
		Octagon	8	8	8
		Nonagon	9	9	9
		Decagon	10	10	10

- 5.5 1. (a) Yes (b) Yes (c) No (d) No 2. Yes
 3. 5-8 : Yes ; 9-10 : No 4. Overlap forms polygon of n sides with $n \leq 6$

5.6 For an n -gon, the sum of the interior angles is $180(n - 2)$; hence the values in the table should be 180, 360, 540, 720, 900, 1080, 1160, 1340.

5.7 1. 18 2. 4 possibilities

3. (a) 4 basic designs (b) none (c) none (d)

5.8 This should give confirmation that, for right angled triangles (I, III and IV),
 $a^2 = b^2 + c^2$.
 (Note that when the angle opposite side A is greater than 90° , then $a^2 > b^2 + c^2$.)

- 5.9 1. (a) No ($l > 600$) (b) No ($l + h + w > 900$) (c) Yes
 2. Many possibilities.
 3. About 671 mm.

TESTS

5.1 Mental Practice

5.2 Mental Practice

5.3 Revision

Answers

Tests 5.1 and 5.2**Mental Practice**

Answer these questions as quickly as you can, but without the use of a calculator.

Test 5.1

1. Two angles in a triangle each measure 35° . What is the size of the third angle?
 2. How many degrees are there between East and South on the points of a compass, measuring clockwise?
 3. What is the name of a six-sided polygon?
 4. What is the size of the third angle in a right-angled triangle if another angle is 42° ?
 5. What is the name given to a quadrilateral which has two pairs of parallel sides of unequal lengths and no right angle?
 6. How many sides does an octagon have?
 7. How many lines of symmetry does a square have?
 8. In a scale drawing, drawn to a scale of 1 in 100, what is the true length of a line of length 4 cm?
 9. In a drawing, 2 cm represents 1 m. What is the scale of the drawing?
 10. If the two smaller sides of a right angled triangle are 6 cm and 8 cm, what is the length of the third side?
-

Test 5.2

1. What is the size of an angle in an equilateral triangle?
 2. How many degrees are there between North and West on the points of a compass?
 3. What is the name of an eight-sided polygon?
 4. In an isosceles triangle, one angle is 66° .
What is the size of the other two equal angles?
 5. What is the name given to a quadrilateral which has two pairs of parallel sides of unequal length and in which all the internal angles are right angles?
 6. How many sides does a pentagon have?
 7. How many lines of symmetry does a regular hexagon have?
 3. If the side of a square of length 3 cm is enlarged to 9 cm, what is the scale factor of the enlargement?
 9. In a drawing, 1 cm represents 2 metres. What is the scale of the drawing?
 10. If the two shorter sides of a right angled triangle are 5 cm and 12 cm, what is the length of the third side?
-

Test 5.3

Revision

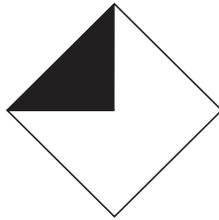
40 minutes are allowed.

1. For each shape below, copy the shape and

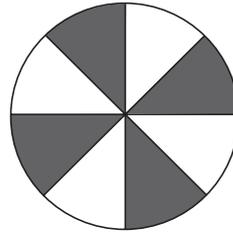
(a) mark all lines of symmetry

(b) state the order of rotational symmetry.

(i)



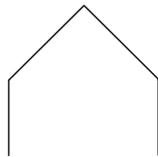
(ii)



(4 marks)

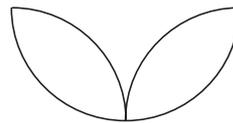
2. Copy and complete each shape below so that they have the stated symmetry.

(a)



One line of symmetry
No rotational symmetry

(b)

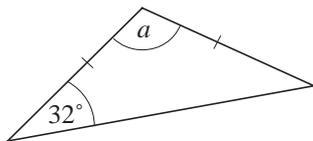


Four lines of symmetry
Rotational symmetry of order 4

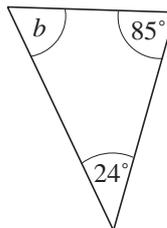
(4 marks)

3. Find the size of the angles marked by letters in each figure.

(a)

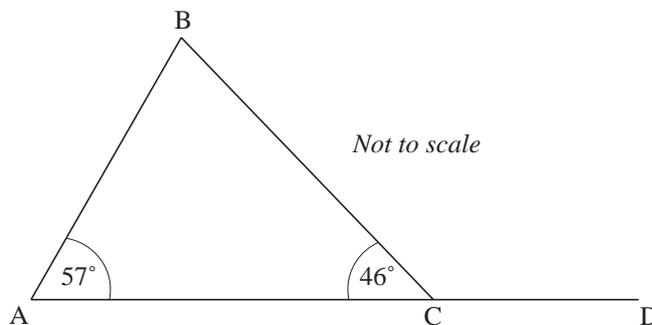


(b)



(2 marks)

4.



(a) Calculate angle BCD, giving a reason for your answer.

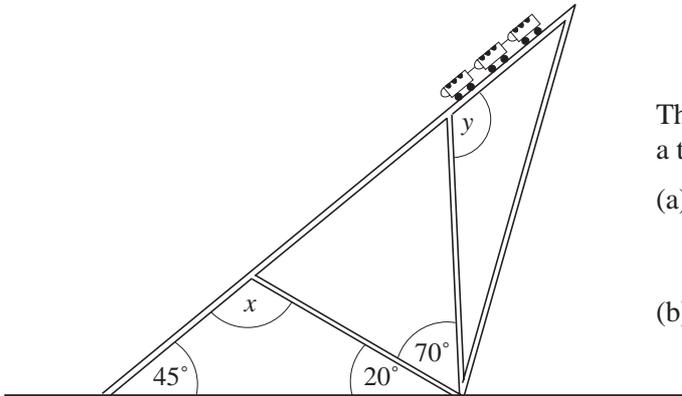
(2 marks)

(b) Calculate angle ABC, giving a reason for your answer.

(2 marks)

Test 5.3

5.

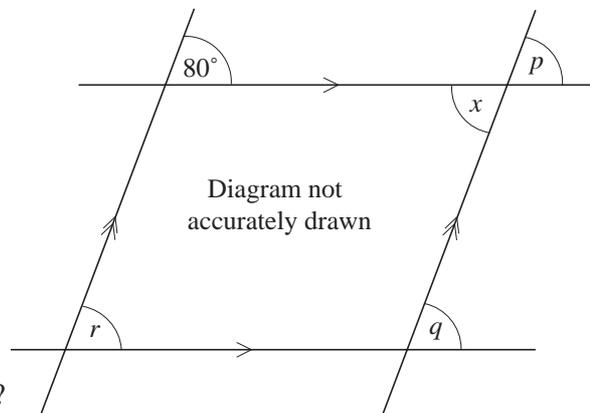


The diagram shows the *Space Shuttle* ride at a theme park.

- (a) Calculate angle x .
Do not measure the angle. (2 marks)
 - (b) Calculate angle y .
Do not measure the angle. (2 marks)
- (SEG)

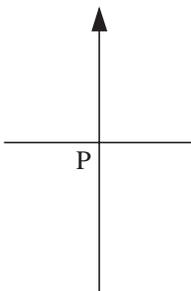
6. The diagram has two pairs of parallel lines. Angles marked p and q are equal.

- (a) What geometrical name is given to this type of equal angles?
- (b) Write down the size of angle r .
- (c) (i) Write down the size of angle x .
- (ii) What geometrical name is given to the pair of angles x and q ?



(4 marks)

7.



John is standing at P. He is facing North. He turns clockwise through 2 right angles.

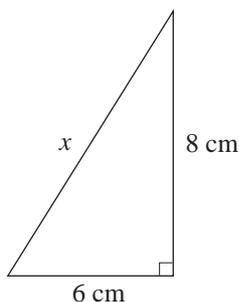
- (a) In what direction is he now facing? (1 mark)

Karen is standing at P. She is facing North. She turns clockwise through 1 right angle.

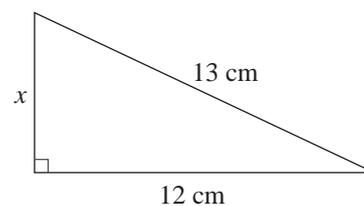
- (b) In what direction is she now facing? (1 mark)
- (NEAB)

8. Find the length of each side marked x .

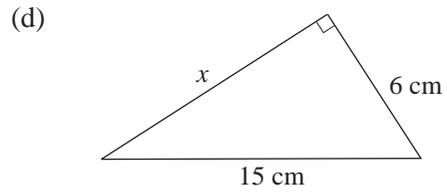
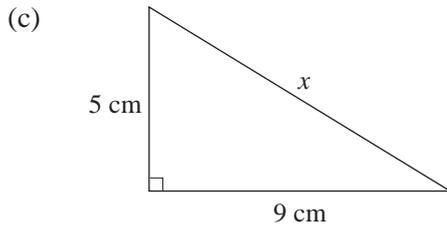
(a)



(b)

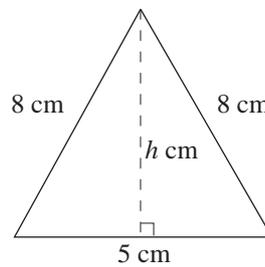


Test 5.3



(8 marks)

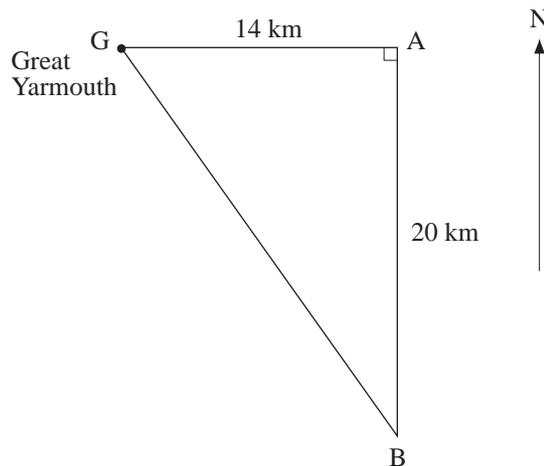
9. An isosceles triangle has sides 8 cm, 8 cm and 5 cm. Use Pythagoras' Theorem to calculate the length of the height marked h on the diagram.



Not to scale

(3 marks)
(NEAB)

10. The diagram represents Nelson's voyage from Great Yarmouth to position B. Nelson's boat sails due east from Great Yarmouth for 14 km to a position A. The boat then changes course and sails for 20 km to a position B.

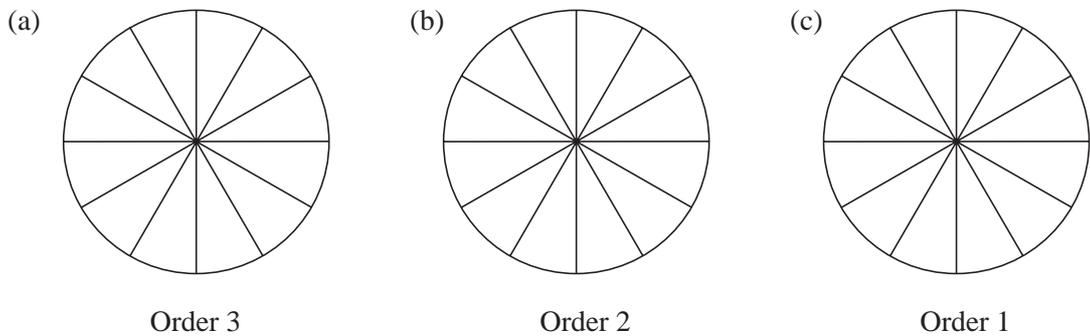


On a map the distance between G and A is 56 cm.

- (a) Work out the scale of the map. Give your answer in the form $1 : n$, where n is an integer. (2 marks)
- (b) Calculate the distance, in km, of B from Great Yarmouth. (2 marks)
- (LON)

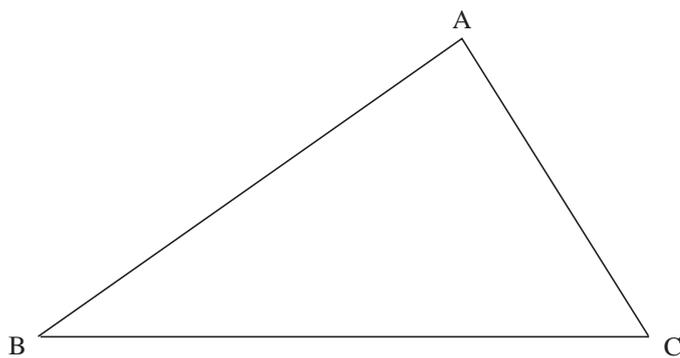
Test 5.3

11. Copy the diagrams and shade *six* sectors of each diagram so that it has the order of rotational symmetry indicated.



(5 marks)
(NEAB)

12. *To score full marks an accurate drawing is required. You may use any of your mathematical instruments to help you do this.*



- (a) Copy the diagram above and on your diagram draw:
- (i) the locus of a point which is the same distance from A as it is from B (2 marks)
 - (ii) the locus of a point which is 6 cm from C. (2 marks)
- (b) P is a point inside the triangle ABC. It is nearer to B than it is to A. It is less than 6 cm from C. Shade the region where P can be. (2 marks)
(SEG)

Tests 5.1 and 5.2**Answers**

Test 5.1

1. 110°
2. 90°
3. hexagon
4. 48°
5. parallelogram
6. 8
7. 4
8. 4 m
9. 1:50
10. 10 cm

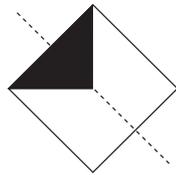
Test 5.2

1. 60°
2. 90°
3. octagon
4. 57°
5. rectangle
6. 5
7. 6
8. 3
9. 1:200
10. 13 cm

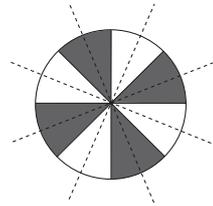
Test 5.3

Answers

1. (a) (i)



(ii)



B1 B1

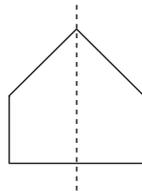
(b) (i) Order 1

(ii) Order 4

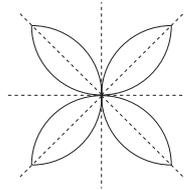
B1 B1

(4 marks)

2. (a)



(b)



B2 B2

(4 marks)

3. (a) 116° (b) 71°

B1 B1

(2 marks)

4. (a) 134° , angles on a straight line are supplementary.

B1 B1

(b) 77° , angles in a triangle are supplementary.

B1 B1

(4 marks)

5. (a) 115° (b) 135°

M1 A1 M1 A1

(4 marks)

6. (a) Corresponding angles (b) 80°

B1 B1

(c) (i) 80° (ii) Alternate angles

B1 B1

(4 marks)

7. (a) South (b) East

B1 B1

(2 marks)

8. (a) 10 cm (b) 5 cm (c) 10.3 cm

M1 A1 M1 A1 M1 A1

(d) 13.7 cm

M1 A1

(8 marks)

9. $h^2 = 8^2 - 2.5^2 \Rightarrow h = 7.60$ cm

M1 A1 A1

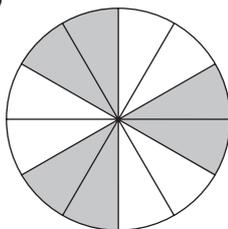
(3 marks)

10. (a) 1 : 25000 (b) 24.4 km

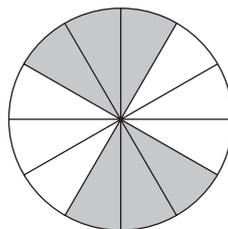
B2 M1 A1

(4 marks)

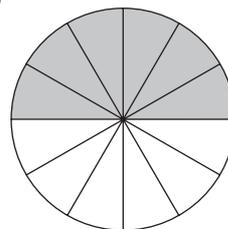
11 (a)



(b)



(c)

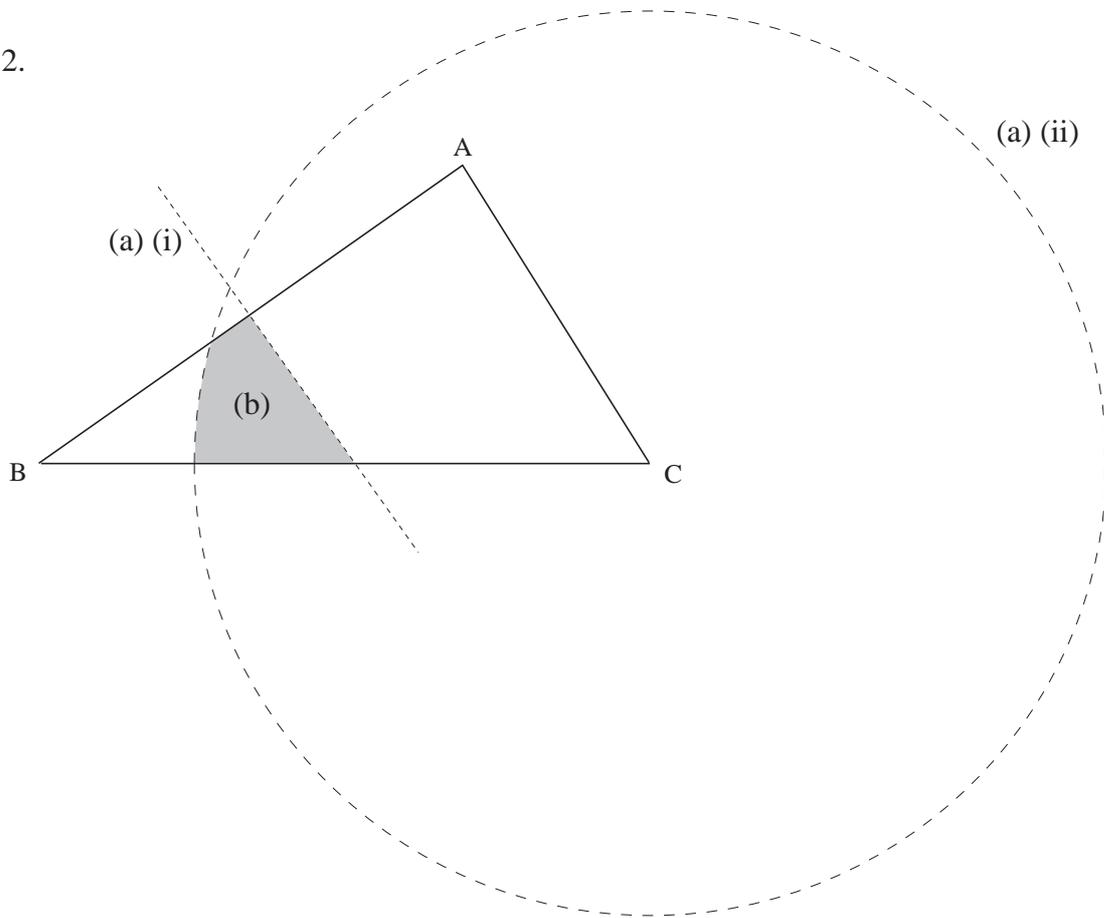


B2 B1 B2

(5 marks)

Test 5.3 Answers

12.



- (a) (i) B2
- (a) (ii) B2
- (b) B2

(6 marks)

(TOTAL MARKS 50)