

Sequences, Series and Sigma Notation

Essential information

- The general form of a **geometric sequence** of n terms is

$$a, ar, ar^2, \dots, ar^{n-1}$$

where a is a constant and r , the ratio of consecutive terms, is called the *common ratio*.

- The sum of n terms of a **geometrical series** is given by

$$S_n = \frac{a(r^n - 1)}{(r - 1)} \quad (r \neq 1)$$

- If $|r| < 1$, the sum to infinity of a geometric series is given by

$$S_n \rightarrow \frac{a}{(1 - r)} \quad \text{as } n \rightarrow \infty$$

- The general **arithmetic sequence** is of the form

$$a, a + d, a + 2d, a + 3d, \dots$$

where a and d are constants.

- The sum of n terms of an arithmetic sequence is given by

$$S_n = \frac{n}{2} (2a + (n - 1)d)$$

If l is the last term, then $l = a + (n - 1)d$ and

$$S_n = \frac{n}{2} (a + l)$$

- Sigma notation** can be used to write a sum of series.

For example, $2 + 4 + 8 + \dots + 2^{12} = \sum_{r=1}^{12} 2^r$

- Important results are

$$\sum_{r=1}^n r = 1 + 2 + 3 + \dots + n = \frac{1}{2}n(n + 1)$$

$$\sum_{r=1}^n r^2 = 1^2 + 2^2 + 3^2 + \dots + n^2 = \frac{1}{6}n(n + 1)(2n + 1)$$

$$\sum_{r=1}^n r^3 = 1^3 + 2^3 + 3^3 + \dots + n^3 = \frac{1}{4}n^2(n + 1)^2$$